Test-Wk-5

Name:- Mahad Hussnain
Class:- 1st year (ICs-physics)
Subject:- Math
Roll NO:- 18
Date:- 23-11-24

Q1

MCQ,s

1- D.
2- D.
3- D.
4- A.

Q2

Short Q/A

(i)

Semi Group:- A non-empty set called a semi group if,

i) It is closed with respect to an operation $*$
ii) The opperation $*$ is associative

As it is obvious from its name, Semi group satifies half of the conditions required for a group.

(ii)

Prepare a table of Multiplication of the element of the set of residue of Modulo 5.

Let $G = \{0, 1, 2, 3, 4\}$
set

		(I)	(II)	(III)	(IV)	(V)
	$\otimes$	0	1	2	3	4
$R_1 \rightarrow$	0	0	0	0	0	0
$R_2 \rightarrow$	1	0	1	2	3	4
$R_3 \rightarrow$	2	0	2	4	1	3
$R_4 \rightarrow$	3	0	3	1	4	2
$R_5 \rightarrow$	4	0	4	3	2	1

$R_3 \rightarrow (IV)$ $2 \times 3 = 6$

$6 \div 5 = 1$

$R_3 \rightarrow (V)$ $2 \times 4 = 8$

$8 \div 5 = 3$

$R_4 \rightarrow (III)$ $3 \times 2 = 6$

$6 \div 5 = 1$

$R_4 \rightarrow (IV)$ $3 \times 3 = 9$

$9 \div 5 = 4$

$R_4 \rightarrow (V)$ $3 \times 4 = 12$

$12 \div 5 = 2$

$R_5 \rightarrow (III)$ $4 \times 2 = 8$

$8 \div 5 = 3$

$R_5 \rightarrow (IV)$ $4 \times 3 = 12$

$12 \div 5 = 2$

$R_5 \rightarrow (V)$ $4 \times 4 = 16$

$16 \div 5 = 1$

(iii)

If $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, Show that $A^4 = I_2$

$$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} i \times i + 0 \times 1 & i \times 0 + 0 \times -i \\ 1 \times i - i \times 1 & 1 \times 0 - i \times -i \end{bmatrix}$$

$$A^2 = \begin{bmatrix} i^2 + 0 & 0 + 0 \\ i - i & 0 - i^2 \end{bmatrix} = \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} \quad \because i^2 = -1$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \times -1 + 0 \times 0 & -1 \times 0 + 0 \times -1 \\ 0 \times -1 + (-1 \times 0) & 0 \times 0 + (-1 \times -1) \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 + 0 & 0 + 0 \\ 0 + 0 & 0 + 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

So, Hence $A^4 = I_2$.

(Section-II)

Long Questions:-

Q#03

Let, G be the Set of all 2x2 non-singular Matrices:

$$G = \{A, B, C, I, \ldots\}$$

→ **Closure:-**

As the product of any 2x2 matrices is again a Matrix of Order 2x2. So, G is Closed under operation.

$$A \cdot B \in G$$

→ **Associative:-**

The operation is associative

e.g $A, B, C, \in G$ then $(AB)C = A(BC)$

→ **Identity:-**

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity in G.

→ **Inverse:-** As G contain non-singular Matrices only so, it contain inverse of each of its element.

The set G is a group under ⊕

$$A \cdot A^{-1} = I = A^{-1} \cdot A$$

→ **Commutative:-** The operation is not commutative As we know that $A \times B \neq B \times A$ in general Particularly for G.

Thus, G is non-Abelian group.