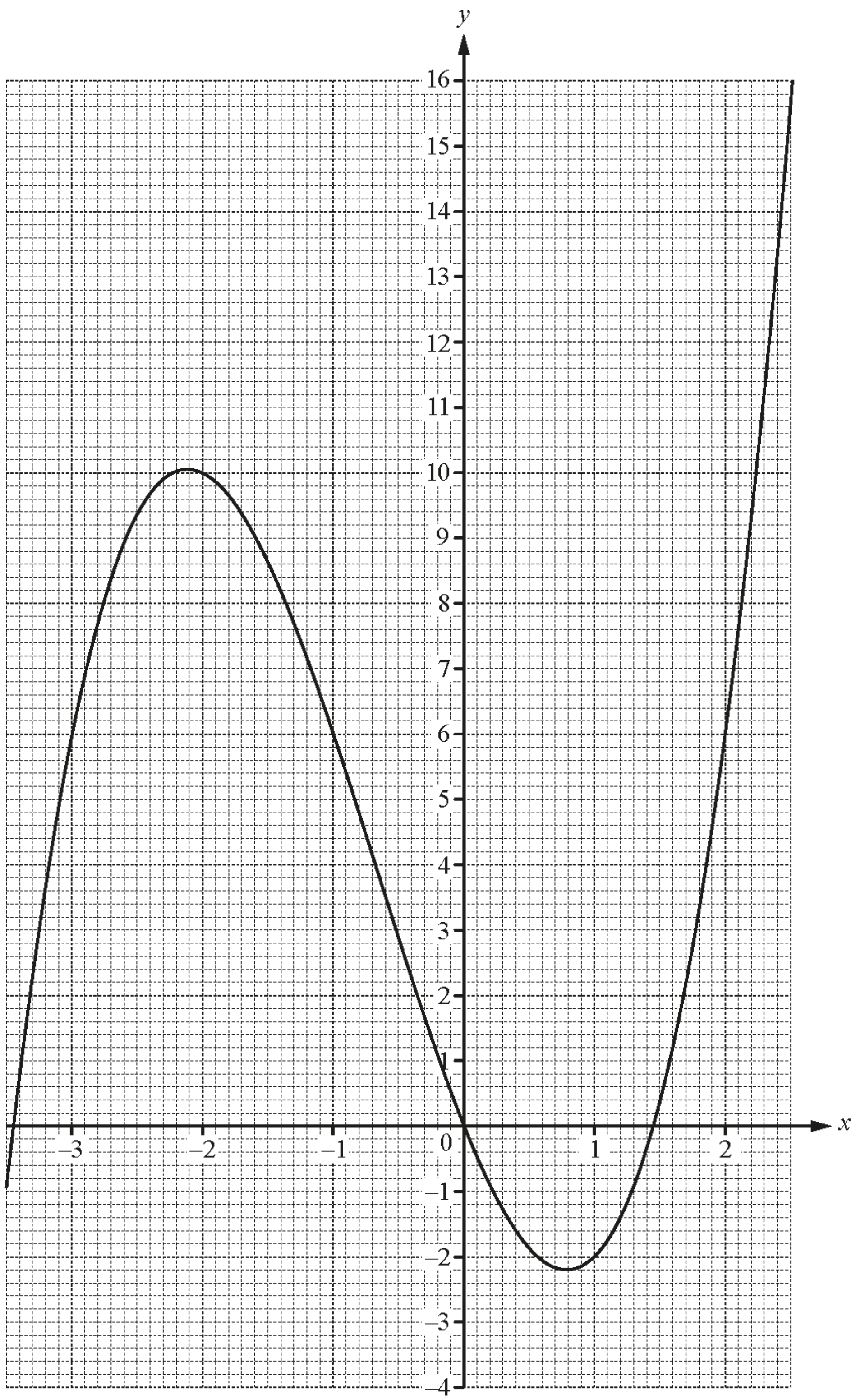
TOPICAL PAST PAPER QUESTIONS - 2016/2017

GRAPHS (Paper 4)

1. **(0580-S 2016-Paper 4/3-Q3)**

The diagram shows the graph of $y = f(x)$ for $-3.5 \leqslant x \leqslant 2.5$.



(a) **(i)** Find f(– 2).

.... [1]

(ii) Solve the equation $f(x) = 2$.

x = ... or x = ... or x = ...[3]

(iii) Two tangents, each with gradient 0, can be drawn to the graph of $y = f(x)$.

Write down the equation of each tangent.

....

.... [2]

(b) **(i)** Complete the table for $g(x) = \frac{2}{x} + 3$ for $-3.5 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 2.5$.

x	–3.5	–3	–2	–1	–0.5		0.5	1	2	2.5
$g(x)$	2.4	2.3		1			7	5		3.8

[3]

(ii) On the grid opposite, draw the graph of $y = g(x)$. [4]

(iii) Use your graph to solve the equation $f(x) = g(x)$.

x = ... or x = ...[2]

(c) Find gf(–2).

.... [2]

(d) Find $g^{-1}(5)$.

.... [1]

