

Internet Economics and Financial Technology

Computer Science COMSM0019

Lectures 5 & 6: Economics of the Internet III & IV

- Part I: Auction Theory

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Today's Topics

- Part I – Auction Theory
 - Four common auction types
 - Strategy equivalence, revenue equivalence, and incentive compatibility
- Part II – Auctions in Practice
 - Collusion and entry attractiveness
 - eBay, Google, the financial markets
- Learning outcomes:
 - define common auction designs, explain how auction design affects behaviours, describe how and why auctions are used online and in financial markets.
- Next week: Markets and Economic Agents

Let's begin by playing a game...

- I first need to assign each of you an independent and identically distributed (iid) random variable...
- Do you all know your own mobile phone number?
 - We will use the last two digits of your phone number, e.g., if your phone number is 07877 654321, then your value is £21,000.
 - All have your value in your head?
 - Keep it private.
- Note: for the game to work, it's important that nobody lies about their value.

Auction Game I

Reminder: your private value is the last two digits of your phone number:

07877 654321 = £21,000

- I'll be the auctioneer
- We will sell one item
- Your private value – your willingness to pay – is the last two digits of your phone number
- This is your maximum amount
- We'll use an open ascending price auction, also called an English Auction
- I'll start by announcing a price. If you are happy to pay that amount raise your hand. I'll confirm the bid is with you and then the price will rise.
- When we reach a price nobody is prepared to accept, the item will be sold to the person who last bid, at the price of that last bid.

Sell with Sotheby's



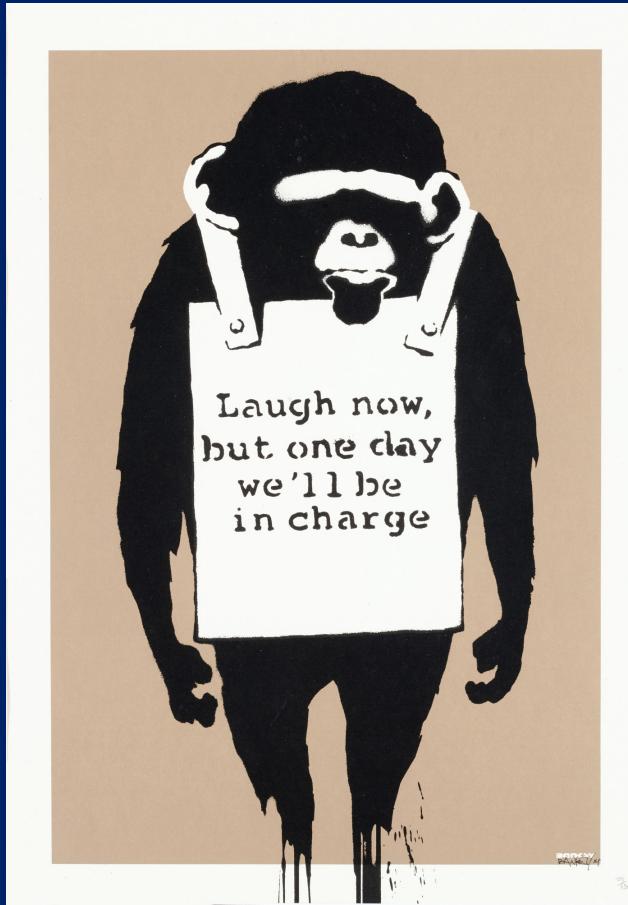
Request an Auction Estimate

- Before we begin...
- Think about your strategy
- You want to win the auction, but as cheaply as possible

English Auction Game

Reminder: your private value is the **last two digits** of your phone number:

07877 654321 = £21,000



Banksy: *Laugh Now*
(screenprint, 2004)

Auction Game II

Reminder: your private value is the 2nd and 3rd last digits of your phone number:

07877 654321 = £32,000

- I'll be the auctioneer
- We will sell one item
- Your private value is now the 2nd and 3rd last digits of your phone number
- This is your maximum amount
- We'll use an open descending price auction, also called a Dutch Auction
- I'll start by announcing a price. I'll keep decreasing the price over time.
- As soon as I say a price you are prepared to accept, quickly raise your hand.
- This will stop the auction
- The item will be sold to the first person who accepts, at the price that was accepted.



