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Constrained Resource Project Scheduling Subject To Material Constraints

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EXECUTIVE SUMMARY

Constrained resource project scheduling techniques schedule project activities subject to finite constraints on the availability of non-storable resources such as labor and equipment without consideration of constraints resulting from material requirements. Projects are frequently delayed and resources are wasted when project activities are delayed due to material shortages. A heuristic procedure is presented here for scheduling large projects subject to the availability of all necessary resources, including materials, manufactured components, facilities, equipment and labor as well as the acquisition lead times required by these resources. Results are listed for tests involving both benchmark and actual problems.

A critical aspect in the scheduling of any large project or series of projects is the acquisition and allocation of resources. The existing models that address resource allocation aspects of the project scheduling problem deal only with non-storable resources that are available in limited quantities, such as labor and equipment (Davis reviews the problem, solution techniques, and technique use in [2] and [3]). However, in practice all projects also require various storable resources such as raw materials and manufactured components, which will be referred to as materials in this article. In a number of settings, including construction, aerospace, research and development, and medical services, projects are frequently delayed because of unforeseen increases in material lead times or the assignment of materials that are common to a number of activities to those that are not the most critical. Such delays will continue to be commonplace if materials acquisitions and inventories are not managed in an integrated fashion with the project schedule.

A heuristic procedure is suggested in this article for scheduling large projects subject to the availability of all necessary resources, including raw materials, manufactured components, facilities, equipment and labor. The technique schedules the activities in a project subject to both resource lead times and availabilities as well as activity durations and precedence constraints. Such an approach should result in the reduction of a number of components of total project cost. Materials costs that should be reduced include materials ordering and holding costs, as well the cost of expediting critical materials. In

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multi-project environments where materials are used in a large number of activities, such a model should facilitate the effective utilization of economic order quantities and quantity discounts. Finally, labor utilization should increase and the cost of the idle time should decrease since workers can be re-assigned to other activities when activities cannot be completed due to materials shortages. These cost reductions are not possible unless all resources constraints, including those generated by materials, are considered during the development of the project schedule.

Because the project scheduling technique described in this article includes aspects of constrained resource project scheduling and materials management in project networks, it was developed in consideration of research in both areas. It was tested for feasibility on a variety of benchmark problems, as well as on an actual application. Previous research, the new technique and the results of the tests are described in succeeding sections of this article.

THE PROBLEM AND PREVIOUS RESEARCH

Constrained resource project scheduling techniques schedule project activities subject to finite constraints on the availability of non-storable resources over the life of a project. The project duration is usually extended beyond the length of the non-resource constrained schedule found by the Critical Path Method (CPM) because activities that are scheduled concurrently in the CPM schedule may generate resource requirements that exceed availabilities. In turn, one of these activities may be delayed until the necessary resources are available, and the critical path of the project may be extended. This scheduling procedure reflects the requirements of a project planning and control system, because while resource levels may be variable in the long run, they are in many cases somewhat fixed in the short run. The project schedule in the near horizon (1 to 3 months) must be continually regenerated so as to meet resource constraints because of changes in a number of factors that affect the project schedule. These factors include changes in activity durations, resource availabilities and precedence relationships between activities.

A variety of techniques are discussed in the literature that will find optimal solutions for projects ranging in size from 50 to 100 activities and requiring up to three different resource types, but these techniques have not yet been proven to be practical for the solution of large problems [2, 4, 12, 15]. Within the group of available techniques, only heuristic procedures have proven to be computationally feasible for the solution of large single and multiple project problems.

Heuristic procedures may be categorized according to the type of rule that they use to prioritize the various project activities for the assignment of resources and as to whether they consider activities for resource assignment serially or in parallel [2]. Serial methods, such as those described by Kelley [6] and Steinberg, *et al.*, [14] schedule activities successfully in order of some pre-determined indice, such as node number or position in the network. Each activity is then scheduled to begin when resources are available and its predecessors have been scheduled. Parallel methods, such as RAMPS [7] and SPAR-1 [17], use a heuristic to select an activity from a list of unscheduled activities whose predecessors have been scheduled. That activity is then scheduled when resources are first available. Pasco [9] concluded that routines that consider activities in parallel for the assignment of resources perform better than serial routines.

The results of studies in heuristic performance are not clear cut. While the best solutions are often found by using a rule that schedules that activity first that has the least slack [5, 10, 11], it has been shown that this may be the worst performing rule in some cases. Patterson [11] found that heuristic performance is correlated with project structure, and he therefore concluded that a number of schedules should be derived for a particular project using a different heuristic for each schedule. Based on the previous research on resource constrained project scheduling, it can be concluded that the preferred method for scheduling large projects under resource constraints is a parallel technique that allows the user to develop a number of schedules using different heuristics, and that the least slack rule should be included as one of the heuristics.

Previous studies, with a few exceptions, have dealt with techniques that only schedule non-storable resources such as labor and equipment. Two recent papers describe techniques that use aspects of Material Requirements Planning (MRP) to find a project schedule. Aquilano and Smith [1] list a set of algorithms for finding project schedules subject to activity durations, precedence constraints, and material lead times and inventories. The technique, which they call CPM-MRP, uses a Materials Requirements Planning-like bill of materials and schedule format to list the project network and project schedule. Requirements for non-storable resources such as labor and equipment are listed, but the project schedule is not found subject to constraints on the availability of these resources. Steinberg, Lee and Khumawala [14] list a Material Requirements Planning-type system that is designed to schedule large projects such as NASA's space shuttle. The authors describe a technique that utilizes a project bill of materials to schedule multiple projects (space shuttle flights) in a serial fashion subject to constraints on non-storable resource availability. Material lead times and inventories are not in the constraint set for this model.

