

# Introduction to Pulsar, Pulsar Timing, and measuring of Pulse Time-of-Arrivals

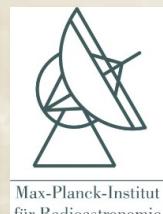
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IPTA 2017 Student Workshop, CIEP, Sèvres, France

26/06/2017



Once upon a time...

Far far away...

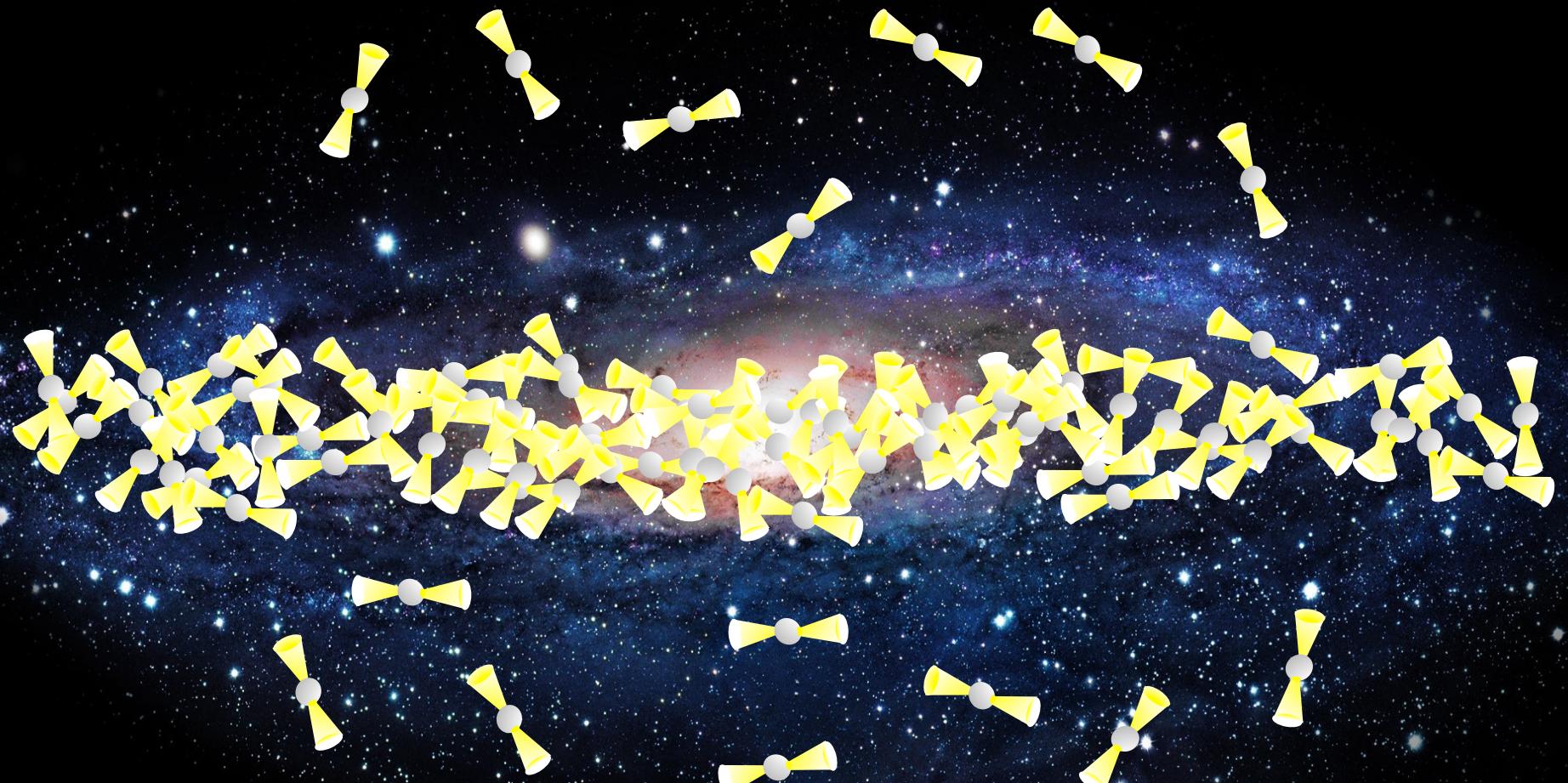
In the Galaxy...



We found a pulsar...



Well...in fact...there are many of them!



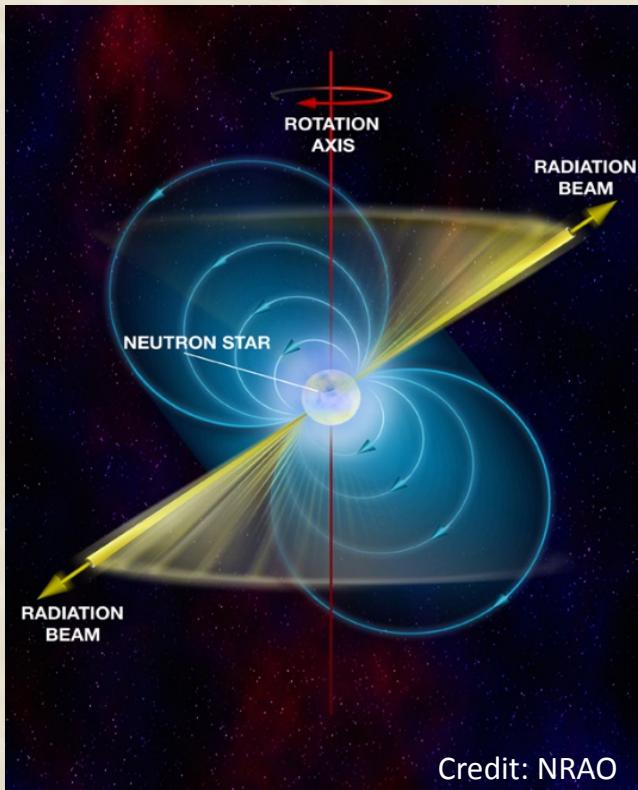
Up to 22/06/2017, there are over 2600 pulsars  
that have been discovered “officially”.

What are pulsars?

# Pulsars

Externally, pulsars are:

- Fast-rotating magnetic dipoles;
- Emitting electromagnetic wave at radio wavelength / X-ray /  $\gamma$ -ray...;
- Cosmic “light houses”;

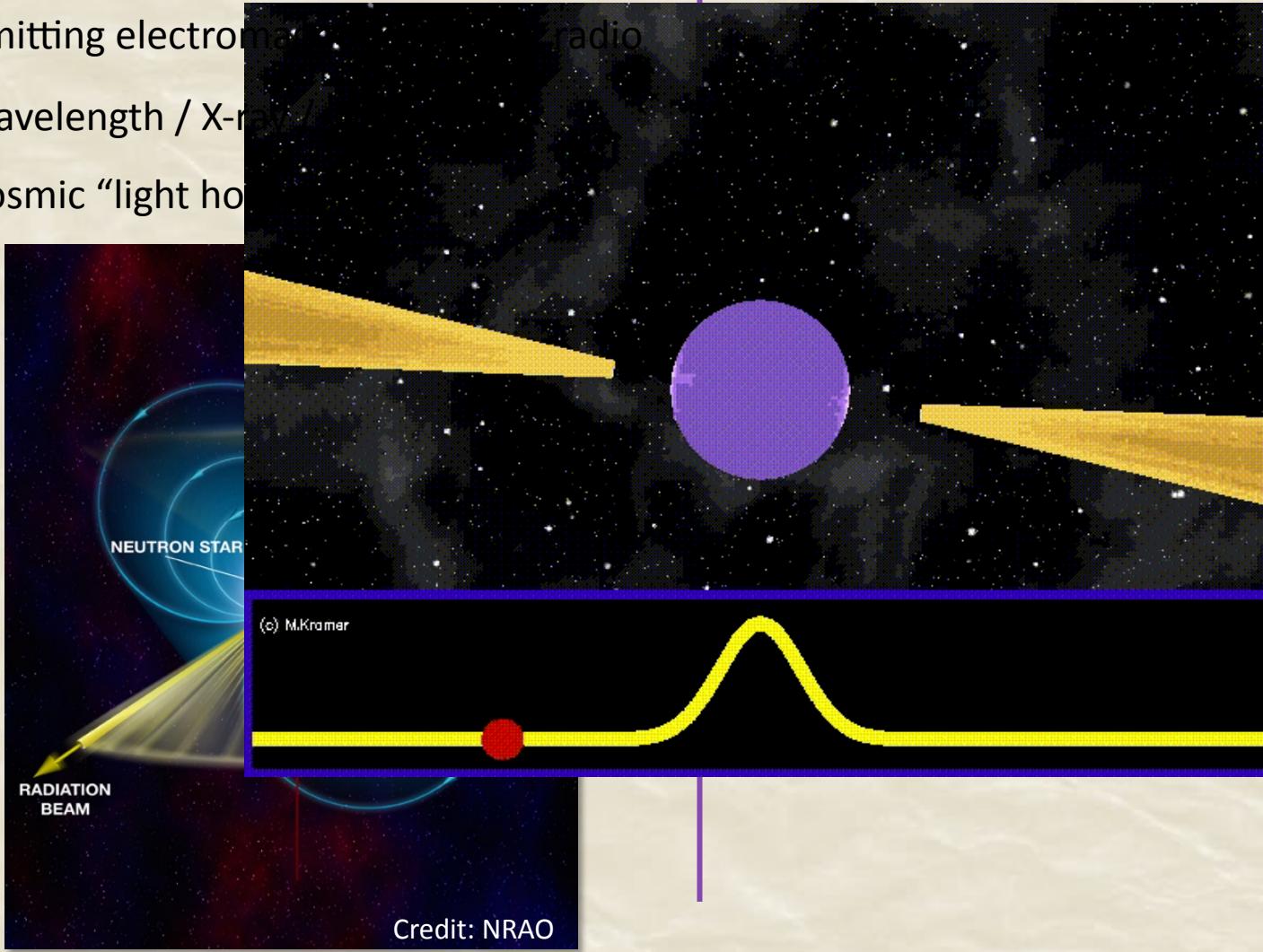


Credit: NRAO

# Pulsars

Externally, pulsars are:

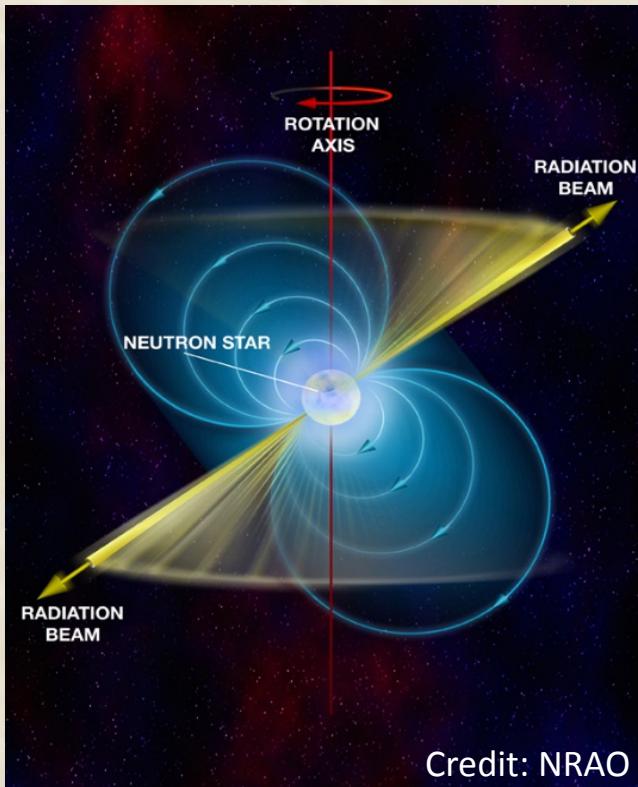
- Fast-rotating magnetic dipoles;
- Emitting electromagnetic radiation at radio wavelength / X-ray wavelengths;
- Cosmic “light house” beacons.