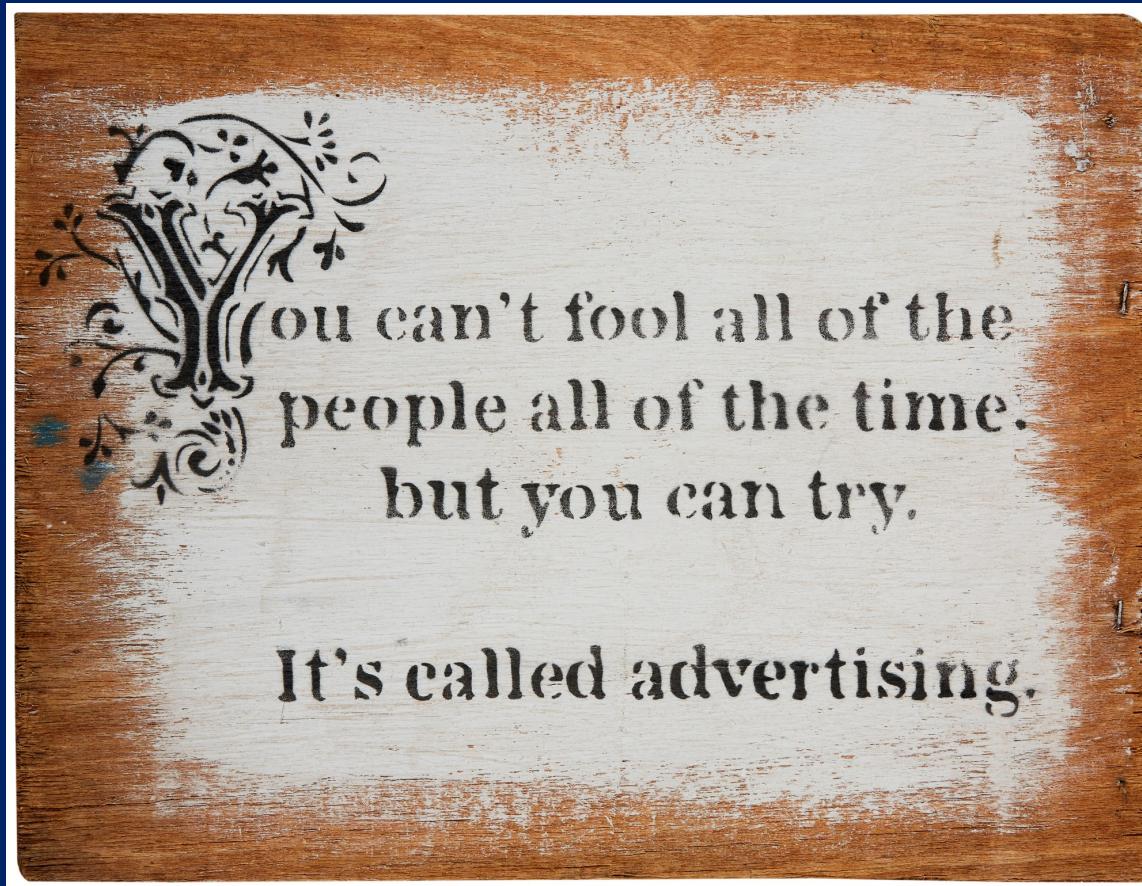
Royal Flora Holland auction hall

- Again, think about your strategy
- You want to win the auction, but as cheaply as possible

Dutch Auction Game

Reminder: your private value is the 2nd and 3rd last digits of your phone number:

07877 654321 = £32,000



Banksy: *You can't fool all of the people...*
(spray paint and emulsion on wood, 2005)

Auction Game III

Reminder: your private value is the 3rd and 4th last digits of your phone number:

07877 654321 = £43,000



- I'll be the auctioneer
- We will sell one item
- Your private value is the 3rd and 4th last digits of your phone number
- This is your maximum amount
- We'll use a first-price sealed bid auction
- This time you each make one bid, in secret. The highest bid will win, and will pay that price (i.e., pay first price)
- Normally, you would submit your bid by letter or email
- Here, you will think of your bid. Then, when ready, I'll ask you to each tell your bid to the person next to you. Then we will find the highest.