The specifications for the ideal constrained resource project scheduling mentioned above should be appended to include consideration of materials lead times and inventories in the development of the project schedule. The objective of the research presented in this paper is to incorporate a series of scheduling heuristics into a requirements planning system such as the one described in [1] so that it will find an early start schedule subject to both storable and non-storable resource constraints in a parallel algorithm.

THE CONSTRAINED RESOURCE CPM-MRP ALGORITHM

The project scheduling algorithm described in this section finds an early start schedule of activities subject to activity durations and precedence constraints as well as non-storable resource availabilities and material lead times and inventories. The algorithm is a parallel, single pass procedure that uses least slack as the primary rule for the allocation of resources to activities, although it is possible to use other rules.

Let:

a = an activity number

ℓ = identifying number of a labor resource

m = identifying number of a material

f = identifying number of a facility input

n = the number of periods in the schedule

A facility input includes such resources as buildings, dry docks, machines, transportation and materials handling equipment or other re-usable capital equipment. This type of resource's availability characteristics are similar to those of a labor input, in that there are only a fixed number of available unit hours in each time period. A differentiation is drawn in the notation between labor and facilities due to the distinctions in ways in which each type of resource is acquired and maintained.

For each activity number a, let:

ES_a = The earliest time at which activity a may be scheduled to start subject to resource and precedence constraints

EF_a = The earliest time at which activity a may be scheduled to be finished subject to resource and precedence constraints

LS_a = The late start date found by the non-resource constrained CPM-MRP algorithm listed in (1)

PT_a = The duration, (or performance time) of activity a

Let resource r be either a material, labor or facility input to the project. Then, let:

EO_r = Earliest time at which resource r should be ordered for use in activity a

EA_r = Earliest time at which resource r should be scheduled to arrive for use in activity a

LT_r = Lead time on an order for this resource

For material m, let:

$ONH_{m,j}$ = Quantity of material m on hand at end of period j

$REQ_{m,a}$ = Amount of material m required by activity a

$TREQ_{m,j}$ = Total amount of material m required to arrive at the beginning of period j

$REC_{m,j}$ = Quantity of material m that has already been ordered and scheduled to arrive at the beginning of period j

$ORD_{m,i}$ = Quantity of material m to be ordered in period i

i = $j - LT_m$, where if $i < 1$ then $i = 1$

Requirements for non-storable resources are compared with availabilities during each period of the activity that requires the resource in order to find a feasible start date for each activity. This approach permits the algorithm to be used in cases where resource availabilities vary over the life of the project, or where resource consumption varies over the duration of an activity. In the simple example used here, it will be assumed that resource consumption remains fixed over the life of the activity. The modification that is required to allow for variable consumption is trivial.

Let:

$AVAIL_{\ell \text{ or } f,j}$ = Quantity of non-storable resource ℓ or f available for use in period j

$REQ_{\ell \text{ or } f,a}$ = Quantity of ℓ or f required for period by activity a

$TREQ_{\ell \text{ or } f,a}$ = Total quantity of ℓ or f required by activity a

$ORD_{\ell \text{ or } f,i}$ = Quantity of ℓ or f to be ordered in period i

i = $j - LT_{\ell \text{ or } f}$, where if $i < 1$ then $i = 1$

To generate the project schedule:

Step 1: a. Schedule the first activity in the project, which must be a dummy activity without resource requirements. Make this activity the first on a list of scheduled activities. Let:

$$ES_a = EF_a = 1$$

b. Calculate the initial on hand inventory for each period, subject to scheduled receipts of materials:

$$ONH_{m,j} = ONH_{m,j-1} + REQ_{m,j} \qquad \text{where} \qquad j = 1, \ldots, n$$

and

$$ONH_{m,0} \text{ is the beginning inventory of material m}$$

Step 2: Derive a list of schedulable activities:

a. Find the list of all activities whose predecessors have been scheduled, and add each of these activities to the list of schedulable activities if they are not on it. Find a tentative early start time, ES'_a for each activity on the list:

$$ES'_a = MAX(EF_a \in \text{predecessors})$$

b. Serially consider each activity with material requirements that is on the list of schedulable activities. Determine whether sufficient materials are on hand to schedule that activity at the time found in Step 2a., or whether the materials must be ordered and the activity start date delayed until materials arrive. For an activity a, if:

$$ES'_a > 1 + LT_m$$

for a material m required by activity a, then sufficient time is available to order the material, and the next material, if any, required by this activity may be considered. If not, determine whether:

$$ONH_{m,j} \geqslant REQ_{m,a}, \qquad \text{where} \qquad ES'_a \leqslant j < 1 + LT_m$$

If so, then set $ES'_a = j$ for the first j where on hand exceeds or equals requirements. If sufficient units are not on hand at any time during this time period, set:

$$ES'_a = 1 + LT_m$$

Continue until each material required by this activity has been considered, then consider each succeeding activity on the list of schedulable activities that has material requirements.