# Pulsars

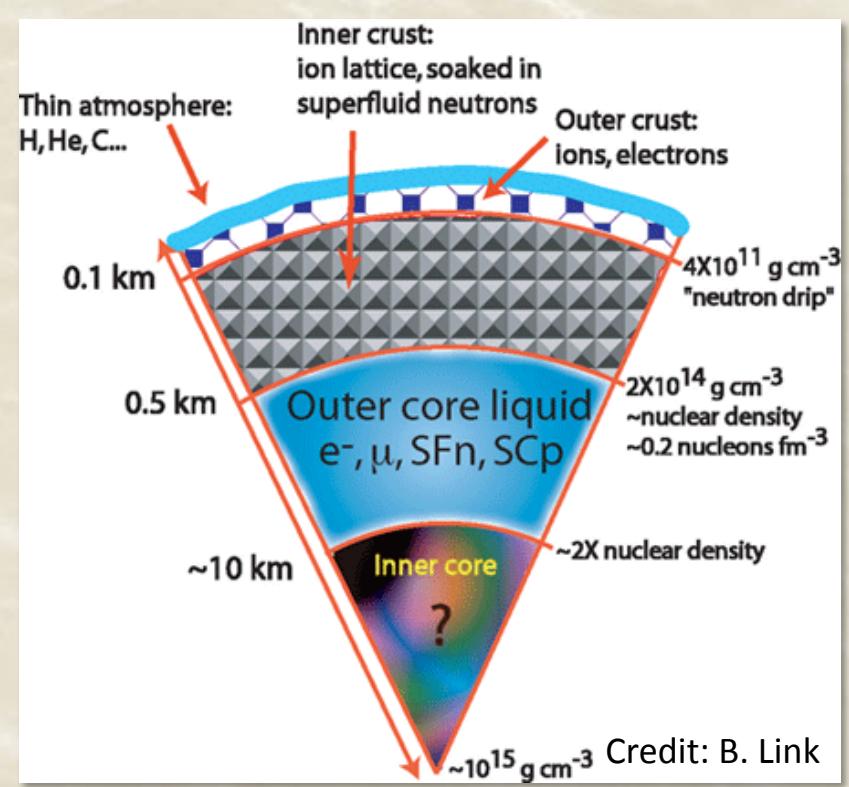
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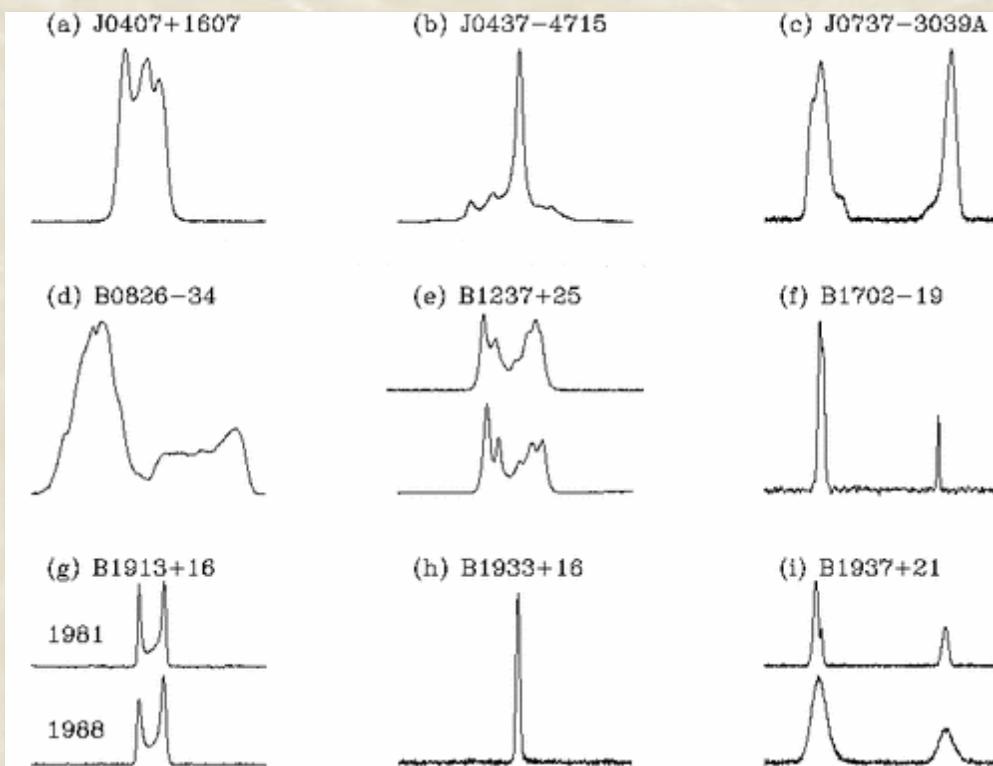
Internally, pulsars are:

- Small objects,  $\sim 20$  km in diameter;
- Heavy objects,  $\sim 1$  solar mass;
- Multi-layer structure;
- Commonly believed to be neutron stars;



## Integrated pulse profile: pulsar's fingerprint

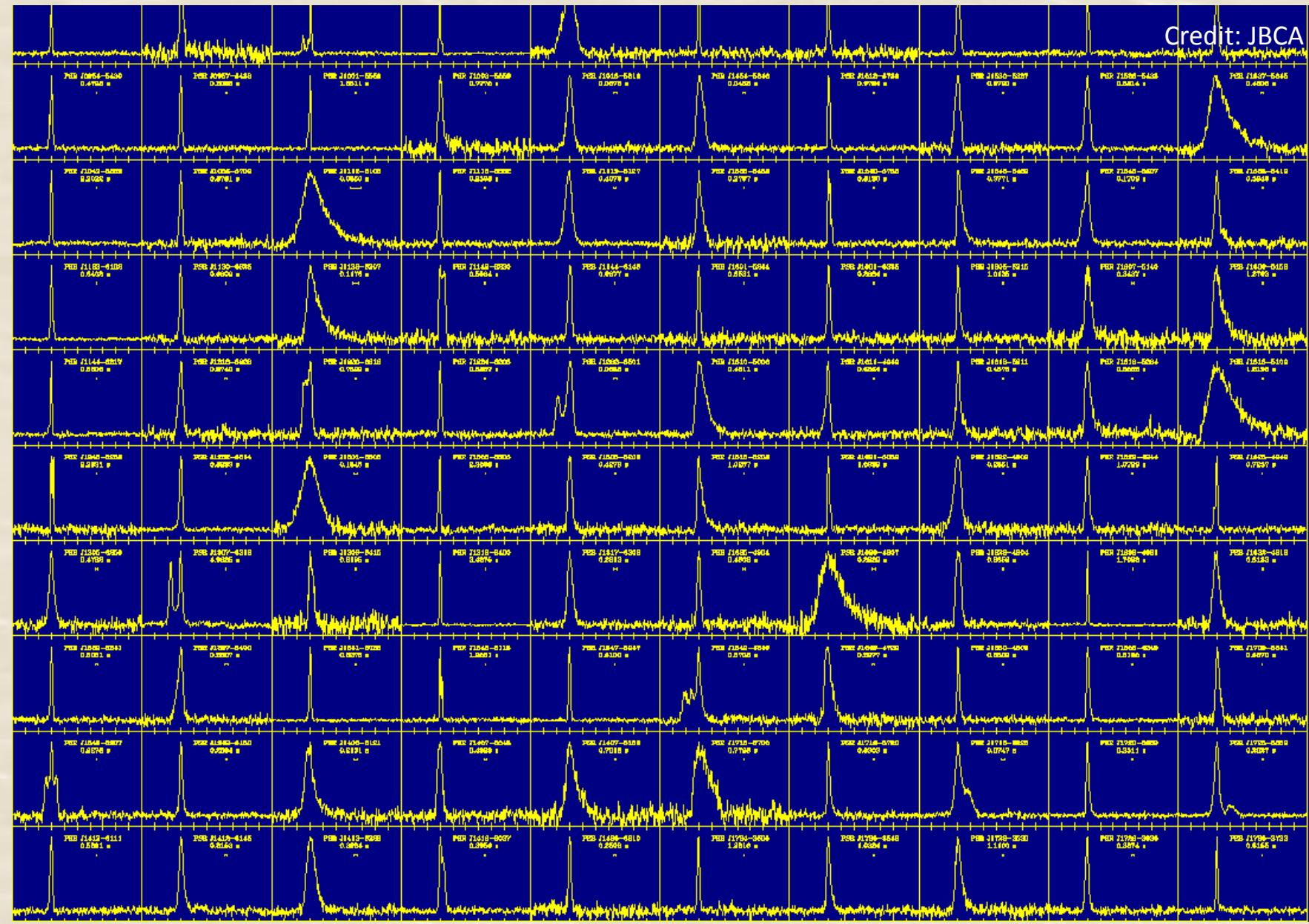
- Pulsar produces **periodic pulsation** signals -> often too **weak** to detect;
- **Fold / Integrated** pulsar signals with respect to its **rotational period** -> increase signal quality and form **integrated pulse profile**;
- Pulsars are **distinguished by their integrated pulse profiles** (not by their names!) -> all pulsars have their **unique** profile shape, just like human's **fingerprint**!



*Not enough samples?  
Get your eyes ready...*

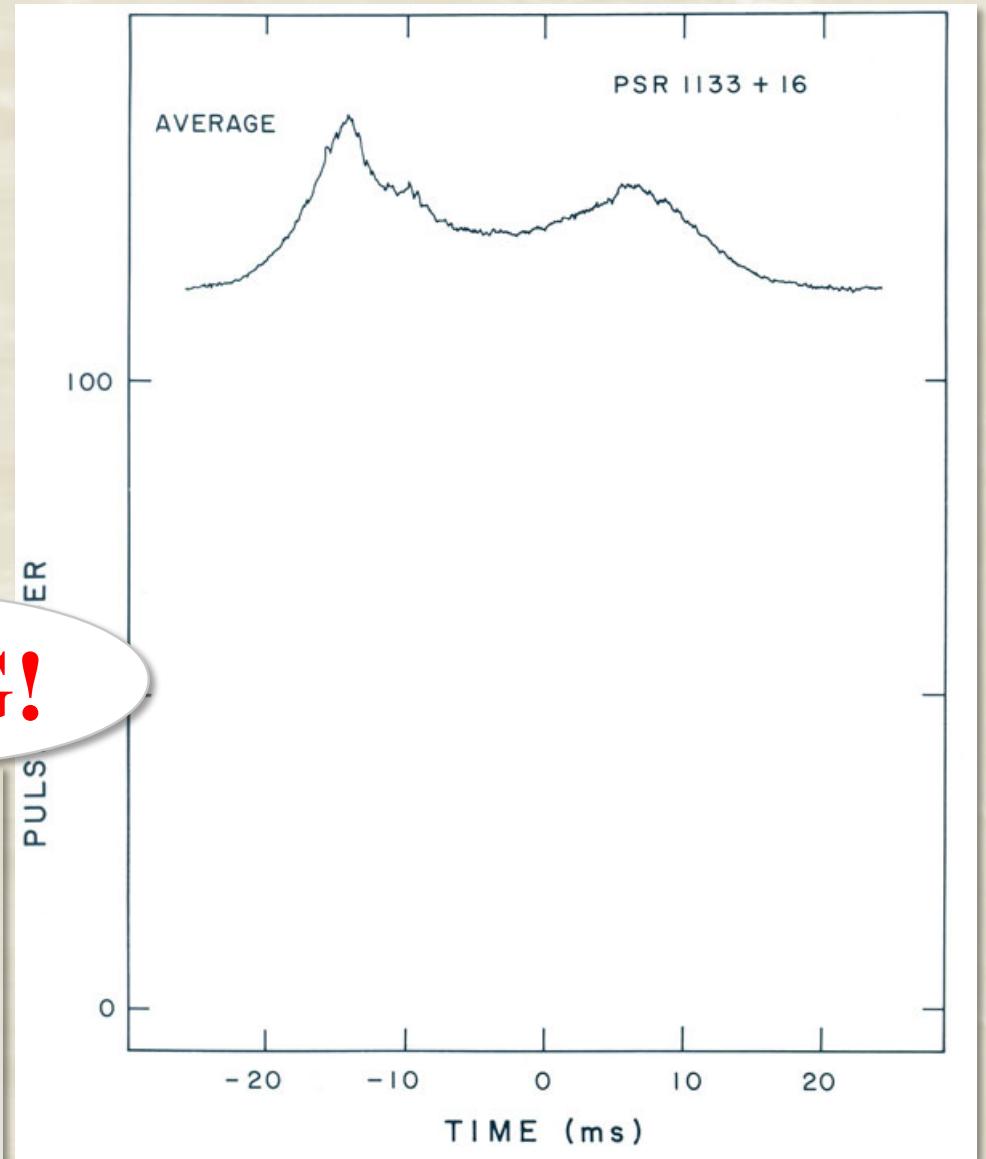
[ Lorimer & Kramer 2005 ]

