M-Zohaib **Math Assignment** Roll 28

Q1 MCQ

1) D
2) D
3) D
4) A

SHORT ANSWER

(i)

Semi Group

A non empty set S is semi group if

1 It is closed with respect to an operation ※

2 The operation ※ is associative

ii

table

⊗	0	1	2	3	4	
0	0	0	0	0	0	
1	0	1	2	3	4	
2	0	2	4	~~6~~	3	
3	0	3	~~6~~ 1	4	~~8~~	2
4	0	4	3	~~4~~ 2	~~0~~	1

$2 \times 3 = 6 = 1$ (remaider ÷ by 5)

$2 \times 4 = 8 = 3$ (remainder ÷ by 5)

$3 \times 3 = 9 = 4$ (remainder ÷ by 5)

$3 \times 4 = 12 = 2$ (remainder ÷ by 5)

$4 \times 4 = 16 = 1$ (remainder ÷ by 5)

(iii)

$$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \quad \text{show } A^4 = I_2$$

$A^4 = I^2$ where $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} i \times i + 0 \times 1, & i \times 0 + 0 \times -i \\ 1 \times i + -i \times 1, & 1 \times 0 + -i \times -i \end{bmatrix}$$

$$= \begin{bmatrix} i + 0 & 0 - 0 \\ i - i & 0 + i^2 \end{bmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A^4 = A^2 \cdot A^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{bmatrix} -1 \times -1 + 0 \times 0 & -1 \times 0 + 0 \times -1 \\ 0 \times -1 + -1 \times 0 & 0 \times 0 + -1 \times -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

Hence proved !

LONG Question

Q3

$G = \{$ All 2×2 non-singular matrices $\}$

$= \{ A, B, C, I, A^{-1}, B^{-1}, C^{-1} \}$

Closure:- set G is closure under operation '.' because

$\forall A, B \in G \quad A \cdot B \in G$

Associative property:- The operation '.' is assciative because

$\forall A, B, C, \in G \quad (A \cdot B) \cdot C = A \cdot (B \cdot C)$

Identity Property:-

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity element under '.' because

$\forall A \in G \quad A \cdot I_2 = A = I_2 A$

Inverse property:

inverse of each element exists in G because all elements i.e matrices are non singular

$\forall A \in G \ \exists \ A^{-1} \in G$

$A \cdot A^{-1} = I = A^{-1} A$.

Commutative Property:

The operation '.' is not commutative because

$\forall A, B \in G \Rightarrow$

$$A \cdot B \neq B \cdot A$$

Hence, G is an non abelian group under multiplication i.e '.'