c. Serially consider all activities on the list of schedulable activities that have non-storable resource requirements. Find the non-storable resource requirements for each activity. For each of the non-storable resources required by an activity determine whether:

$$REQ_{\ell \text{ or } f,a} \leqslant AVAIL_{\ell \text{ or } f,j}, \qquad \text{where} \qquad j = ES'_a, \ldots, EF'_a - 1$$

where the tentative early finish time, $EF'_a = ES'_a + PT_a$. If this is true, then the tentative early start time remains unchanged. If not, set:

$$ES'_a = ES'_a + 1$$

and continue to compare requirements with availability until a feasible tentative early start time is found. If sufficient resources are never available, stop the execution of the algorithm and revise either resource requirements or availabilities. If resources are available consider each succeeding requirement for non-storable resources for this activity, and then consider each succeeding activity with non-storable resource requirements on the list of schedulable activities.

d. Compute new slack values for all schedulable activities:

$$\text{Slack}_a = LS_a - ES'_a$$

Step 3: Select an activity for scheduling from the list of schedulable activities according to the following heuristics:

a. Schedule the activity with minimum slack and set:

$$ES_a = ES'_a$$

If there is a tie, *go to b.*

b. Select that activity with the highest identifying number and schedule it such that $ES_a = ES'_a$.

Step 4: Revise the inventory records to reflect the quantities of resources available after an activity is scheduled in Step 3, and generate orders for resources.

a. For all materials required by the scheduled activity:

$\overline{ONH}_{m,j} = ONH_{m,j} - \underline{REQ_{m,a}}$

where $j = ES_a, \ldots, n$ and $\overline{ONH}_{m,j}$ is the revised quantity on hand. Then, if $\overline{ONH}_{m,j} \geq 0$, do not generate any orders. If $\overline{ONH}_{m,j} < 0$ then generate an order for the amount required:

$$\overline{ORD}_{m,i} = |ONH_{m,j}|$$

where $i = ES_a - LT_m$ and $\overline{ORD}_{m,j}$ is the revised order quantity.

b. For all non-storable resources required by the scheduled activity set:

$$\overline{AVAIL}_{\ell \text{ or } f,j} = Avail_{\ell \text{ or } f,j} - REQ_{\ell \text{ or } f,a}$$

where $j = ES_a, \ldots, EF_a - 1$ and $\overline{AVAIL}_{\ell \text{ or } f,j}$ is the revised quantity available. Then, generate orders for non-storable resource and add them to previous orders, if there are any:

$$\overline{ORD}_{\ell \text{ or } f,i} = TREQ_{\ell \text{ or } f,a} + ORD_{\ell \text{ or } f,i}$$

where $i = ES_a - LT_{\ell \text{ or } f}$, and $\overline{ORD}_{\ell \text{ or } f,i}$ is the revised order quantity.

c. Restore the inventory related variables to their original form:

$$\begin{aligned} ONH_{m,j} &= \overline{ONH}_{m,j} \\ ORD_{m,i} &= \overline{ORD}_{m,i} \\ AVAIL_{\ell \text{ or } f,j} &= \overline{AVAIL}_{\ell \text{ or } f,j} \end{aligned}$$

