IIE Transactions
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/uiie20

Optimal Project Scheduling with Materials Ordering

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Published online: 09 Jul 2007.

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Abstract: A mixed integer $0-1$ programming formulation of a Project Scheduling-Materials Ordering Problem (PSMOP) is presented that provides an optimal schedule of project activities and materials orders. It is proven that an optimal solution may be found by decomposing the problem into the derivation of the project schedule and the subsequent derivation of materials lot sizes.

■ All projects must be scheduled subject to constraints on resource availability, activity precedence relationships and resource acquisition lead time. Resources required in a project include both non-storable resources such as labor and equipment as well as storable resources such as materials, component parts and assemblies. While the effects of non-storable resource constraints on project schedules are well documented in the literature on the resource constrained project scheduling problem [5, 6, 10], the effects of materials constraints on project cost have yet to be ascertained in the form of a mathematical formulation of the problem or experimentation with solution methods. A formulation of a Project Scheduling-Materials Ordering Problem (PSMOP) is presented here along with a proof of how the PSMOP may be decomposed so as to allow the derivation of an optimal solution. It is shown that the Wagner-Whitin lot-sizing algorithm [16] may be used in conjunction with the late start schedule of project activities as the optimal project scheduling and materials ordering procedure for this project environment. Computational experience illustrates that the optimal solution may be found relatively quickly. These developments have implications for managers both in the traditional project industries as well as in the make-to-order manufacturing environments.

Background

Products in a large number of environments are produced in a project mode on a made-to-order basis. In industries such as shipbuilding, research and development, aerospace [15], manufacturing [2], and commercial and residential construction, a wide variety of complex products are produced that require materials acquired from a number of different subcontractors and vendors. In addition, a particular type of material, component part or test subject may be required at a number of points throughout the production of the finished product. In practice, materials are usually either acquired entirely at the beginning of a project, using a one-lot ordering strategy, or throughout the project on a lot-for-lot basis.

Although a number of authors have described possible approaches for integrating the materials function into the development of a project schedule [1, 2, 4, 13, 15], previous research has not discussed the possible monetary advantages or disadvantages of these project management systems. For instance, both [2] and [15] describe systems developed for particular applications. In both cases it is illustrated how non-storable resource constraints must be integrated with materials constraints in project and made-to-order environments if proper coordination is to be maintained across the various project activities. While it is important to maintain coordination between the schedule of project activities and the acquisition of materials, it is also necessary to ascertain what benefits may be obtained from this approach other than just the assurance that materials will arrive such that activities may begin on schedule. A major question that should be addressed is how the introduction of materials constraints and costs into the project scheduling problem will effect the project schedule and total project costs.

The research on project scheduling subject to monetary objectives is also limited in scope. In early formulations of

Received July 1985; revised March 1986. Handled by a Senior Editor

the problem, Russell [12] and Grinold [9] provided formulations for the maximization of the net present value of project cash flows under the assumptions that both positive and negative cash flows occur over the course of a project network, and all resources, including capital, are unconstrained in availability. Doersch and Patterson [7] extended the problem to the case where capital is constrained in availability. In this paper we solve for the project schedule subject to the various costs associated with the materials management function under the assumption that payment for the project is received upon the completion of all project activities, all cash flows during the project are negative, and all resources are unconstrained in their availability.

Problem Formulation

The mixed-integer $0-1$ programming formulation listed in this section integrates materials management constraints and costs with the timing and precedence relationships that are frequently encountered in project environments. This formulation yields an explicit schedule of project activities, material order dates and planned inventory levels. The objective function includes the costs of ordering and holding materials inventories as well as the cost of holding completed project activities and a fixed charge delay penalty for each period that a project exceeds its due date. The cost of holding completed project activities includes the cost of capital invested in the activity in the form of materials and labor. This capital will not be recouped until payment is received for the project upon its completion. Both materials and activity holding costs include out-of-pocket costs such as insurance, spoilage and other variable costs incurred in their storage and upkeep.

