Modified Grey Wolf Optimization to Solve Traveling Salesman Problem

Dibbendu Singha Sopto
Dept. of Computer Science and Engineering
Khulna University of Engineering & Technology
Khulna, Bangladesh
Email: dibbendu.kuet@gmail.com

Safial Islam Ayon
Dept. of Computer Science and Engineering
Khulna University of Engineering & Technology
Khulna, Bangladesh
Email: safialislam302@gmail.com

M. A. H. Akhand
Dept. of Computer Science and Engineering
Khulna University of Engineering & Technology
Khulna, Bangladesh
Email: akhand@cse.kuet.ac.bd

N. Siddique
School of Computing, Engineering and Intelligent Systems
Ulster University
Northern Ireland, United Kingdom
Email: nh.siddique@ulster.ac.uk

***Abstract*—Traveling Salesman Problem (TSP) is arguably the most familiar combinatorial optimization problem. TSP is also a very popular benchmark problem for performance verification of new optimization algorithms even though the algorithms are developed for different problems such as numerical optimization. This paper proposes a new method to solve TSP based on a recently developed optimization method inspired by the hunting and social behavior of the grey wolf pack. Standard Grey Wolf Optimization (GWO) is developed for numerical optimization; GWO is modified and adapted in this study considering Swap Operator (SO) and Swap Sequence (SS) implication for TSP. SS is an arrangement of several SOs in which each SO holds two particular positions in a TSP solution that can be swapped to make a new solution. In the proposed modified GWO, each grey wolf is considered as a TSP solution and SS is considered to update the solution. In every iteration, a new tour is formed by swapping positions indicated by SOs of the SS to the previous solution. The proposed technique is tested on several instances of the benchmark TSP and the final results are compared with the other well-known methods. Experimental results show that the proposed method is a decent technique to resolve TSP.**

Keywords—Traveling Salesman problem, Swap Sequence, Swap Operator, Grey Wolf Optimization

I. Introduction

Travelling Salesman Problem (TSP) is probably the most familiar and largely used real-world combinatorial optimization problem. TSP is the problem faced by a salesman who makes a journey from a specific city and travels all other cities in the shortest possible path. The salesman is not allowed to visit any city twice before returning to the starting city [1, 2]. There are numerous real-life applications of TSP such as vehicle routing, computer wiring, X-Ray crystallography, printed circuit panel, and order selecting problem in warehouses [1].

TSP is also used as a benchmark test problem for verification of new algorithms. Nearly every year new methods are developed for solving various engineering optimization problems, which are verified on the TSP considering it as a standard test problem. Ant Colony Optimization (ACO) [3] is a metaheuristic algorithm which has been applied to TSP and is shown to provide quality solutions to TSP. The optimization methods developed for numerical optimization problems are required modification to be applicable to TSP [4, 10, 11]. A number of methods have been developed for TSP based on Particle Swarm Optimization (PSO) [4, 10, 11]; and Swap Operator (SO) and Swap Sequence (SS) based PSO methods have been proved to work well for TSP.

This paper proposes a modified method for solving the TSP based on a recent optimization algorithm inspired by the hunting and social behaviour of the grey wolf pack, called Grey Wolf Optimization (GWO) [6]. GWO is developed for numerical optimization problems, which has been used by many researchers in the various fields in the recent years. GWO has been applied to multi-verse optimizer [7], non-convex economic load dispatch problem [8], hypercube interconnection network [9], and many other domains. In this study, GWO is modified and updated to solve TSP, and SO and SS are introduced to modify GWO for TSP.

GWO algorithm is a new addition to the population-based meta-heuristic algorithm family [6]. Like all others meta-heuristic algorithms, GWO starts with random positions of the grey wolves distributed uniformly over the search space. The positions are updated in every iteration. A group of wolves living together is called a pack. The grey wolf pack maintains a social hierarchy of leadership as a hunting strategy. The leader of the grey wolf pack is called Alpha wolf, the wolf next to the hierarchy is a Beta wolf, and then there is Delta wolf in the hierarchy. The leading three wolves play the key roles for making decision or updating the current position of the wolves while hunting. In the society of grey wolves, the hierarchy is maintained strictly and the best hunting agents Alpha, Beta and Delta work together to make the decisions. The existing best agents are evaluated and positions are updated every iteration. The rest of the wolves that work under the command of the leaders are called Omega wolves. Omega wolves update their positions or values based on the locations of the Alpha, Beta and Delta wolves.

