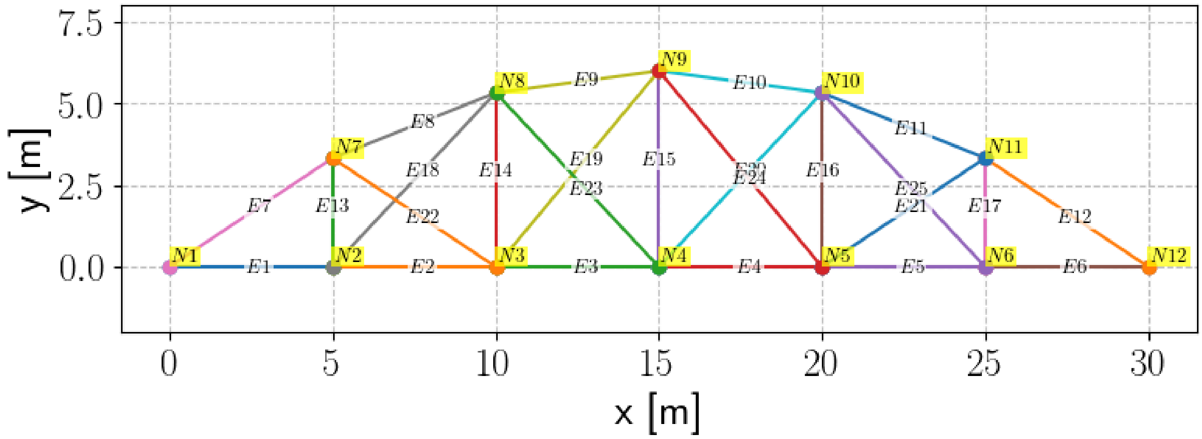
Dynamic simulation of a Bridge

In this example, we consider the simulation of the same bridge to some initial conditions.

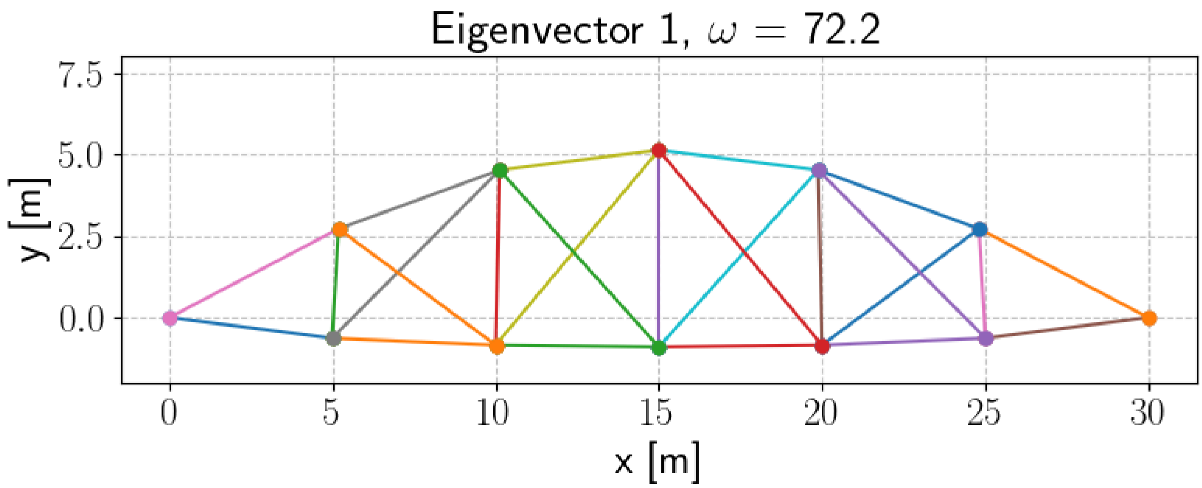
In [5]:
```
# As usual we start by generating the coordinates and mesh (same as before)
import numpy as np
from src.postprocessing.plot_mesh import plot_truss_structure_2d
import matplotlib.pyplot as plt


# Define parameters
E = 200e3 # [Pa]
rho = 1.0 # [kg/m^3]
A = 1     # [m^2]

EA = E * A
rhoA = rho * A

l = 5.0
h1 = 3.33
h2 = 5.33
h3 = 6.0

# Node coordinates
bottom_indices = np.arange(1, 6)
top_indices = np.arange(6, 11)

# Create coordinates matrix
n_nodes = 12
coordinates = np.zeros((n_nodes, 2))

# Assign coordinates
# Left (first node) and right (last) points
left_id = 0
right_id = 11
coordinates[left_id] = [0.0, 0.0]
coordinates[right_id] = [6*l, 0.0]

# Bottom points
for i in bottom_indices:
    coordinates[i] = [i*l, 0.0]

# Top points
for i, coord_id in enumerate(top_indices):
    h_i = [h1, h2, h3, h2, h1][i]
    coordinates[coord_id] = [(i+1)*l, h_i]


# Create connectivity table
num_bottom_lines = 6        # left + 5 bottom points + right = 7 points, 6 l
num_top_lines = 6           # left + 5 top points + right = 7 points, 6 line
```

