LGS GROUP OF COLLEGES
A PROJECT OF LAHORE GRAMMAR SCHOOL

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Subject: Matnematics Test No. W-5 Date: 23-11-24

	A	B	C	D
1				●
2				●
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Marks Obtained

-: SHORT QUESTION:-

1) Semi-group:-

(Ans) A non-empty set S is semi group if:-

1) It is closed w.r.t to an operation $*$.

2) The operation $*$ is associative, $(N,+)$ is a semi-grоup.

2)

$*$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

3) $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$

Sol:- $A \cdot A = A^2 = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$

$= \begin{bmatrix} i(i) + 0(1) & i(0) + 0(-i) \\ 1(i) + (-i) & 1(0) + (-i)(-i) \end{bmatrix}$

$= \begin{bmatrix} i^2 & 0-0 \\ i-i & 0+i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 0+(-1)^2 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 \\ 0 & 0+(1) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2$

Now $A^2 \times A^2 = A^4 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} (-1)(-1)+0(0) & 0(0)+0(1) \\ 0(0)+1(0) & 0(0)+1(1) \end{bmatrix} = \begin{bmatrix} +1 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} +1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

-: LONG QUESTION:-

Q:- Solution:-

let M_2 represent all 2x2 matrices over the real field.

$$\therefore M_2 = \left\{ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \text{where } a_{11}, a_{12}, a_{21}, a_{22} \in R \text{ and } \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0 \right\}$$

i) Closure property:-

For all $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \in M_2$ where $|A| \neq 0$ & $|B| \neq 0$

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{12} \end{bmatrix}$$

Hence $AB \in M_2$

ii) Associative Property:-

$\forall\ A, B, C \in M_2 \Rightarrow A(BC) = (AB)C$

iii) Identity element:-

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a 2x2 non-singular matrix hence it belongs to M_2 and $\forall\ A \in M_2$

$\rightarrow IA = A = AI$

Thus $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity element in M_2.

(iv) Inverse element:-

As inverse of a non-singular square matrix is again a non-singular matrix of that order therefore,

If $A \in M_2, \exists\ A^{-1} \in M_2$ such that $A \cdot A^{-1} = A^{-1} \cdot A = I$

★ Inverse of each matrix A exists in M_2

(V) Commutative Property:-

Since $AB \neq BA$ in general so it does not hold commutative property.

★ **Hence proved**