Standard GWO is developed for numerical optimization. The components of GWO are updated and hence a new method is proposed to solve TSP in this study. The position of each grey wolf represents a TSP solution. In the modified GWO, SS is a set of SOs that is used to update the solution. Each SO holds two particular positions of a solution that undergo swap operation involving swap of the two positions. All the calculated SOs of an SS are used in a selected solution preserving the order and thus the application of the SS converts the TSP tour into a new tour. The SOs are generated using the exact location of a specific grey wolf as well as the location of the other best hunting agents (Alpha wolf, Beta wolf, and Delta wolf) of the wolf pack.

Rest of the paper has been organized as follows. Section II describes the proposed modified GWO algorithm. Section III elaborates the experimental outcomes of the proposed method and compares the performance with other methods. Finally, Section IV presents a brief conclusion of the paper.

II. Modified Grey Wolf Optimization to Solve TSP

Employment of the SO and SS operations are the main contributions of this study in proposed modified GWO for solving TSP. Following subsections contain explanation of the operators and the algorithm in detail.

A. Swap Operator in Solving TSP

An *SO(i,j)* is a Swap Operator with a pair of indexes $< i, j >$ representing two cities i and j which may swap their positions in a tour of a TSP solution [4, 10, 11]. Assume a solution of a TSP containing four cities in a sequence as defined by S = (d – a – c – b) and the SO is *SO*(1,3), then the modified solution S' is

$$S' = (d - a - c - b) + SO(1,3) = (c - a - d - b), \quad (1)$$

here '+' means the application of the swap sequence to the solution.

An Swap Sequence (SS) is a set of one or more SO(s) that may be applied to a specific solution one after another consecutively [4, 10, 11] as described by:

$$SS = (SO_1, SO_2, SO_3, \cdots, SO_n) \quad (2)$$

Here $SO_1, SO_2, SO_3, \cdots, SO_n$ are SOs. The SS may also be derived from solutions S_1 and S_2 as described by the following equation.

$$SS = S_2 - S_1 = (SO_1, SO_2, SO_3, \cdots, SO_n) \quad (3)$$

Here '-' indicates extraction of *SS* with one or more SOs to get S_2 from S_1. Suppose two solutions are S_1 = (a – b – c – d – e) and S_2 = (b – c – a – e – d) then *SS* = {*SO* (1,2), *SO* (2,3), *SO* (4,5)}.

It is to be noted that various SSs acting on the same solution may generate the same or similar solution. All these SSs are called the equal set of SSs. Among all the SSs, the SS which has the minimum SOs is called Basic Swap Sequence (BSS). Suppose two SSs are SS_1 = *{SO (2,3), SO (3,2), SO (4,1), SO (5,3), SO (3,4)}* and SS_2 = *{SO (4,1), SO (5,3), SO (3,4)}*. When both are applied on S_1 = (e-a-b-c-d) independently, the result is S_2 = (c-a-e-d-b). Consequently, SS_2 is the BSS and it will also be found using $S_2 - S_1$.

Partial Search (PS) is a recently developed improved SS implication technique in which one SO is applied to calculate the result and the next SO is applied and continued with the result. Here, every individual result is remembered and the last result is selected as the best solution among them [15]. PS approach is adapted in this study.

B. Modified GWO to Solve TSP

Like all other population-based swarm algorithms, the proposed modified GWO initiates a population of random solutions and tries to improve these solutions at each iteration step. Algorithm 1 shows the main steps of the method to solve TSP and a brief description of the algorithm is given below.