# Integrated pulse profile: pulsar's fingerprint



## Integrated pulse profile vs single pulses

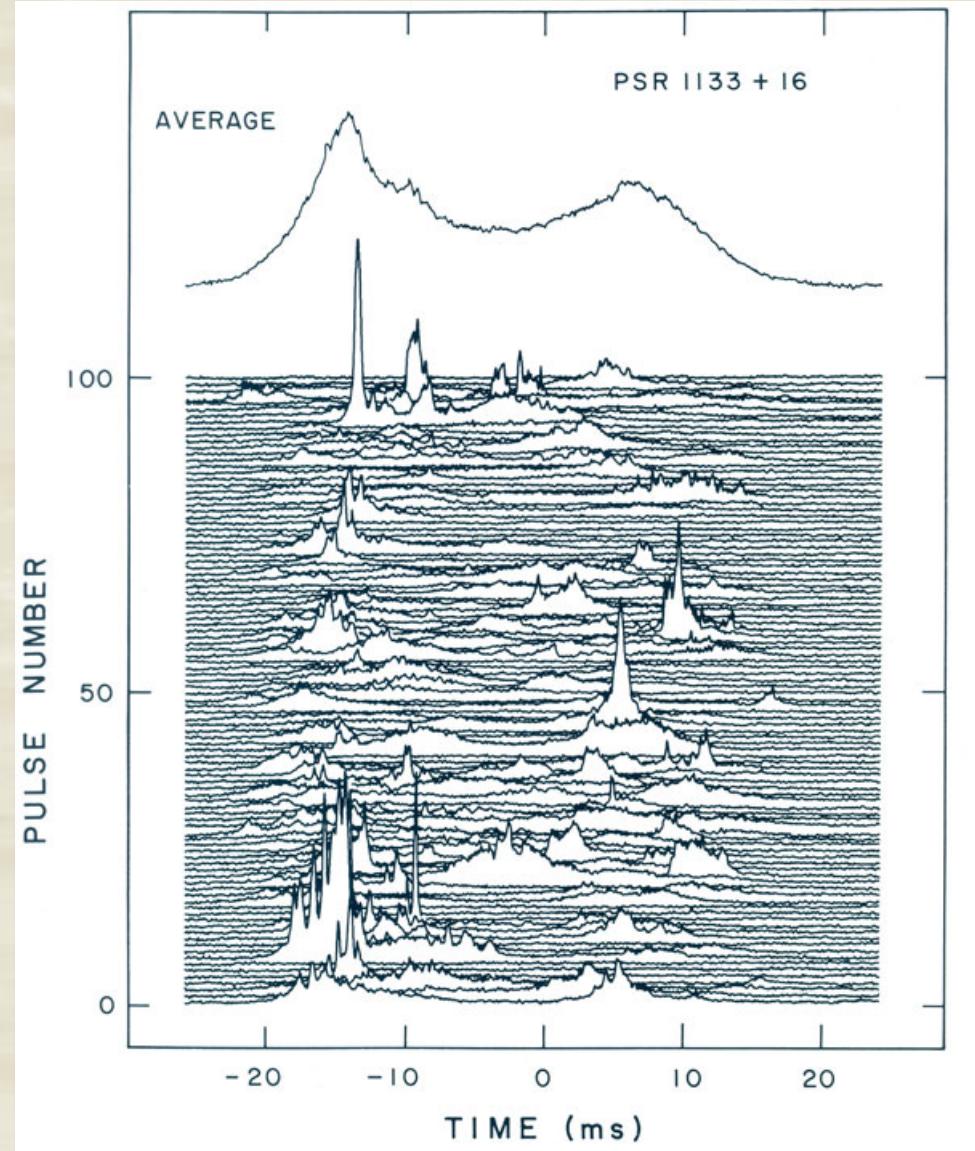
- Integrated profiles are seen to be consistent from different observations and in general **stable** in time;
- Pulse emissions from each individual rotations, i.e., **single pulses**, are seen to be highly **variable from pulse to pulse!**



Credit: A. Lyne

## Integrated pulse profile vs single pulses

- Integrated profiles are seen to be consistent from different observations and in general **stable** in time;
- Pulse emissions from each individual rotations, i.e., **single pulses**, are seen to be highly **variable from pulse to pulse!**
- Variation of single pulses can be both **stochastic** and **systematic** (periodic intensity modulation, drifting sub-pulse, mode-changing, nulling, etc.);

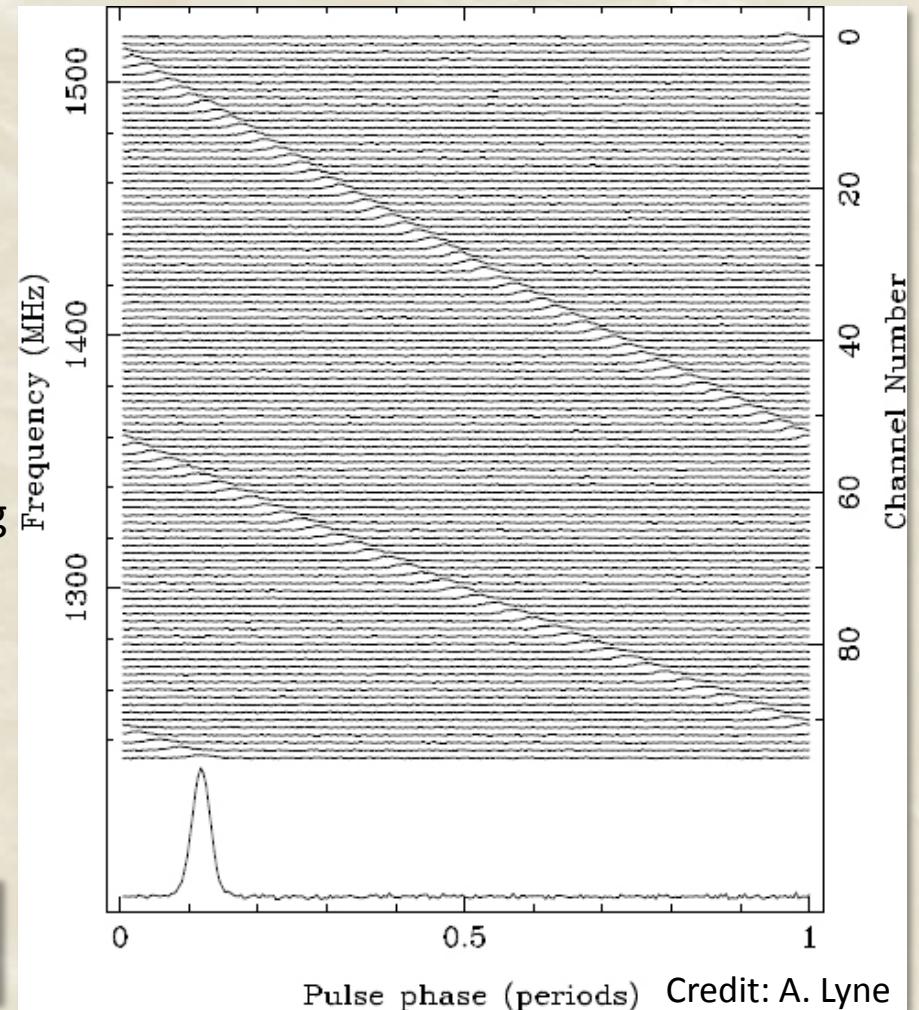


Credit: A. Lyne

## Pulsar signal and dispersion delay

- In between the pulsar and the earth, there is **interstellar medium (ISM)**, containing cold plasma of ionized free electrons, etc.;
- Electromagnetic waves in radio frequency propagating through the ISM will endure a **time delay** (smaller group velocity) depending on their frequencies.
- The difference in time delay between signals at two different frequencies are given by:

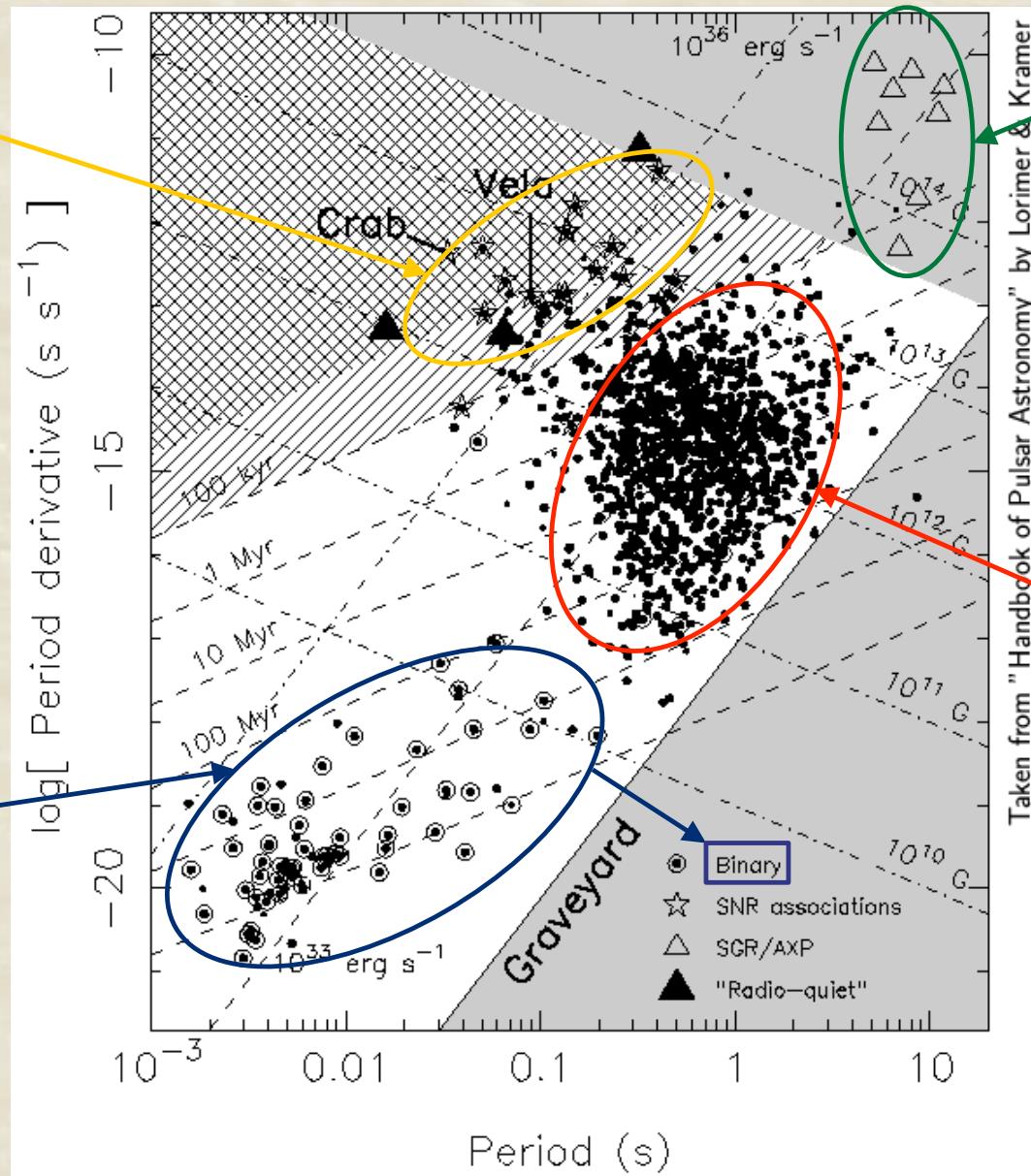
$$\Delta t = 4.15 \text{ ms} \times \left[ \left( \frac{\nu_{\text{lo}}}{\text{GHz}} \right)^{-2} - \left( \frac{\nu_{\text{hi}}}{\text{GHz}} \right)^{-2} \right] \times \left( \frac{\text{DM}}{\text{cm}^{-3} \text{ pc}} \right)$$



where the dispersion measure (DM) is defined by the column density of free electrons along the line of sight:

$$\text{DM} = \int_0^d n_e dl$$

# P-Pdot diagram and classification



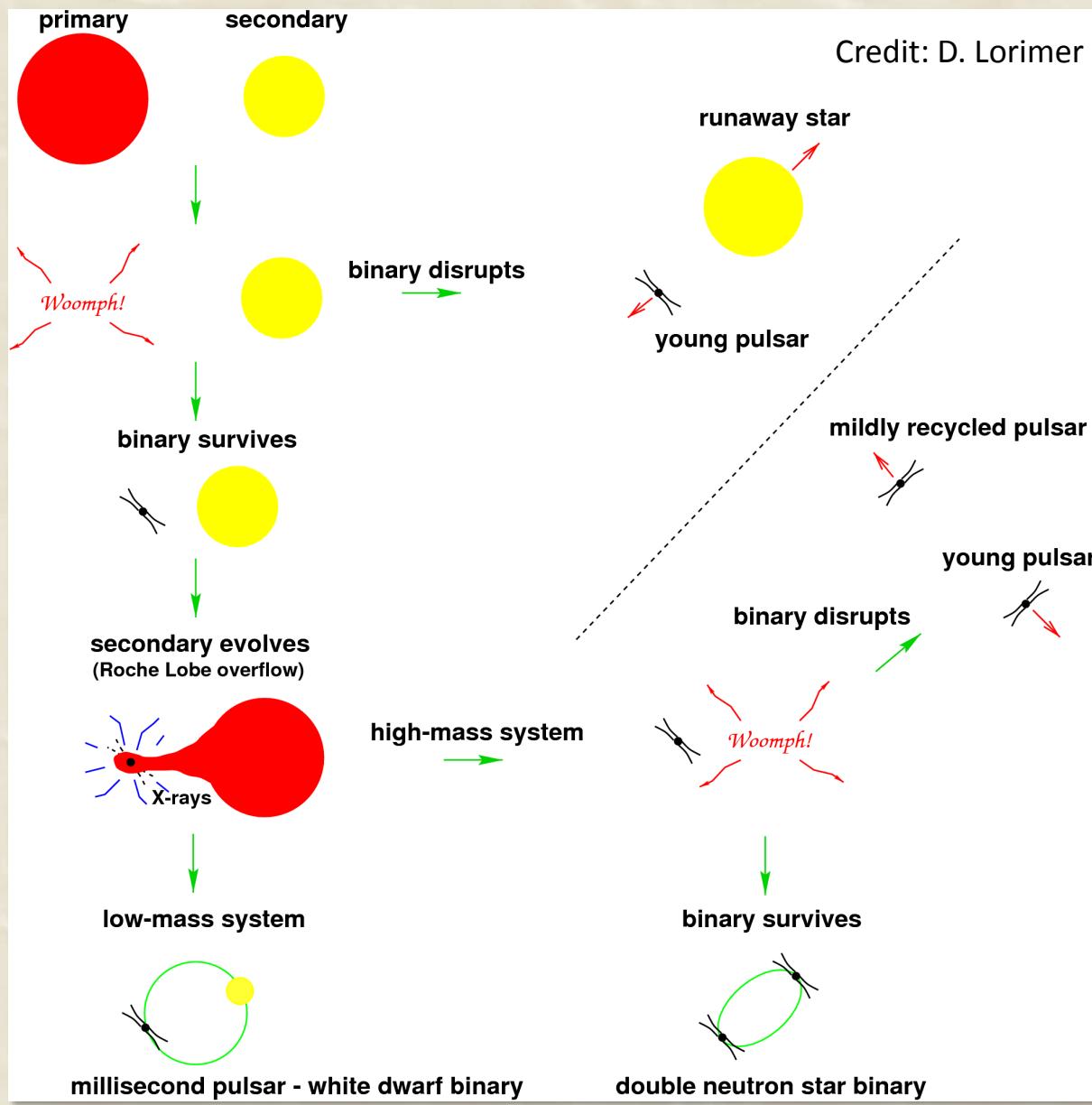
Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

Recycled /  
millisecond  
pulsars  
(MSPs)

Magnetars

Canonical /  
slow  
pulsars

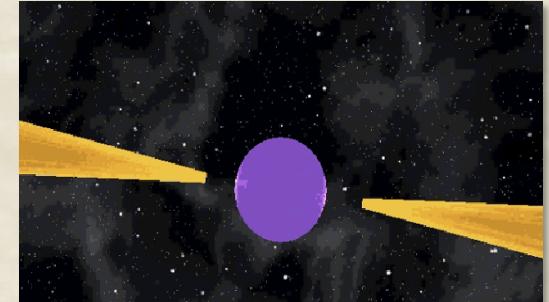
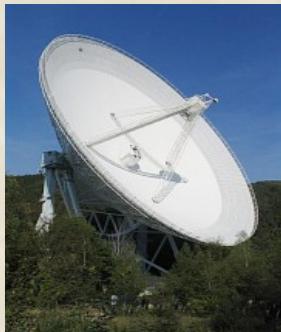
# Formation and evolution of binary pulsar



- Binary pulsar in the end:
    - 1). Mildly recycled pulsars ( $P > 20$  ms) with heavy companion (neutron star);
    - 2). Fully recycled pulsars ( $P < 20$  ms), i.e., MSPs, with light companion (white dwarf);
    - 3). Pulsar-black hole possible;
  - Companion of binary pulsars found so far: white dwarf, neutron star, pulsar, main-sequence star, planet;
- Only black hole missing!**

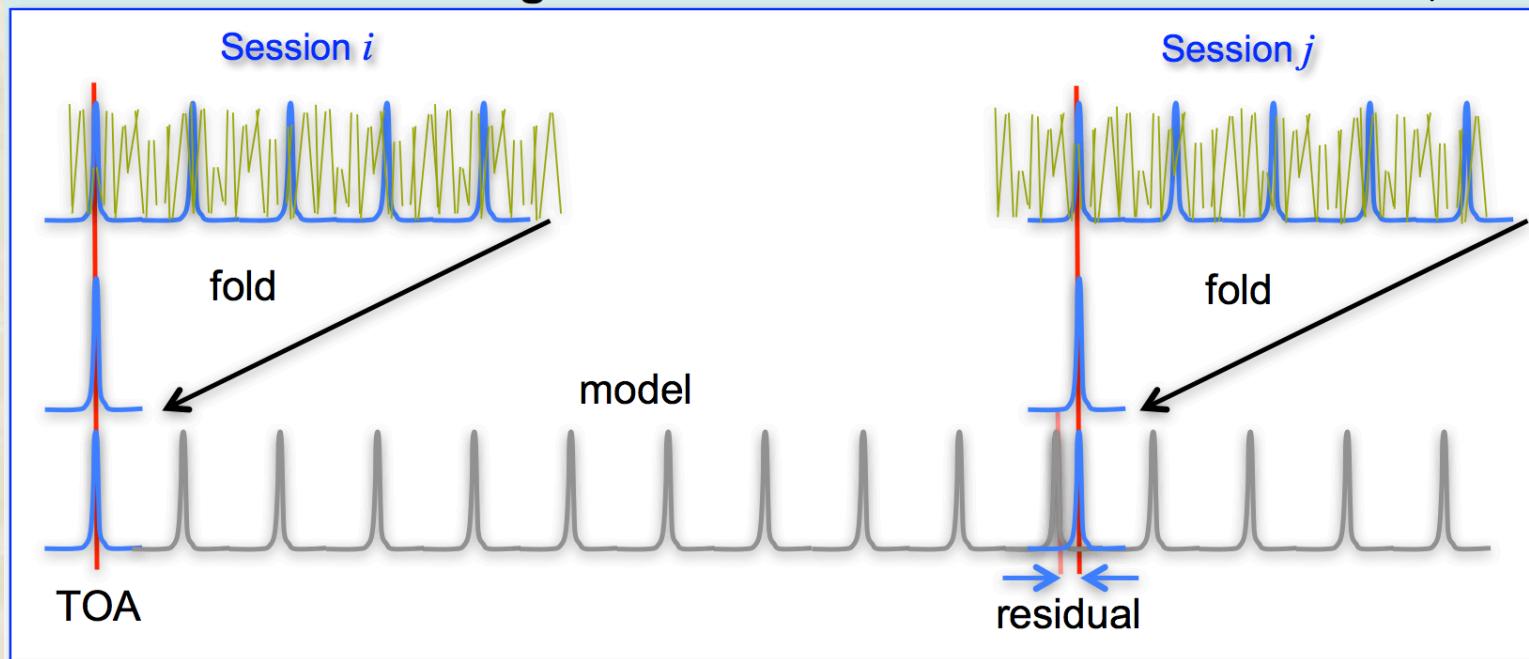
Pulsar timing

# The first principle of timing experiment



Phase-connected timing solution:

Credit: D. Champion



PSR J1012+5307:  $P = 0.0052557490101970103(19)$  s (Desvignes et al. 2016);

<- By counting all pulses ( $3 \times 10^{13}$  rotations!!) in **20 years!!**

## Time transformation in timing experiment

- Pulse phase / “counts” of pulses at pulsar proper time:

$$\phi(t) = \sum_{n \geq 1} \frac{\nu^{(n-1)}}{n!} (t_e^{\text{psr}} - t_p)^n + \phi_0$$

The fractional part of  $\phi(t)$  is the **timing residual**.

**Pulsar is intrinsically a precise clock -> It is a clock ONLY WHEN we measure with respect to its proper time!**

- Top-level timing formula:

$$t_e^{\text{psr}} = t_a^{\text{obs}} - \Delta_{\odot} - \Delta_{\text{IS}} - \Delta_B$$

Pulsar proper time (at the centre of the pulsar)

TOA at observatory time (after clock correction to TT)

Transfer to solar-system barycentre

Transfer to barycentre of the pulsar (system)

Transfer to pulsar centre (if in a binary system)

## Time model

- Components of time transformation:

$$\Delta_{\odot} = \Delta_A + \Delta_{R\odot} + \Delta_p + \Delta_{D\odot} + \Delta_{E\odot} + \Delta_{S\odot}$$

$$\Delta_{IS} = \Delta_{VP} + \Delta_{ISD} + \Delta_{FDD} + \Delta_{ES}$$

$$\Delta_B = \Delta_{RB} + \Delta_{AB} + \Delta_{EB} + \Delta_{SB}$$

[ Edwards et al. 2006 ]

- **Timing model parameters:**

a). **Spin parameters:** period, period derivative, glitches, spin noise, ...

b). **Astrometry parameters:** RA, DEC, proper motion, parallax, ...

c). **ISM parameters:** DM, derivative(s) of DM, ...

d). **Keplerian binary parameters:** orbital period ( $P_b$ ), projected semi-major axis ( $x$ ), longitude of periastron ( $\omega$ ), eccentricity ( $e$ ), epoch of periastron passage ( $T_0$ );

e). **Post-Keplerian (relativistic) parameters:** advance of periastron ( $\dot{\omega}$ ), second Doppler & gravitational time dilation (Einstein delay,  $\gamma$ ), orbital decay ( $\dot{P}_b$ ), curvature of space-time (Shapiro delay,  $\sin i$ ,  $M_2$ ), variation of projected semi-major axis ( $\dot{x}$ ), ...

To better understand the gravitational time dilation...

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## Timing residuals

- For the  $N_i$ th TOA, given the values of the timing parameters, one can calculate its corresponding timing residual in pulsar proper time, by subtracting the integer part of  $\phi_i$  (number of rotations):

$$R_i = \frac{\phi_i - N_i}{\nu}$$

**Note:** You may well have ambiguity of an integer number (i.e., lose coherency in pulse phase) if the initial values of the timing parameters are not good enough to keep residuals within  $\pm$  half a period;

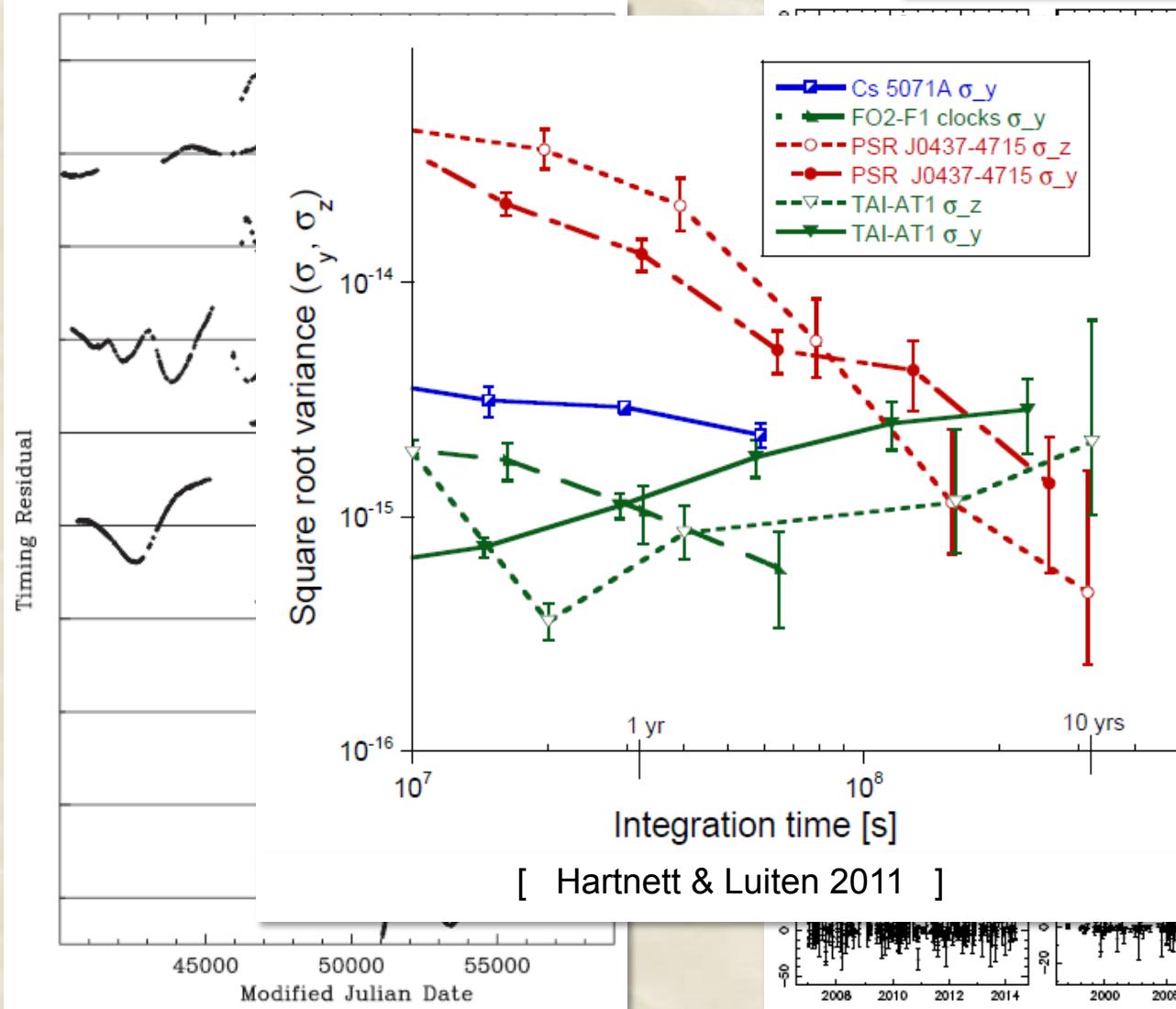
- The timing parameters are fitted based on a linear singular-value decomposition, weighted least-squares algorithm, minimising:

