Sheet # ______

LGS GROUP OF COLLEGES
A PROJECT OF LAHORE GRAMMAR SCHOOL

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Subject: Math's Test No. W5 Date: 23-11-24

	A	B	C	D
1				●
2	●			
3				●
4				●
5				

Marks Obtained

Subjective Type

Short Questions

(i)

Semi-Group:-

A non-empty set S is semi-group if;

(i) It is closed with respect to an operation $*$.

(ii) The operation $*$ is associative.

As it is obvious from its very name, a semi-group satisfies half of the conditions required for a group.

(ii)

The set of residue classes modulo 5:

$*$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

The above table shows the multiplication of the elements of the set of residue classes modulo 5 because in table

$2 \times 3 = 6 = 1$ (remainder after dividing 6 by 5)
$2 \times 4 = 8 = 3$ (remainder after dividing 8 by 5).
$3 \times 3 = 9 = 4$ (remainder after dividing 9 by 5).
$3 \times 4 = 12 = 2$ (remainder after dividing 12 by 5).
$4 \times 4 = 16 = 1$ (remainder after dividing 16 by 5).

(iii)

$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, **Show that** $A^4 = I_2$.

Sol;

$$A^4 = I^2 \quad \text{where} \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} (i)(i)+(0)(1) & (i)(0)+(0)(-i) \\ (1)(i)+(-i)(1) & (1)(0)+(-i)(-i) \end{bmatrix}$$

$$= \begin{bmatrix} i^2+0 & 0-0 \\ i-i & 0+i^2 \end{bmatrix} \quad \because i^2 = -1$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Now,

$$A^4 = A^2 . A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(-1)+(0)(0) & (-1)(0)+(0)(-1) \\ (0)(-1)+(-1)(0) & (0)(0)+(-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2.$$

Student Name: ______

LONG QUESTIONS

(i)

Prove that all 2 x 2 non-singular matrices over the real field from a non-abelian group under multiplication:

Sol:-

Let $G\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix}, a,b,c,d \in R, ad-bc \neq 0\right\}$

C-1 let $A, B \in G$.

Show that

$A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$, $B = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$

$= \begin{bmatrix} a_1a_2 + b_1c_2 & a_1b_2 + b_1d_2 \\ c_1a_2 + d_1c_2 & c_1b_2 + d_1d_2 \end{bmatrix} \in G$

$\Rightarrow$ G is closed under multiplication.

C-2 '$\cdot$' is associative in G.

because in matrices, $\forall A, B, C \in G$

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

C-3 $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in G$ which is an identity element in G such that $\forall A \in G$ $AI_2 = A = I_2A$.

$\Rightarrow$ Identity element exists in G.

C-4 $\forall A \in G$ $\exists A^{-1} \in G$ such that

$A \cdot A^{-1} = I_2 = A^{-1}A$.

We can check it as

if

$$A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

then

$$A^{-1} = \frac{adj\, A}{|A|} = \frac{1}{a_1d_1 - b_1c_1}\begin{bmatrix} d_1 & -b_1 \\ -c_1 & a_1 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{d_1}{a_1d_1 - b_1c_1} & \dfrac{-b_1}{a_1d_1 - b_1c_1} \\ \dfrac{-c_1}{a_1d_1 - b_1c_1} & \dfrac{a}{a_1d_1 - b_1c_1} \end{bmatrix}$$

$\Rightarrow$ Inverse of each element in G exist in G

C-5 In matrices, we know that:

$\forall\ A, B \in G$

$A \cdot B \neq B \cdot A$

$\Rightarrow$ Commutative law does not hold in G.

$\Rightarrow$ 'G' form a non-abelian group under multiplication.