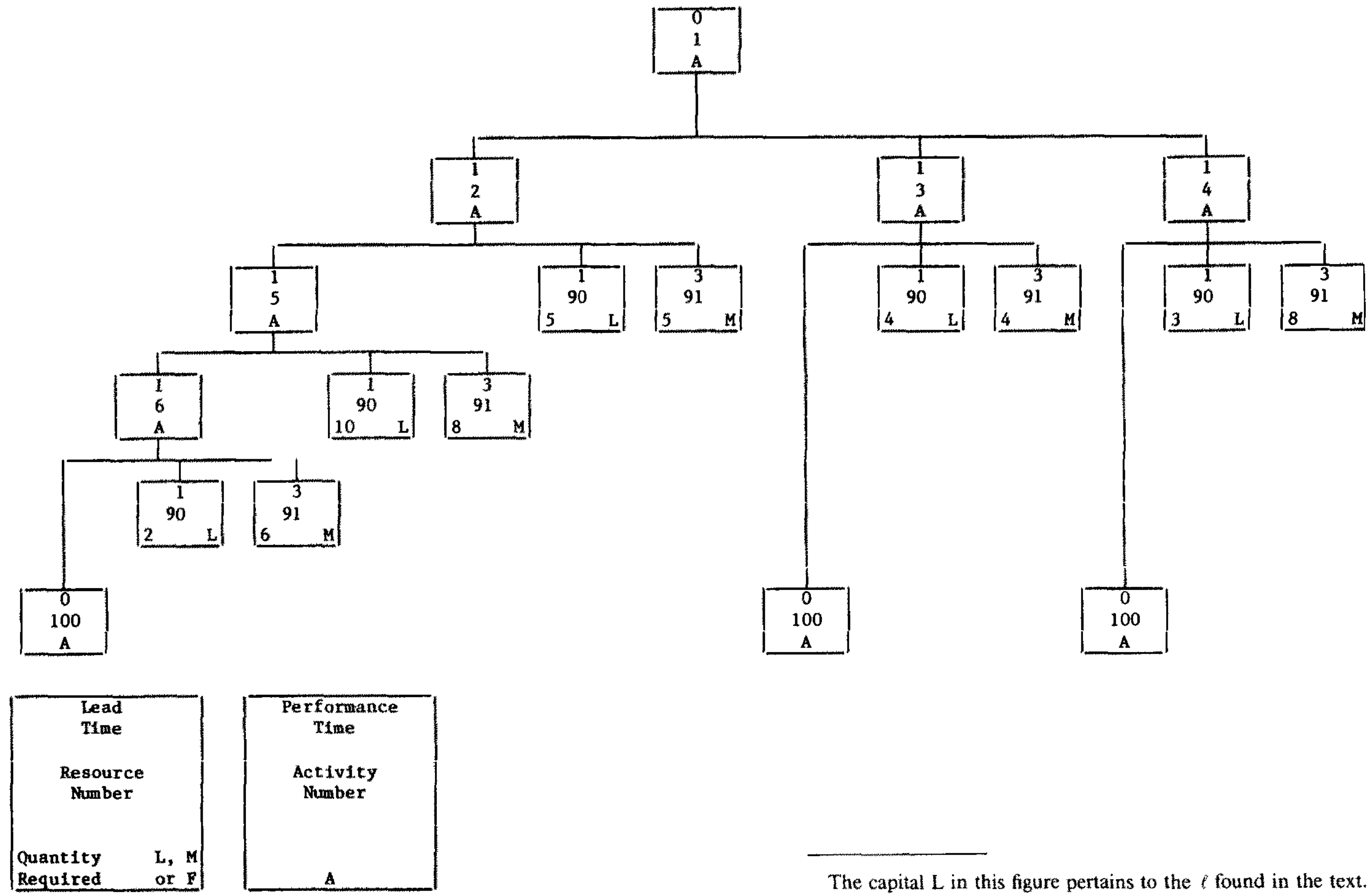
For $AVAIL_{\ell \text{ or } f,j}$, $j = ES_a, \ldots, ES_a - 1$ where a is the activity that was just scheduled.

Step 5: If all activities have been scheduled, *stop.* If not, *return to Step 2.*

AN EXAMPLE SCHEDULING PROBLEM

The short project that will be used to demonstrate the algorithm listed in the previous section is based on a problem described by Wiest [16], that has been expanded to include material requirements. This example was chosen because it facilitates the demonstration of features of the algorithm. The project is displayed using a project structure tree [1] in

FIGURE 1
Project Structure Tree for the Example Project



The capital L in this figure pertains to the ℓ found in the text.

Figure 1, which displays both the project activities and resource requirements. While the original Wiest problem involved only non-storable resources, and no storable resources, the expanded project used here requires one non-storable resource (90) and one storable resource (91), which is a material. There are eight units of material 91 on hand, and twelve units of the labor resource, 90, are available per day. The materials could be on hand either because they were acquired, but not used on an earlier project, or because they were ordered so as to take advantage of economic order quantities or quantity discounts. Any additional requirements above the eight units must be fulfilled through orders. Activities 1 and 100 have been added to the Wiest problem to give it starting and ending nodes.

The non-constrained CPM-MRP schedules [1] are derived first to provide the late start times required to calculate the slack values for each activity. These schedules are displayed in Figures 2 and 3. Each activity's scheduled start and finish dates are marked with a one, since they only take place once. The planned early finish classification can be used after the project is underway to indicate that an activity has been started and that it should be finished by the planned early finish date. This classification functions much the same as a planned early receipt would in MRP. After an activity has been started, a one would no longer appear in the early start row, but a one would appear in the planned early finish row on the appropriate date. The planned early receipt category functions as in MRP, that is, as an indication of what orders have been released for materials and when they are scheduled to arrive.

The project due date is set at day ten. The project would require three days to complete if there were no requirements for materials, and the critical path would include activities 2, 5 and 6. However, since each activity requires certain amounts of material 91, which has a three day lead time, the project duration is five days for the late start schedule and six days for the early start schedule. The difference in length between the two non-constrained resource schedules exists because resource allocation does not take place in the non-resource constrained algorithms. In the early start schedule either activity 3, activity 4, or activity 6 could be scheduled to start on the first day, as each of the three activities requires less than eight units of material 91 that are on hand, and the non-storable resource is assumed to be available in unlimited amounts. However, there are only enough units of material 91 on hand to schedule one of the three activities, and a decision must be made as to how this scarce resource should be allocated. The original CPM-MRP algorithms did not include a rule for resource allocation, therefore the start date for all three activities is delayed until period four, which is when a unit of resource 91 that is ordered on day one will arrive. It will be shown how the duration of the early start schedule can be shortened to five days through proper allocation of resources. In the late start schedule, both activities 3 and 4 are left shifted to day ten, so that activity 6 is the only activity scheduled for day seven. Therefore, no resource allocation decision is necessary and the activities in the project may be scheduled such that the project duration is only five days.