2. **(0580-S 2016-Paper 4/2-Q4)**

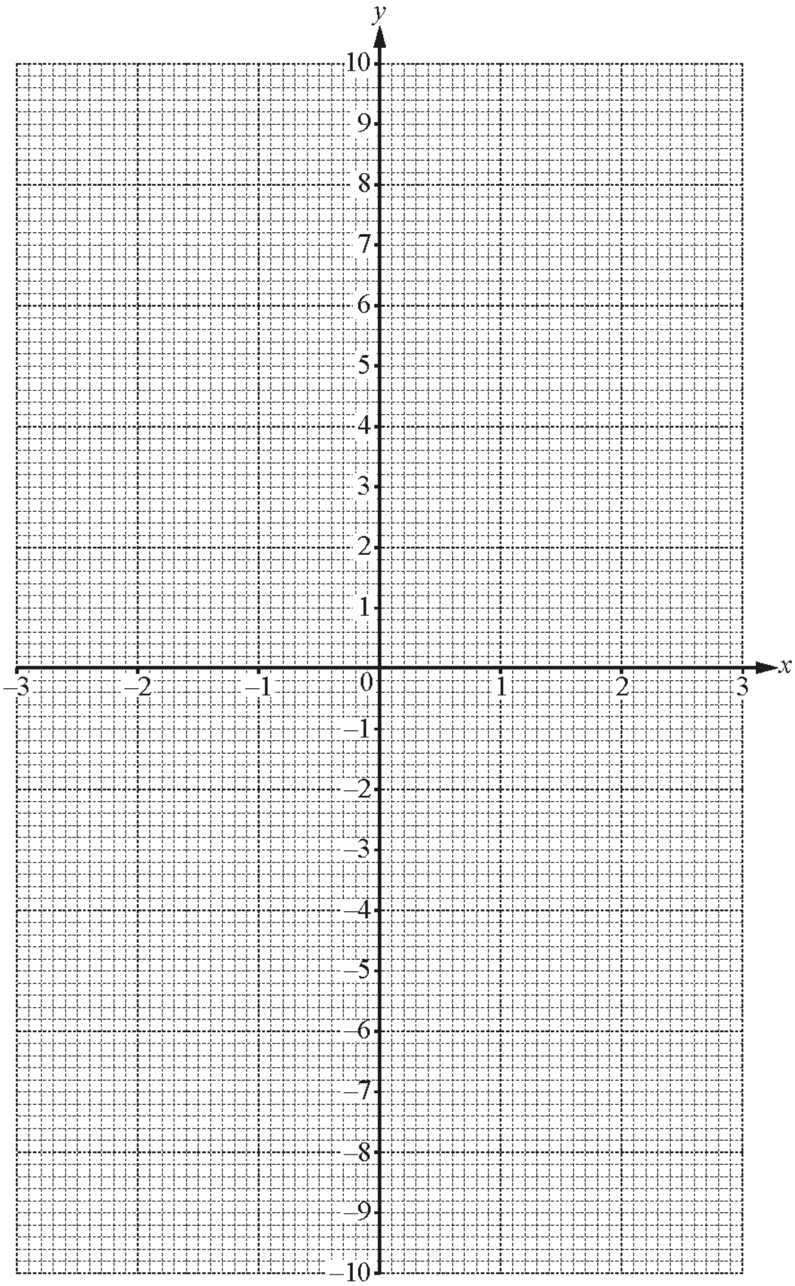
$$f(x) = x^2 - \frac{1}{x} - 4\,,\quad x \neq 0$$

(a) **(i)** Complete the table.

x	–3	–2	–1	–0.5	–0.1		0.2	0.5	1	2	3
f(x)	5.3	0.5		–1.8	6.0		–9.0	–5.8	–4		4.7

[2]

(ii) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.1$ and $0.2 \leqslant x \leqslant 3$.



[5]

(b) Use your graph to solve the equation $f(x) = 0$.

x = ... or x = ... or x = ... [3]

(c) Find an integer k, for which $f(x) = k$ has one solution.

k = ... [1]

(d) **(i)** By drawing a suitable straight line, solve the equation $f(x) + 2 = -5x$.

x = ... or x = ... [4]

(ii) $f(x) + 2 = -5x$ can be written as $x^3 + ax^2 + bx - 1 = 0$.

Find the value of a and the value of b.

a = ...

b = ... [2]

3. (0580-S 2016-Paper 4/1-Q5)

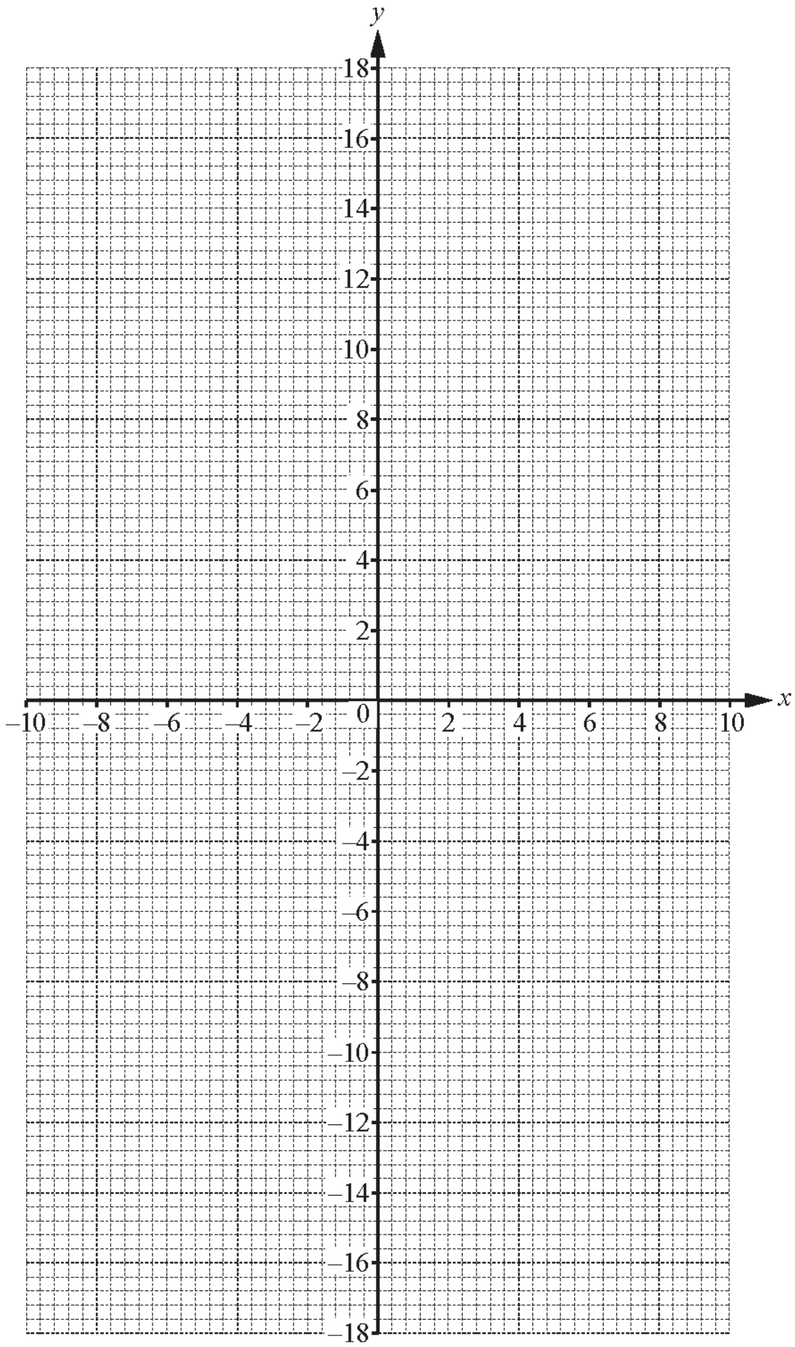
$$f(x) = \frac{20}{x} + x, \quad x \neq 0$$