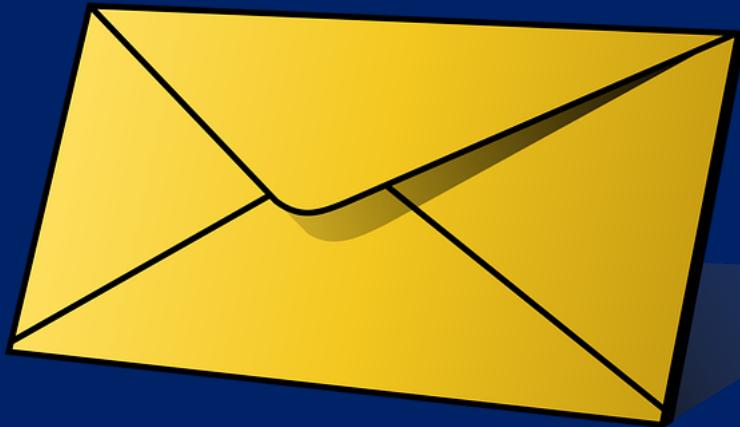
First-Price Sealed Bid Auction

Reminder: your private value is the 3rd and 4th last digits of your phone number:

07877 654321 = £43,000



Banksy: *No Ball Games (Green)*
(screenprint, 2009)



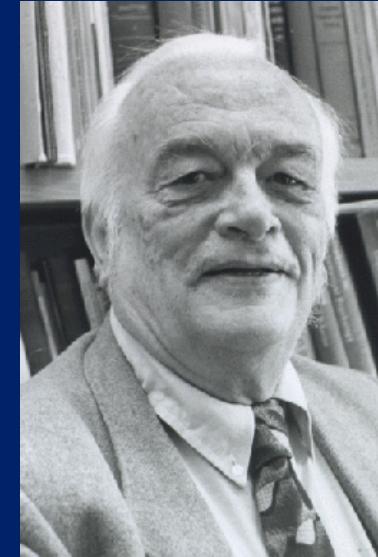
- Normally, you would submit your bid by letter or email
- Have you thought of your bid?
- When ready, I'll ask you to each tell your bid to the person next to you. Then we will find the highest bid to discover the winner and the price to pay.

Auction Game IV

Reminder: your **private value** is the 4th and 5th last digits of your phone number:

07877 654321 = £54,000

- I'll be the **auctioneer**
- We will sell one item
- Your **private value** is the 4th and 5th last digits of your phone number
- This is your maximum amount
- We'll use a **second-price sealed bid auction**, also called a **Vickrey auction**
- You each make one bid, in secret.
The highest bid will win, and will pay the price of the **second highest** bid
- Normally, you would submit your bid by letter or email
- Think of your bid. When ready, I'll ask you to each tell your bid to the person next to you. Then we will find the two highest bids to find the winner and the price to pay.



William Vickrey

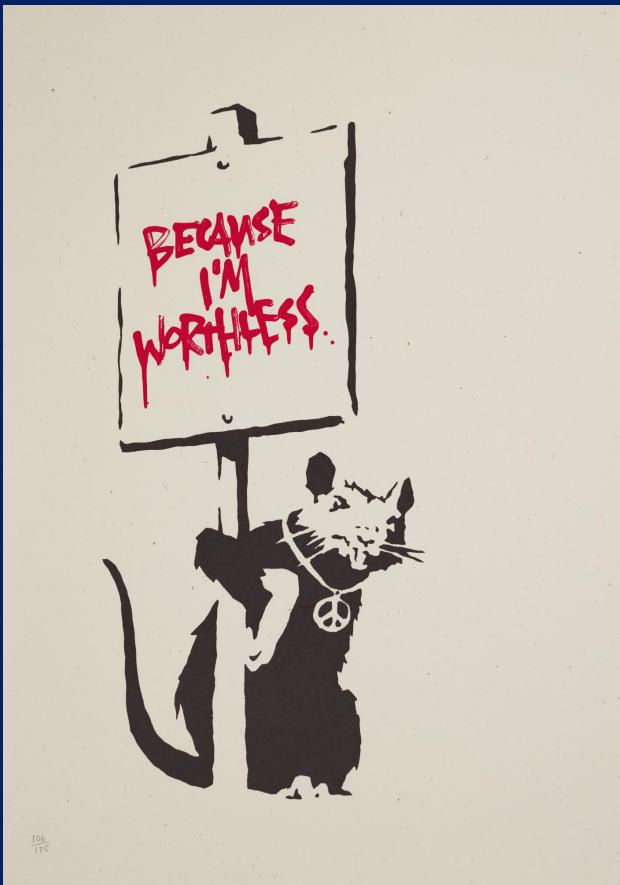
1961 – work on second-price sealed bid auctions
1996 - Nobel Memorial Prize in Economic Sciences

