Google Pagerank Algorithm Application of Eigenvectors

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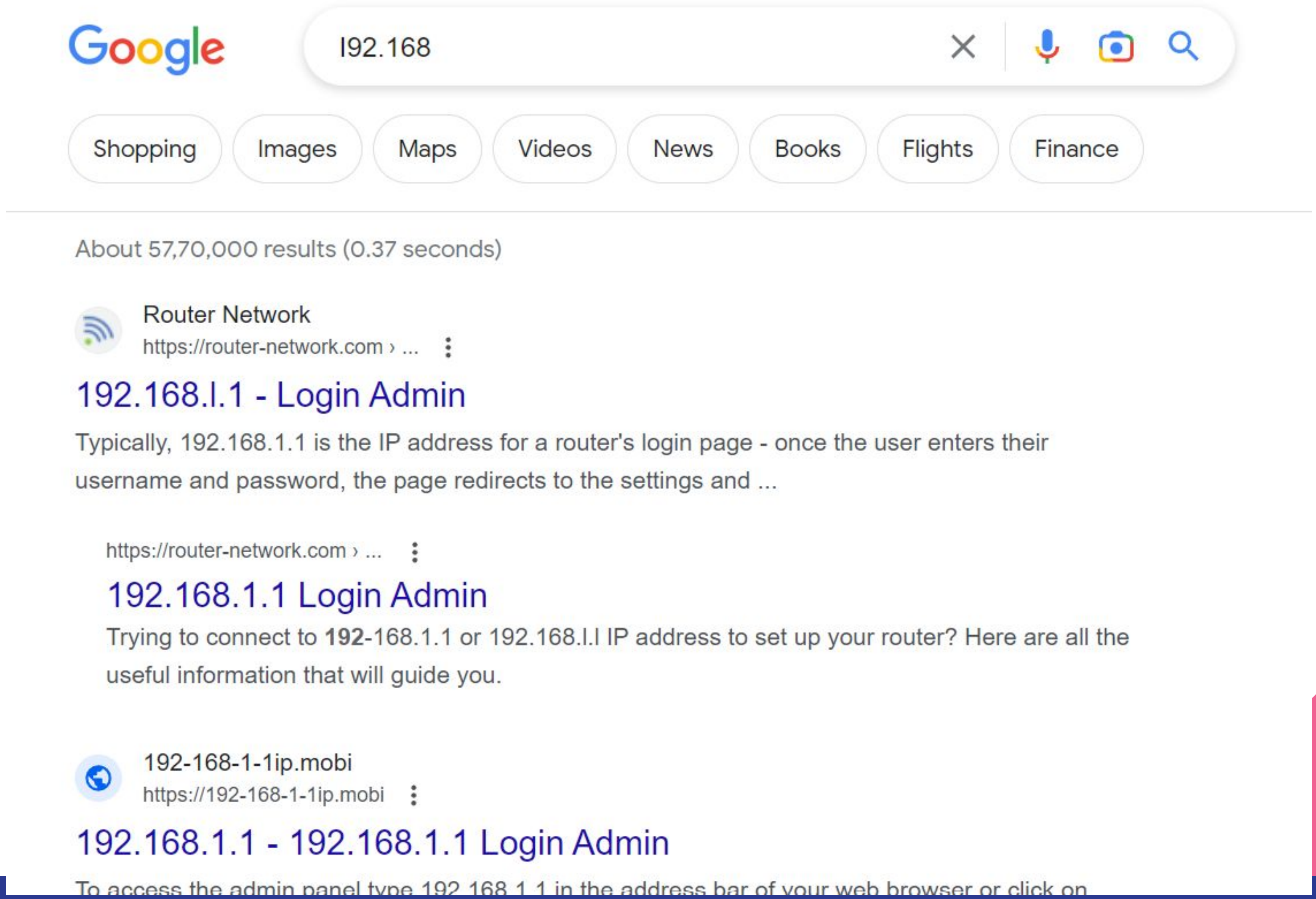


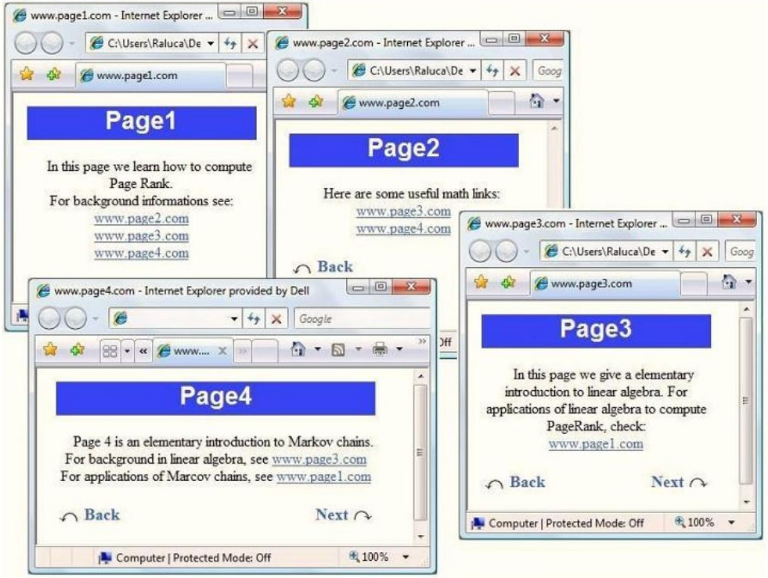
Have you ever wondered how Google Ranks the relevant pages to display on a search?

What is Google pagerank algorithm?

Google PageRank algorithm is a ranking system used by Google to determine the importance and relevance of a website. It assigns a score to each web page based on the number and quality of links pointing to that page.

The higher the PageRank score, the more likely a page is to appear at the top of search results for relevant queries.