The decision variables and parameters are listed below, followed by the objective function and constraints:

Decision Variables:

$$X_{jt} = \begin{cases} 1 & \text{if activity } j \text{ begins in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$Z_{mt} = \begin{cases} 1 & \text{if material } m \text{ is ordered in period } t \\ 0 & \text{otherwise} \end{cases}$$

Y_{mt} = Quantity of material m ordered in period t

I_{mt} = Inventory of material m on hand at the end of period t

C_t = Value of project activities completed through period t

Parameters:

d_j = Duration of activity j

R_{jm} = Amount of material m required by activity j

V_j = Value of activity j upon completion, including material and labor costs

S_m = Cost of placing an order for material m

h_m = Holding cost per unit per period for material m

P_j = The set of predecessors of activity j

T_m = Lead time for material m

S_{1j} = Early start time for activity j in a time-only schedule

S_{2j} = Late start time for activity j in a time-only schedule

D = Penalty cost per period of delaying the project

H = Cost per period of holding completed activities stated as a percent of activity value

Q = A very large number

T = An upper bound on the project duration, defined as the critical path plus the maximum allowable delay in the project

N = The final activity in the project

The objective is to minimize the total of penalty costs due to delays in the project, materials and activity holding costs, and materials ordering charges:

$$\text{Minimize } Z = \sum_{t=S_{2N}+1}^{T} (t - S_{2N}) D X_{Nt} + \sum_{t=1}^{T}\sum_{m=1}^{M} S_m Z_{mt} + \sum_{t=1}^{T}\sum_{m=1}^{M} h_m I_{mt} + \sum_{t=1}^{T} HC_t. \qquad (1)$$

The completion date of the last activity in the project may be either its completion date derived using the Critical Path Method, or it may be a fixed deadline for the completion of the project. Although the former will be assumed in the remainder of this research, the extension to the latter case is minor.

The constraint set is designed to capture precedence constraints, ordering requirements and inventory computations. All activities must be performed in the correct sequence [11]:

$$\sum_{t=1}^{T} tX_{it} + d_i \leq \sum_{t=1}^{T} tX_{jt} \quad \text{for all } i \in P_j \quad j = 1, 2, \ldots, N. \qquad (2)$$

All activities must be performed [11]:

$$\sum_{t=S_{1j}}^{T} X_{jt} = 1 \qquad j = 1, 2, \ldots, N. \qquad (3)$$

The inventory balance constraints are:

$$I_{m(t-1)} + Y_{m(t-T_m)} - \sum_{j=1}^{N} R_{jm} X_{jt} = I_{mt} \qquad m = 1, 2, \ldots, M; \quad t = 1, 2, \ldots, T. \qquad (4)$$

An order must be placed for each requisition of materials:

$$QZ_{mt} - Y_{mt} \geq 0 \qquad \begin{array}{l} m = 1, 2, \ldots, M \\ t = 1, 2, \ldots, T. \end{array} \tag{5}$$

Project value is increased upon the completion of each activity in the project:

$$C_{t-1} + \sum_{j=1}^{N} V_j X_{j(t-d_j)} = C_t \qquad t = 1, 2, \ldots, T. \tag{6}$$

Decomposition Procedure

In this section, it is shown that the optimal solution of the Project Scheduling-Materials Ordering Problem (PSMOP) will be one where each activity is scheduled to begin at its late start date. Given the fixed timing of the materials requirements, the PSMOP reduces to M single item lot-sizing problems under time-varying demand.