$$\chi^2 = \sum_{i=1}^N \left( \frac{R_i}{\sigma_i} \right)^2$$

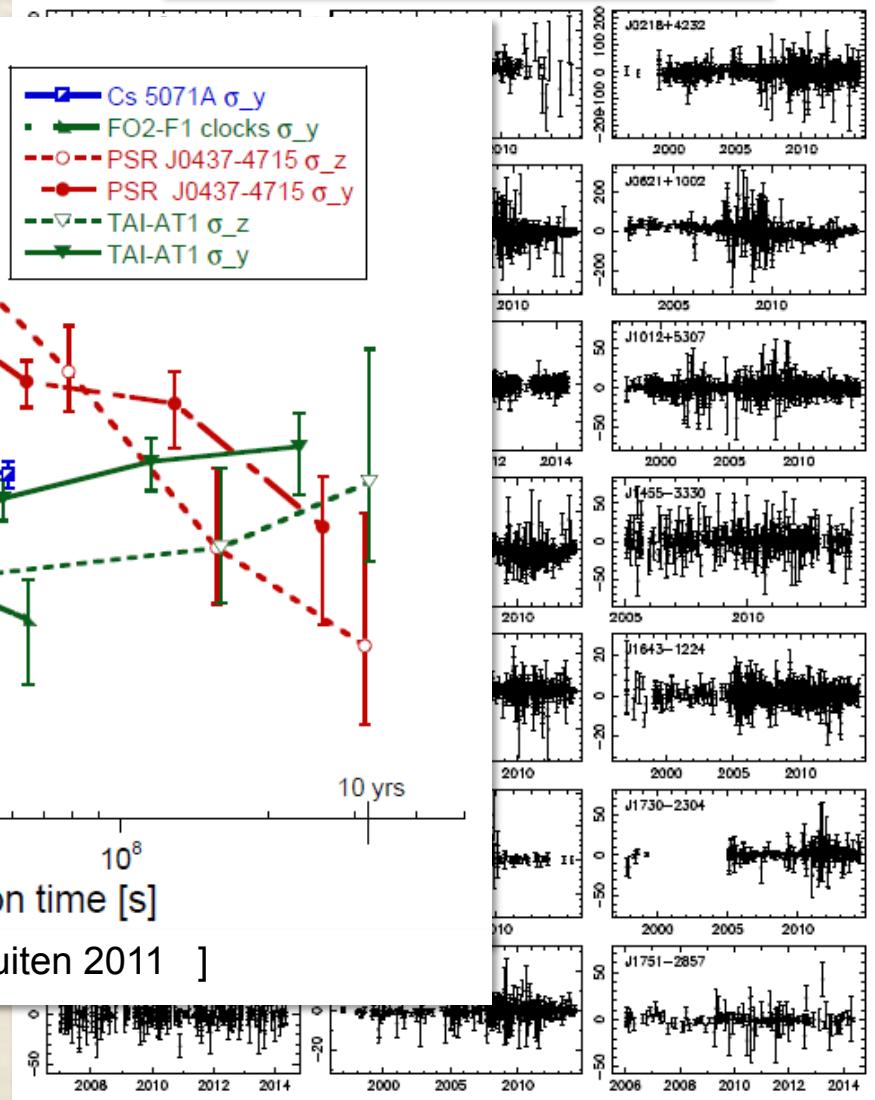
- The timing residuals are supposed to be Gaussian / white noise when the model & model parameters describe the data perfectly.

# Timing stabilities

## Canonical Pulsars

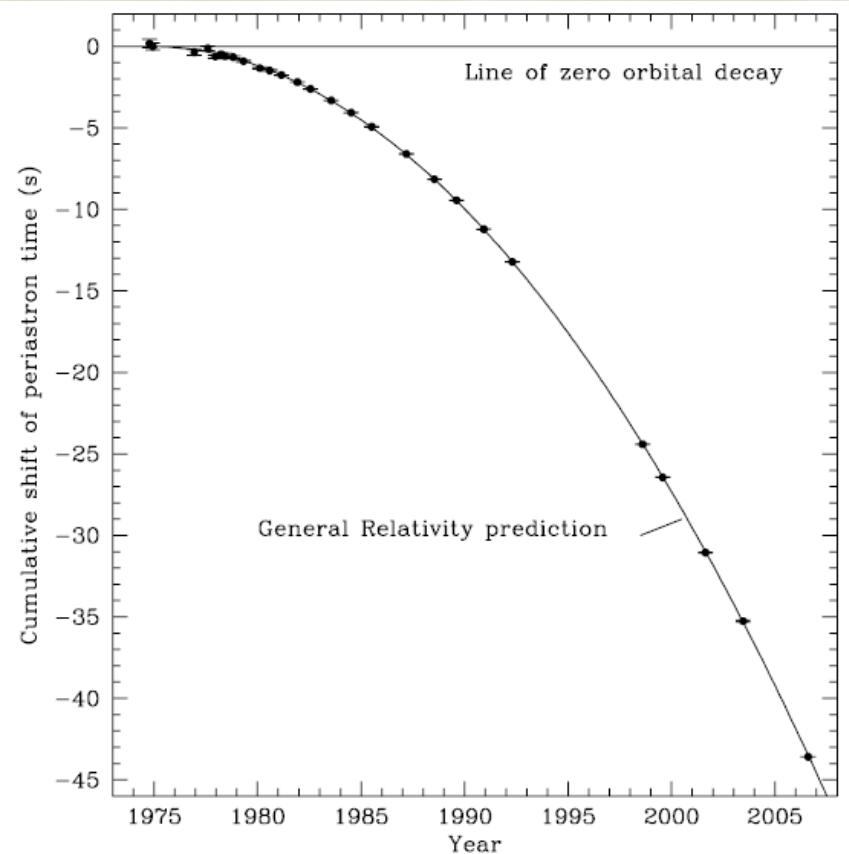


## Millisecond pulsars



# Testing General Relativity with pulsar timing

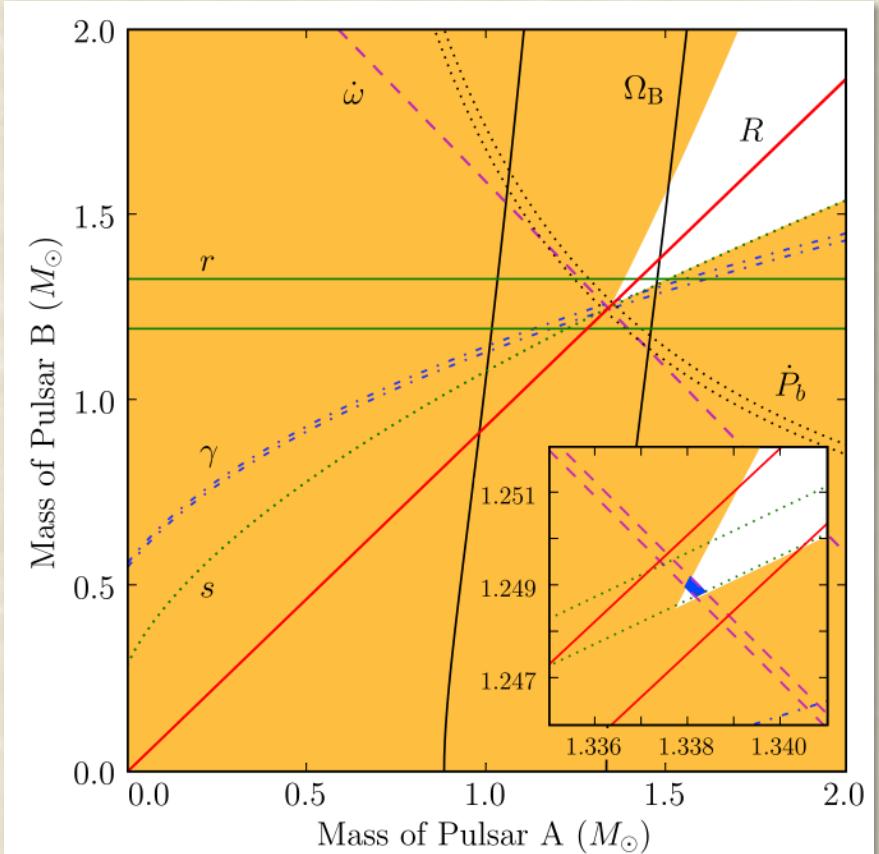
Hulse-Taylor Pulsar



[ Weisberg et al. 2010 ]

Physics 1993

The double-pulsar system



[ Kramer et al. 2006 ]

The most constraining test of General Relativity (better than 99.95%) !

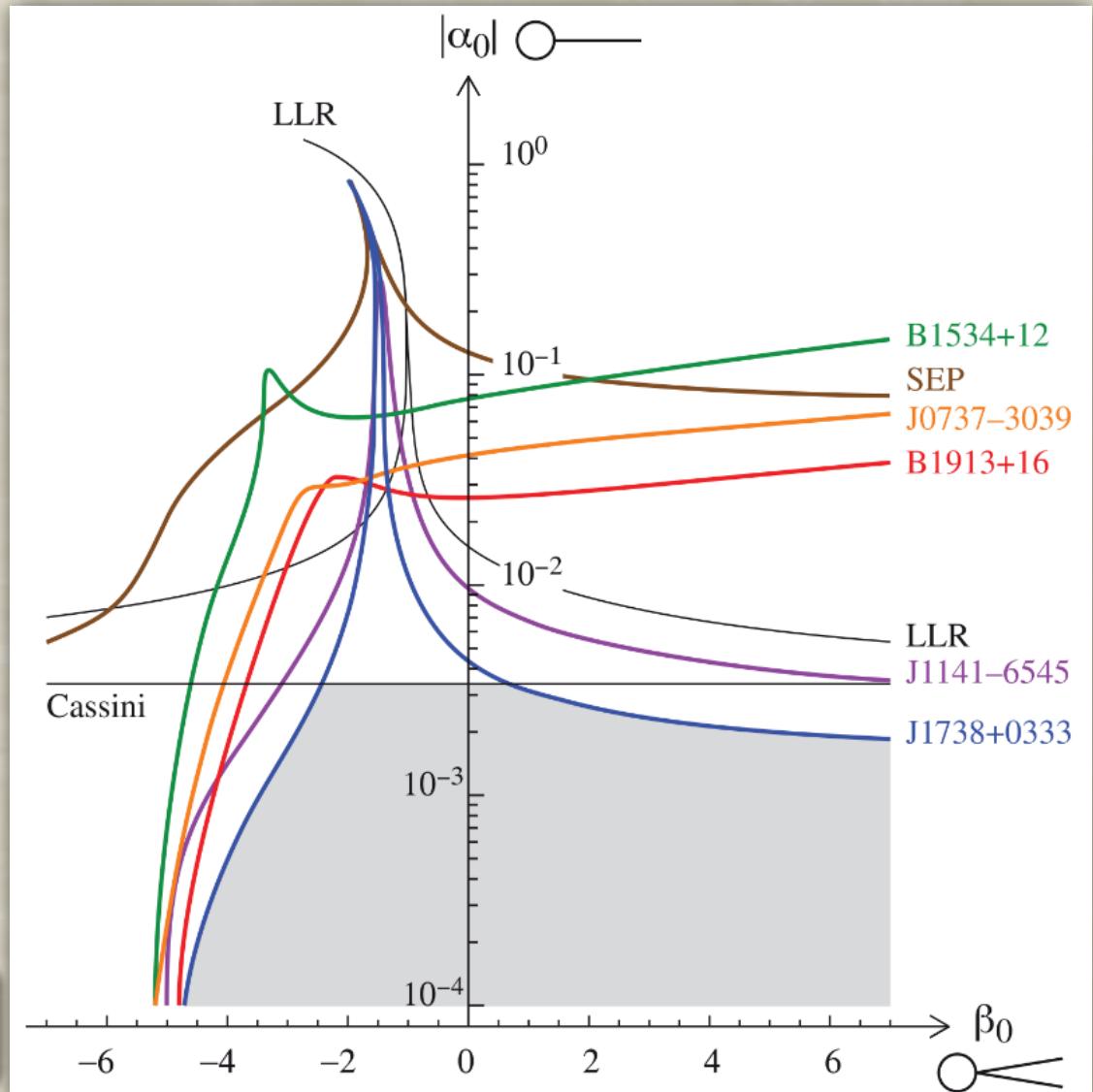
# Constraining alternative theory of gravity