Algorithm 1: Modified GWO Algorithm

Step 1: Initialization

Step 2: For each particle X_t in the population,

a. Find SS considering X_α, X_β, and X_δ using Eq. (4)
b. Calculate the BSS of the generated SS
c. Update X_t using PS manner using Eq. (5)
d. Update the best hunt agents X_α, X_β, and X_δ considering updated agents.

Step 3: If (termination criterion is not met),
Goto **Step 2**

Step 4: Return the best solution (X_α) as outcome.

1) Initialization

In the initialization stage, the number individuals (i.e., the grey wolves) in the population are defined and a termination criterion is set as the maximum number of iterations. A random solution (i.e., TSP tour) is assigned to each of the wolf in the population and fitness values (i.e., tour cost) of each wolf is calculated. The best three wolves are considered as the best hunt agents, i.e., Alpha, Beta, and Delta, respectively.

2) Solution Update of the Grey Wolf

In each iteration step, GWO updates the position of each wolf (i.e., Omega wolf) based on the position of the best hunt agents by generating a velocity (i.e., Swap Sequence) and applying the SS to the current solution. The velocity D of SS is computed according to

$$D = c_1 \times |X_\alpha - X_t| + c_2 \times |X_\beta - X_t| + c_3 \times |X_\delta - X_t|, \quad (4)$$

here X_α, X_β, and X_δ represent Alpha wolf, Beta wolf and Delta wolf, respectively. The computed SS is determined from the three best wolves considering the random variables c_1, c_2 and c_3 to select a portion of the calculated SS.

Calculated SS D is applied to a solution with PS technique using Eq. (5) to get a new solution.

$$X_{t+1} = X_t + D \quad (5)$$

With the PS, each SO of SS makes an individual tour solution, and the wolf takes the best one as its position among those.

The tour cost of each new solution (X_{t+1}) of a wolf is compared with X_α, X_β, and X_δ. The best agents are updated if X_{t+1} is shown to be better than one or more. Taking the global best agents as three unique wolves, the process based on the chosen wolves should be helpful and useful to improve the search outcomes of GWO with a better solution.

3) Termination Criteria and Final Outcome

GWO examines termination criterion after the finishing of each iteration and terminates if the criterion is satisfied. In the propsoed method, a defined maximum number of iterations is taken as the termination criteria. If the termination criterion is not met, GWO continuously updates the positions of the wolves over the iterations as indicated in the looping process from Step 3 to Step 2 in Algorithm 1. Finally, the best agent X_α is returned as the outcome of the whole optimisation proceure.

III. Experimental Studies

In this section, the proposed GWO method is applied to solve benchmark TSPs and the results are compared to ACO, Genetic Algorithm (GA) [5, 14] and Producer Scrounger Method (PSM) [12]. ACO and GA are two well-studied methods that have been applied to TSP; on the other hand, PSM is a recently method for TSP. For a fair comparison, the experimental methodology is chosen carefully, e.g. same population size and max iterations.

A. Benchmark TSP Data and Experimental Methodology

In the above-mentioned examination, a set of 15 benchmark instances are taken from TSPLIB [13] where the number of cities varies from 14 to 100. The experiment was developed on Visual C++ of Visual Studio 2017. For appropriate understanding, tests have been performed on a particular computer (HP ProBook 450G2, Intel(R) Core (TM) i5-5200U, CPU @ 2.20 GHz CPU, RAM 4 GB) with Windows10.

B. Experimental Analyses

In this section, the experimental analyses of GWO were compared to ACO, GA and PSM on eil51 problem. We considered two parameters: i) the effect of population size, and ii) the effect of maximum iterations. The results were taken as the average value of 10 individual runs.

Figure 1 shows the tour cost of various population sizes. Here, the population size is varied from 5 to 500. For fair observation, the maximum number of iterations was fixed at 500. Standard deviation (SD) for 10 runs for a particular setting is placed as vertical bar on the average tour cost. For ACO, the number of ants and the number of cities is set equal (i.e. 51) as it is desired. It was observable from the Fig. 1 that for small population size (e.g., 5 or 10) all the methods gave worst tour costs and increasing the population size the results were improved. Meanwhile, for a specific problem, population size was fixed in ACO; it demonstrated no change in the performance in Fig. 1 and achieved the worst performance among the four methods. On the other hand, proposed GWO outperformed other three methods for different population size.

