



LGS GROUP OF COLLEGES

A PROJECT OF LAHORE GRAMMAR SCHOOL

Sheet # _____

Name: Rida Fatima Class: 2-year Roll No. _____
 Subject: Maths Test No. WT-5 Date: 23.11.2024

A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D	Marks Obtained				
1	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	6	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	11	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	16		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	7	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	12	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	17		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	8	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	13	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	18		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
4	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	9	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	14	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	19		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
5	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	10	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	15	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	20	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	

* SHORT QUESTION

1) Semi-group:-

Ans: A non-empty set S is semi group if :-

- 1) It is closed w.r.t to an operation $*$
- 2) The operation $*$ is associative, $(N, +)$ is a semi-group.

2)

$*$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

3)

$$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

Sol:-

$$A \cdot A = A^2 = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} i(i) + 0(1) & i(0) + 0(-i) \\ 1(i) + (-i) & 1(0) + (-i)(-i) \end{bmatrix}$$

$$= \begin{bmatrix} i^2 & 0-0 \\ i-i & 0+i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 0+(-1)^2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 0+1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = A^2$$

$$\text{Now, } A^2 \times A^2 = A^4 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$= \begin{bmatrix} (-1)(-1) + (0)(0) & (0)(0) + (0)(1) \\ (0)(0) + (1)(0) & (0)(0) + (1)(1) \end{bmatrix} = \begin{bmatrix} +1 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

LONG QUESTION

Q3. let M_2 represent all 2×2 matrices over the real field

$$\therefore M_2 = \left\{ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \text{ where } a_{11}, a_{12}, a_{21}, a_{22} \in \mathbb{R} \text{ and } \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0 \right\}$$

ci) Closure property:-

$$\text{For all } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \in M_2 \text{ where } |A| \neq 0 \text{ \& } |B| \neq 0$$

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix} \text{ Hence } AB \in M_2$$

ii) Associative property:

$$\forall A, B, C \in M_2 \Rightarrow A(BC) = (AB)C$$

iii) Identity element:

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ is a } 2 \times 2 \text{ non-singular matrix hence it belongs to } M_2 \text{ and } \forall A \in M_2$$

$$\rightarrow IA = A = AI$$

$$\text{Thus, } I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ is the identity element in } M_2$$

iv) Inverse element:

As inverse of a non-singular square matrix is again a non-singular matrix of that order therefore,

$$\text{If } A \in M_2, \exists A^{-1} \in M_2 \text{ such that } A \cdot A^{-1} = A^{-1} \cdot A = I$$

* Inverse of each matrix A exists in M_2

v) Commutative property:

Since $AB \neq BA$ in general so it does not hold commutative property.

(Hence proved).