LGS GROUP OF COLLEGES
A PROJECT OF LAHORE GRAMMAR SCHOOL

Sheet #

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Class: 11th A
Roll No.

Subject: Mathematics
Test No.
Date: 22-11-2024

Q	A	B	C	D
1				●
2		●		
3				●
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Marks Obtained

Question: 1

SHORT Questions

(i)

Semi group:

A semi group is an aljebric stucture consisting of set S equipped with binary operation denoted * st satisfied the operations.

Properties:

Closure, Associative

ii

Prepare a table:

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

$2\times3-1$ remainder when 6 is divided by 5

$3\times4-2$ remainder when 12 is divided 5

$2\times4=3$ remainder when 8 is divided 5

The set of residue classes modulo 5

Question: 2

Long

$M_2 = \left\{ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ where a } a_{11}, a_{12}, a_{21}, a_{22} \in R \text{ and } \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0 \right\}$

(i) closure property:

for all A

$A \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$AB \quad \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12}a_{22}b_{22} \end{bmatrix}$

Hence $AB \in M_2$

Associative

$\forall A, B, C, \in M_2 \Rightarrow A(BC) = (AB)C$

identity element

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is 2×2 non singular

Thus $I_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Inverse

As inverse of non singular square matrix is again a non singular matrix of that $A \in M_2 \ \exists A^{-1} M^2$ such that

$= A^{-1} A = I$

iii

$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$ show $A^4 = I_2$

$$A^2 = A \cdot A \quad \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} (i)(i) + (0)(1) & (i)(0) + (0)(-i) \\ (1)(i) + (-i)(1) & (1)(0) + (-i)(-i) \end{bmatrix}$$

$$= \begin{bmatrix} i^2 + 0 & 0 + 0 \\ i - i & 0 + i^2 \end{bmatrix}$$

$$= \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= A^4 = A^2 \cdot A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(-1) + (0)(0) & (-1)(0) + (0)(-1) \\ (0)(-1) + (-1)(0) & (0)(0) + (-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 0 & 0 + 0 \\ 0 + 0 & 0 + 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$A^4 = I_2$