Figure 2 demonstrates the incurred tour costs of ACO, GA, PSM, and GWO for different iterations. Here the population size was fixed at 100 but the iteration number was varied from 20 to 1000 for all methods. It was seen from the figure that GWO, GA and PSM showed the worse results in the case of small number of iterations and improve with increased iteration numbers up to a certain number. After that certain number, the change of result was not significant. According to the figure, proposed modified GWO is the best and ACO is the worst among the investigated methods. In short, modified GWO has demonstrated the capacity to accomplishing better outcome in varying iteration number and population size.

C. Experimental Results and Comparison

This section compares the experimental results of GWO to ACO, GA, and PSM. In the above section, Fig. 1 and Fig. 2 show that when the population size was greater than 100 and the iteration number was greater than 500, the result did not change significantly. So, in comparison, the iteration number was fixed at 500 and population size was 100 for all the problems.

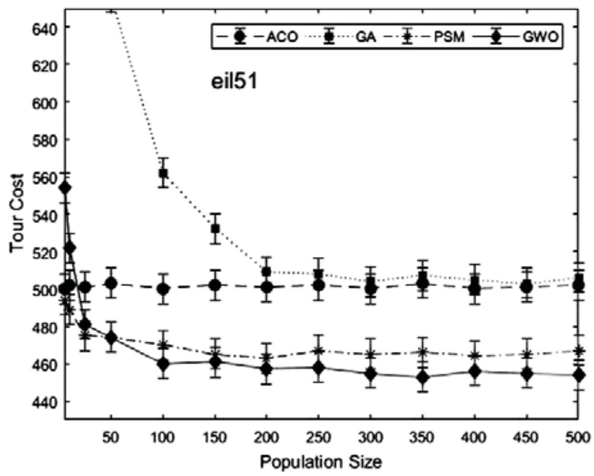

Figure 1. Variation effect of population size on tour cost.

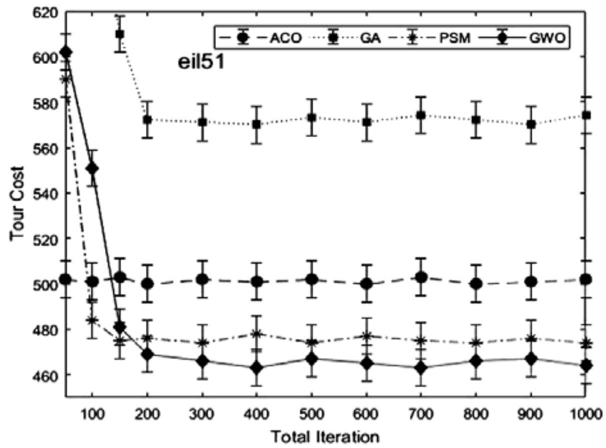

Figure 2. Variation effect of total iteration on tour cost.

Table I compares the results of ACO, GA, PSM, and GWO for 15 different benchmark TSPs. Here all the problems were run 10 independent times. In the table, among the four methods, the best tour cost (i.e., smallest value) is indicated as the bold-face type and the worst tour cost (i.e., biggest value) is indicated as underlined. In the lower part of the table, a summary of the comparison was shown.

According to the results presented in Table I, average tour costs for all the problems demonstrate that GWO was better than GA, and PSM, but worse than ACO. The average tour cost for GWO was 4512.08 for the selected 15 problems. The achieved average tour costs for GA, ACO and PSM were 7063.50, 4349.97 and 4858.51, respectively. ACO and GA showed the worst result for instances 2, and 13, respectively. On the other hand, PSM and GWO did show good results for all instances. On the other hand, GWO showed the best result in the nine instances out of 15.