(a) Complete the table.

x	-10	-8	-5	-2	-1.6		1.6	2	5	8	10
$f(x)$	-12	-10.5	-9	-12	-14.1		14.1	12			12

[2]

(b) On the grid, draw the graph of $y = f(x)$ for $-10 \leqslant x \leqslant -1.6$ and $1.6 \leqslant x \leqslant 10$.



(c) Using your graph, solve the equation $f(x) = 11$.

$x =$ .... or $x =$ .... [2]

(d) k is a prime number and $f(x) = k$ has no solutions.

Find the possible values of k.

.... [2]

(e) The gradient of the graph of $y = f(x)$ at the point (2, 12) is -4.

Write down the co-ordinates of the other point on the graph of $y = f(x)$ where the gradient is -4.

(..................... , ...) [1]

(f) **(i)** The equation $f(x) = x^2$ can be written as $x^3 + px^2 + q = 0$.

Show that $p = -1$ and $q = -20$.

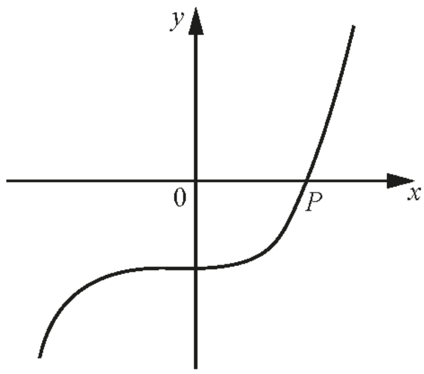
[2]

(ii) On the grid opposite, draw the graph of $y = x^2$ for $-4 \leqslant x \leqslant 4$. [2]

(iii) Using your graphs, solve the equation $x^3 - x^2 - 20 = 0$.

$x =$ .... [1]

(iv)



NOT TO SCALE

The diagram shows a **sketch** of the graph of $y = x^3 - x^2 - 20$.
P is the point $(n, 0)$.

Write down the value of n.

$n =$ .... [1]

4. **(0580-S 2016-Paper 4/3-Q7)**

Alfonso runs 10 km at an average speed of x km/h.
The next day he runs 12 km at an average speed of $(x - 1)$ km/h.

The time taken for the 10 km run is 30 minutes less than the time taken for the 12 km run.

(a) **(i)** Write down an equation in x and show that it simplifies to $x^2 - 5x - 20 = 0$.

[4]

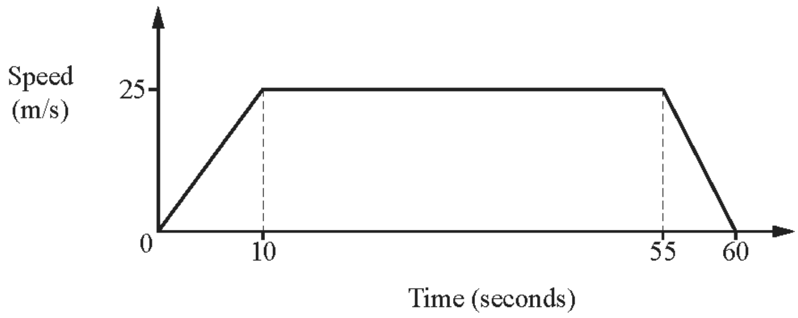
(ii) Use the quadratic formula to solve the equation $x^2 - 5x - 20 = 0$.
Show your working and give your answers correct to 2 decimal places.

x = ... or x = ... [4]

(iii) Find the time that Alfonso takes to complete the 12 km run.
Give your answer in hours and minutes correct to the nearest minute.

.... hours .... minutes [2]

(b) A cheetah runs for 60 seconds.
The diagram shows the speed-time graph.



NOT TO SCALE

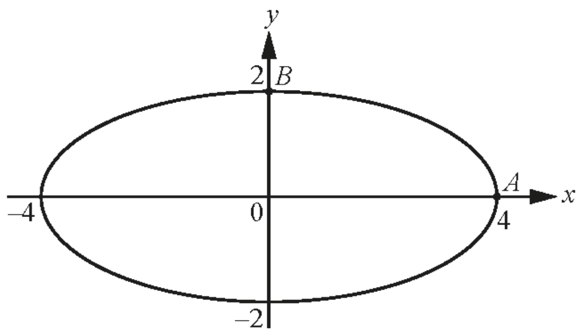
(i) Work out the acceleration of the cheetah during the first 10 seconds.

.... m/s^2 [1]

(ii) Calculate the distance travelled by the cheetah.

.... m [3]

5. (0580-S 2016-Paper 4/1-Q9)



NOT TO SCALE

The diagram shows a curve with equation $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$.

(a) A is the point (4, 0) and B is the point (0, 2).

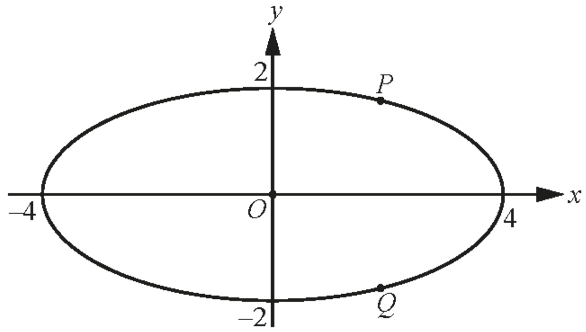
(i) Find the equation of the straight line that passes through A and B.
Give your answer in the form $y = mx + c$.