Proof:

Assume that the penalty cost for delay of the project, D, is extremely large, that is $D = \infty$. Under this condition, the start time for each activity will be:

$$X_{jt} = 1 \qquad S_{1j} \leq t \leq S_{2j}.$$

For those activities on the critical path, by definition, $S_{1j} = S_{2j}$ and the optimal start date is specified as $X_{jS_{2j}} = 1$. The PSMOP can then be stated as:

$$\text{Minimize } Z = \sum_{t=1}^{T} \sum_{m=1}^{M} S_m Z_{mt} + \sum_{t=1}^{T} \sum_{t=1}^{M} h_m I_{mt} + \sum_{t=1}^{T} HC_t \tag{7}$$

Subject to:

$$I_{m(t-1)} + Y_{m(t-T_m)} - \sum_{j \in CP} R_{jm} X_{jt} - \sum_{j' \notin CP} R_{j'm} X_{j't} = I_{mt} \qquad \begin{array}{l} m = 1, 2, \ldots, M \\ t = 1, 2, \ldots, T \end{array} \tag{8}$$

where CP = the set of critical path activities.

$$C_{t-1} + \sum_{j \in CP} V_j X_{j(t-d_j)} + \sum_{j' \notin CP} V_{j'} X_{j'(t-d_{j'})} = C_t \qquad t = 1, 2, \ldots, T \tag{9}$$

and (2), (3) and (5).

Constraint sets (8) and (9) have been modified such that they discriminate between the set of activities on the critical path, whose start dates are fixed, and the set of non-critical activities, whose start dates may range between $S_{1j'}$ and $S_{2j'}$ for each activity j'. The specific start dates remain to be determined for each j'.

Since the optimal start dates of all activities on the critical path are known, the term $\sum_{j \in CP} V_j X_{j(t-d_j)}$ is a constant. The objective function in (7) can now be restated as:

$$\sum_{t=1}^{T} \sum_{m=1}^{M} S_m Z_{mt} + \sum_{t=1}^{T} \sum_{m=1}^{M} h_m I_{mt} + \sum_{t=1}^{T} H \left[C_{t-1} + \sum_{j \in CP} V_j X_{j(t-d_j)} - \sum_{j' \notin CP} V_{j'} X_{j'(t-d_{j'})}\right]. \tag{10}$$

In order to minimize (10), the start dates of the noncritical activities must be evaluated between $S_{1j'}$ and $S_{2j'}$. Let t' represent any time period before $S_{2j'}$. If $X_{j't'} = 1$ for all j'pr$\notin CP$, then the PSMOP reduces to M single item lot-sizing problems where the material requirements occur at t' for the activities not on the critical path and at S_{2j} for all $j \in CP$. If j' is started at t' instead of at its late start date, then the objective function will increase by the cost of holding the completed activity j' from t' until $S_{2j'}$:

$$H\left[V_{j'}(S_{2j'} - t')\right]. \tag{11}$$

In turn, a decrease in total costs, as a result of starting an activity any earlier than $S_{2j'}$, is a function of when the order is placed for materials required by the activity, as described in cases I, II and III below:

Case I: Orders arrive for all materials required by j' on or before $S_{1j'}$. If j' begins at $S_{1j'}$ instead of $S_{2j'}$, the corresponding decrease in inventory holding costs is $\sum_m h_m R_{j'm}(S_{2j'} - S_{1j'})$.

For it to be optimal to begin j' on $S_{1j'}$, it must be the case that:

$$H\left[V_{j'}(S_{2j'} - S_{1j'})\right] < \sum_m h_m R_{j'm}(S_{2j'} - S_{1j'}). \tag{12}$$

However, investigation of the components of the cost of holding activity j' for one period, $HV_{j'}$, will illustrate that it is highly unlikely that condition (12) will hold:

Define:

b_m = cost per unit for material m

$a_{j'}$ = cost of labor for j'

E_m = out-of-pocket costs for holding a unit of material one period, stated as a percentage of unit cost

E_a = out-of-pocket costs for holding an activity one period, stated as a percentage of the cost of the activity

E_c = cost of capital.