Since a total listing of the solution to the example project would be much too lengthy, only the most essential steps in the development of the project schedule will be covered here. The first of these is the creation of a list of schedulable activities after the first activity in the project, activity 100, has been scheduled. These are activities 3, 4 and 6. After the successors of activity 100 have been identified, the next step is to review the

FIGURE 2
CPM-MRP Early Start Non-resource Constrained Schedule

EARLY START SCHEDULE

ACTIVITY

1 LT = 0	1	2	3	4	5	6	7	8	9	10
EARLY FINISH							1			
PLANNED E. F.							0			
EARLY START							1			

ACTIVITY

2 LT = 1	1	2	3	4	5	6	7	8	9	10
EARLY FINISH							1			
PLANNED E. F.							0			
EARLY START						1				

ACTIVITY

3 LT = 1	1	2	3	4	5	6	7	8	9	10
EARLY FINISH					1					
PLANNED E. F.					0					
EARLY START				1						

ACTIVITY

4 LT = 1	1	2	3	4	5	6	7	8	9	10
EARLY FINISH					1					
PLANNED E. F.					0					
EARLY START				1						

ACTIVITY

5 LT = 1	1	2	3	4	5	6	7	8	9	10
EARLY FINISH						1				
PLANNED E. F.						0				
EARLY START					1					

ACTIVITY

6 LT = 1	1	2	3	4	5	6	7	8	9	10
EARLY FINISH					1					
PLANNED E. F.					0					
EARLY START				1						

LABOR

90 LT = 0	1	2	3	4	5	6	7	8	9	10
REQUIREMENTS				9.00	10.00	5.00				
PLANNED RECEIPTS										
ENDING INVENTORY				-9.00	-19.00	-24.00	-24.00	-24.00	-24.00	-24.00
ORDER RELEASE				9.00	10.00	5.00				

MATERIAL

91 LT = 3	0	1	2	3	4	5	6	7	8	9	10
REQUIREMENTS					18	8	5				
PLANNED RECEIPTS											
ENDING INVENTORY	8	8	8	8	-10	-18	-23	-23	-23	-23	-23
ORDER RELEASE		10	8	5							

ACTIVITY

100 LT = 0	1	2	3	4	5	6	7	8	9	10
EARLY FINISH	1									
PLANNED E. F.										
EARLY START	1									

FIGURE 3
CPM-MRP Non-resource Constrained Late Start Schedule

LATE START SCHEDULE

ACTIVITY

1 LT = 0	1	2	3	4	5	6	7	8	9	10
LATE FINISH										1
PLANNED L. F.										0
LATE START										1

ACTIVITY

2 LT = 1	1	2	3	4	5	6	7	8	9	10
LATE FINISH										1
PLANNED L. F.										0
LATE START									1	

ACTIVITY

3 LT = 1	1	2	3	4	5	6	7	8	9	10
LATE FINISH										1
PLANNED L. F.										0
LATE START									1	

ACTIVITY

4 LT = 1	1	2	3	4	5	6	7	8	9	10
LATE FINISH										1
PLANNED L. F.										0
LATE START									1	

ACTIVITY

5 LT = 1	1	2	3	4	5	6	7	8	9	10
LATE FINISH									1	
PLANNED L. F.									0	
LATE START								1		

ACTIVITY

6 LT = 1	1	2	3	4	5	6	7	8	9	10
LATE FINISH								1		
PLANNED L. F.								0		
LATE START							1			

LABOR

90 LT = 0	1	2	3	4	5	6	7	8	9	10
REQUIREMENTS							2.00	10.00	12.00	
PLANNED RECEIPTS										
ENDING INVENTORY							-2.00	-10.00	-12.00	-12.00
ORDER RELEASE							2.00	10.00	12.00	

MATERIAL

91 LT = 3	0	1	2	3	4	5	6	7	8	9	10
REQUIREMENTS								6	8	17	
PLANNED RECEIPTS											
ENDING INVENTORY	8	8	8	8	8	8	8	2	-6	-23	-23
ORDER RELEASE						6	17				

ACTIVITY

100 LT = 0	1	2	3	4	5	6	7	8	9	10
LATE FINISH					1					
PLANNED L. F.					0					
LATE START					1					

material requirements for each of these activities and compare the requirements to the initial inventory records for material 91:

Material 91 LT = 3	Day 0	1	2	3	4	5	6	7	8	9	10
Requirements		0	0	0	0	0	0	0	0	0	0
Planned Receipts		0	0	0	0	0	0	0	0	0	0
Ending Inventory	8	8	8	8	8	8	8	8	8	8	8
Planned Orders		0	0	0	0	0	0	0	0	0	0

All three activities can be scheduled in day one, as their immediate predecessor, activity 100, is complete at the beginning of day one. Therefore:

$$ES'_3 = ES'_4 = ES'_6 = 1 = \text{MAX}(EF_a \in \text{predecessors}) = EF_{100}$$

From the inventory records and bill of materials:

$ONH_{91,1} = 8$

$REQ_{91,3} = 4 \qquad REQ_{91,4} = 8 \qquad REQ_{91,6} = 6$

Since $ONH_{91,1}$ is sufficient to schedule any one of the three activities on day one, ES'_a remains equal to day one for all three activities.

