CSC236 2017 Summer Assignment 3

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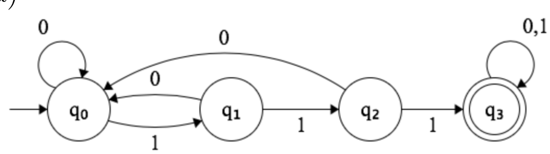
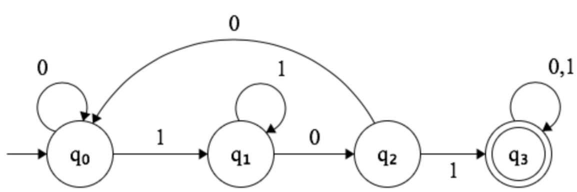
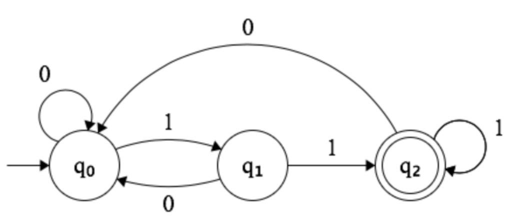
Q1.(a)



(b)



(c)



Q2.Proof by simple induction.

Define $P(n) = (q, w) \vdash^* (q, \varepsilon), |w| = n$.

We want to prove $\forall n \in N, P(n)$.

Base Case: Let $n = 0$, we want to prove P(0).

We have $w = \varepsilon$.

Then $(q, w) \vdash^* (q, \varepsilon), |w| = 0$.

Then P(0) hold.

Induction Step:Assume P(k),i.e, $(q, w) \vdash^* (q, \varepsilon), |w| = k$.

We want to prove P(k+1), i.e, $(q, w') \vdash^* (q, \varepsilon), |w'| = k + 1$.

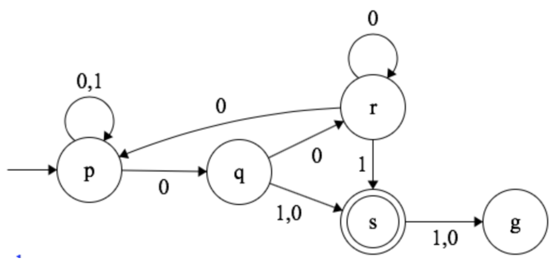
Let $w' = wa$. Since $\delta(q, a) = q$ for all input symbols a.

Then we have $(q, w') \vdash^0 (q, wa) \vdash^* (q, w)$.

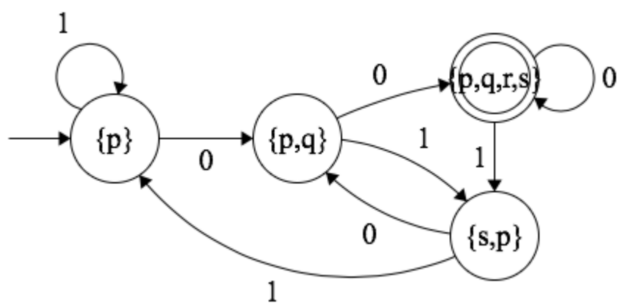
Then we have $(q, w') \vdash^0 (q, wa) \vdash^* (q, w) \vdash^* (q, \varepsilon)$ (By Induction Hypothesis).

Hence $\forall n \in N, (q, w) \vdash^* (q, \varepsilon), |w| = n.$

Q3.This is the NFA diagram.



Note: g is empty.
This is the DFA diagram.



Then L(w) accepts all strings w whose second symbol from the right end is 0.

Q4. Write the regular expression.

a) (1+0)*1(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)

b) 0*(100*)*(ε+1+11)(00*1)*0*

c) (0+10)*1*

Q5. Proof or Disproof

a) Disprove for (R+S)* = R* + S*
Let the counter example be: "SR"
We know left side can generate "SR" easily, but the right side can't generate "SR" since it can only contains "R"s or "S"s but not the concatenation.
Hence, (R+S)* $\neq$ R* + S*

b)Disprove for (RS+R)*RS = (RR*S)*
Let the counter example be: "ε"

We know left side must end with "RS", and it can not generate ε. But the right side can be form ε with the kleene start.
Hence, (RS+R)*RS $\neq$ (RR*S)*

c) Disprove for (R+S)* = (R*S)*
Let the counter example be:"SR"
We know left side can generate "SR" easily, but the right side can't generate "SR" since it must be end with "S" not "R".
Hence, (R+S)* $\neq$ (R*S)*

d) Disprove for S(RS+S)*R = RR*S(RR*S)*
Let the counter example be: "SR"
We know left side can generate "SR" easily, but the right side can't generate "SR" since it must be start with "R".
Hence, S(RS+S)*R $\neq$ RR*S(RR*S)*

