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Class: 1^{st} year
Subject: Maths
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Q#1

MCQs

1) B
2) D
3) D
4) D

SECTION-1

Q #2

(i) SEMI GROUP

Defination:

A non-empty set S is semi-group if,

- It is closed with respect to an operation $*$.
- The operation is associative.

EXAMPLE:

- N is closed w.r.t addition

Also $\forall\, a,b,c \in N$, $a+(b+c) = (a+b)+c$

Therefore, it is a semi group

(iii) $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$

Solution:

$$A^4 = I_2$$

$$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \times \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} i^2+0 & 0-i \\ i-i & 0+i^2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}$$

$$A^3 = A \times A^2 = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \times \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}$$

$$= \begin{bmatrix} i^3+0 & 0+0 \\ i^2-0 & 0-i^3 \end{bmatrix}$$

$$= \begin{bmatrix} i^3 & 0 \\ i^2 & i^3 \end{bmatrix}$$

$$A^4 = A \times A^3 = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \times \begin{bmatrix} i^3 & 0 \\ i^2 & i^3 \end{bmatrix}$$

$$= \begin{bmatrix} i^4+0 & 0+0 \\ i^3-i^3 & 0-i^4 \end{bmatrix}$$

$$= \begin{bmatrix} i^4 & 0 \\ 0 & i^4 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

(ii) MODULO 5

Solution:

x	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

This is the modulo of 5

SECTION-II

Q# 3

Solution:

let G be the all non-singular 2×2 matrices over the real field.

i) let $A, B \in G$ then $A_{2\times2} \times B_{2\times2} = C_{2\times2} \in G$

Thus closure law holds in G under multiplication

ii) Associative law in matrices of same order under multiplication holds

therefore for $A, B, C \in G$

$$A\times(B\times C) = (A\times B)\times C$$

(iii) $I_{2\times2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a non-singular matrix

$$A_{2\times2}\times I_{2\times2} = I_{2\times2}\times A_{2\times2} = A_{2\times2}$$

Thus $I_{2\times2}$ is identity element in G.

(iv) Since inverse of non-singular square matrix exists, therefore $A \in G$ there exists $A^{-1} \in G$ such that $AA^{-1} = A^{-1}A = I$

(v) As we know for any two matrices $A, B \in G$, $AB \neq BA$ in general.

Therefore commutative law does not hold.

Hence the set of all 2×2 non-singular matrices over a real field is a non-abelian group under multiplication.

2024/11/23 20:32

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