



# LGS GROUP OF COLLEGES

A PROJECT OF LAHORE GRAMMAR SCHOOL

Sheet # \_\_\_\_\_

me: Waniha

Class: 11 A

Roll No. 06

ject: Maths

Test No. WK 5

Date: 23-11-2024

| A | B | C | D | A  | B | C | D | A  | B | C | D | A  | B | C | D | Marks Obtained |
|---|---|---|---|----|---|---|---|----|---|---|---|----|---|---|---|----------------|
| 1 |   |   |   | 6  |   |   |   | 11 |   |   |   | 16 |   |   |   |                |
| 2 |   |   |   | 7  |   |   |   | 12 |   |   |   | 17 |   |   |   |                |
| 3 |   |   |   | 8  |   |   |   | 13 |   |   |   | 18 |   |   |   |                |
| 4 |   |   |   | 9  |   |   |   | 14 |   |   |   | 19 |   |   |   |                |
| 5 |   |   |   | 10 |   |   |   | 15 |   |   |   | 20 |   |   |   |                |

## Assignment :- (WK-5)

(i)

Semi Group:-

A semi group is an algebraic structure consisting of a set equipped with an associative binary operation.

(ii)

The set of residue classes of modulo 5 is 0, 1, 2, 3, 4.

|   | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | 4 |
| 2 | 0 | 2 | 4 | 1 | 3 |
| 3 | 0 | 3 | 1 | 4 | 2 |
| 4 | 0 | 4 | 3 | 2 | 1 |



### 3 Identity Element:-

The identity matrix  $I_2$  satisfies  $IA = AI = A$

### 4. Inverse Element:-

For every non-singular matrix  $A$ , there exists  $A^{-1}$  such that  $AA^{-1} = A^{-1}A = I_2$ .

### 5. Non-Abelian Property:-

Matrix multiplication is not commutative in general i.e.  $AB \neq BA$  for some matrices.

Sol:-  $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$  where  $a, a_{11}, a_{12}, a_{13}, a_{22} \in \mathbb{R}$  and  $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \neq$

(i) Closure property:-  $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$   $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Hence  $AB \in M_2$

(ii) Associative

$$\forall A, B, C \in M_2 \Rightarrow A(BC) = (AB)C$$

(iii) Identity element:-  $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  is  $2 \times 2$  non



(iii)

$$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

To prove:  $A^2$ , then  $A^4 =$

$$A^2 = A \cdot A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I_2$$

$$A^4 = (A^2)^2 = \cancel{-1}$$

$$\begin{bmatrix} (i)(i) + (0)(1) & (i)(0) + (0)(-i) \\ 1(i) + (-i)(1) & (1)(0) + (-i)(-i) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I_2$$

$$A^4 = -(A^2) \cdot (A^2) = (-I_2)(-I_2) = I_2$$

## LONG Question

1. Closure:-

If  $A$  and  $B$  are  $2 \times 2$  non-singular matrices, Their product  $AB$  is also  $2 \times 2$  non singular.

2. Associativity:- Matrix multiplication is associative.

$$(AB)C = A(BC)$$





singular.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Inverse:- As inverse of non singular square matrix is again a non singular matrix such that  $A \in M_n$ ,  $\exists A^{-1} \in M_n$ ,  $AA^{-1} = A^{-1}A = I$