Sheet # _____

LGS GROUP OF COLLEGES
A PROJECT OF LAHORE GRAMMAR SCHOOL

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Subject: Maths Test No. WP-5 Date: 23-11-24

	A	B	C	D		A	B	C	D		A	B	C	D		A	B	C	D
1	○	●	○	○	6	○	○	○	○	11	○	○	○	○	16	○	○	○	○
2	○	○	○	●	7	○	○	○	○	12	○	○	○	○	17	○	○	○	○
3	○	○	○	●	8	○	○	○	○	13	○	○	○	○	18	○	○	○	○
4	○	○	○	●	9	○	○	○	○	14	○	○	○	○	19	○	○	○	○
5	○	○	○	○	10	○	○	○	○	15	○	○	○	○	20	○	○	○	○

Marks Obtained

(Subjective Type)
(Short Questions)

(i)
Semi-Group:-

- A non-empty set S is semi group if;
  It is closed with respect to an operation ※
- The operation ※ is associative.

A semi-group satisfies half of the conditions required for a group.

(ii)

※	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

(iii) $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, show $A^4 = I_2$

Solution:-

$$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \Rightarrow A^2 = A \cdot A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} (i)(i)+0(1) & (i)(0)+(0)(-i) \\ 1(i)+(-i)(1) & (1)(0)+(-i)(-i) \end{bmatrix} \Rightarrow \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(-1)+(0)(0) & (-1)(0)+0(-1) \\ 0(-1)+(-1)(0) & (0)(0)+(-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow I_2.$$

Que3:-

Solution:-

Let G be the non-singular matrices over the real field 2×2 matrices.

(i) Let $A, B \in G$ then $A_{2\times2} \times B_{2\times2} = C_{2\times2} \in G$
Thus closure law holds in G under multiplication.

(ii) Associative law in matrices of same order under multiplication

holds.

therefore for $A, B, C \in G$

$$A \times (B \times C) = (A \times B) \times C$$

(iii) $I_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a non-singular matrix such that,

$$A_{2\times2} \times I_{2\times2} = I_{2\times2} \times A_{2\times2} = A_{2\times2}$$

Thus $I_{2\times2}$ is an identity element in G.

(iv) Since inverse of non-singular square matrix exists, therefore for $A \in G$ there exist $A^{-1} \in G$ such that $AA^{-1} = A^{-1}A = I$

(v) As we know for any two matrices $A, B \in G$, $AB \neq BA$ in general. Therefore commutative law does not holds in G under multiplication. Hence set of all 2x2 non-singular matrices over a real field is a non-abelian group under multiplication.