$y =$ .... [3]

(ii) Show that $a^2 = 16$ and $b^2 = 4$.

[2]

(b)



NOT TO SCALE

$P\,(2, k)$ and $Q\,(2, -k)$ are points on the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

(i) Find the value of k.

$k =$ .... [3]

(ii) Calculate angle POQ.

Angle $POQ =$ .... [3]

(c) The area enclosed by a curve with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

(i) Find the area enclosed by the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

Give your answer as a multiple of π.

.... [1]

(ii) A curve, mathematically similar to the one in the diagrams, intersects the x-axis at $(12, 0)$ and $(-12, 0)$.

Work out the area enclosed by this curve, giving your answer as a multiple of π.

.... [2]

6. **(0580-S 2016-Paper 4/2-Q9)**

A line joins the points A (–2, –5) and B (4, 13).

(a) Calculate the length AB.

AB = ... [3]

(b) Find the equation of the line through A and B.
Give your answer in the form $y = mx + c$.

y = ... [3]

(c) Another line is parallel to AB and passes through the point (0, –5).

Write down the equation of this line.

.... [2]

(d) Find the equation of the perpendicular bisector of AB.

.... [5]

7. (0580-W 2016-Paper 4/2-Q2)

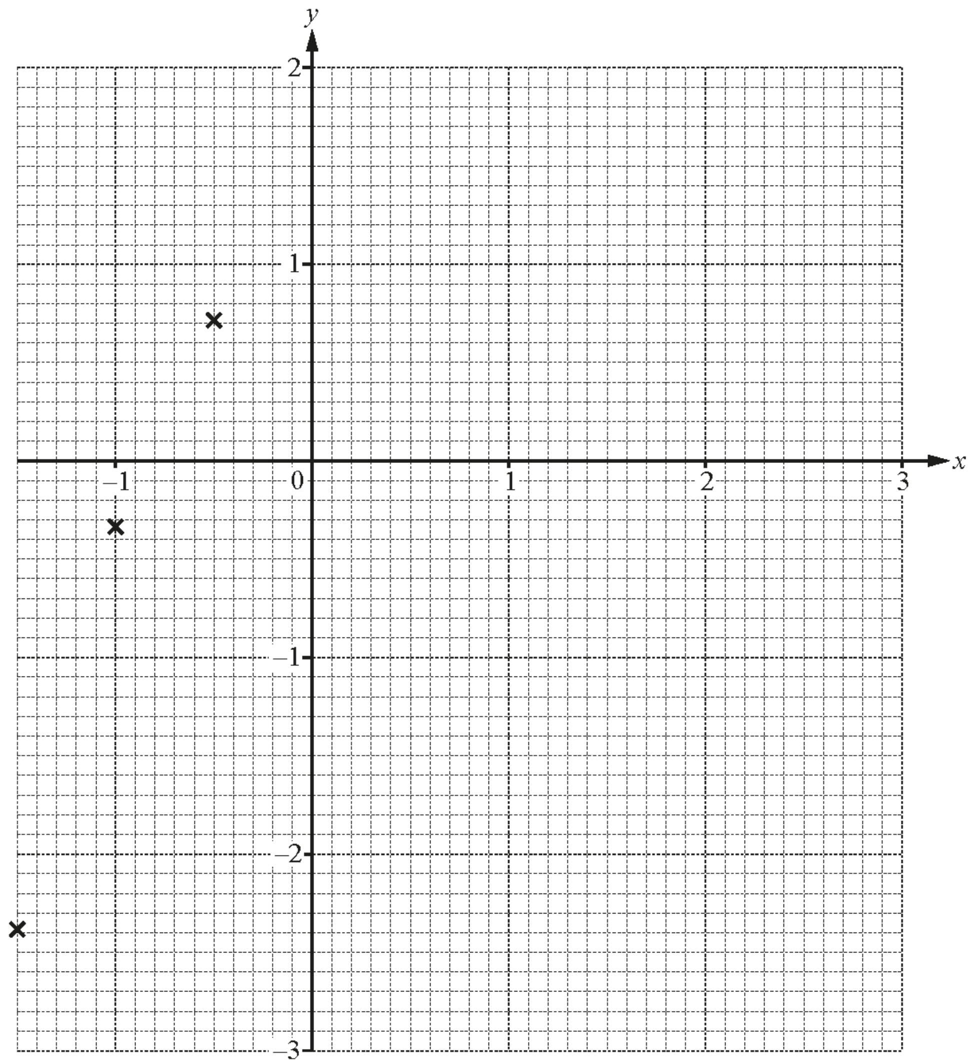
(a) Complete the table of values for $y = \frac{x^3}{3} - x^2 + 1$.

x	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5	3
y	–2.38	–0.33	0.71		0.79	0.33	–0.13	–0.33	–0.04	

[2]

(b) Draw the graph of $y = \frac{x^3}{3} - x^2 + 1$ for $-1.5 \leqslant x \leqslant 3$.

The first 3 points have been plotted for you.



[4]

(c) Using your graph, solve the equations.

(i) $\frac{x^3}{3} - x^2 + 1 = 0$

x = ... or x = ... or x = ...[3]

(ii) $\frac{x^3}{3} - x^2 + x + 1 = 0$

x = ... [2]

(d) Two tangents to the graph of $y = \frac{x^3}{3} - x^2 + 1$ can be drawn parallel to the x-axis.

(i) Write down the equation of each of these tangents.

....