- Alternative theories of gravity predict a variety of deviations from General Relativity.
- Mono-scalar-tensor theory introduces scalar field ( $\phi$ ) which couples with matter (strength denoted by  $\alpha_0, \beta_0$ ):

$$\square_{g_*} \phi = -\frac{4\pi G_*}{c^4} (\alpha_0 + \beta_0 \phi) T_*$$

- Dipole gravitational radiation when asymmetry in mass components (e.g., NS-WD):

$$\dot{P}_b^{\text{dipolar}} \simeq -\frac{4\pi^2 G}{c^3 P_b} \frac{M_{\text{PSR}} M_{\text{WD}}}{M_{\text{PSR}} + M_{\text{WD}}} (\alpha_{\text{PSR}} - \alpha_{\text{WD}})^2$$



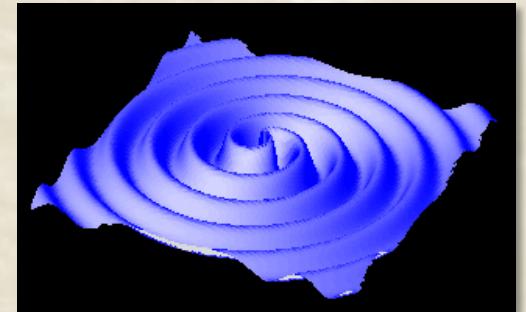
[ Freire et al. 2012 ]

# The gravitational wave astronomy

- Gravitational waves are:

- a). Ripples in the curvature of space-time propagating as a wave;
- b). Predicted by General Relativity and alternative theories;

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

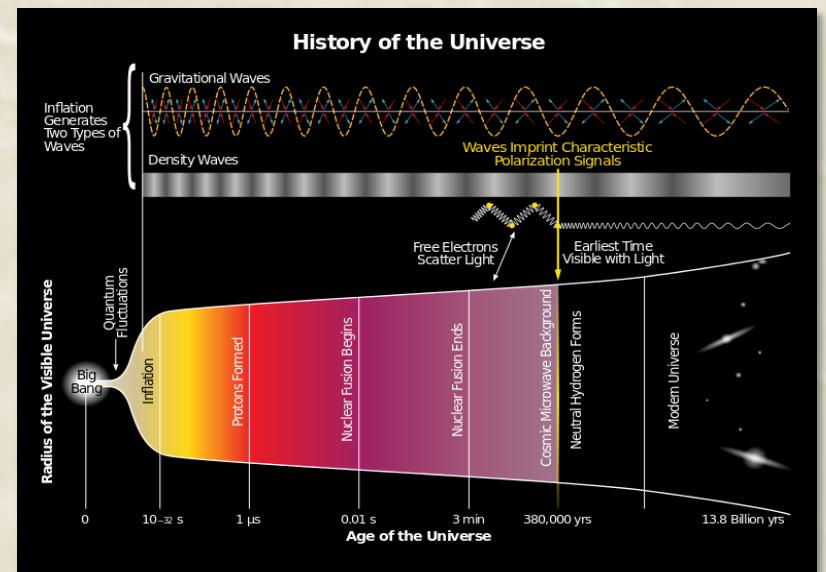


- c). Indirect evidence found from the **Hulse-Taylor pulsar**'s orbital decay;
- d). Direct detection can be from both stochastic background (GWB) and single sources (e.g., supermassive black hole pairs);

e). **Window now opened by the LIGO detection!**

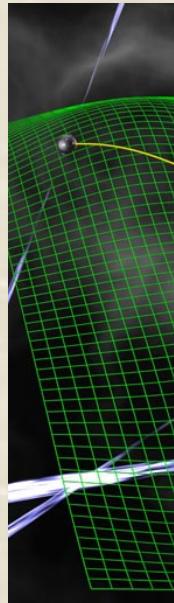
- The GWB:

- a). Generated from the **early universe** !
- b). Origin of major component: still not clear !  
(large number of supermass black hole binaries? Cosmic strings? ...?)

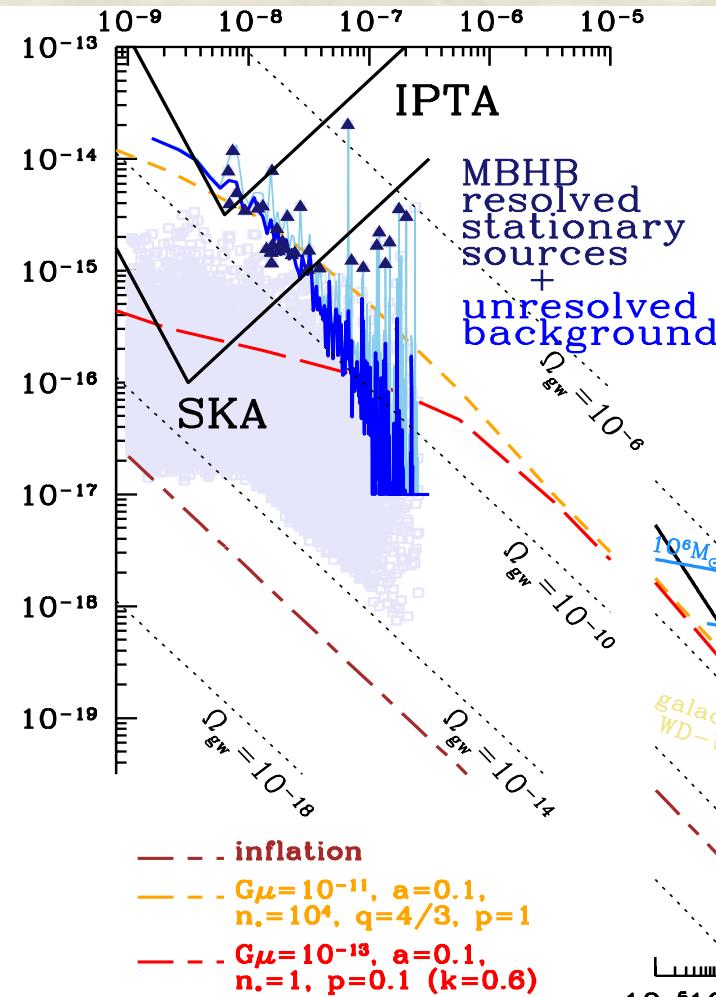


# Detecting gravitational wave with a pulsar timing array

- A pulsar timing array
- a). Conservation of angular momentum
- b). Look for periodic signals
- c). A correlation analysis
- d). The correlation function

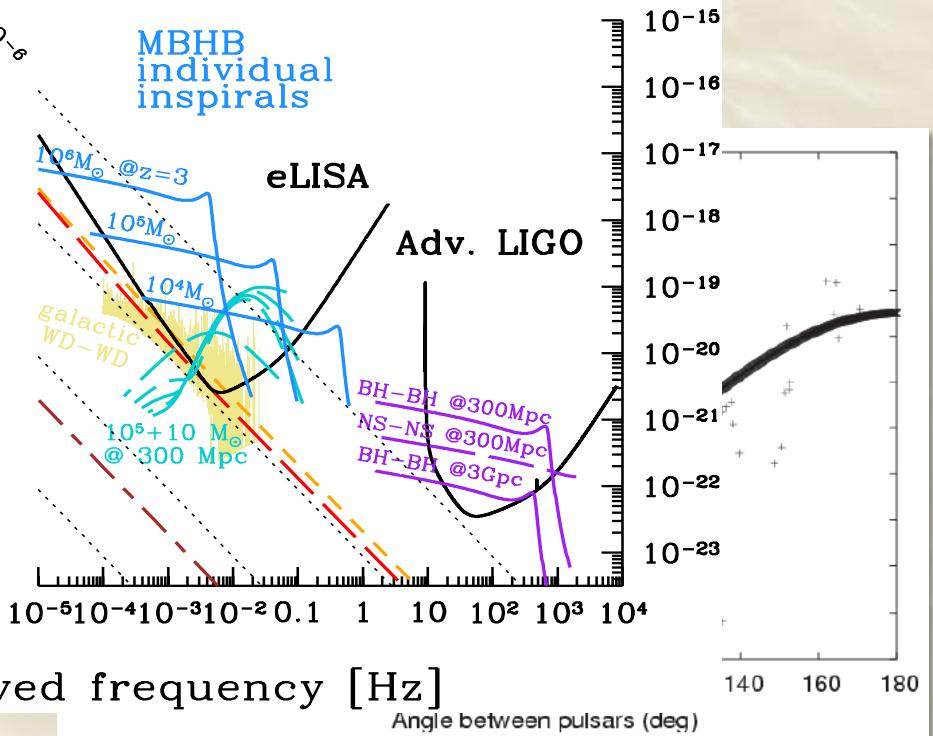


characteristic amplitude



[ Janssen et al. 2015 ]

## The gravitational wave landscape



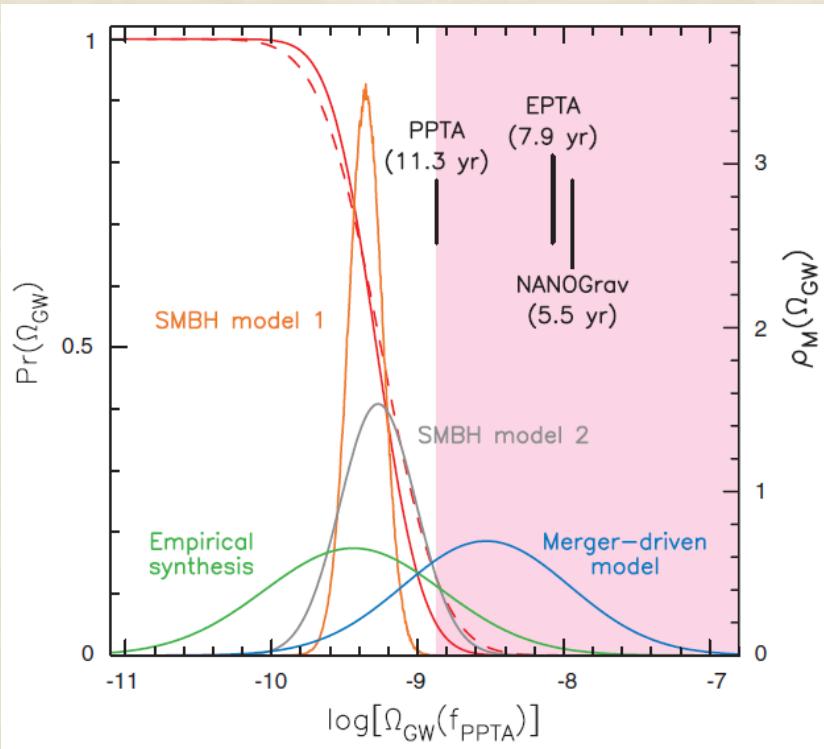
[ Helling & Downs 1983 ]

Credit: D. Champion

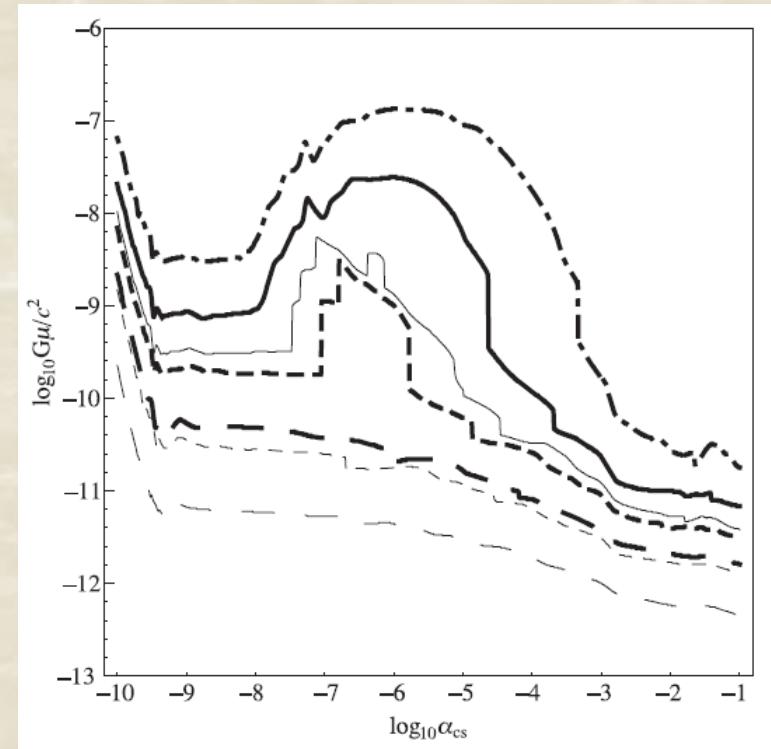
# Gravitational wave experiments with PTA

