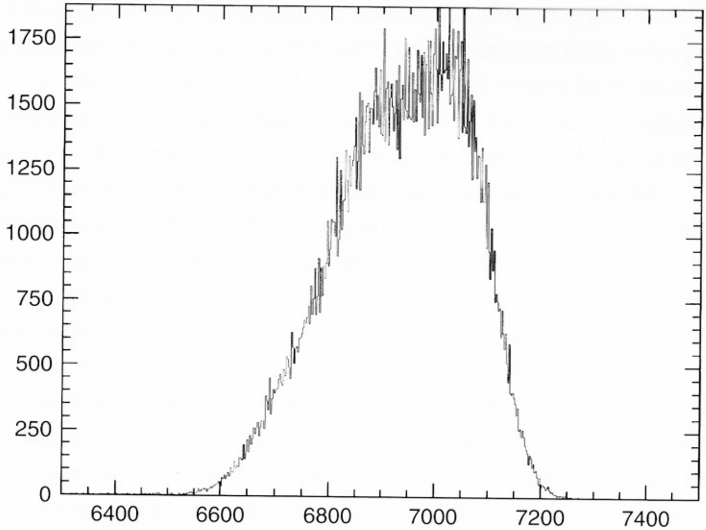

Fig. 4.1. Histogram of a typical flat field image. Note the fairly Gaussian shape of the histrogram and the slight tail extending to lower values. For this R-band image, the filter and dewar window were extremely dusty leading to numerous out of focus "doughnuts" (see Figure 4.4), each producing lower than average data values.

is $\bar{F} = 6950$ ADU and its width (assuming it is perfectly Gaussian (Massey & Jacoby, 1992)) will be given by

$$\sigma_{\mathrm{ADU}} = \frac{\sqrt{\bar{F} \cdot \mathrm{Gain}}}{\mathrm{Gain}}.$$

We have made the assumption in this formulation that the Poisson noise of the flat field photons themselves is much greater than the read noise. This is not unreasonable at all given the low values of read noise in present day CCDs.

Let us now look at how bias frames and flat field images can be used to determine the important CCD properties of read noise and gain. Using two bias frames and two equal flat field images, designated 1 and 2, we can proceed as follows. Determine the mean pixel value within each image.[1] We will call the mean values of the two bias frames $\bar{B}_1$ and $\bar{B}_2$ and likewise $\bar{F}_1$ and $\bar{F}_2$ will be the corresponding values for the two flats. Next, create two difference images ($B_1 - B_2$ and $F_1 - F_2$) and measure the standard deviation

[1] Be careful here not to use edge rows or columns, which might have very large or small values due to CCD readout properties such as amplifier turn on/off (which can cause spikes). Also, do not include overscan regions in the determination of the mean values.

of these image differences: $\sigma_{B_1-B_2}$ and $\sigma_{F_1-F_2}$. Having done that, the gain of your CCD can be determined from the following:

$$\text{Gain} = \frac{(\bar{F}_1 + \bar{F}_2) - (\bar{B}_1 + \bar{B}_2)}{\sigma^2_{F_1-F_2} - \sigma^2_{B_1-B_2}},$$

and the read noise can be obtained from

$$\text{Read noise} = \frac{\text{Gain} \cdot \sigma_{B_1-B_2}}{\sqrt{2}}.$$

4.4 Signal-to-noise ratio

Finally we come to one of the most important sections in this book, the calculation of the signal-to-noise (S/N) ratio for observations made with a CCD.

Almost every article written that contains data obtained with a CCD and essentially every observatory user manual about CCDs contains some version of an equation used for calculation of the S/N of a measurement. S/N values quoted in research papers, for example, do indeed give the reader a feel for the level of goodness of the observation (i.e., a S/N of 100 is probably good while a S/N of 3 is not), but rarely do the authors discuss how they performed such a calculation.

The equation for the S/N of a measurement made with a CCD is given by

$$\frac{\text{S}}{\text{N}} = \frac{N_*}{\sqrt{N_* + n_{\text{pix}}(N_S + N_D + N_R^2)}},$$

unofficially named the "CCD Equation" (Mortara & Fowler, 1981). Various formulations of this equation have been produced (e.g., Newberry (1991) and Gullixson (1992)), all of which yield the same answers of course, if used properly. The "signal" term in the above equation, N_*, is the total number of photons[1] (signal) collected from the object of interest. N_* may be from one pixel (if determining the S/N of a single pixel as sometimes is done for a background measurement), or N_* may be from several pixels, such as all of those contained within a stellar profile (if determining the S/N for the

[1] Throughout this book, we have and will continue to use the terms photons and electrons interchangeably when considering the charge collected by a CCD. In optical observations, every photon that is collected within a pixel produces a photoelectron; thus they are indeed equivalent. When talking about observations, it seems logical to talk about star or sky photons, but for dark current or read noise discussions, the number of electrons measured seems more useful.

measurement of a star), or N_* may even be from say a rectangular area of X by Y pixels (if determining the S/N in a portion of the continuum of a spectrum).

