Question no: 02

i

Semi - Group:

- A non-empty set is called a semi group if,

i. It is closed with respect to an operation *
ii. The operation should hold closure and associative properties. only.

- A semi-group satisfies half of the conditions required for a group.

ii

The Table of multiplication of the elements of the set of residue classes modulo 5.

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	01	93	2	1

$$A = \begin{bmatrix} i & 0 \\ 01 & -i \end{bmatrix}, \quad A^4 = ?$$

$A^2 = A \cdot A$

$$= \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \times \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} i^2 + 0 & 0 + 0 \\ i - i & 0 + i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$A^4 = A^2 \cdot A^2$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0 \cdot 0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus,

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Question no 03

Let,
$$G = \{All\ 2 \times 2\ non\ singular\ matrices\}$$
$$= \{...A, B, C, I, A^{-1}, B^{-1}, C^{-1}, ...\}$$

Closure :

Set 'G' is closure under operation '.' because

$$\forall\ A, B \in G \qquad A.B \in G.$$

Associative :

The operation '.' is Associative because

$$\forall\ A, B, C \in G$$
$$(A.B).C = A.(B.C)$$

Identity :

~~(The operation '×' is)~~

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ is the Identity element

under '.' because

$$\forall\ A \in G$$
$$A.I_2 = A = I_2 A$$

Inverse :

Inverse of each element exists in G because all elements i.e matrices, are non singular

$$\forall\ A \in G \qquad \exists\ A^{-1} \in G$$
$$A.A^{-1} = I = A^{-1}A$$

Commutative :

The operation is not commutative because

$$\forall \; A, B \in G$$

and

$$A.B \neq B.A$$

Hence,

G is an non-abelian group under multiplication.