- Detect gravitational wave / place an upper limit on GWB can:

a). Constrain cosmological models of supermassive black hole population;



[ Shannon et al., 2013 ]

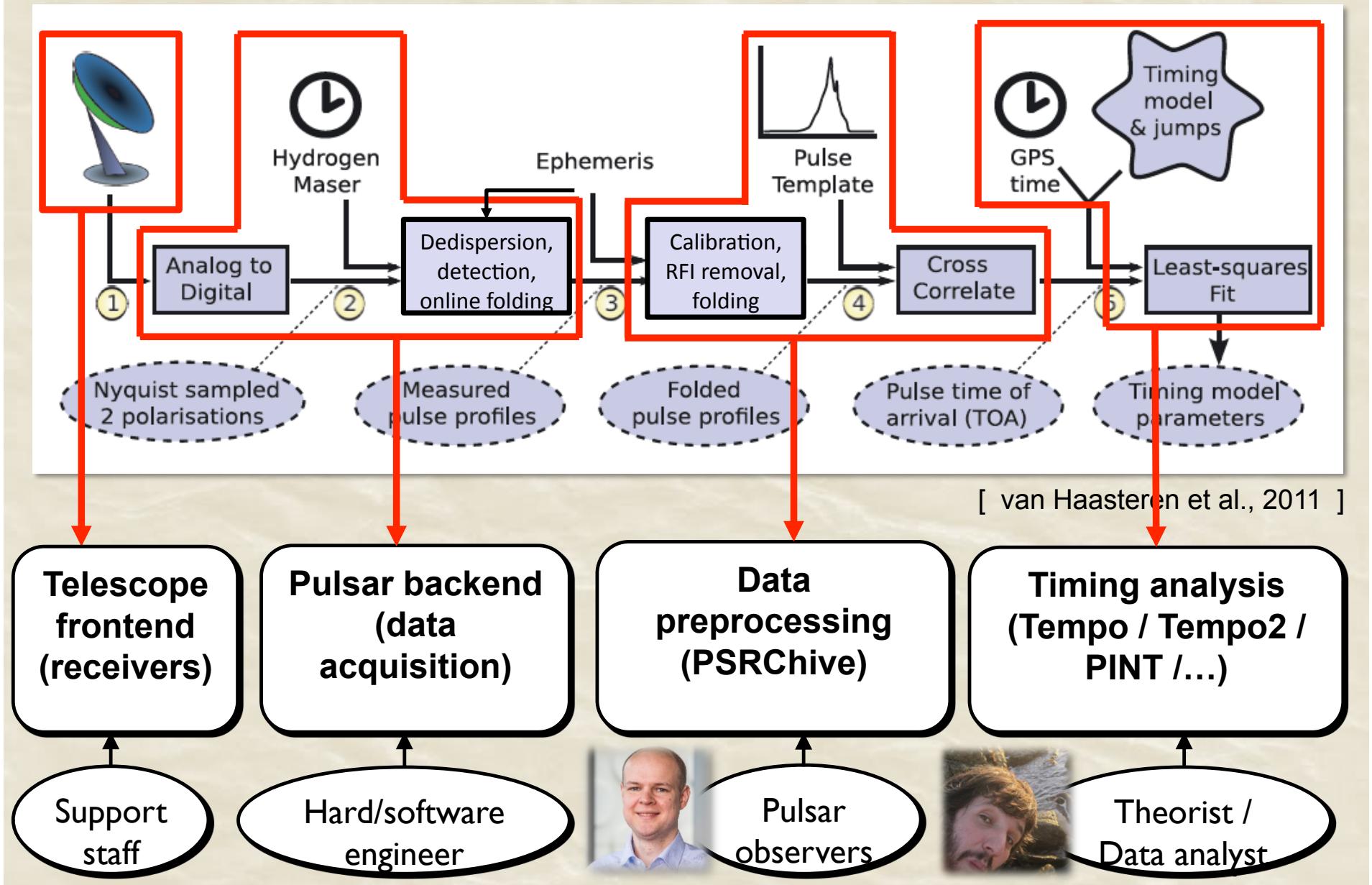


[ Lentati et al., 2015 ]

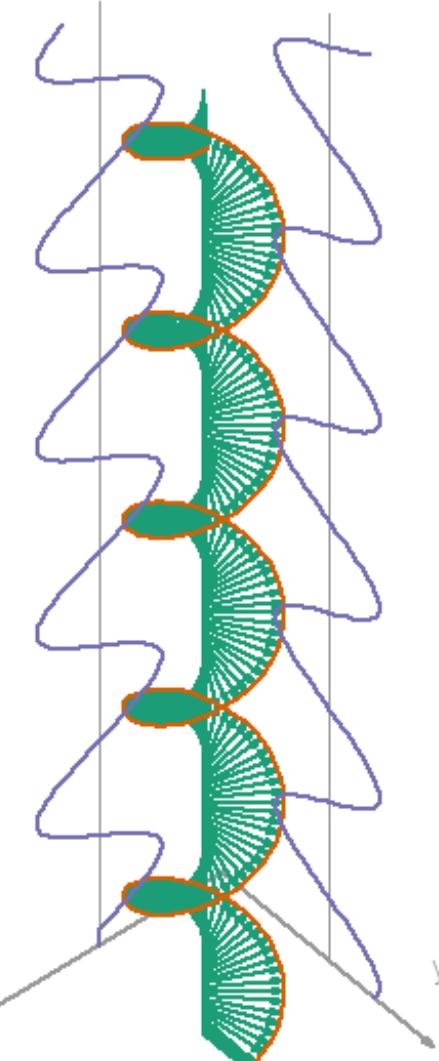
- b). Constrain string tension of cosmic strings and models of the early universe;
- c). Measure properties (e.g., polarisation, speed) of gravitational wave and test General Relativity;

Processing pulsar data  
to obtain pulse time-of-  
arrival

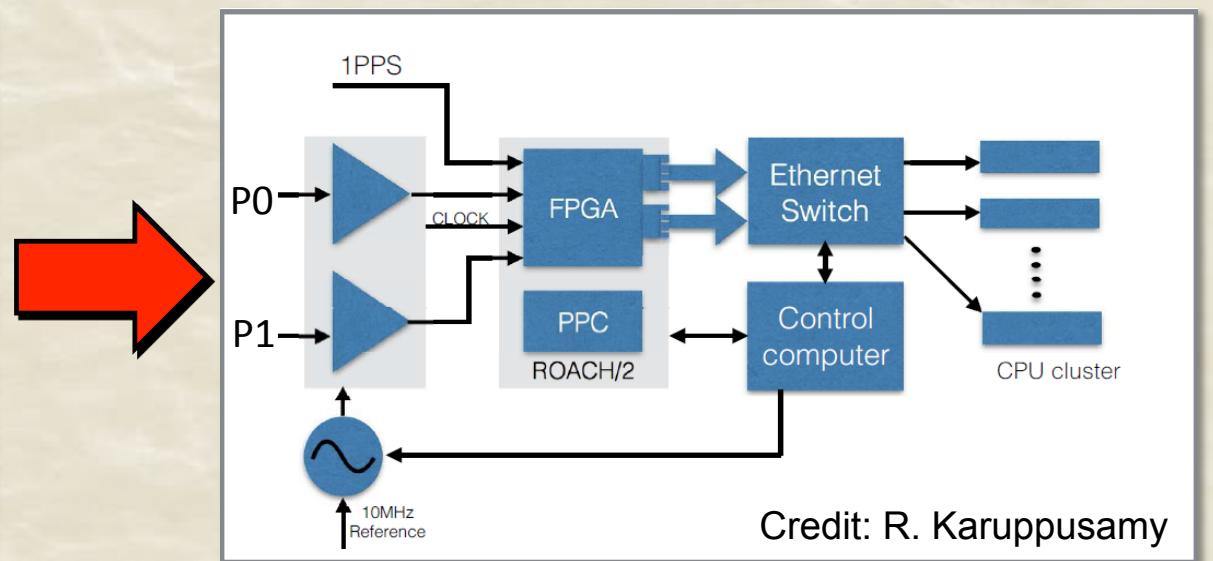
# The pulsar timing signal chain



# Signal with polarisation



Credit: wikipedia



Credit: R. Karuppusamy

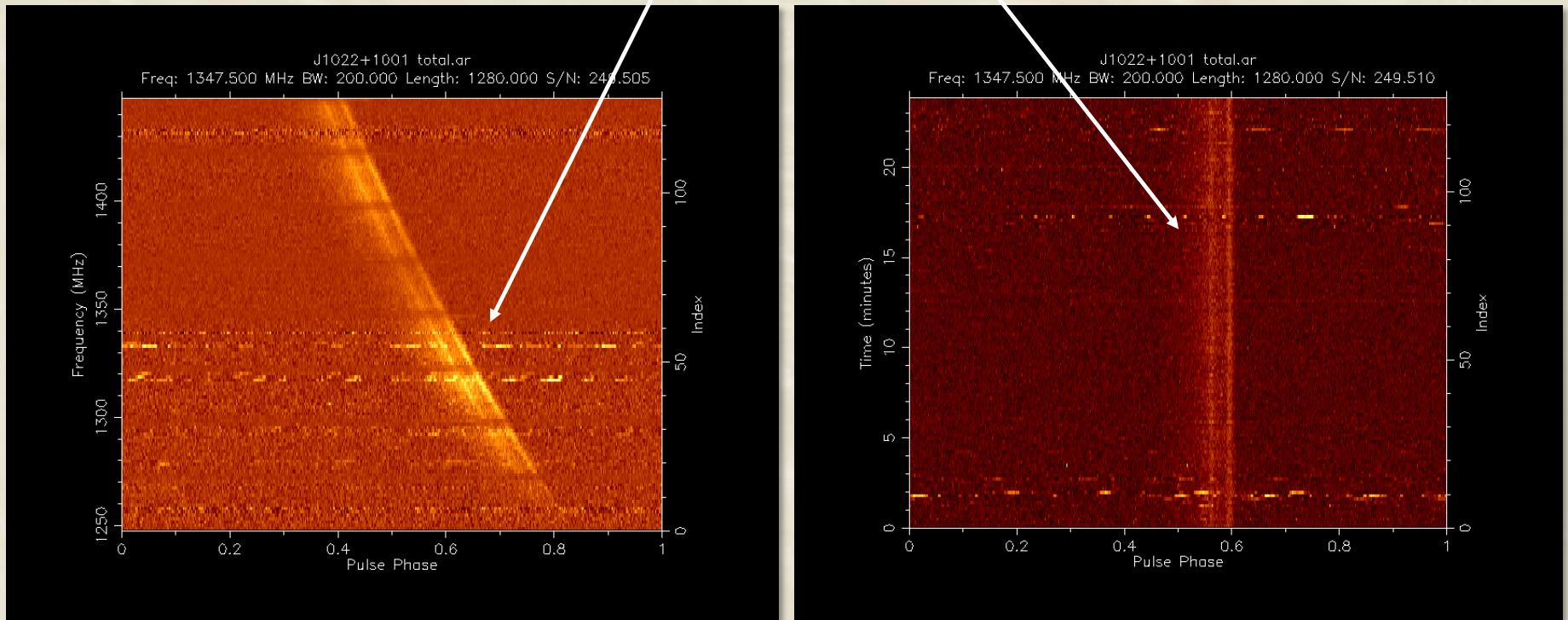
- Pulsar EM signal usually has significant **polarisations** (specific orientation of oscillation during propagation);
- The signal can be fully represented by sampling in dual polarisation on either **linear** or **circular** basis, and is detected in the form of **stokes parameters (I, Q, U, V)**;
- Calibration needed to obtain the original polarized signal (correct for feed rotation & receiver imperfection):

$$\rho' = \mathbf{J} \rho \mathbf{J}^\dagger$$

$$\mathbf{M} = \mathbf{A}(\mathbf{J} \otimes \mathbf{J}^*)\mathbf{A}^{-1}$$

## Radio interference

- Radio interference (RFI) is common in observations of Radio Astronomy!
- Terrestrial artificial radio signal, many possible origins: satellite, plane, radio broadcast, wifi, cell phone, lightening, microwave, ...
- "Most common" feature: strong, narrow band, time-variant, with zero DM, appear in multiple beams (if any) simultaneously (near-field), ...