The "noise" terms in the above equation are the square roots of N_*, plus n_{pix} (the number of pixels under consideration for the S/N calculation) times the contributions from N_S (the total number of photons per pixel from the background or sky), N_D (the total number of dark current electrons per pixel), and N_R^2 (the total number of electrons per pixel resulting from the read noise.[1]

For those interested in more details of each of these noise terms, how they are derived, and why each appears in the CCD Equation, see Merline & Howell (1995). In our short treatise, we will remark on some of the highlights of that paper and present an improved version of the CCD Equation. However, let's first make sense out of the equation just presented.

For sources of noise that behave under the auspices of Poisson statistics (which includes photon noise from the source itself), we know that for a signal level of N, the associated 1 sigma error (1σ) is given by $\sqrt{N}$. The above equation for the S/N of a given CCD measurement of a source can thus be seen to be simply the signal (N_*) divided by the summation of a number of Poisson noise terms. The n_{pix} term is used to apply each noise term on a per pixel basis to all of the pixels involved in the S/N measurement and the N_R term is squared since this noise source behaves as shot noise, rather than being Poisson-like (Mortara & Fowler, 1981). We can also see from the above equation that if the total noise for a given measurement $\sqrt{N_* + n_{\mathrm{pix}}(N_S + N_D + N_R^2)}$ is dominated by the first noise term, N_* (i.e., the noise contribution from the source itself), then the CCD Equation becomes

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{N_*}{\sqrt{N_*}} = \sqrt{N_*},$$

yielding the expected result for a measurement of a single Poisson behaved value.

This last result is useful as a method of defining what is meant by a "bright" source and a "faint" source. As a working definition, we will use the term bright source to mean a case for which the S/N errors are dominated by the source itself (i.e., S/N $\sim \sqrt{N_*}$), and we will take a faint source to be the case in which the other error terms are of equal or greater significance compared with N_*, and therefore the complete error equation (i.e., the CCD Equation) is needed.

[1] Note that this noise source is not a Poisson noise source but a shot noise; therefore it enters into the noise calculation as the value itself, not the square root of the value as Poisson noise sources do.

The CCD Equation above provides the formulation for a S/N calculation given typical conditions and a well-behaved CCD. For some CCD observations, particularly those that have high background levels, faint sources of interest, poor spatial sampling, or large gain values, a more complete version of the error analysis is required. We can write the complete CCD Equation (Merline & Howell, 1995) as

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{N_*}{\sqrt{N_* + n_{\mathrm{pix}}\left(1 + \frac{n_{\mathrm{pix}}}{n_B}\right)\left(N_S + N_D + N_R^2 + G^2\sigma_f^2\right)}}.$$

This form of the S/N equation is essentially the same as that given above, but two additional terms have been added. The first term, $(1 + n_{\mathrm{pix}}/n_B)$, provides a measure of the noise incurred as a result of any error introduced in the estimation of the background level on the CCD image. The term n_B is the total number of background pixels used to estimate the mean background (sky) level. One can see that small values of n_B will introduce the largest error as they will provide a poor estimate of the mean level of the background distribution. Thus, very large values of n_B are to be preferred but clearly some trade-off must be made between providing a good estimate of the mean background level and the use of pixels from areas on the CCD image that are far from the source of interest or possibly of a different character.

The second new term added into the complete S/N equation accounts for the error introduced by the digitization noise within the A/D converter. From our discussion of the digitization noise in Chapter 3, we noted that the error introduced by this process can be considerable if the CCD gain has a large value. In this term, $G^2\sigma_f^2$, G is the gain of the CCD (in electrons/ADU) and σ_f is an estimate of the 1 sigma error introduced within the A/D converter[1] and has a value of approximately 0.289 (Merline & Howell, 1995).

In practice for most CCD systems in use and for most observational projects, the two additional terms in the complete S/N equation are often very small error contributors and can be ignored. In the instances for which they become important – for example, cases in which the CCD gain has a high value (e.g., 100 electrons/ADU), the background level can only be estimated with a few pixels (e.g., less than 200), or the CCD data are of poor pixel

[1] The parameter σ_f^2 and its value depend on the actual internal electrical workings of a given A/D converter. We assume here that for a charge level that is half way in between two output ADU steps (that is, 1/2 of a gain step), there is an equal chance that it will be assigned to the lower or to the higher ADU value when converted to a digital number. See Merline & Howell (1995) for further details.

sampling (see Section 5.9) – ignoring these additional error terms will lead to an overestimation of the S/N value obtained from the CCD data.

