GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING (Autonomous)
MCA I SEMESTER
Mathematical Foundations of Computer Applications (MFCA)
(20BM3101)

UNIT 1: Mathematical Logic

1. Show that $(\neg P \wedge [\neg Q \wedge R]) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$
2. Determine whether $[\neg P \wedge (\neg Q \wedge R)] \vee (Q \wedge R) \vee (P \wedge R)$ is a tautology
3. Determine the PDNF and PCNF of the formula $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$
4. Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q, Q \rightarrow R, P \rightarrow M$ and $\neg M$
5. Show that $R \rightarrow S$ is a valid conclusion from the premises $P \rightarrow (Q \rightarrow S), \neg R \vee P$ and Q
6. Prove that $(\exists x) M(x)$ follows logically from the premises $(\exists x) H(x)$ and $(x)[H(x) \rightarrow M(x)]$

UNIT 2: Relations

7. Let $A = \{1,2,3,4,5\}, R = \{(1,1), (1,4), (2,2), (2,5), (3,3), (4,1), (4,4), (5,2), (5,5)\}$. Write the matrix and draw the graph of the relation R.
8. Let $R = \{(1,2), (2,2), (3,4)\}$ and $S = \{(1,3), (2,5), (3,1), (4,2)\}$ be the relations on $A = \{1,2,3,4,5\}$. Find (i) $R \circ S$ (ii) $S \circ R$ (iii) $R \circ R$ (iv) R^3 (v) $S \circ S$ (vi) $R \circ (S \circ R)$ (vii) $(R \circ S) \circ R$
9. Let $A = \{a,b,c\}$ and $R = \{(a,b), (b,c), (c,a)\}$ be a relation on A. Find the transitive closure of R
10. Let $R = \{(1,1), (1,2), (2,1), (3,2), (3,3)\}$ and $S = \{(1,2), (2,1), (2,2)\}$ be relations on the set $A = \{1,2,3\}$. Find (i) M_R (ii) M_S (iii) $M_R \circ M_S$ (iv) $R \circ S$ (v) $M_{R \circ S}$
11. Prove that $R = \{(x, y) \mid x - y \text{ is divisible by } 3\}$ is an equivalence relation on the set $A = \{1,2,3,4\}$.
12. If R and S are both equivalence relations, show that $R \cap S$ is also equivalence relation
13. Prove that $\leq = \{(x, y) \mid x \text{ divides } y\}$ is a partial order relation on the set $P = \{1,2,3,6\}$
14. Draw the Hasse diagram for the poset $(P, \leq)$, where $P = \{1,2,3,4,6,12\}$ and $\leq$ is divides relation

UNIT 3: Lattice and Boolean algebra

15. Define and give an example (i) Lattice (ii) Bounded lattice (iii) Distributive lattice
16. In a lattice $(L, \leq)$, prove that (i) $a \leq b \Leftrightarrow a \wedge b = a$ (ii) $a \leq b \Leftrightarrow a \vee b = b$
17. In a lattice $(L, \leq)$, prove that (i) $b \leq c \Rightarrow a \wedge b \leq a \wedge c$ (ii) $b \leq c \Rightarrow a \vee b \leq a \vee c$
18. Define Boolean algebra and write the properties
19. In a distributive lattice, prove the following. $a \wedge b = a \wedge c$ and $a \vee b = a \vee c \Rightarrow b = c$
20. Determine (i) the sum of products canonical form (ii) the product of sums canonical form of the Boolean expression $(x_1 * x_2') \oplus x_3$

UNIT 4: Recurrence relations

21. Determine a generating function for the sequence a_r given by the number of integer solutions to the equation $e_1 + e_2 + e_3 = r$ with $0 \leq e_1 \leq 6$, $2 < e_2 \leq 7$, $5 \leq e_3 \leq 7$, e_1 is even and e_2 is odd.
22. Find the generating function for the sequence a_r given by the number of integer solutions to the equation $e_1 + e_2 + e_3 + e_4 + e_5 = r$ with $0 \leq e_1 \leq 3$, $0 \leq e_2 \leq 3$, $2 \leq e_3 \leq 6$, $2 \leq e_4 \leq 6$, $0 \leq e_5 \leq 9$ and e_5 is odd.

23. Determine the coefficient of x^r in the expansion of $\dfrac{1}{x^2-5x+6}$

24. Find the coefficient of (i) x^{16} (ii) x^{18} (iii) x^{20} in the product $(x+x^2+x^3+x^4+x^5)(x^2+x^3+\cdots)^5$

25. Solve the recurrence relation $a_n = a_{n-1}+n(n-2)$ for $n \geq 1,\ a_0 = 2$ by the substitution method

26. Solve the recurrence relation $a_n - 7a_{n-1} + 12a_{n-2} = 0$ for $n \geq 2,\ a_0 = 2,\ a_1 = 5$ by using the method of characteristic roots

27. Solve the recurrence relation $a_n - 10a_{n-1} + 21a_{n-2} = 0$ for $n \geq 2,\ a_0 = \dfrac{10}{21},\ a_1 = 2$ using generating functions

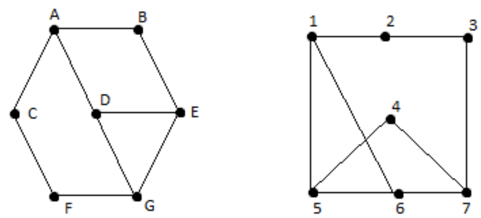
UNIT 5: Graph Theory

28. Define and give an example for each (i) Graph (ii) Directed graph (iii) Subgraph (iv) Spanning subgraph

29. Define and give an example for each (i) Complete graph (ii) Regular graph (iii) Complete bipartite graph (iv) Connected graph

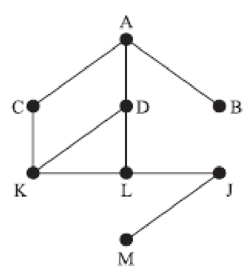
30. Show that in any non directed graph there is even number of vertices of odd degree

31. Define isomorphism of graphs and show that the following graphs are isomorphic



32. Show that a tree with n vertices has exactly $n-1$ edges

33. Determine a spanning tree from the following graph by using (i) BFS algorithm (ii) DFS algorithm



34. Using Kruskal's algorithm, determine a minimal spanning tree from the following graph