**Data needs to be cleaned to minimize systematics !!!**

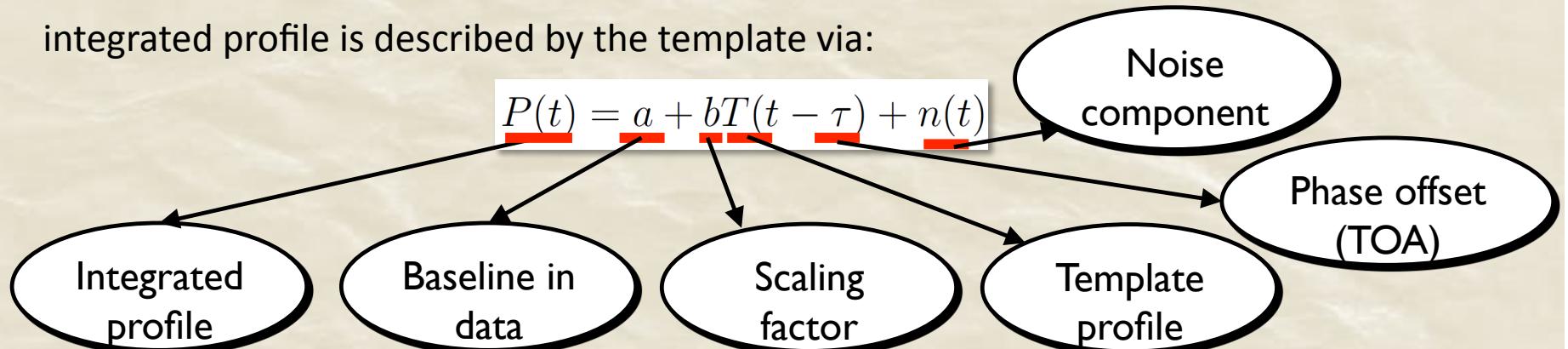
## Measuring TOA with template-matching

- Expected uncertainty of TOA given the signal-to-noise (S/N) of detection of the integrated profile (averaged N pulses), when only radiometer noise (white noise) is on top of the profile:

$$\sigma_{\text{rn}} = \frac{1}{\beta \times S/N_1} \sqrt{\frac{\Delta}{N}}$$
$$\beta = \sqrt{\int [U'(t)]^2 dt}$$

[ Downs & Reichley, 1983; Liu et al., 2011 ]

- In practice, TOAs are measured by cross-correlating the integrated profile with a **template profile** (normally formed from independent observations), assuming the integrated profile is described by the template via:



- The template profile needs to be of **high S/N**, or **noise free** (analytic), or **after noise-removal** technique.

## Frequency-domain fitting algorithm

- The template-matching is normally carried out in the frequency-domain, after Discrete-Fourier-Transform of the data:

$$P_k \exp(i\theta_k) = \sum_{j=0}^{N-1} p_j e^{i2\pi j k / N}$$
$$S_k \exp(i\phi_k) = \sum_{j=0}^{N-1} s_j e^{i2\pi j k / N}$$

- The model becomes:

$$P_k \exp(i\theta_k) = aN + bS_k \exp[i(\phi_k + k\tau)] + G_k, \quad k = 0, \dots, (N-1),$$

- The parameters ( $b, \tau$ ) are then obtained by minimising the goodness-of-fit:

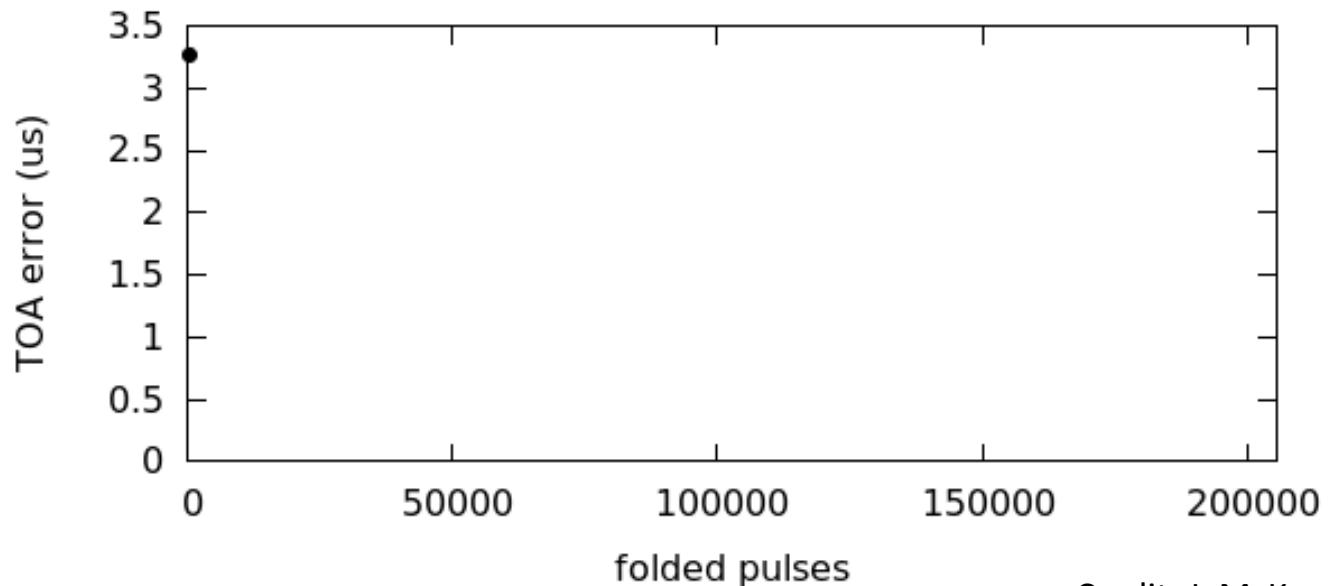
$$\chi^2(b, \tau) = \sum_{k=1}^{N/2} \left| \frac{P_k - bS_k \exp[i(\phi_k - \theta_k + k\tau)]}{\sigma_k} \right|^2$$

- The errors are obtained from standard error propagation (covariance matrix):

$$\sigma_\tau^2 = \left( \frac{\partial^2 \chi^2}{\partial \tau^2} \right)^{-1} = \frac{\sigma^2}{2b \sum k^2 P_k S_k \cos(\phi_k - \theta_k + k\tau)},$$
$$\sigma_b^2 = \left( \frac{\partial^2 \chi^2}{\partial b^2} \right)^{-1} = \frac{\sigma^2}{2 \sum S_k^2}.$$

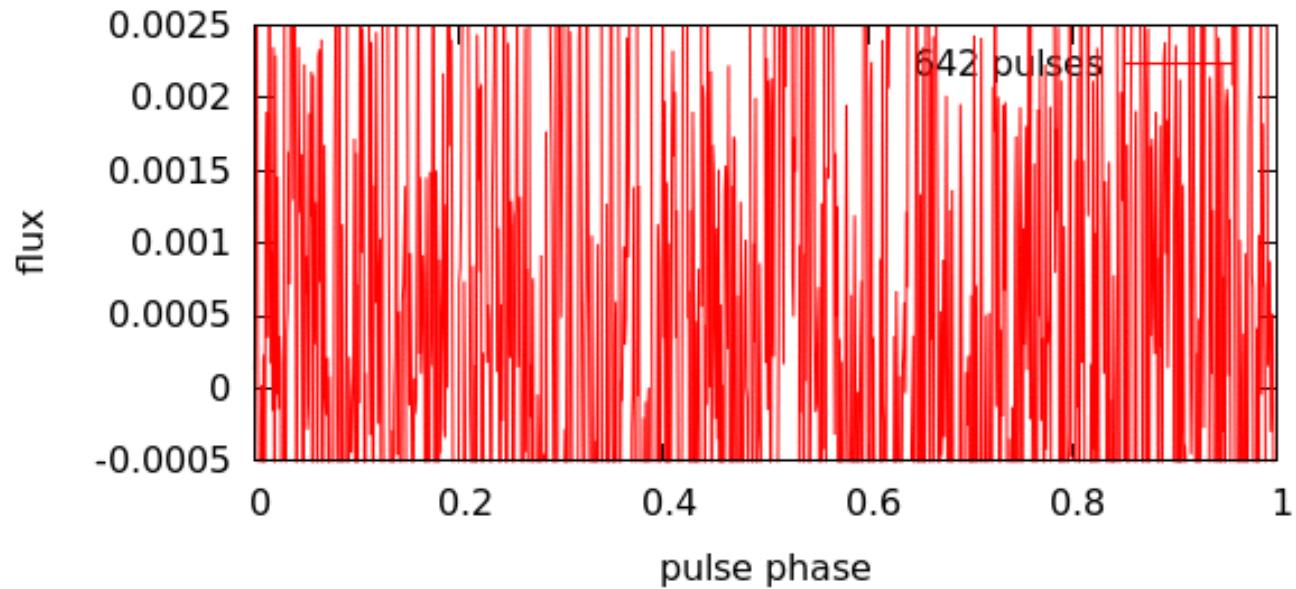
## Frequency-domain fitting algorithm

$$\sigma_{TOA} \propto \frac{1}{\sqrt{N}}$$



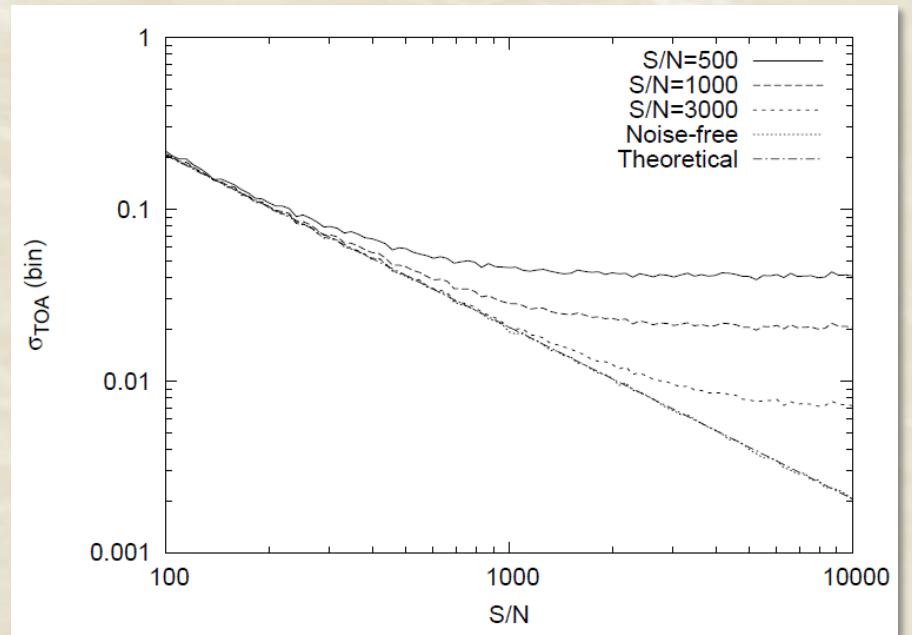
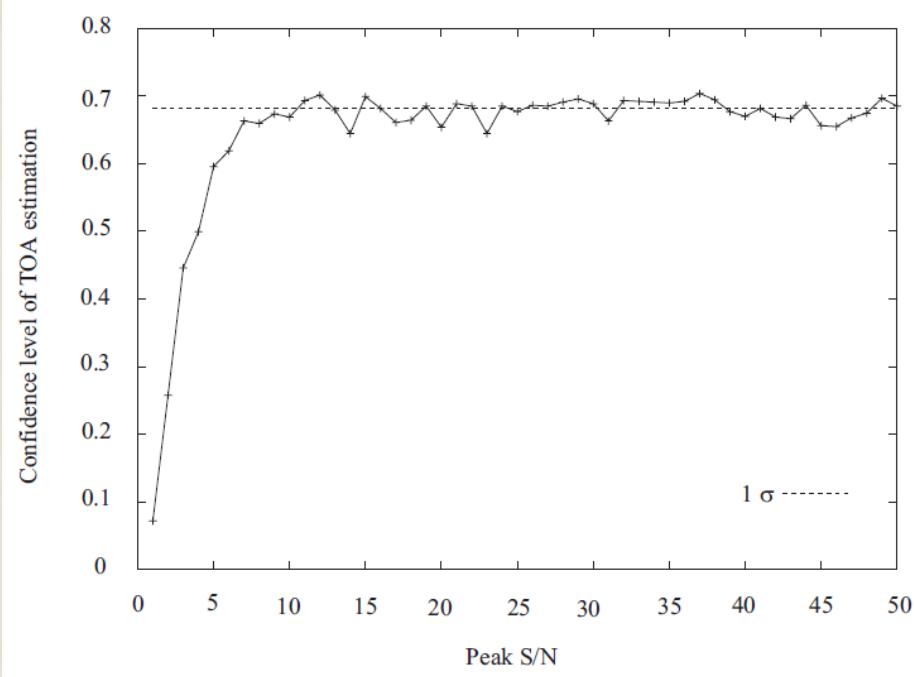
Credit: J. McKee

$$S/N \propto \sqrt{N}$$



## Frequency-domain fitting algorithm

- If the template profile used is not perfect, e.g., of a different shape from the integrated profile or of significant noise, the accuracy of the TOA will be affected, i.e., less than expected from theory.



- In low S/N region, the error scales non-linearly with S/N, and the standard error propagation in template-matching underestimate the uncertainty;
- There are other approaches (e.g., the FDM method) that can be used to obtain a more reliable error estimate.