.... [2]

(ii) For $0 \leqslant x \leqslant 3$, write down the smallest possible value of y.

y = ... [1]

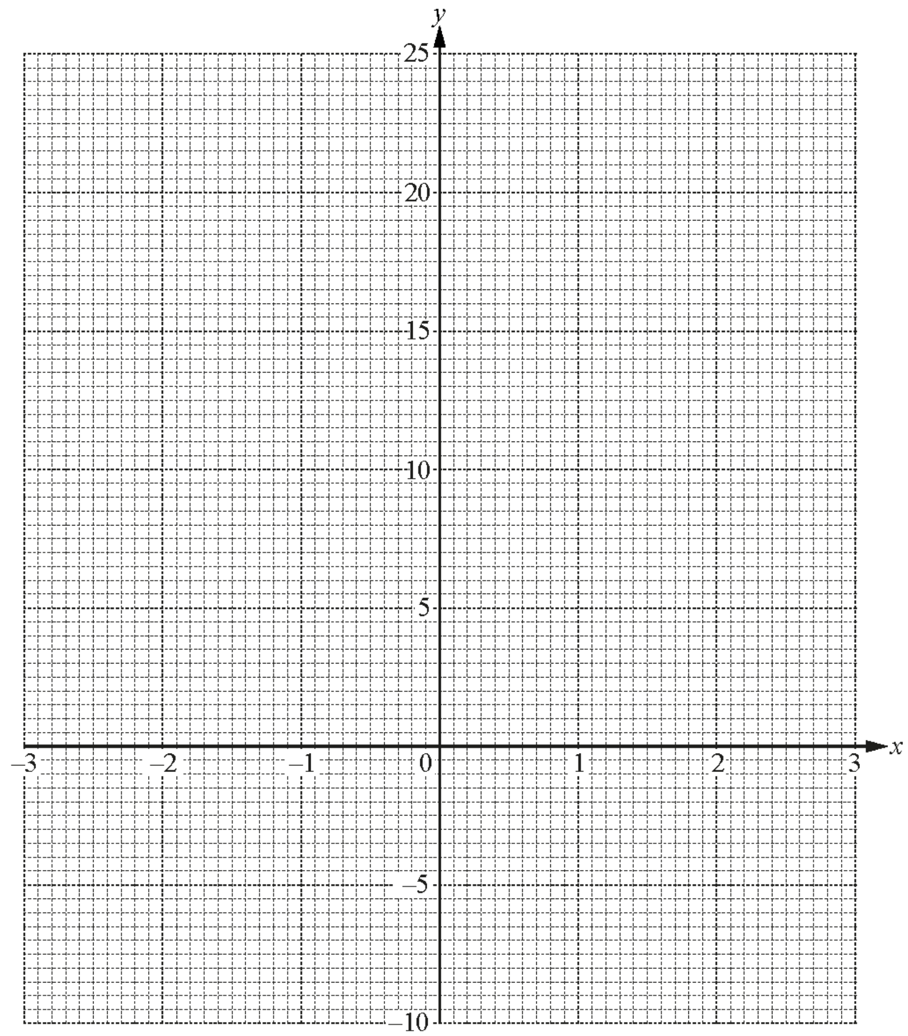
8. **(0580-W 2016-Paper 4/3-Q2)**

(a) Complete the table for $y = 3x + \frac{2}{x^2} + 1,\ x \neq 0$.

x	-3	-2	-1	-0.5	-0.3		0.3	0.5	1	2	3
y	-7.8		0	7.5	22.3		24.1		6	7.5	10.2

[2]

(b) On the grid, draw the graph of $y = 3x + \frac{2}{x^2} + 1$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 3$.



[5]

(c) Write down the value of the largest integer, k, so that the equation $3x + \frac{2}{x^2} + 1 = k$ has exactly one solution.

$k =$ .... [1]

(d) **(i)** By drawing a suitable straight line on the grid, solve $3x + \frac{2}{x^2} + 1 = 15 - 3x$.

x = ... or x = ... or x = ... [4]

(ii) The equation $3x + \frac{2}{x^2} + 1 = 15 - 3x$ can be written in the form $ax^3 + bx^2 + cx + 2 = 0$, where a, b and c are integers.

Find a, b and c.

a = ...

b =..

c =.. [3]

9. **(0580-W 2016-Paper 4/1-Q4)**

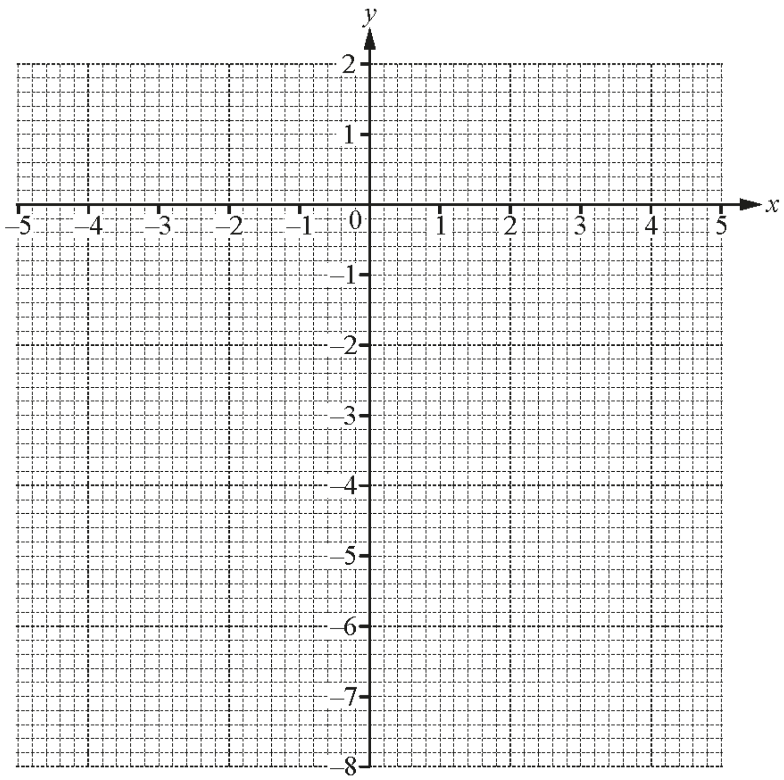
$$y = 1 - \frac{2}{x^2}, \; x \neq 0$$

(a) Complete the table.

x	–5	–4	–3	–2	–1	–0.5		0.5	1	2	3	4	5
y		0.88	0.78			–7		–7			0.78	0.88	

[3]

(b) On the grid, draw the graph of $y = 1 - \frac{2}{x^2}$ for $-5 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 5$.