Then:

$$V_{j'} = \sum_m b_m R_{j'm} + a_{j'}$$

$$h_m = E_m b_m + E_c b_m$$

$$H = E_a + E_c.$$

Substituting in (12) for $V_{j'}$, h_m and H:

$$\sum_m E_a b_m R_{j'm} + (E_a + E_c)a_{j'} < \sum_m E_m b_m R_{j'm}. \qquad (13)$$

In Case I, (10) will be minimized if $X_{j'S_{2j'}} = 1$ for all activities and condition (13) does not hold. It would be highly unlikely that condition (13) would hold, since for this to be the case the out-of-pocket holding costs for the materials required by activity j' would have to exceed both the total costs of holding the portion of the complete j' representing labor costs as well as the out-of-pocket costs of holding the materials incorporated into the completed activity. In the unlikely event that this is the case, then the optimal solution would be for j' to begin on $S_{1j'}$ if an order for material m is expected to arrive on or before $S_{1j'}$.

Case II: Materials are required by activity j' at period t' where $S_{1j'} < t' < S_{2j'}$. Condition (13) must hold for it to be beneficial to begin j' on t' instead of its late start date. As in Case I, if condition (13) dominates, then the time periods between $S_{2j'}$ and $S_{2j'} - 1$ must be evaluated as possible start dates, subject to (7), (8) and (9).

Case III: Materials required by activity j' arrive on $S_{2j'}$ and j' begins on $S_{2j'}$. This case will minimize (10) under all conditions.

Let it next be assumed that the delay penalty is no longer so large that the possibility of increasing the project duration beyond the critical path duration will be ignored. Consider the possibility that $X_{jt} = 1$ for some $t > S_{2j}$ for any activity j. Let t^* represent one time period following S_{2j}, that is, $t^* = S_{2j} + 1$. Assume that for another parallel activity i that requires materials that are also used by j that $S_{2i} = t^*$. It is shown in cases IV, V and VI that the savings generated from lot-sizing activities never exceed the delay penalties, that is, it is optimal for j to start at S_{2j}.

Case IV: Orders for material m required by activities i and j arrive in period t^*, thus both i and j are jointly replenished. In this case, ordering charges are decreased by placing orders that supply both i and j. For it to be cost effective to delay the project beyond its critical path, it must be the case that:

$$D + HC_{(t^*-1)} + \sum_m h_m R_{im} < \sum_m S_m. \qquad (14)$$

Note that the materials required by activity i would still be held for one period, since the delay in activity j would cause an increase in the late start period of i by one period, as dictated in Case I above. It is assumed in equation (14) that all of the activities that contribute to $HC_{(t^*-1)}$ are scheduled to begin at their original late start times. However, they could now be rescheduled to begin one period later, thus removing $HC_{(t^*-1)}$ from the inequality. In that case, if equation (14) holds, then the ordering costs for the required materials exceed the delay penalty cost plus the cost of holding materials for activity i. It would not, however, be economical to delay either activity j or the project, because project costs could be reduced further by merely scheduling j to begin at its original late start time, and then holding the materials required by i one period, thereby foregoing any delay penalty.

Case V: The materials required by activity j arrive on or before S_{2j}. Materials required by activity i are not included in this replenishment. The increase in costs include the delay penalty, the cost of holding materials for i and j an additional period, and the cost of holding the activities completed as of period $t^* - 1$ an additional period. There is not a corresponding decrease in materials or activity holding costs in this case. The change in (1) as a result of starting j on t^* is an increase of at least:

$$D + \sum_m h_m R_{jm} + HC_{(t^*-1)} + \sum_m h_m R_{im}. \qquad (15)$$

Since j is delayed by one period, an additional period of slack is added for i, and as shown in Case I, it is not optimal to start an activity before its late start date, as this results in an increase in total costs.

Case VI: Orders for materials m arrive before t^* for both activities i and j. As in Case IV, there is not a reduction in total costs, j would be delayed, and the materials required by i would be held an additional period.

Note that cases IV through VI would hold regardless of the precedence relationships between activities i and j. In particular, if i were a successor of j along a connected path, the proof is trivial.

It can be concluded from cases I through VI that the optimal solution to the PSMOP would be to schedule each project activity at its late start date, and to then derive lot sizes for each material independently. The appropriate optimal solution procedure for this lot-sizing problem is the Wagner-Whitin Procedure.

