Sheet # 1

LGS GROUP OF COLLEGES
A PROJECT OF LAHORE GRAMMAR SCHOOL

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Subject: MATHEMATICS Test No. WT-8 Date:

	A	B	C	D
1		●		
2		●		
3		●		
4				●
5				

Marks Obtained

Subjective Type

Question no 2:

(i)

Consider

$\underline{a} = -\underline{i} + \underline{j} + \underline{k}$

$|\underline{a}| = \sqrt{(-1)^2 + (1)^2 + (1)^2} = \sqrt{3}$

$\boxed{|\underline{a}| = \sqrt{3}}$

Now

$\hat{\underline{a}} = \dfrac{\underline{a}}{|\underline{a}|} = \dfrac{-\underline{i} + \underline{j} + \underline{k}}{\sqrt{3}}$

$\hat{\underline{a}} = \dfrac{-\underline{i}}{\sqrt{3}} + \dfrac{\underline{j}}{\sqrt{3}} + \dfrac{\underline{k}}{\sqrt{3}}$

let $\underline{b}$ be a vector with magnitude '2'

Since

$\underline{b}$ is parallel to $\underline{a}$

$\because \hat{\underline{b}} = \hat{\underline{a}}$

$\underline{b} = |\underline{b}|\,\hat{\underline{b}}$

$= 2\left[\dfrac{-1}{\sqrt{3}}\underline{i} + \dfrac{1}{\sqrt{3}}\underline{j} + \dfrac{1}{\sqrt{3}}\underline{k}\right]$

$$= -\frac{2}{\sqrt{3}}\underline{i} + \frac{2}{\sqrt{3}}\underline{j} + \frac{2}{\sqrt{3}}\underline{k}$$ Ans

(ii)

$\underline{a} = \underline{i} - \underline{k}$

$\underline{b} = \underline{j} + \underline{k}$

$|\underline{a}| = \sqrt{1^2 + (0)^2 + (-1)^2} = \sqrt{1+1}$

$\boxed{|\underline{a}| = \sqrt{2}}$

$|\underline{b}| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{1+1}$

$\boxed{|\underline{b}| = \sqrt{2}}$

$\underline{a} \cdot \underline{b} = (\underline{i} - \underline{k}) \cdot (\underline{j} + \underline{k})$

$= (1)(0) + (0)(1) + (-1)(1)$

$= 0 + 0 - 1$

$= -1$

Since

$\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}| \cos\theta$

so

projection of a along $\underline{b}$ $= |\underline{a}| \cos\theta$

$$= \frac{\underline{a} \cdot \underline{b}}{|\underline{b}|}$$

$$= \frac{-1}{\sqrt{2}}$$

Also

projection of $\underline{b}$ along $\underline{a} = |\underline{b}| \cos\theta$

$$= \frac{a \cdot b}{|\underline{a}|}$$

$$= \frac{-1}{\sqrt{2}}$$

Ans

(iii)

$A(1,-1)$; $B(2,0)$
$C(-1,3)$; $D(-2,2)$

$$\vec{AB} = \vec{OB} - \vec{OA}$$
$$= (2-1)\underline{i} + [0-(-1)]\underline{j}$$

$$\boxed{\vec{AB} = \underline{i} + \underline{j}}$$

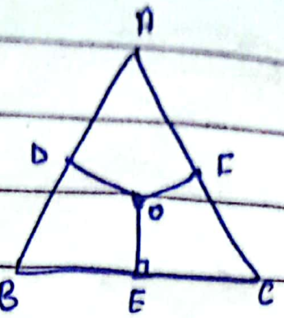
$$\vec{CD} = \vec{OD} - \vec{OA}$$
$$= (-2+1)\hat{i} + (2-3)\underline{j}$$

$$\boxed{\vec{CD} = -\underline{i} - \underline{j}}$$

$$\vec{AB} + \vec{CD} = \cancel{\underline{i}} + \cancel{\underline{j}} - \cancel{\underline{i}} - \cancel{\underline{j}}$$
$$= 0\underline{i} + 0\underline{j}$$
$$= 0$$

Ans

let
A, B and C be
the vertices of a
triangle having
position vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ respectively