The non-storable resource requirements for each activity are then compared with the quantity available. The availability of the non-storable resource, labor 90, is displayed below:

Labor 90 LT = 0	Day 1	2	3	4	5	6	7	8	9	10
Requirements	0	0	0	0	0	0	0	0	0	0
Available Res.	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0
Remaining Res.	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0
Planned Orders	0	0	0	0	0	0	0	0	0	0

The requirements for resources cannot exceed the quantities listed in the available resources row. The row labeled "Remaining Resources" lists the quantity of resources left unused after the requirements for the resources used in each period are subtracted from "Available Resources." These classifications are equivalent to the ending inventory and planned receipts categories in MRP. The quantities on hand exceed requirements generated by activities 3, 4, and 6. It is assumed that activities begin at the beginning of a day and end at the beginning of the day listed as the finish date. Therefore, non-storable resource requirements are compared with availabilities beginning with the early start period through the end of period $EF'_a - 1$. ES'_a remains at day one for all three activities.

The application of the first priority rule (least slack) necessitates the computation of slack values for activities 3, 4 and 6:

$Slack_6 = LS_6 - ES'_6 = 7 - 1 = 6$ (minimum)

$\text{Slack}_3 = LS_3 - ES'_3 = 9 - 1 = 8$

$\text{Slack}_4 = LS_4 - ES'_4 = 9 - 1 = 8$

Activity 6 should be scheduled first, since it has the least slack. Activity 6 is scheduled on day one and the resource records are revised:

	Day										
Labor 90 LT = 0	0	1	2	3	4	5	6	7	8	9	10
Requirements		2.0	0	0	0	0	0	0	0	0	0
Available Res.		12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0
Remaining Res.		10.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0
Planned Orders		2.0	0	0	0	0	0	0	0	0	0
Material 91 LT = 3											
Requirements		6	0	0	8	0	0	0	0	0	0
Planned Receipts		0	0	0	0	0	0	0	0	0	0
Ending Inventory	8	2	2	2	−6	−6	−6	−6	−6	−6	−6
Planned Orders		6	0	0	0	0	0	0	0	0	0

The next step is to add any successors of activity 6 to the list of schedulable activities and then find a revised list of possible early start times for the schedulable activities. Activity 5 may be added to the list of schedulable activities, since its predecessor, activity 6, has been scheduled. ES'_5 is set equal to day two, the early finish time of activity 6. The inventory records show that:

$ONH_{91,1} = 2$

$REQ_{91,3} = 4 \qquad REQ_{91,4} = 8 \qquad REQ_{91,5} = 8$

An order placed for material 91 in day one would not arrive until day four. Therefore, since the quantity on hand is less than the requirements generated by all of the schedulable activities, none of the four activities can be scheduled until an order arrives on day four. The revised early start times would be:

$$ES'_3 = ES'_4 = ES'_5 = 4$$

Sufficient quantities of resource 90 are available such that any of the three activities can be scheduled on day four, therefore the tentative early start date remains unchanged.

The application of the least slack heuristic yields the following:

$\text{Slack}_5 = LS_5 - ES'_5 = 8 - 4 = 4$ (minimum)

$\text{Slack}_3 = LS_3 - ES'_3 = 9 - 4 = 5$

$\text{Slack}_4 = LS_4 - ES'_4 = 9 - 4 = 5$

Activity 5 is scheduled to start next, because it has the minimum slack value. Activity 5 is scheduled to start on day four, and the inventory records are revised:

Labor 90 LT = 0	Day 0	1	2	3	4	5	6	7	8	9	10
Requirements		2.0	0	0	10.0	0	0	0	0	0	0
Available Res.		12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0
Remaining Res.		10.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0	12.0
Planned Orders		2.0	0	0	10.0	0	0	0	0	0	0
Material 91 LT = 3											
Requirements		6	0	0	8	0	0	0	0	0	0
Planned Receipts		0	0	0	0	0	0	0	0	0	0
Ending Inventory	8	2	2	2	−6	−6	−6	−6	−6	−6	−6
Planned Orders		6	0	0	0	0	0	0	0	0	0

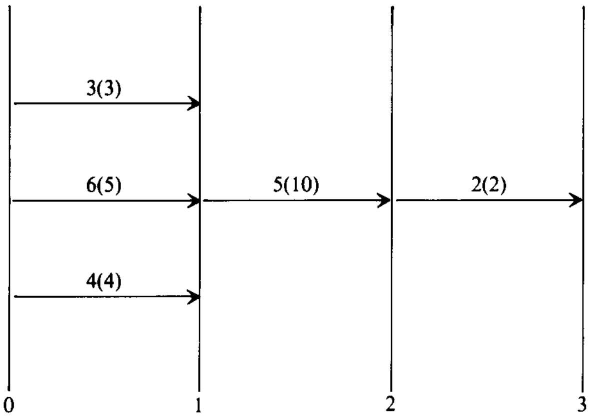
Only six units of material 91 need to be ordered, because there are two units available on day four before activity 5 is scheduled.

A totally different project schedule is generated when material constraints are added to the example project. If material lead time constraints had been ignored, activities 3 and 4 could have been scheduled on day one. There are sufficient non-storable resources available such that this is possible (Table 1(a). When material constraints are added to the problem, activities 3 and 4 cannot be scheduled on the first day, since the material requirements for both activities exceed the two units that are available after activity 6 is scheduled. They cannot be scheduled to occur until day four, when an order for material placed on day one will arrive. Then, on day four, activities 3, 4 and 5 must all compete for the ten units of the non-storable resource that are available. Since activity 5 is scheduled first using the least slack rule and it requires all ten units of the non-storable resource, activities 3 and 4 cannot be scheduled until day 5 at the earliest. The project schedule with the material constraints added is shown in Table 1(b). The gap between activity 6 and activity 5 exists due to the material lead time constraint mentioned above.