Let us work through an example of a S/N calculation given the following conditions. A 300-second observation is made of an astronomical source with a CCD detector attached to a 1-m telescope. The CCD is a Thomson 1024×1024 device with 19-micron pixels and it happens that in this example the telescope has a fast f-ratio such that the plate scale is 2.6 arcsec/pixel.[1] For this particular CCD, the read noise is 5 electrons/pixel/read, the dark current is 22 electrons/pixel/hour, and the gain (G) is 5 electrons/ADU. Using 200 background pixels surrounding our object of interest from which to estimate the mean background sky level, we take a mean value for N_B of 620 ADU/pixel. We will further assume here (for simplicity) that the CCD image scale is such that our source of interest falls completely within 1 pixel (good seeing!) and that after background subtraction (see Section 5.1), we find a value for N_* of 24 013 ADU. Ignoring the two additional minor error terms discussed above (as the gain is very small and $n_B = 200$ is quite sufficient in this case), we can write the CCD Equation as

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{24\,013(\mathrm{ADU}) \cdot G}{\sqrt{24\,013(\mathrm{ADU}) \cdot G + (1) \cdot (620(\mathrm{ADU}) \cdot G + 1.8 + 5^2(e^-))}}.$$

Note that all of the values used in the calculation of the S/N are in electrons, *not* in ADUs. The S/N value calculated for this example is ~ 342, a very high S/N. With such a good S/N measurement, one might suspect that this is a bright source. If we compare $\sqrt{N_*}$ with all the remaining error terms, we see that indeed this measurement has its noise properties dominated by the Poisson noise from the source itself and the expression $\mathrm{S/N} \sim \sqrt{N_*} = 346$ works well here.

While the S/N of a measurement is a useful number to know, at times we would prefer to quote a standard error for the measurement as well. Using the fact that $\mathrm{S/N} = 1/\sigma$, where σ is the standard deviation of the measurement, we can write

$$\sigma_{\text{magnitudes}} = \frac{1.0857\sqrt{N_* + p}}{N_*}.$$

In this expression, p is equal to $n_{\text{pix}}(1 + n_{\text{pix}}/n_B)(N_S + N_D + N_R^2 + G^2\sigma_f^2)$, the same assumptions apply concerning the two "extra" error terms, and the value of 1.0857 is the correction term between an error in flux (electrons) and that same error in magnitudes (Howell, 1993). We again see that if the Poisson error of N_* itself dominates, the term p can be ignored and this equation

[1] Using the results from Section 4.1, what would be the f-ratio of this telescope?

reduces to that expected for a 1σ error estimate in the limiting case of a bright object.

Additionally, one may be interested in a prediction of the S/N value likely to be obtained for a given CCD system and integration time. N_* is really $N \cdot t$, where N is the count rate in electrons (photons) per second (for the source of interest) and t is the CCD integration time. Noting that the integration time is implicit in the other quantities as well, we can write the following (Massey, 1990):

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{Nt}{\sqrt{Nt + n_{\mathrm{pix}}(N_S t + N_D t + N_R^2)}},$$

in which we have again ignored the two minor error terms. This equation illustrates a valuable rule of thumb concerning the S/N of an observation: $\mathrm{S/N} \propto \sqrt{t}$, not to t itself. Solving the above expression for t we find

$$t = \frac{-B + (B^2 - 4AC)^{1/2}}{2A},$$

where $A = N^2$, $B = -(\mathrm{S/N})^2(N + n_{\mathrm{pix}}(N_S + N_D))$, and $C = -(S/N)^2\ n_{\mathrm{pix}} N_R^2$. Most instrument guides available at major observatories provide tables that list the count rate expected for an ideal star (usually 10th magnitude and of 0 color index) within each filter and CCD combination in use at each telescope. Similar tables provide the same type of information for the observatory spectrographs as well. The tabulated numeric values, based on actual CCD observations, allow the user, via magnitude, seeing, or filter width, to scale the numbers to a specific observation and predict the S/N expected as a function of integration time.

4.5 Basic CCD data reduction

The process of standard CCD image reduction makes use of a basic set of images that form the core of the calibration and reduction process (Gullixson, 1992). The types of images used are essentially the same (although possibly generated by different means) in imaging, photometric, and spectroscopic applications. This basic set of images consists of three calibration frames – bias, dark, and flat field – and the data frames of the object(s) of interest. Table 4.1 provides a brief description of each image type and Figures 4.2–4.5