The best result from different runs may use the outcome of a method. Therefore, minimum tour cost achieved from 10 runs is also compared in Table I for better understanding. On the basis of average results, GWO outperformed ACO, GA and PSM based on the lowest tour costs showed in the table. GWO showed the average lowest tour cost 4251.30 for the 15

TABLE I. OUTCOMES OF THE PROPOSED GWO COMPARISON WITH GA, ACO AND PSM TO SOLVE BENCHMARK TSPS.

		Average Tour Cost (Standard Deviation)				Best Tour Cost			
Sl.	**Problem**	**GA**	**ACO**	**PSM**	**GWO**	**GA**	**ACO**	**PSM**	**GWO**
1	burma14	31.83 (0.89)	31.21 (0.0)	30.89 (0.07)	**30.87** (0.00)	**30.87**	31.21	**30.87**	**30.87**
2	ulysses16	74.79 (0.73)	77.13 (0.0)	74.2 (0.24)	**73.99** (0.00)	74.0	77.13	**73.99**	**73.99**
3	gr17	2458.36 (157.25)	**2332.58** (0.0)	2375.39 (66.24)	**2332.58** (0.00)	2332.58	2332.58	2332.58	2332.58
4	gr21	3033.82 (337.79)	2954.58 (2.0)	2838.22 (248.07)	**2714.65** (127.32)	**2672.27**	2949.81	**2672.27**	**2672.27**
5	ulysses22	79.62 (4.29)	86.81 (0.08)	76.68 (1.0)	**76.08** (0.28)	76.09	86.74	**75.51**	**75.51**
6	gr24	1402.01 (122.53)	**1267.13** (0.0)	1372.57 (71.18)	1289.23 (35.13)	**1249.82**	1267.13	**1249.82**	**1249.82**
7	fri26	689.49 (27.43)	646.48 (0.0)	675.24 (36.24)	**644.67** (13.19)	647.78	646.48	**635.58**	**635.58**
8	bays29	9981.49 (490.87)	9964.78 (0.0)	9917.59 (391.84)	**9219.40** (146.52)	9336.82	9964.78	**9076.98**	**9076.98**
9	hk48	16033.31 (1170.13)	12731.07 (81.41)	13870.94 (952.95)	**12117.05** (472.46)	14040.66	12699.86	12239.3	**11183.46**
10	eil51	592.3 (31.61)	504.83 (3.07)	474.58 (21.23)	**463.29** (8.61)	524.18	499.92	**438.7**	455.24
11	berlin52	10413.61 (690.02)	**8088.95** (11.58)	8865.08 (407.97)	8289.11626 (212.57)	9184.19	**8046.06**	8109.91	8048.91
12	st70	1203.35 (76.16)	**748.65** (7.23)	845.4 (47.65)	800.14 (27.68)	1015.0	**734.19**	767.65	752.84
13	eil76	926.4 (48.47)	**601.77** (3.12)	631.58 (27.84)	629.24 (18.58)	805.78	595.58	**591.89**	604.32
14	gr96	1092.04 (75.36)	**590.67** (7.61)	618.68 (37.33)	660.48 (28.49)	950.18	567.52	**564.47**	593.38
15	kroa100	57940 (3568.51)	**24623.01** (80.58)	30210.57 (2164.08)	28340.42 (1645.64)	51446.8	**24504.9**	26419.8	25983.8
	Average	**7063.50**	**4349.97**	**4858.51**	**4512.08**	**6288.47**	**4333.59**	**4351.96**	**4251.30**
	Best /Worst	**0/13**	**7/2**	**0/0**	**9/0**	**3/8**	**3/5**	**10/0**	**8/0**

instances. The average minimum tour costs of ACO, GA, and PSM were 4333.59, 6288.47 and 4351.96, respectively. But GWO showed best results for eight cases among the 15 instances. On the other hand, PSM showed best results for 10 cases and GWO and PSM showed the same performance for seven cases. Finally, experimental evidences show that the proposed GWO is a novel technique to solve TSP.