- Think about your **strategy**
- You want to win the auction, but as cheaply as possible

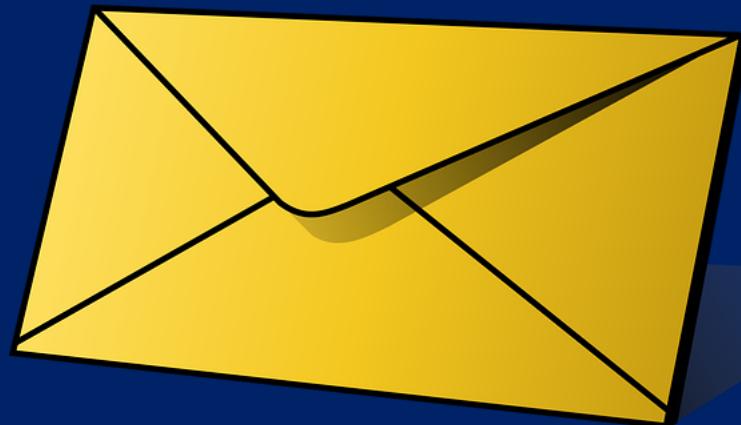
Second-Price Sealed Bid Auction

Reminder: your private value is the 4th and 5th last digits of your phone number:

07877 654321 = £54,000



Banksy: *Because I'm Worthless*
(screenprint, 2004)

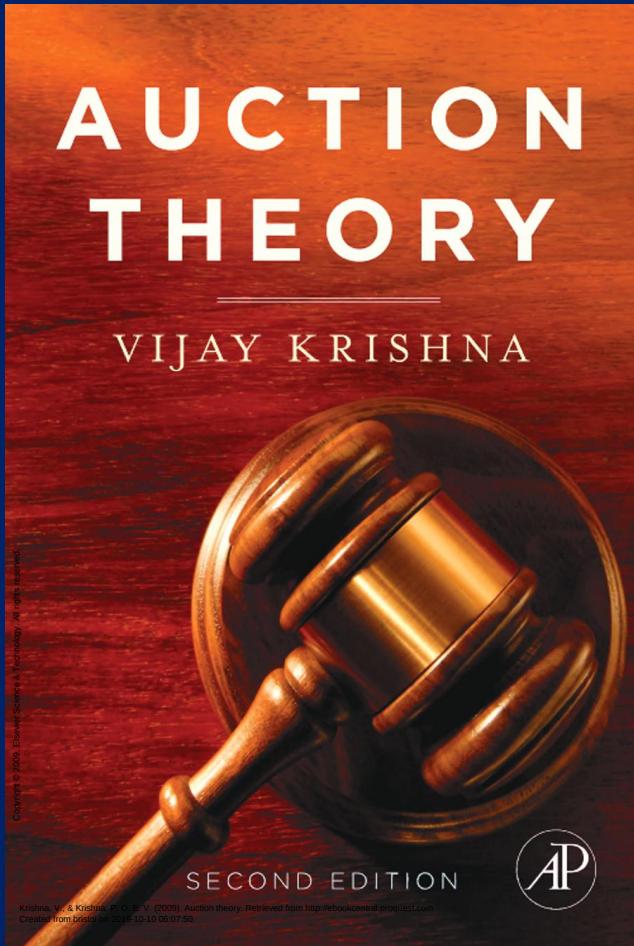


- Normally, you would submit your bid by letter or email
- Have you thought of your bid?
- When ready, I'll ask you to each tell your bid to the person next to you. Then we will find the two highest bids to find the winner and the price to pay.

