WT-5

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Roll no:-

MCQ's

1- B

2- D

3- D

4- D

(Subjective type)

(Short Questions)

(i)

Semi Group

A non-empty set S is semi group if;

- It is closed with respect to an operation $\dot{\times}$
- The operation $\dot{\times}$ is associative.

A semi group satisfies half of the conditions required for a group

(ii)

$\dot{\times}$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

(iii)

$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, show $A^4 = I_2$

$$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \Rightarrow A^2 = A \cdot A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} (i)(i) + 0(1) & (i)(0) + (0)(-i) \\ 1(i) + (-i)(1) & (1)(0) + (-i)(-i) \end{bmatrix} \Rightarrow \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(-1) + (0)(0) & (-1)(0) + 0(-1) \\ 0(-1) + (-1)(0) & (0)(0) + (-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow I_2$$

Q no 3:-

Solution:-

Let G be the non-Singular matrics over the real field 2×2 matrices.

(i) Let $A, B \in G$ then $A_{2\times2} \times B_{2\times2} \times C_{2\times2} \in G$ Thus closure law holds in G under multiplication.

(ii) Associative law in matrices of Same order under multiplation holds.

therefore

for $A, B, C, \in G$

$A \times (B \times C) = (A \times B) \times C$

(iii) $I_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a non-singular matrix such that,

$A_{2\times2} \times I_{2\times2} = I_{2\times2} \times A_{2\times2} = A_{2\times2}$

Thus $I_{2\times2}$ is an identity element in G.

(iv) Since inverse of non-singular square matrix exists,

therefore for $A \in G$ there exit $A^{-1} \in G$ such that $AA^{-1} = A^{-1}A = I$

(v) As we know for any two matrices $A, B, \in G$, $AB \neq BA$ in general.

Therefore communicative law does not holds in G under multiplication. Hence set of all 2×2 non-singular matrices over a real field is a non-abelian group under multiplication.