

Physics Assignment WK-5

Question no. 1

When an object rolls without slipping, its total mechanical energy is conserved. This includes both translational and rotational kinetic energy.

1) Initial Energy at Height

- The sphere starts from rest, so its initial kinetic energy is zero.
- Its potential energy is mgh , where m is the mass, g is acceleration due to gravity and h is height.

2) Energy at the bottom

- At the bottom, the potential energy is zero.
- The kinetic energy is a combination of translational kinetic energy ($\frac{1}{2}mv^2$) and the rotational kinetic energy ($\frac{1}{2}I\omega^2$) where I is the moment of inertia and ω is the angular velocity.

For a solid sphere, the moment of Inertia I is $\frac{2}{5}mr^2$ and the relationship between the linear velocity, the moment of inertia ω is $\omega = \frac{v}{r}$.

The total energy at the bottom is:

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2$$

Simplifying the rotational kinetic energy term:

$$\frac{1}{2} \cdot \frac{2}{5} mr^2 \cdot \frac{v^2}{r^2} = \frac{1}{5} mv^2$$

So, the total kinetic energy at the bottom is:

$$\frac{1}{2} mv^2 + \frac{1}{5} mv^2 = \frac{7}{10} mv^2$$

Question no. 2

For an object to be in a stable low Earth orbit, this velocity can be calculated using the formula for orbital velocity:

$$v_{\text{critical}} = \sqrt{\frac{GM}{r}}$$

Where:

- G is the gravitational constant, $6.674 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$.
- M is the mass of Earth, approximately $5.972 \times 10^{24} \text{ kg}$.
- r is the distance from the center of the Earth to the object, which for a low Earth orbit is roughly equal to the Earth's radius plus the altitude of the orbit (ranging from about 200 km to 2,000 km above the Earth's surface). However, to simplify, we often use the Earth's radius ($6.371 \times 10^6 \text{ m}$) for basic

calculations.
Using these approximate values, we can calculate

$$v_{\text{critical}} \approx \sqrt{\frac{(6.674 \times 10^{-11})(5.972 \times 10^{24})}{6.371 \times 10^6}}$$

When you compute this, you find that $v_{\text{critical}} = 7900 \text{ m/s}$

Since 7900 m/s is essentially 7.9 km/s , this is the critical velocity required to maintain a stable.

Question no. 3

We can use the concept of apparent weight.

1. Understanding Weight in Physics

- The actual weight W of a person is the gravitational force acting on them, which is mg , where m is the mass of a person and g is the acceleration due to gravity.

2. Apparent weight in an Elevator

- When the elevator is at rest or moving at a constant velocity, the apparent weight is equal to actual weight.
- When the elevator accelerates, the apparent weight is the normal

Force N exerted by the floor. This changes due to the acceleration of the elevator.

3. Elevator Accelerating Upward

• If the elevator accelerates upward with acceleration a , the apparent weight can be expressed using Newton's second law:

$$N = m(g + a)$$

• In this case, a is equal to g .
Therefore,

$$N = m(g + g) = 2mg$$

• Thus, the apparent weight N becomes $2W$, since $W = mg$ the person weight

Thus, when in the elevator will be $2W$ when the elevator accelerating upward with an acceleration equal to g .

Question no. 4

Given :

- Speed of the satellite, $v = 1.01 \text{ km/s}$
- Radius of the orbit, $r = 39,400 \text{ km}$

To find the time T it takes for the satellite to complete one revolution, we use the formula for the circumference of the orbit and the relationship between distance, speed and time.

$$C = 2\pi r$$

$$T = \frac{C}{v} = \frac{2\pi r}{v}$$

First let's calculate the circumference:

$$C = 2\pi \times 390,400 \text{ km} \approx 2,452,629 \text{ km}$$

Now, the time to complete one revolution:

$$T = \frac{2,452,629 \text{ km}}{1.01 \text{ km/s}}$$

$$T \approx 2,428,347 \text{ s}$$

To convert seconds into days:

$$T \approx \frac{2,428,347 \text{ s}}{86400} \approx 28.11 \text{ days}$$

Therefore, the satellite will complete one revolution approximately every 28.11 days.

Question no. 5

The formula for the orbital radius r of a geostationary satellite is given by:

$$r = \left(\frac{GMT^2}{4\pi^2} \right)^{1/3}$$

Where,

• G is the gravitational constant ($6.674 \times 10^{-11} \text{ m}^3/\text{kg s}^2$).

• M is the mass of earth ($5.972 \times 10^{24} \text{ kg}$)

• T is the orbital period (one sidereal day in seconds: 86164 s).

Given these values, we compute the orbital radius r :



$$r = \frac{(1.6 \times 10^{-11}) \times (1.5 \times 10^{24}) \times (86164)}{4\pi^2}$$

Upon calculating this, we find:
 $r = 42,164 \text{ km.}$

This is the distance from the center of the Earth.

Question no. 6

Given:-

Diameter of beam = length of arc = $S = 2.50 \text{ m}$

Distance of moon from the earth = $r = 3.8 \times 10^8 \text{ m}$

Find:-

Divergence angle = $\theta = ?$

Calculation:-

$$\text{As } S = r\theta$$
$$\theta = \frac{S}{r}$$

Putting values, we get

$$\theta = \frac{2.50}{3.8 \times 10^8}$$

$$\theta = 6.6 \times 10^{-9} \text{ rad.}$$

Question no. 7

Given:-

Distance between Earth and the Moon = $r_o = 3.85 \times 10^8 \text{ m}$

Radius of the moon = $r_s = 1.74 \times 10^6 \text{ m}$

Find:-

Ratio of spin and orbital angular = $\frac{L_s}{L_o} = ?$

Calculation:-

The spin angular momentum of the Moon about its own axis is

$$L_s = I_s \omega$$

$$L_s = \frac{2}{5} m r_s^2 \omega \dots (1)$$

The orbital angular momentum is given by

$$L_o = I_o \omega$$

$$L_o = m r_o^2 \omega \dots (2)$$

Dividing equation (1) by equation (2) we get

$$\frac{L_s}{L_o} = \frac{\frac{2}{5} m r_s^2 \omega}{m r_o^2 \omega}$$

$$\frac{L_s}{L_o} = \frac{2}{5} \frac{r_s^2}{r_o^2}$$

$$\frac{L_s}{L_o} = \frac{2}{5} \frac{r_s^2}{r_o^2}$$

putting values, we get

$$\frac{L_s}{L_o} = \frac{2}{5} \frac{(1.74 \times 10^6)^2}{(3.85 \times 10^8)^2}$$

$$\frac{L_s}{L_o} = 8.2 \times 10^{-6}$$

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$$\frac{L_s}{L_o}$$