Auction games – wash up

- What were the **key features** of the different auctions?
 - Which auction was **quickest**?
- What **strategy / strategies** did you employ?
 - Did your **strategy change** between auctions – if so, why?
- Which auction mechanism did you **prefer** – and why?
 - Were you a **low value** or a **high value player**?
- Which auction made you bid **highest** (relative to your value)?
 - Did you consider what **other bidders** were doing?
- Let's try to formalize some of this...

Auction Theory



- eBook available to read online
- Recommend reading Chapters 1-4.
- Mathematical, but no need to follow in depth (this is not an Game Theory economics / mathematics unit).

We are going to take a theoretical look at **4 common auction types**:

- Open Ascending Price (English)
- Open Descending Price (Dutch)
- First-Price Sealed-Bid
- Second-Price Sealed-Bid (Vickrey)

Krishna, Auction Theory (2nd Ed), Elsevier, pp.1–59

Recommended: eBook available UoB Library online.

Auction Theory: Helps us Consider Questions...

- Why are auctions and competitive bidding so prevalent?
- Are there situations to which an auction is particularly suited as a selling mechanism as opposed to, say, a fixed, posted price?
- From the point of view of the bidders, what are good bidding strategies?
- From the point of view of the sellers, are particular forms of auctions likely to bring greater revenues than others?

Auctions: A history of uncertainty

- Auctions have been used since antiquity (Herodotus: Babylon c.500BC).
 - Used because the **seller** is **unsure** about the values that bidders attach to the object being sold.
 - If the seller knew bidder values, he could just offer the object to the bidder with the highest value, **v**, at or just below **v**.
 - The **uncertainty** regarding values facing both sellers and buyers is an inherent feature of auctions.
- Let's introduce some terminology for bidder values: **private** (independent) and **interdependent**...

Terminology: *Private values* (independence)

- (independent) *private values*:
 - each bidder knows the value of the object to him/herself at the time of bidding
 - no bidder knows with certainty the values attached by *other* bidders; and knowledge of other bidders' values has no affect
- Question: when is this a plausible assumption?
 - Assumption is most **plausible** when the value of the object to a bidder is derived from its **consumption or use** alone (e.g., a pizza, or a bicycle)
 - E.g., you value a pizza because you are hungry and (don't) like the ingredients; you care little what other people value the pizza at.

Terminology: *Interdependence*

- How much the object is worth may be **unknown** at the time of the auction to the bidder.
 - Bidder may have only an **estimate** or some private **signal**—e.g., an expert's estimate—that is **correlated** with the true value.
 - Other bidders may possess **information**—e.g., additional estimates—that if known, would affect the value that a particular bidder attaches to the object.
- We call this specification *interdependent values*:
 - values are **unknown at the time of the auction** and may be **affected by information** available to other bidders.
- Question: when is this a **plausible assumption**?
 - Assumption most **plausible** for situations in which the object being sold is an asset that can possibly be **resold** after the auction (e.g., Harry Potter first Edition)
 - You might be a fan of Harry Potter, but does that make you willing to pay thousands of pounds for a book when you can read it free from the library? No – it's likely you would only do this because you see other people are very interested in paying lots for first editions so you think it might go up in value.

Equivalent Auctions

- First-price sealed-bid (FPSB) auction:
 - Bidder's strategy maps private information to a bid
 - No information about other bidders available as auction is sealed
- Dutch auction:
 - Conducted in the open, but offers no useful information to bidders
 - The only information available is that some bidder has agreed to buy at the current price, but that causes the auction to end
- Equivalence:
 - Bidding a certain amount in a first-price sealed-bid auction is equivalent to offering to buy at that amount in a Dutch auction
 - Therefore, Dutch open descending price auction is strategically equivalent to the first-price sealed-bid auction.
 - Definition - Strategic equivalence: for every strategy in one game, a player has a strategy in the other game, which results in the same outcomes.