An Example

The important features of the proof will be illustrated with the aid of the project network shown in Figure 1. All of the relevant costs for the project are listed in Figure 1, including the materials ordering and holding costs as well as the holding cost for each completed activity.

Cases I, II and III may be illustrated with the aid of activities 3 and 4. It will be assumed, as noted above, that the delay penalty is exceedingly large. Therefore, the project activities will not be delayed beyond their late start dates.

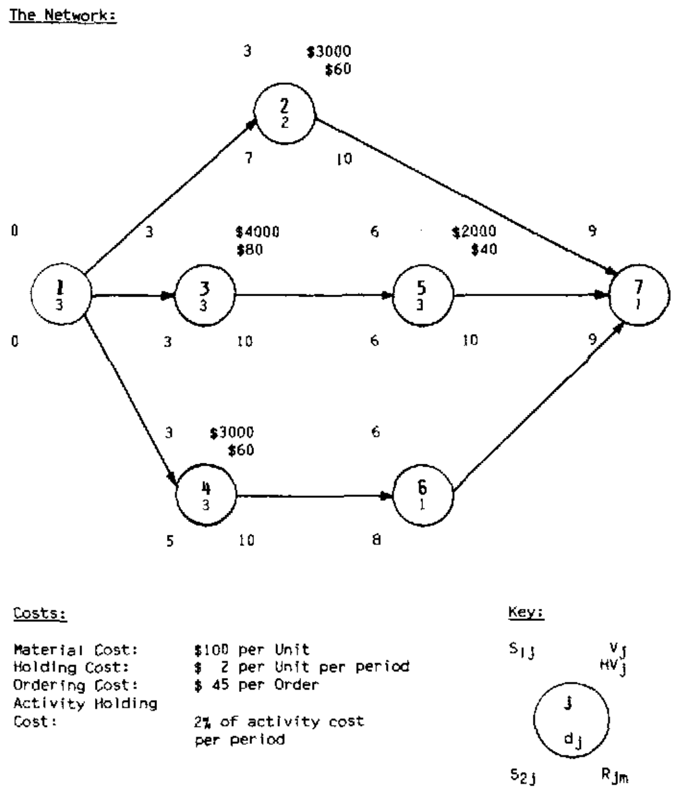

Figure 1. Example Project Network

Note that while activity 3 is on the critical path and should therefore begin at period 3, activity 4 has two periods of slack. Both 3 and 4 require 10 units of the same material. If the material requirements for 4 are met by an order covering both activities 3 and 4, then the order should arrive in period 3. If 4 is scheduled to start at S_{14} in period 3, then it would cost $120 to hold the completed activity 4 for two extra periods after it has been completed, regardless of whether or not activity 6 is begun immediately after activity 4 has been completed. On the other hand, if activity 4 is not scheduled to start until period 5, then the 10 units of material required by 4 would be held from period 3 until period 5 at a cost of $40. Thus, a total of $80 would be saved by delaying the start of activity 4 until period 5. As in Case I above, the savings in materials holding costs resulting from beginning activity 4 in period 3 would be exceeded by the costs of holding activity 4 two periods. In turn, it can be seen that Case II would also hold, since the costs would again be higher if activity 4 were started at period 4 instead of period 5, and Case III would hold because $S_{24} = 5$ yields the lowest cost solution.

Cases IV, V and VI may be illustrated by changing the assumptions for the project slightly. A delay penalty of $5 will be incurred each period that the project is delayed. To illustrate Case IV, it will be assumed that activities 2 and 5 share requirements for the same material. If activity 5 is delayed until period 7, such that a material replenishment is shared with activity 2, then the entire project would be delayed one period. In addition, in accordance with Case I above, activity 2 would be scheduled to start its late start time in period 8. If activities 3 and 4 remain scheduled to begin at the late start times indicated, additional activity holding costs of $140 would be incurred, since both activities would be held an additional period upon their completion. A delay penalty of $5 would also be incurred. Thus, the total costs would be:

Materials Ordering Cost:	$ 45
Delay Penalty:	5
Cost of Holding Activities 3 and 4 one period:	140
Cost of Holding the Materials for Activity 2 One Period:	20
Total Cost:	$210

If all activities start at their original late start times, then the total costs would be:

Materials Ordering Cost:	$ 45
Cost of Holding the Materials for activity 2 One Period:	20
Total Cost:	$ 65

Even if activities 3 and 4 were rescheduled to begin at revised late start times ($S_{23} = 4$, $S_{24} = 6$), thus reducing the total costs of the first strategy to $70, the net result would only be to incur an unnecessary delay penalty of $5. Thus, the optimal solution is a schedule where each activity is started at its late start time and materials orders would then be generated independently, as in Cases I, II and III.

Computational Experience

The viability of the decomposition procedure was tested using a series of project test problems that represent combinations of four experimental factors. The projects displayed different levels of project network structure, coefficient of variation, the percentage of activities requiring a material, and the ratio of ordering to holding costs. The solutions for the decomposition procedure are compared to the results for a Lot-for-Lot ordering strategy below. Although the test problems required only one material input, the effect of additional material types would be predictable, since it has been shown that the optimal solution is for lot sizes to be generated independently for each material.

Project Network Structure (NET). The three project networks are illustrated in Figure 2. Each structure results in a *distinctly different* material requirements pattern. Activity durations were randomly assigned, with a minimum of one period, a maximum of four and an average of 2.5 periods.

Percentage of Activities with Materials Requirements (PERC). This factor determines the degree of commonality of material requirements across a project network. It was anticipated that in situations where there were both low common requirements and an excessively large number of consecutive periods with zero requirements that a different materials order plan might be derived. Therefore, PERC was