A schedule was also generated by a computer program that is based on the algorithm listed in this article (Figure 4). The activity start and finish dates and resource requirements are listed in a form similar to the MRP-like output used by CPM-MRP (1). While the activity dates are listed in an MRP format, they could be listed in a more compact format. The MRP format is used here so that the relationships between resources and activities are evident.

The three day schedule listed in Table 1(a) should be treated as a lower bound on the project's duration before material constraints have been added. This is the resource constrained schedule before material constraints are added. If 31 units of material 91 are available in day one, then the project could be scheduled to be completed in three days. If there are no materials on hand, then all activities will be delayed until the first acquisition of materials takes place in day four, and the project duration will be six days. In the example there are at least enough materials such that one of the activities on the critical path, activity 6, can start on day one. The project duration is then five days, so

Table 1(a)
Schedule Chart for Wiest's Problem—Resource Limit for Resource 90 Equals 12 Units



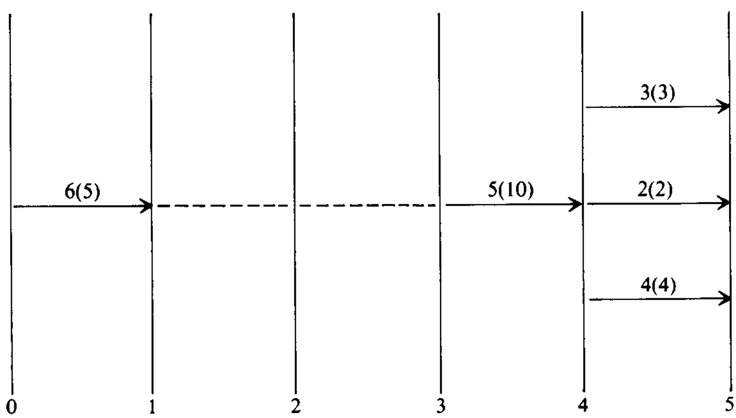
Activity number's are placed on the arrows, and labor requirements per day in the parentheses.

that the project is completed by the beginning of day six. Thus, it can be seen that the project duration is dependent not only on activity durations, precedence constraints and the availability of non-storable resources, but also on material lead times and availabilities.

COMPUTATIONAL EXPERIENCE WITH THE CONSTRAINED RESOURCE ALGORITHM

The constrained resource algorithm has been incorporated into a FORTRAN computer program that has been run on both CDC Cyber 175 and IBM 370 computers. The output appears in the form shown in Figure 4. While the schedule of both activities and resources

Table 1(b)
Schedule Chart for the Modified Problem



Activity 5 is delayed by lead time on material 92.

FIGURE 4
Early Start Constrained Resource Schedule

EARLY START CONSTRAINED RESOURCE SCHEDULE

ACTIVITY

1 LT = 0	1	2	3	4	5	6	7	8	9	10
EARLY FINISH						1				
PLANNED E. F.						0				
EARLY START						1				

ACTIVITY

2 LT = 1	1	2	3	4	5	6	7	8	9	10
EARLY FINISH						1				
PLANNED E. F.						0				
EARLY START					1					

ACTIVITY

3 LT = 1	1	2	3	4	5	6	7	8	9	10
EARLY FINISH						1				
PLANNED E. F.						0				
EARLY START					1					

ACTIVITY

4 LT = 1	1	2	3	4	5	6	7	8	9	10
EARLY FINISH						1				
PLANNED E. F.						0				
EARLY START					1					

ACTIVITY

5 LT = 1	1	2	3	4	5	6	7	8	9	10
EARLY FINISH					1					
PLANNED E. F.					0					
EARLY START				1						

ACTIVITY

6 LT = 1	1	2	3	4	5	6	7	8	9	10
EARLY FINISH		1								
PLANNED E. F.		0								
EARLY START	1									

LABOR

90 LT = 0	1	2	3	4	5	6	7	8	9	10
REQUIREMENTS	2.00			10.00	12.00					
AVAILABLE RESOURCE	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
REMAINING RESOURCE	10.00	12.00	12.00	2.00	0.00	12.00	12.00	12.00	12.00	12.00
ORDER RELEASE	2.00			10.00	12.00					

MATERIAL

91 LT = 3	0	1	2	3	4	5	6	7	8	9	10
REQUIREMENTS		6			8	17					
PLANNED RECEIPTS											
ENDING INVENTORY	8	2	2	2	-6	-23	-23	-23	-23	-23	-23
ORDER RELEASE		6	17								

ACTIVITY

100 LT = 0	1	2	3	4	5	6	7	8	9	10
EARLY FINISH	1									
PLANNED E. F.	0									
EARLY START	1									

is shown in an MRP format, for operational purposes and the sake of brevity it is only really necessary to show the resource schedule in this format, while the activities could be listed in a more compact format.

The computer program was used to schedule a variety of projects. The projects and results appear in Table 2. The list of projects includes benchmark problems found in Martino [8] and Wiest [17], which were expanded to include material requirements for all project activities. Execution times are reasonable for all of the projects. While the relationship between execution time and project size is essentially linear, this is a limited sample and a larger sample would be necessary to predict the relationship between project size and heuristic performance.