Q6.
To find the recurrence N(k),According to figure1.When k = 0,1,3. The figure 1 accepts no words which is N(k) = 0. When k = 2, the figure1 accepts 10 and 01, then N(k) = 2. When n $\geq$ 4,Firstly, we need to consider how many ways we can go back to the initial state. Since We have two ways to go back to the final state by 3 steps, where is starting from D, 001 and 010. we just have one way to go back to the final state by 2 steps, which is 11.Then N(k) = N(k-2) + 2N(k-3).

$$\text{N(k)} = \begin{cases} 0, & \text{if } k = 0, 1, 3 \\ 2, & \text{if } k = 2 \\ N(k-2) + 2N(k-3), & \text{if } k \geq 4 \end{cases}$$

It is easier to calculate N(4) = 2,N(5) = 4,N(6) = 2. Then N(14) = N(12) + 2N(11)
= N(10) + 2N(9) + 2N(9) + 4(8)
= N(10) + 4N(9) + 4(8)
= N(8) + 2N(7) + 4N(7) + 8N(6) + 4N(6) + 8N(5)
= N(8) + 6N(7) + 12N(6) + 8N(5)
= N(6) + 2N(5) + 6N(5) + 12N(4) + 12N(6) + 8N(5)
= 13N(6) + 16N(5) + 12N(4)
= 114

Q7.(a)
$L = \{0^n 1^m 0^n \mid m, n \geq 0\}$.
Proof by controdiction:
Assume L is regular languages.
Then we can use pumping lemma theorem.

Let p be pumping lemma constant.
Let $w = 0^p1^m0^p$ (such that $w \in L, |w| \geq p$)
We can be factored into xyz,such that:(By pumping lemma)
1.$|xy| \leq p$
2.$|y| \geq 0$
3.for all $i \geq 0, xy^iz \in L$
Let x = $0^{p-k}, y = 0^k, z = 1^m0^p$
Let i = 0, Then $xy^iz = xz = 0^{p-k}1^m0^p$.
Then we have $0^{p-k} \neq 0^p$.
Then $0^{p-k}1^m0^p \notin L$(Since $0^{p-k} \neq 0^p$).

(b)
$L = \{w \in \{0,1\}^* \mid w \text{ is palindrome}\}$
Also,L = $\{w \in \{0,1\}^* \mid w = w^R\}$(Since w is palindrome, then w = w^R).
Proof by controdiction:
Assume L is regular languages.
Then we can use pumping lemma theorem.
Let p be pumping lemma constant.
Let $w = 0^p1^m0^p$ (such that $w \in L$,(since $w = w^R$), $|w| \geq p$)
We can be factored into xyz,such that:(By pumping lemma)
1.$|xy| \leq p$
2.$|y| \geq 0$
3.for all $i \geq 0, xy^iz \in L$
We want to prove xy$^iz \notin L$.
Then L is not regular.(By(a)), we prove it already)

Q8. Want to show that the regular languages are closed under the following operation.

$$\frac{1}{2}(L) = \{x \in \Sigma^* \mid \text{there exists } y \in \Sigma^* \text{ with } |y| = |x| \text{ such that } xy \in L\}.$$

Proof: Assume L is a regular language.
Want to show $\frac{1}{2}(L)$ is also regular.
Let D={Q, Σ, δ, s, F} be the DFA that represent L.
We need to generate an new DFA named D' from D which represent $\frac{1}{2}(L)$ in order to show that $\frac{1}{2}(L)$ is also regular.
Let D'={Q', Σ', δ', s', F'}
Q' will be formed by Q x 2^Q, which means each state in D' will be a combination of an original state from D and Sn. i.e. Q' = (q, S_n)
Define Sn: The set of all states that will reach the original final state after n transitions.
Σ' = Σ
Since we know S_n represent the state of that will reach the original final state after n transitions.
Therefore the previous states of S_n should be S_{n+1} i.e. B(S_n) = S_{n+1}

δ': δ((q, S_n), x) = (δ(q, x), B(S_n))
Define B(S): The set of all states that can reach S by only one transition.
s' = (q_0, S_0), and we know the state that reach the final states with 0 transition must be themselves. So we can say S_0 = F, s' = (q_0, F)
F':{(q, S_n) | q $\in$ Q, $S_n \in 2^Q$, q $\in S_n$}
Now, we have generated the DFA that represents $\frac{1}{2}(L)$
Hence that $\frac{1}{2}(L)$ is regular.