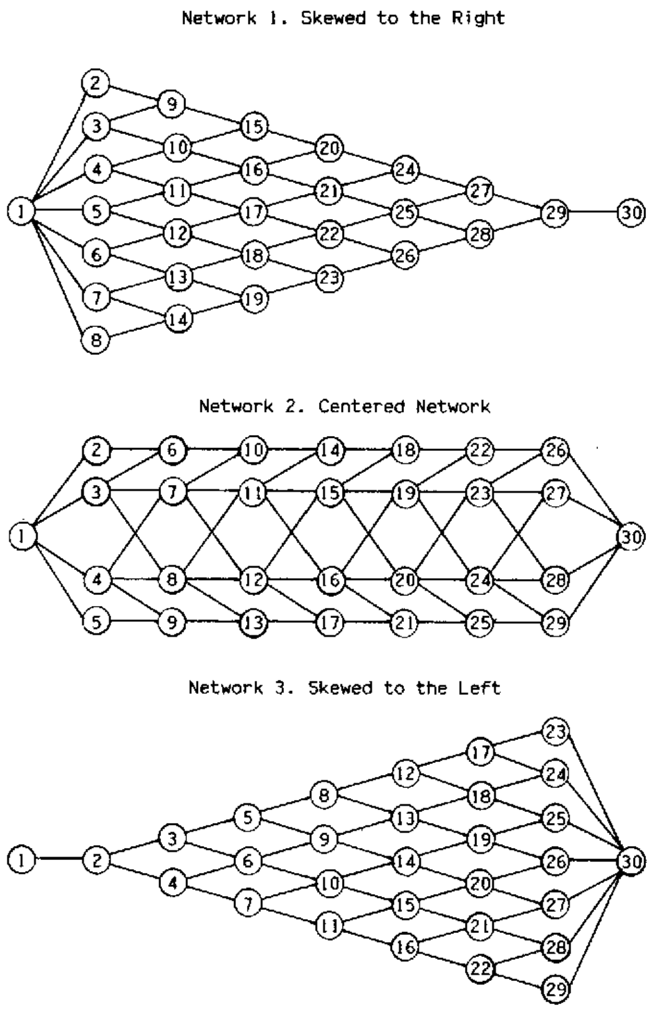

Figure 2. Project Network Structures Networks are Activity-On-Node

Table 1. Activity Durations and Material Requirements					
		Material Requirements Per Activity:			
Activity:	Duration:	20% Low CV	20% High CV	40% Low CV	40% High CV
1	4	0	0	117	66
2	3	239	130	0	0
3	2	0	0	0	0
4	4	0	0	0	0
5	3	0	0	135	155
6	3	0	0	0	0
7	3	0	0	0	0
8	2	0	0	122	203
9	2	0	0	0	0
10	2	0	0	0	0
11	4	258	363	125	91
12	1	268	162	115	253
13	1	0	0	127	45
14	2	0	0	0	0
15	3	229	444	0	0
16	4	0	0	135	176
17	3	0	0	129	165
18	3	0	0	0	0
19	4	0	0	125	83
20	3	256	99	0	0
21	2	0	0	0	0
22	1	0	0	119	128
23	1	0	0	0	0
24	2	0	0	0	0
25	4	250	302	0	0
26	3	0	0	0	0
27	4	0	0	0	0
28	3	0	0	121	73
29	3	0	0	130	62
30	1	0	0	0	0
Total Requirements:		1500	1500	1500	1500

set at a low level of 20% and a high level of 40%. Material requirements for each activity were determined by a random assignment. Total requirements across each project were held at a constant level of 1500 units, resulting in an average material demand of 250 units per activity under the low level (20%) and 125 units per activity at the high level (40%).

Coefficient of Variation (CV). The coefficient of variation (CV) was redefined as the average requirement per activity for only those activities requiring materials divided into the standard deviation of material requirements per activity. CV was set at a low level of .05 and a high setting of .50. If CV covers all activities, as in the demand streams studied in the lot-sizing literature, the coefficient will be much higher for the projects in this study. The set of activity durations and the material requirements for each activity given the various levels of CV and PERC are shown in Table 1.

Ordering to Holding Cost Ratio (O/H). Ordering and holding cost parameters were selected for 3 and 5 period order cycles. In particular, Blackburn and Millen [3] show that an order cycle of T periods is achieved by selecting the smallest integer T where:

$$T(T+1) \geq \frac{S_m}{h_m \text{ (Average Demand per period)}}.$$

Ordering costs were \$520 and \$1300 at their low and high levels, respectively. Holding cost per unit per period was 1.

Materials lot sizes were found using an improved form of the Wagner-Whitin algorithm listed by Evans [8]. The algorithm is preferable to earlier procedures in that its computer execution time is a polynomial function of T^2, where T is the number of periods in the planning horizon. Utilizing the notation listed earlier, for material m:

Define: F_t = Minimum cost lot sizing solution for periods 0 through t

M_{kt} = Cost incurred by ordering in period k for all periods k through t

The recursive equations for the Wagner-Whitin algorithm for the PSMOP are:

$$F_t = \underset{1 \leq k \leq t}{\text{Minimum}} (F_{k-1} + M_{kt}) \; t = 1, 2, \ldots, T \quad (16)$$

and:

$$M_{kt} = S_m + \sum_{l=k}^{t-1} h_m \sum_{n=l+1}^{t} \sum_{j=1}^{N} R_{jm} X_{jn}.$$

Evan's modified algorithm relies on the following observations to reduce computational complexity:

$$M_{kk} = S_m \qquad \text{for all } k = 1, 2, \ldots, T \tag{17}$$

$$M_{k(t+1)} = M_{kt} + \sum_{j=1}^{N} R_{jm} X_{j(t+1)} (t - k) h_m \qquad \text{for all } k \leq t. \tag{18}$$

All computations were performed using a compiled BASIC program that was executed on an IBM PC. The average total computation time required to find the project schedule and materials lot sizes was 4.53 seconds, with a maximum of 4.9 seconds and a minimum of 3.8.

The results for the 24 different project test problems are shown in Table 2. The number of orders placed by the Lot-for-Lot and Wagner-Whitin procedures are also listed. Note that even at the lowest levels of PERC that the Wagner-Whitin procedure generated fewer orders than Lot-for-Lot.