Equivalent Auctions:

- English auction:
 - offers information when other bidders drop out. By observing this, it may be possible to infer something about their privately known information.
 - with **private values**, however, this information is of no use.
- English auction **strategy**:
 - it *cannot be optimal* to:
 - stay in after the price, p , exceeds the value, v , i.e., $p > v$ (this causes a loss)
 - drop out before the price reaches the value, $p < v$ (thus forgoing potential gains)
 - therefore, *optimal strategy is to bid up to the value*, $p = v$

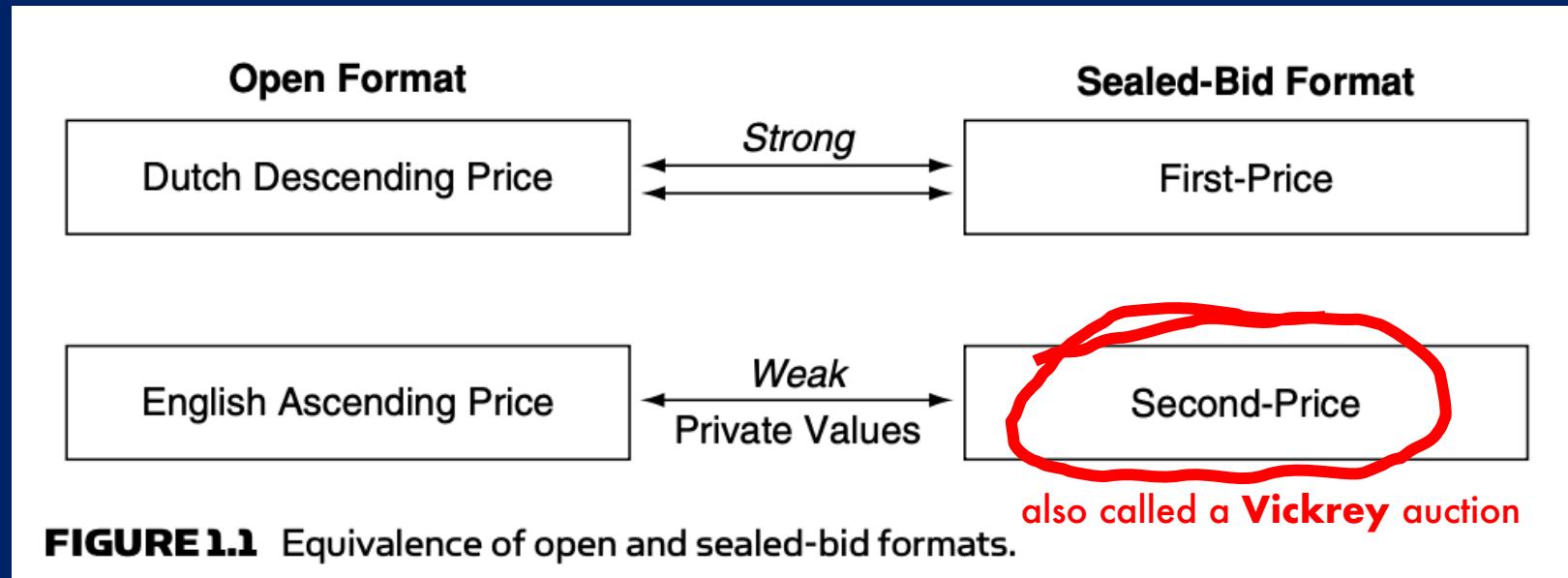
Equivalent Auctions:

- Second-price sealed-bid (SPSB, or Vickrey) auction. Assume:
 - bidder **B**, with private value v , bids a price p ;
 - the **highest** competing bid has price c
- Let's consider optimal bidding strategy...
- If **B** bids at $p = v$
 - **B** wins if $v > c$ (making profit $v - c$) and does not win if $v < c$
 - (if $v = c$, assume **B** indifferent between winning/losing)
- Alternatively, if **B** bids p lower than v , i.e., $p < v$, then
 - If $v > p > c$ then **B** still wins, and profit is still $v - c$
 - If $c > v > p$, then **B** still loses, so also no change, but
 - If $v > c > p$, then **B** now loses, so makes less profit
 - Therefore, bidding $p < v$ can never increase profit for **B**, but can decrease profit.
- A similar argument shows it is not profitable to bid $p > v$
- Therefore, *optimal strategy is to bid the value*, $p = v$.

