ASSIGNMENT
MATHS

LARAIB ASIF
IA- B

Question#1:

MCQS

1) 2×3
2) Monoid
3) Singular
4) $a^{-1}b$

Question#2.

Short Questions

(i)

Semi-Group

A non-empty set is semi-group if it is closed with ~~propert~~ respect to an operation $*$ and this operation is associative, $(N,+)$ is a semi group.

(iii)

$A^4 = I_2$

$$A^2 = A\cdot A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} (i)(i)+(0)(1) & (i)(0)+(0)(-i) \\ (1)(i)+(-i)(1) & (1)(0)+(-i)(-i) \end{bmatrix}$$

$$= \begin{bmatrix} i^2+0 & 0+0 \\ i-i & 0+i^2 \end{bmatrix} = \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} (-1)(-1) + (0)(0) & (-1)(0) + (0)(-1) \\ (0)(-1) + (-1)(0) & (0)(0) + (-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = I_2$$

(ii)

$\times$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	$2\cancel{4}$	1

From Table

$3 \times 2 = 1$ (Remainder of 5/6)

$3 \times 4 = 2$ (5/12)

$2 \times 4 = 3$ (5/8)

Question #3:

LONG QUESTION

Let

$$M_2 = \left\{ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ where } a_{11}, a_{12}, a_{21}, a_{22} \in R \text{ \& } \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0 \right\}$$

i) Closure Property

For All $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

ii) Associative Property

$M_2 \Rightarrow A(BC) = (AB)C$

iii) Identity Element

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is

$IA = A = AI$

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity element in M_2

(iv) Inverse Element

As inverse ~~element~~ of a non-singular square matrix is again non-singular matrix of that order before.
if $A \in M_2$ $\exists$ $A^{-1} \in M_2$ such that $AA^{-1} = A^{-1}A = I$
Inverse of each Matrix A exists in M_2.

(v) Commutative Property

For any two matrices A & B, $AB \neq BA$ in general
Therefore set M_2 does not posses the commutative property

Hence M_2 the set of all 2×2 non-singular matrices over the real field is a non abelian group under multiplication.