[5]

(c) **(i)** On the grid, draw the graph of $y = -x - 1$ for $-3 \leqslant x \leqslant 5$. [2]

(ii) Solve the equation $1 - \frac{2}{x^2} = -x - 1$.

$x =$ .... [1]

(iii) The equation $1-\frac{2}{x^2}=-x-1$ can be written in the form $x^3+px^2+q=0$.

Find the value of p and the value of q.

p = ...

q = ... [3]

(d) The graph of $y=1-\frac{2}{x^2}$ cuts the positive x-axis at A.

B is the point (0, – 2).

(i) Write down the co-ordinates of A.

(..................... , ...) [1]

(ii) On the grid, draw the straight line that passes through A and B. [1]

(iii) Complete the statement.

The straight line that passes through A and B is a ....

at the point .... [2]

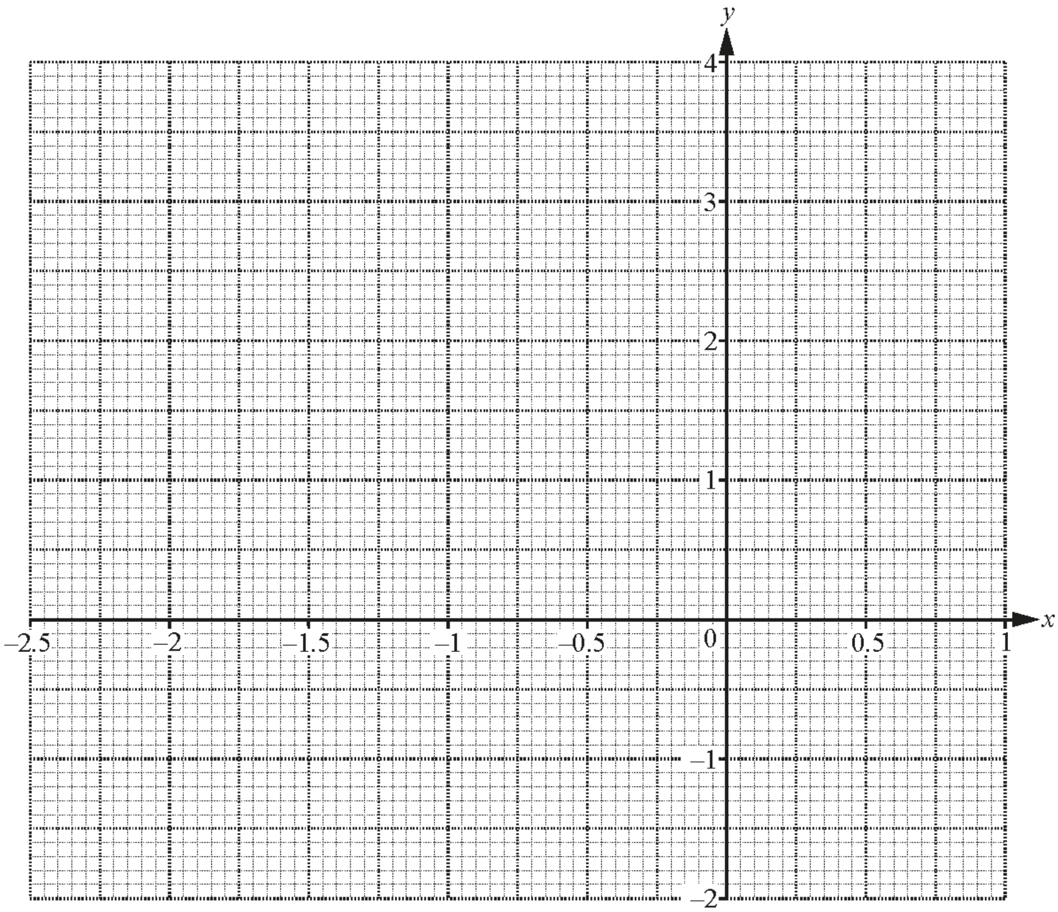
10. **(0580-S 2017-Paper 4/3-Q3)**

The table shows some values for $y = 2x^3 + 4x^2$.

x	-2.2	-2	-1.5	-1	-0.5	0	0.5	0.8
y	-1.94				0.75	0		3.58

(a) Complete the table. [4]

(b) Draw the graph of $y = 2x^3 + 4x^2$ for $-2.2 \leqslant x \leqslant 0.8$.



[4]

(c) Find the number of solutions to the equation $2x^3 + 4x^2 = 3$.

.... [1]

(d) **(i)** The equation $2x^3 + 4x^2 - x = 1$ can be solved by drawing a straight line on the grid.

Write down the equation of this straight line.

$y =$ .... [1]

(ii) Use your graph to solve the equation $2x^3 + 4x^2 - x = 1$.

$x =$ .... or $x =$ .... or $x =$ ....[3]