Equivalent Auctions:

- Optimal strategy:
 - English (with private values): *bid up to the value*, $p = v$
 - Second-price sealed-bid: *bid up to the value*, $p = v$
- English and second-price sealed auctions are *weakly equivalent*
 - the two auctions are not strategically equivalent.
 - optimal strategies in the two are the same only if values are private.
- With interdependent values:
 - information available to others in the open auction is relevant to a bidder's evaluation of worth.
 - seeing some other bidder drop out early may bring bad news that may cause a bidder to reduce his own estimate of the object's value.
 - thus, *if values are interdependent, the two auctions may not be equivalent* from the perspective of the bidders.

Equivalent Auctions



*English and Sealed Second-Price are not strategically equivalent, and optimal strategies in the two are the same only if values are private and not interdependent.

Incentive Compatibility

- An auction is said to be *incentive compatible* if it encourages bidders to bid their **true value** of the good (i.e., bid *truthfully*).
- We have seen **English** and **Vickrey** auctions are incentive compatible: when values are private, optimal strategy is bid: $p = v$

Question: What about the other auctions?

- **Dutch** and **First-price sealed-bid** are not. The optimal strategy is to *shade*, by bidding lower than the true value, i.e., bid: $p < v$.
 - E.g., in Dutch auction, bidders try to guess what others value the good at, and attempt to stop the auction just before that price.

Incentive compatible auctions stop game-playing between bidders

Revenue Equivalence Theorem (RET)

- In English/Vickrey auctions, bidders bid price $p = v$
- In Dutch/FPSB, bidders bid price $p < v$
- However, the *expected revenue* in a first-price auction is the *same* as the expected revenue in a second-price auction.
- We can go further...
- **Revenue Equivalence Theorem:**
 - *If private values are iid and all bidders are risk neutral, then any standard auction (such that the person who bids the highest amount is awarded the item) yields the same expected revenue to the seller.*
 - *So, e.g., a third-price auction would also generate the same revenue [Auction Theory p.32 – optimal strategy to bid higher than one's value.]*
- Surprised? Is this realistic?

Risk Aversion

- Revenue Equivalence Theorem requires *risk neutral* bidders:
 - i.e., all bidders only seek to maximise expected profits
- In reality, many bidders are *risk averse*:
 - effect of a slightly lower winning bid on wealth level has a smaller utility consequence than the possible loss if this lower bid were to result in losing the auction.
 - Compared to a risk-neutral bidder, a risk-averse bidder will thus bid higher: as it were, they “buy” insurance against the possibility of losing.
 - E.g., is losing the home of your dreams in an auction by bidding £1 less worse than the benefit you would get from that £1 for winning the auction for £1 less?
- When we take risk aversion into account, we get...

Proposition 4.1. *Suppose that bidders are risk-averse with the same utility function. With symmetric, independent private values, the expected revenue in a first-price auction is greater than that in a second-price auction.*

Interdependence

- Revenue Equivalence Theorem also requires (*independent*) **private values**:
 - i.e., all bidders have an intrinsic value that does not depend on other bidders' values
- In reality, values are often **interdependent**:
- When we take **interdependence** into account, we find...
 - *ordinary ascending auctions are more profitable than standard (first-price) sealed-bid auctions, in expectation (Milgrom and Weber, 1982)*

Auction Theory: Questions and Answers

- Why are auctions and competitive bidding so prevalent?
 - Simple to run, easy to automate, a form of differential pricing.
- Are there situations to which an auction is particularly suited as a selling mechanism as opposed to, say, a fixed, posted price?
 - Price discovery: when you don't know how to estimate (highest) bidders' values.
- From the point of view of the bidders, what are good bidding strategies?
 - It depends on the auction mechanism. If auction is incentive compatible, then just bid honestly. If it's not, then being strategic (shading your bid) can help.
- From the point of view of the sellers, are particular forms of auctions likely to bring greater revenues than others?
 - If buyers are risk averse (i.e., don't want to lose the opportunity of buying) then First-Price Sealed Bid can increase revenues over a Vickrey auction.
 - When values are interdependent, open-auctions can be better than sealed

Auction Theory

- So, with some fancy mathematics, we can understand everything that is needed to know about auctions...
- *Not quite...*