Also,
consider D, E, and F are midpoints of sides $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$, then

point vector of $D = \overrightarrow{OD} = \dfrac{\underline{a}+\underline{b}}{2}$

point vector of $E = \overrightarrow{OE} = \dfrac{\underline{b}+\underline{c}}{2}$

point vector of $F = \overrightarrow{OF} = \dfrac{\underline{c}+\underline{a}}{2}$

let right bisector on $\overrightarrow{AB}$ and $\overrightarrow{BC}$ intersect at point O (which is an origin)

since $\overrightarrow{OD}$ is $\perp$ to $\overrightarrow{AB}$

Therefore

$$\overrightarrow{OD} \cdot \overrightarrow{AB} = 0$$

$$\Rightarrow \left(\frac{\underline{a}+\underline{b}}{2}\right) \cdot (\underline{b}-\underline{a}) = 0$$

$$\Rightarrow \frac{1}{2}(\underline{b}+\underline{a})(\underline{b}-\underline{a}) = 0$$

$\Rightarrow (\underline{b}+\underline{a})(\underline{b}-\underline{a}) = 0$

$\Rightarrow \underline{a}(\underline{b}-\underline{a}) + \underline{b}(\underline{b}-\underline{a}) = 0$

$\Rightarrow \underline{a}\cdot\underline{b} - \underline{a}\cdot\underline{a} + \underline{b}\cdot\underline{b} - \underline{b}\cdot\underline{a} = 0$

$\Rightarrow \underline{a}\cdot\underline{b} - |\underline{a}|^2 + |\underline{b}|^2 - \underline{b}\cdot\underline{a} = 0$

$\boxed{\because \underline{a}\cdot\underline{b} = \underline{b}\cdot\underline{a}}$

$\Rightarrow |\underline{b}|^2 - |\underline{a}|^2 = 0 \longrightarrow (1)$

Also

$\overrightarrow{OE}$ is $\perp$ to $\overrightarrow{BC}$

Therefore

$\overrightarrow{OE}\cdot\overrightarrow{BC} = 0$

$\Rightarrow \left(\frac{\underline{b}+\underline{c}}{2}\right)\cdot(\underline{c}-\underline{b}) = 0$

$\Rightarrow \frac{1}{2}(\underline{b}+\underline{c})(\underline{c}-\underline{b}) = 0$

$\Rightarrow (\underline{b}+\underline{c})(\underline{c}-\underline{b}) = 0$

$\Rightarrow (\underline{c}+\underline{b})(\underline{c}-\underline{b}) = 0$

$\Rightarrow \underline{c}(\underline{c}-\underline{b}) + \underline{b}(\underline{c}-\underline{b}) = 0$

$\Rightarrow \underline{c}\cdot\underline{c} - \underline{c}\cdot\underline{b} + \underline{b}\cdot\underline{c} - \underline{b}\cdot\underline{b} = 0$

$\Rightarrow |\underline{c}|^2 - \underline{c}\cdot\underline{b} + \underline{b}\cdot\underline{c} - |\underline{b}|^2 = 0$

$\Rightarrow |\underline{c}|^2 - |\underline{b}|^2 = 0 \longrightarrow (2)$ $\boxed{\because \underline{c}\cdot\underline{b} = \underline{b}\cdot\underline{c}}$

Adding (1) and (2), we have

$|\underline{b}|^2 - |\underline{a}|^2 + |\underline{c}|^2 - |\underline{b}|^2 = 0 + 0$

$\Rightarrow |\underline{c}|^2 - |\underline{a}|^2 = 0$

$\Rightarrow (\underline{c}+\underline{a})(\underline{c}-\underline{a}) = 0$

$\Rightarrow \left(\frac{\underline{c}+\underline{a}}{2}\right)\cdot(\underline{c}-\underline{a}) = 0$

$\Rightarrow \vec{OF} \cdot \vec{AC} = 0$

$\Rightarrow \vec{OF}$ is $\perp$ to $\vec{AC}$

$\Rightarrow \vec{OF}$ is also right bisector of $\vec{AC}$

Hence perpendicular bisector of the sides of the triangle are concurrent.