IV. CONCLUSION

A modified GWO is investigated in this study to solve TSP. Mainly standard GWO is a function optimization algorithm, but the algorithm is modified in such a way so that it can be applied to TSP which is a popular combinatorial optimization problem. In GWO, individual grey wolf holds a TSP solution. In every iteration, the grey wolf tries to update his position. The result is updated when Swap Sequence (SS) is applied to a particular solution. The proposed GWO showed an improved performance in most of the cases compared to GA, ACO and PSM. A different method to identify an optimal subset of SOs owing to achieve the better result is also an open challenge, which can be addressed in a future research.

REFERENCES

[1] R. Matai, S. P. Singh, and M. L. Mittal, "Traveling Salesman Problem: An Overview of Applications, Formulations, and Solution Approaches," Traveling Salesman Problem, Theory and Applications, Edited by D. Davendra, InTech, pp. 1-24, 2010.

[2] K. L. Hoffman, M. Padberg, and G. Rinaldi, "Traveling Salesman Problem," Encyclopedia of Operations Research and Management Science, pp. 1573-1578, 2013.

[3] Z. Chi, S. S. Hlaing, and M. A. Khine, "Solving Traveling Salesman Problem by using Improved Ant Colony Optimization Algorithm," International Journal of Information and Education Technology, vol. 1, pp. 404-409, December 2011.

[4] K. P. Wang, L. Huang, C. G. Zhou, and W. Pang, "Particle swarm optimization for traveling salesman problem," Proc. International Conference on Machine Learning and Cybernetics, pp. 1583–1585, November 2003.

[5] Z. Hua, and F. Huang, "A variable-grouping based genetic algorithm for large-scale integer programming," Information Science, vol. 176, pp. 2869–2885, 2006.

[6] S. Mirjalili, S. M. Mirjalili, and A. Lewis, "Grey Wolf Optimizer," Advances in Engineering Software, vol. 69, pp. 46-61, 2014.

[7] S. Mirjalili, S. M. Mirjalili, and A. Hatamlou, "Multi-Verse Optimizer: a nature-inspired algorithm for global optimization", The Natural Computing Applications Forum, 2015

[8] V. K. Kamboj, S. K. Bath, and J. S. Dhillon, "Solution of non-convex economic load dispatch problem using Grey Wolf Optimizer, "The Natural Computing Applications Forum, 2015.

[9] A. Shaheen, A. Sleit, and S. Al-Sharaeh, "Travelling Salesman Problem Solution Based-on Grey Wolf Algorithm over Hypercube Interconnection Network," Modern Applied Science, vol. 12, no. 8, 2018.

[10] X. Wei, Z. Jiang-Wei, and Z. Hon-lin, "Enhanced Self-Tentative Particle Swarm Optimization Algorithm for TSP," Journal of north China electric power university, vol. 36, no. 6, pp. 69–74, 2009.

[11] J. Zhang, and W. Si, "Improved Enhanced Self-Tentative PSO Algorithm for TSP," Proc. Sixth IEEE International Conference on Natural Computation, Yantai, Shandong, pp. 2638–2641, August 2010.

[12] M. A. H. Akhand, Pintu Chnadra Shill, Md. Forhad Hossain, and A. B. M. Junaed, "Producer-Scrounger Method to Solve Traveling Salesman Problem," Intelligent systems and applications, 2015.

[13] TSPLIB – a library of sample instances for the TSP. Available: http://comopt.ifi.uni-heidelberg.de/software/TSPLIB9/

[14] S. Alharbi and I. Venkat, "A Genetic Algorithm Based Approach for Solving the Minimum Dominating Set of Queens Problem.,"Journal of Optimization, 2017.

[15] M. A. H. Akhand, Shahina Akter, M. A. Rashid, and S. B. Yaakob, "Velocity Tentative PSO: An Optimal Velocity Implementation based Particle Swarm Optimization to Solve Traveling Salesman Problem," IAENG International Journal of Computer Science , vol. 42, no.3, pp 221-232, 2015.