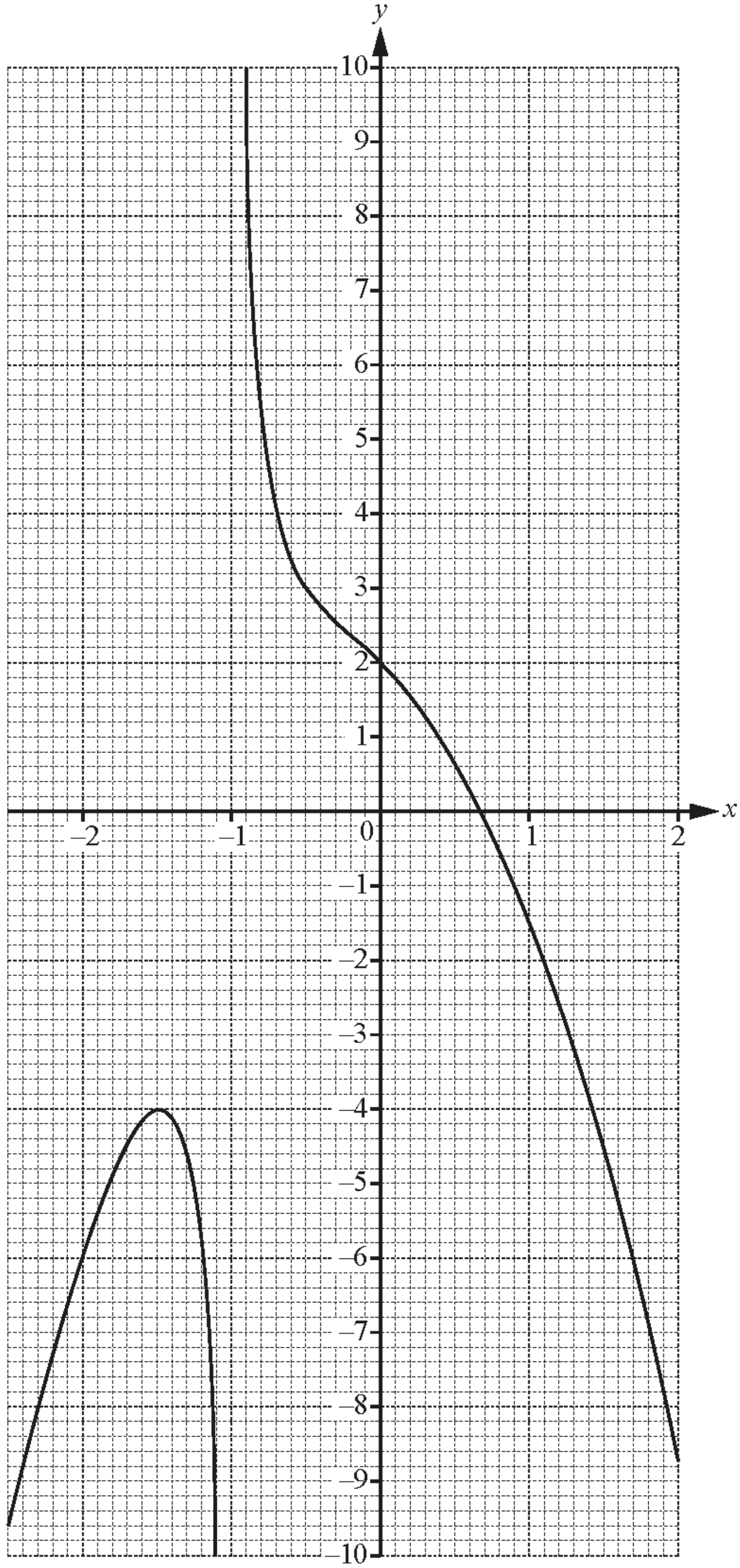
(e) The tangent to the graph of $y = 2x^3 + 4x^2$ has a negative gradient when $x = k$.

Complete the inequality for k.

.... $< k <$ ....[2]

11. (0580-S 2017-Paper 4/1-Q4)

The diagram shows the graph of $y = f(x)$ for $-2.5 \leqslant x \leqslant 2$.



(a) Find f(1).

.... [1]

(b) Solve $f(x) = 3$.

$x =$ .... [1]

(c) The equation $f(x) = k$ has only one solution for $-2.5 \leqslant x \leqslant 2$.

Write down the range of values of k for which this is possible.

.... [2]

(d) By drawing a suitable straight line, solve the equation $f(x) = x - 5$.

$x =$ .... or $x =$ .... or $x =$ .... [3]

(e) Draw a tangent to the graph of $y = f(x)$ at the point where $x = 1$.

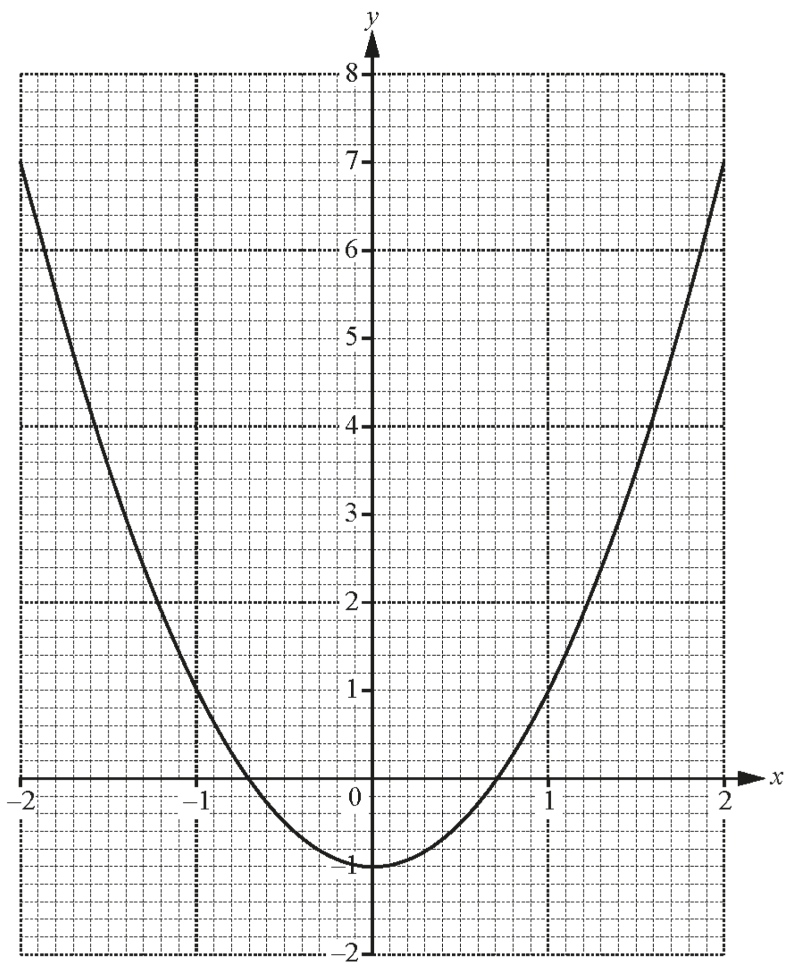
Use your tangent to estimate the gradient of $y = f(x)$ when $x = 1$.

.... [3]

12. **(0580-S 2017-Paper 4/2-Q4)**

$$f(x) = 2x^2 - 1$$

The graph of $y = f(x)$, for $-2 \leqslant x \leqslant 2$, is drawn on the grid.