The project labeled "Engineering Network" was a large research and development project executed by one of the big three automobile manufacturers in the United States. Personnel involved with the project provided data on the major resource inputs to the project, as well as their opinion on the results. While they were not in position to implement the technique at the time (1979) due to economic conditions, they felt that materials requirements had often been the source of delays in projects and that they should be explicitly included in the project network.

CONCLUSIONS AND IMPLICATIONS

The constrained resource algorithm listed here integrates the materials function into the project schedule so that changes in any one of the project variables, including activity durations, resource availabilities and material lead times, will be rejected in terms of their effect on the entire schedule of activities and resource acquisitions. Thus the purchasing and scheduling functions may be integrated for the purpose of planning materials requirements and availabilities over multiple projects. A number of aspects of the problem

TABLE 2
Execution Time Summary and Linear Regression Results For the Seven Projects Scheduled With the Constrained Resource Algorithm

	Number of Activities	Number of Materials	Number of Non-Stor. Resources	Execution Time (in Seconds)
Weist Example	11	1	3	.290
Martino Project No.				
327	21	1	1	1.193
327 doubled	41	1	1	3.321
327 tripled	61	1	1	6.649
736	16	1	1	.702
821	27	1	4	1.524
836	16	1	3	.819
Engineering Network	41	5	4	4.283

Regression Equation:
X = Number of Activities and Resources
Y = Execution Time
$Y = -1.808037 + .12559X$ $\qquad$ $R = .9689$

need to be explored before this can be done effectively, however. These include both the theoretical underpinnings of the model and the mechanics of applying the model to real-world problems. They will be discussed in this final section.

Although lead time uncertainty has been dealt with extensively in the production and inventory control literature, the effect of variability in resource lead times in project applications has not been discussed. The algorithm listed in this paper should be expanded to include consideration of lead time uncertainty, and it should be determined how activities should be scheduled subject to the different degrees of uncertainty in various material lead times. Although PERT has been used to deal with uncertainty in activity durations, materials lead times and safety stocks are beyond the capabilities of this model, therefore there is a need for a model that includes these considerations as well as resource constraints.

All large projects must be re-scheduled due to changes in activity durations, resource lead times and resource availabilities. In practice, much of this re-scheduling is done by hand or without recognition of the changes in resource requirements that will take place when activities are re-scheduled. Such practices may result in overtime and expediting costs that were not reflected in the original project schedules. In the example listed in this paper it could be seen that materials lead times would not cause the delay of activities after the expiration of the longest material lead time (the first day in the project plus the lead time). However, many activities may be delayed after the project is in progress due to the type of uncertainty in lead times discussed previously, or due to changes in the availability of the various types of non-storable resources required by an activity. Because these changes in availability and lead time will take place, it is important to evaluate the value of heuristics that may be used to re-schedule the project after these changes take place. This problem is different from the one that has been explored previously in the literature on resource constrained project scheduling, in that stochastic variables are now introduced into the problem. The problem now becomes one of scheduling the activities that are remaining in a project subject to the uncertainty in the lead times of those materials that are required to complete the project.

While many materials are only required by one or two activities in a project, there are usually a number of components that are used in a large number of activities. Examples of such items include structural materials, electrical components or fasteners that are used in the construction industry. Obviously this will require that the producer develop some sort of inventory policy for stocking those items that are commonly used over a variety of activities, so that these materials might be purchased in the most economical quantities. In a recent Business Week article a project manager in the U.S. Naval ship yards points out that the Navy is now taking advantage of quantity discounts on purchased parts that are used on a variety of different projects [19]. An economical purchasing strategy would require that the scheduler link the demand for parts with requirements if such discounts are to be utilized effectively. It would be useful to determine whether the findings in the literature on lot sizing in an MRP environment are applicable in a project setting.

The constrained resource algorithm should be tested on additional real-world projects so as to determine its applicability in a variety of industries. The solution generated for the engineering project discussed in the previous section was acceptable to the engineering personnel who would use it and they felt that it would be useful for project control. They felt that it would improve their utilization of resources and enable more efficient rescheduling. The bill of materials approach used in generating the project schedule requires documentation of estimates of materials and labor requirements in projects. Particularly in the construction industry, this would provide the contractor with a method for improving estimating performance and control of material and labor usage. It would also allow the contractor to integrate the costing function with the project schedule so that progress payments could be more easily supported and justified. The authors are familiar with a number of large construction projects where the justification of progress payments has been impeded by a lack of adequate documentation on the part of the prime contractor. The ability of the contractor to trace the resource of materials back to particular arrival dates and production dates was necessary to the justification of progress payments and the avoidance of performance penalties.

Each of the four extensions discussed here deals with the materials constraints that are introduced into the constrained resource project scheduling problem in the algorithm listed in this paper. The major implication of this technique is that it allows the user to integrate the acquisition and storage of materials with the acquisition of other resources so that each is available when it is required by a particular activity in a project. As shown in the example, if this is not done, activities may be delayed and the project may be unexpectedly extended if the project schedule is derived first and then material requirements are considered later. This will result in higher project costs due to expediting and overtime, and perhaps an extension in the project duration.

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