LGS GROUP OF COLLEGES
A PROJECT OF LAHORE GRAMMAR SCHOOL

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Subject: Math Test No. 5 Date: 23-11-2024

	A	B	C	D
1				●
2				●
3				●
4	●			
5				

Marks Obtained

- **Short Questions:**

(i)

- **Semi Group:**
A semi group is a set with an operation that is associative.
- E.g:
Set $N = \{0, 1, 2, 3 \ldots\}$

- In a semigroup, the operation is associative, but you don't need an element that acts as a neutral or identity for the operation.

(ii)

- **Modulo 5**
$A = \{0, 1, 2, 3, 4\}$

x	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

(iii)

- $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$

Show that $A^4 = I_2$

$$A^2 = A \times A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \times \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$A^2 = \begin{bmatrix} i^2 + 0 & 0 + 0 \\ i - i & 0 + i^2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^2 \times A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^4 = I_2$$

- **Long Questions:**

(i)

- 2x2 matrices

Matrix multiplication is associative. This is a well-known property of matrices in general.

$$(AB)C = A(BC)$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A \times I_2 = I_2 \times A = A$$

- I_2 is the identity element for matrix multiplication.

- Every non-singular matrix A has an inverse matrix A^{-1}, such that:

$$A \times A^{-1} = A^{-1} \times A = I_2$$

$$AB \neq BA$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1+1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

- It is non-abelian.

- The set of all 2x2 non-singular matrices over R forms a non-abelian group under matrix multiplication.