	A	B	C	D			A	B	C	D			A	B	C	D			A	B	C	D
1	○	○	○	●		6	○	○	○	○		11	○	○	○	○		16	○	○	○	○
2	○	●	○	○		7	○	○	○	○		12	○	○	○	○		17	○	○	○	○
3	○	○	○	●		8	○	○	○	○		13	○	○	○	○		18	○	○	○	○
4	●	○	○	○		9	○	○	○	○		14	○	○	○	○		19	○	○	○	○
5	○	○	○	○		10	○	○	○	○		15	○	○	○	○		20	○	○	○	○

Marks Obtained

Q # 2 → (i)
Semi Group

A non-empty set called a semi group if,

(i) It is closed with respect to an operation ·×·

(ii) The operation ·×· is associative.

A semi-group statisfies half of the conditions required for a group.

Q # 2 → (ii)

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

$$Q\# \; 2 \rightarrow (iii)$$

$$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}, \qquad A^4 = ?$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \cdot \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} i^2+0 & 0+0 \\ i-i & 0+i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

hence $A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Long Question

Q# 3

let $G = \{$ All 2×2 non-singular matrices $\}$

$= \{A, B, C, I, A^{-1}, B^{-1}, C^{-1}, \dots\}$

i.e $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Closure:

set G is closure under operation '.'

because:

$\forall \overset{A}{B}, B \in G \qquad \partial A \cdot B \in G$

Associative property.

The operation '.' is associative because

$\forall A, B, C \in G \qquad (A \cdot B) C = A \cdot (B \cdot C)$

Identity property:

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity element

under '.' because.

$\forall A \in G \qquad A \cdot I_2 = A = I_2 A$

Inverse property:

Inverse of each element exists in

G because all elements, i.e matrices, are non-singular

$$\forall \ A \in G \ \exists \ A^{-1} \in G$$

$$A \cdot A^{-1} = I = A^{-1} A$$

Commutative property:

The operation '·' is not commutative because

$$\forall \ A, B \in G \Rightarrow \ \cancel{A \cdot B}$$
$$A \cdot B \neq B \cdot A$$

Hence, G is an non-abelian group under multiplication i.e '·'.