Working


www.page1.com - Internet Explorer ...
Page1
In this page we learn how to compute Page Rank.
For background informations see:
www.page2.com
www.page3.com
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www.page2.com - Internet Explorer ...
Page2
Here are some useful math links:
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Page3
In this page we give a elementary introduction to linear algebra. For applications of linear algebra to compute PageRank, check:
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www.page4.com - Internet Explorer provided by Dell
Page4
Page 4 is an elementary introduction to Markov chains.
For background in linear algebra, see www.page3.com
For applications of Marcov chains, see www.page1.com
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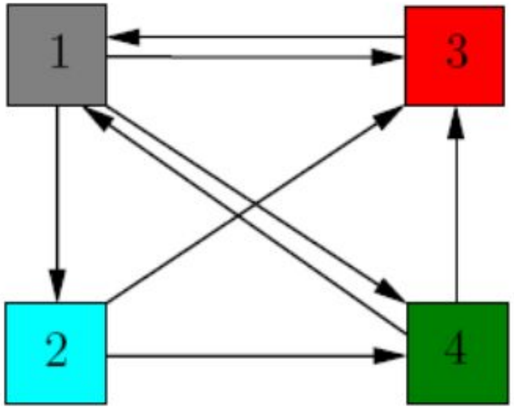
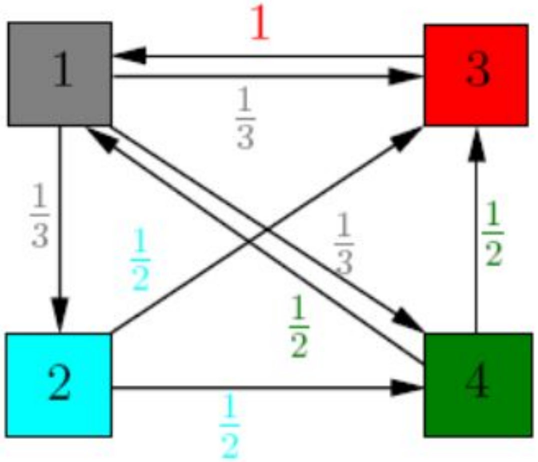

1
3
2
4



$$A = \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix}.$$

Two methods:

1. Dynamical systems point of view
2. Linear algebra point of view

Dynamical systems point of view

$$v = \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{pmatrix}, \quad Av = \begin{pmatrix} 0.37 \\ 0.08 \\ 0.33 \\ 0.20 \end{pmatrix}, \quad A^2 v = A(Av) = A \begin{pmatrix} 0.37 \\ 0.08 \\ 0.33 \\ 0.20 \end{pmatrix} = \begin{pmatrix} 0.43 \\ 0.12 \\ 0.27 \\ 0.16 \end{pmatrix}$$

$$A^3 v = \begin{pmatrix} 0.35 \\ 0.14 \\ 0.29 \\ 0.20 \end{pmatrix}, \quad A^4 v = \begin{pmatrix} 0.39 \\ 0.11 \\ 0.29 \\ 0.19 \end{pmatrix}, \quad A^5 v = \begin{pmatrix} 0.39 \\ 0.13 \\ 0.28 \\ 0.19 \end{pmatrix}$$

$$A^6 v = \begin{pmatrix} 0.38 \\ 0.13 \\ 0.29 \\ 0.19 \end{pmatrix}, \quad A^7 v = \begin{pmatrix} 0.38 \\ 0.12 \\ 0.29 \\ 0.19 \end{pmatrix}, \quad A^8 v = \begin{pmatrix} 0.38 \\ 0.12 \\ 0.29 \\ 0.19 \end{pmatrix}$$

We notice that the sequences of iterates v, Av, ..., $A^k v$ tends to the equilibrium value $v^* = \begin{pmatrix} 0.38 \\ 0.12 \\ 0.29 \\ 0.19 \end{pmatrix}$. We call this the PageRank vector of our web graph.

Linear algebra point of view

Let us denote by x_1, x_2, x_3, and x_4 the importance of the four pages. Analyzing the situation at each node we get the system:

$$\begin{cases} x_1 = 1 \cdot x_3 + \frac{1}{2} \cdot x_4 \\ x_2 = \frac{1}{3} \cdot x_1 \\ x_3 = \frac{1}{3} \cdot x_1 + \frac{1}{2} \cdot x_2 + \frac{1}{2} \cdot x_4 \\ x_4 = \frac{1}{3} \cdot x_1 + \frac{1}{2} \cdot x_2 \end{cases} \qquad A = \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix}.$$

This is equivalent to asking for the solutions of the equations $A \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$. From Example 6 in Lecture 1 we know that the eigenvectors corresponding to the eigenvalue 1 are of the form $c \cdot \begin{bmatrix} 12 \\ 4 \\ 9 \\ 6 \end{bmatrix}$. Since PageRank should reflect only the relative importance of the nodes, and since the eigenvectors are just scalar multiples of each other, we can choose any of them to be our PageRank vector. Choose v^* to be the unique eigenvector with the sum of all entries equal to 1. (We will sometimes refer to it as the probabilistic eigenvector corresponding to the eigenvalue 1). The eigenvector $\frac{1}{31} \cdot \begin{bmatrix} 12 \\ 4 \\ 9 \\ 6 \end{bmatrix} \sim \begin{bmatrix} 0.38 \\ 0.12 \\ 0.29 \\ 0.19 \end{bmatrix}$ is our PageRank vector.

References

http://pi.math.cornell.edu/~mec/Winter2009/RalucaRemus/Lecture3/lecture3.html

https://en.wikipedia.org/wiki/PageRank#:~:text=PageRank%20(PR)%20is%20an%20algorithm,the%20importance%20of%20website%20pages.