(a) Use the graph to solve the equation $f(x) = 5$.

$x =$ .... or $x =$ .... [2]

(b) **(i)** Draw the tangent to the graph of $y = f(x)$ at the point $(-1.5, 3.5)$. [1]

(ii) Use your tangent to estimate the gradient of $y = f(x)$ when $x = -1.5$.

.... [2]

(c) $g(x) = 2^x$

(i) Complete the table for $y = g(x)$.

x	-2	-1	0	1	2
y	0.25	0.5		2	4

[1]

(ii) On the grid opposite, draw the graph of $y = g(x)$ for $-2 \leqslant x \leqslant 2$. [3]

(d) Use your graphs to solve

(i) the equation $f(x) = g(x)$,

$x =$ .... or $x =$ .... [2]

(ii) the inequality $f(x) < g(x)$.

.... [1]

(e) **(i)** Write down the three values.

$g(-3) =$ .... $g(-5) =$ .... $g(-10) =$ .... [1]

(ii) Complete the statement.

As x decreases, $g(x)$ approaches the value .... [1]

13. **(0580-S 2017-Paper 4/1-Q7)**

A line joins the points $A(-3, 8)$ and $B(2, -2)$.

(a) Find the co-ordinates of the midpoint of AB.

(...................... , ...) [2]

(b) Find the equation of the line through A and B.
Give your answer in the form $y = mx + c$.

$y =$ .... [3]

(c) Another line is parallel to AB and passes through the point (0, 7).

Write down the equation of this line.

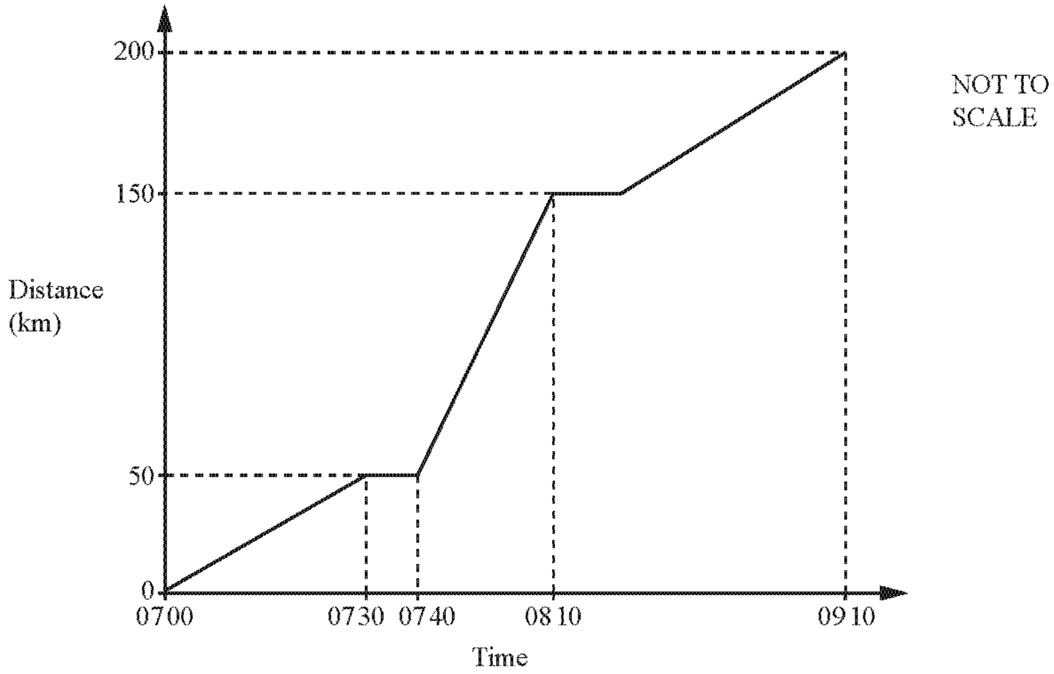
.... [2]

(d) Find the equation of the line perpendicular to AB which passes through the point (1, 5).
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

.... [4]

14. **(0580-S 2017-Paper 4/2-Q9)**

(a)



The distance-time graph shows the journey of a train.

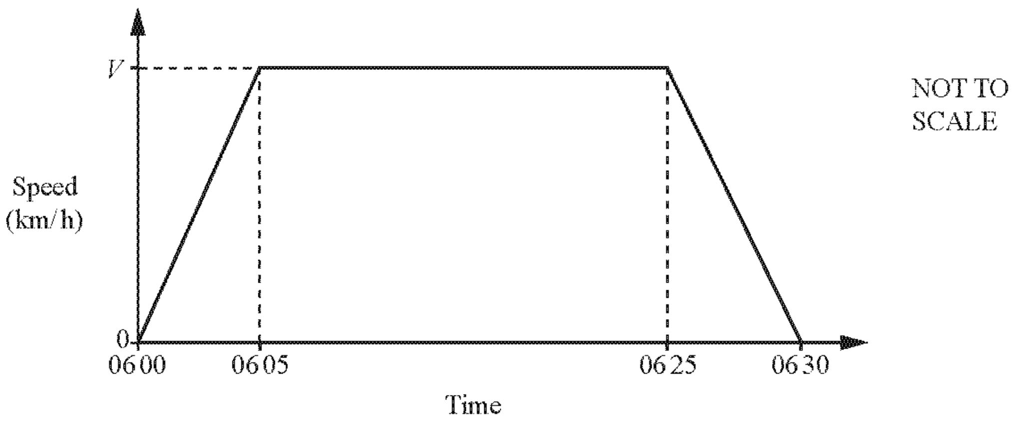
(i) Find the speed of the train between 0700 and 0730.

.... km/h [1]

(ii) Find the average speed for the whole journey.

.... km/h [3]

(b)



The speed-time graph shows the first 30 minutes of another train journey.
The distance travelled is 100 km.
The maximum speed of the train is V km/h.

(i) Find the value of V.

V = ... [3]

(ii) Find the acceleration of the train during the first 5 minutes.
Give your answer in m/s^2.

.... m/s^2 [2]