The Wagner-Whitin algorithm yielded results that were on the average 42.17% less costly than those provided by the Lot-for-Lot strategy. However, Lot-for-Lot did provide very good results in a limited number of situations, particularly when CV and O/H were at their low levels. However, under the high setting of CV Lot-for-Lot did not perform well, since it often placed two orders when the optimal strategy was to place one order covering two or more replenishments. Thus, even in cases where requirements were sparse, the Wagner-Whitin algorithm outperformed the simple Lot-for-Lot strategy, and its performance improved as conditions became less favorable for the Lot-for-Lot strategy.

Although Wagner-Whitin has often been discarded for its computational complexity and excessive computational requirements, the low execution times in this research for the Evans algorithm suggest that it may in fact be a practical solution procedure for large problems. In those cases where projects are excessively large and complex, a number of heuristic procedures are available that have been shown to perform well where demand is highly variable. It has been shown in [14] that a number of heuristic lot-sizing procedures may yield optimal or near optimal lot-sizing solutions to the PSMOP.

Conclusions and Implications for Future Research

It has been proven that the PSMOP may be decomposed into two components in the case where payment is received at the end of the project and resource availability is unconstrained. Each activity is scheduled at its late start date and materials ordered so as to arrive on or before that date. The second part of the PSMOP, materials lot-sizing, is equivalent to the single item time varying demand lot-sizing problem, since lot sizes for one material are independent of those for other items. This decomposition of the PSMOP significantly reduces the computational complexity of the problem. While orders for more than one type of material are accounted for

Table 2. Performance of Lot-for-Lot Vs. Wagner Whitin

Experimental Factor:				Results:		
				Percent Above Optimal for	Number of Orders:	
NET:	CV:	O/H:	PERC:	Lot-for-Lot	Wagner-Whitin	Lot-for-Lot
1	L	H	20%	26.66	4	6
			40	79.42	4	6
		L	20	2.70	4	6
			40	30.39	4	6
	H	H	20	28.25	4	11
			40	86.15	6	11
		L	20	5.33	4	11
			40	43.86	6	11
2	L	H	20	60.92	3	6
			40	65.46	3	6
		L	20	24.45	2	6
			40	19.79	4	6
	H	H	20	59.74	4	10
			40	72.46	5	10
		L	20	24.45	3	10
			40	28.78	6	10
3	L	H	20	28.46	3	6
			40	84.63	5	6
		L	20	9.24	4	6
			40	39.75	5	6
	H	H	20	34.00	4	10
			40	86.70	5	10
		L	20	15.60	4	10
			40	54.99	5	10
Average:				42.17	4.21	8.17
Number of Optimal Solutions:				0		

in the formulation, it has been shown that the optimal solution is to treat each set of material orders independently using the Wagner-Whitin procedure. Moreover, the formulation could be expanded to multiple projects through the establishment of independent schedules and completion dates for each project.

It may be argued that the risk of project delay would be increased by scheduling each activity at its late start date, since any delay in any activity would cause a delay in a project. Chance constraints could be added to the model so that activities with particularly uncertain durations are begun before their late start dates. Such an approach would be useful in situations involving dependence upon uncertain technologies or environmental factors such as the weather or labor unrest. This approach is somewhat comparable to the finding of Whybark and Williams [17] that the use of safety lead time is appropriate in dependent demand inventory systems in cases where the timing of demand is uncertainty.

Future research on this problem is possible in a number of areas. The PSMOP should be examined in the resource constrained environment, including cases where labor or equipment is constrained in availability, or cases where capital is constrained and the objective is to minimize the net present value of a project [7]. It would then remain to be seen whether the effects of material costs would have an impact on the scheduling of activities subject to constraints on the availability of labor, equipment or capital. In both cases the PSMOP has significant implications for project managers in that materials decisions should be an intergral component of the project planning and scheduling process if a project is to be completed in a timely and economical manner.

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