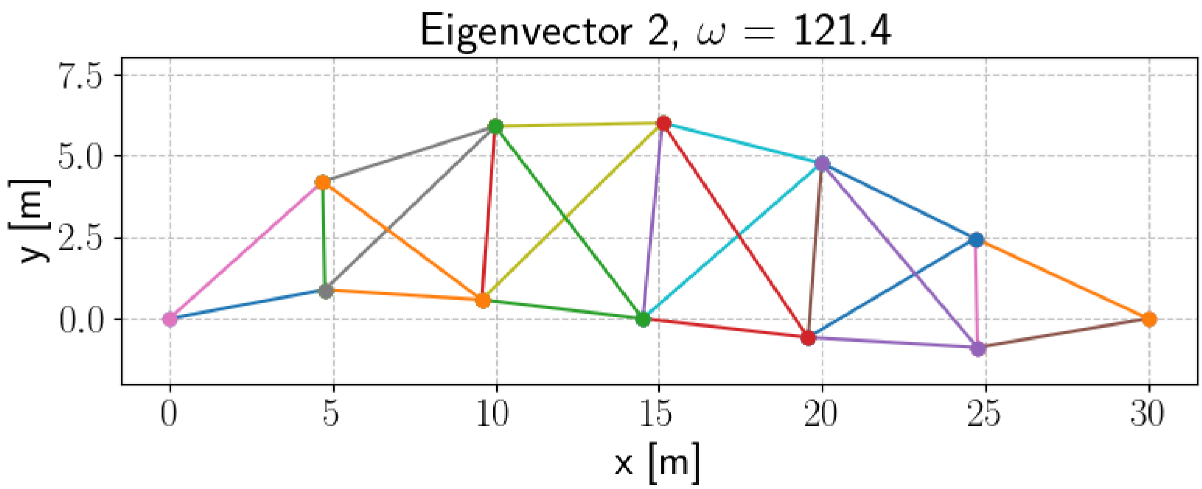
```python
num_vertical_lines = 5
num_diagonal_lines = 4 * 2  # 4 left diagonals + 4 right diagonals

n_elements = num_bottom_lines + num_top_lines + num_vertical_lines + num_dia
connectivity_table = np.zeros((n_elements, 2), dtype=int)

element_id = 0
# Bottom lines
for i in range(num_bottom_lines):
    if i == 0:
        # First bottom line connects left to first bottom point
        connectivity_table[element_id] = [left_id, bottom_indices[0]]
    elif i == num_bottom_lines - 1:
        # Last bottom line connects last bottom point to right
        connectivity_table[element_id] = [bottom_indices[-1], right_id]
    else:
        # Middle bottom lines connect adjacent bottom points
        connectivity_table[element_id] = [bottom_indices[i-1], bottom_indice
    element_id +=1

# Top lines
for i in range(num_top_lines):
    if i == 0:
        # First top line connects left to first top point
        connectivity_table[element_id] = [0, top_indices[0]]
    elif i == num_top_lines - 1:
        # Last top line connects last top point to right
        connectivity_table[element_id] = [top_indices[-1], right_id]
    else:
        # Middle top lines connect adjacent top points
        connectivity_table[element_id] = [top_indices[i-1], top_indices[i]]
    element_id +=1

# Vertical lines
for i in range(num_vertical_lines):
    connectivity_table[element_id] = [bottom_indices[i], top_indices[i]]
    element_id +=1

# Right diagonal lines (bottom to top)
for i in range(num_vertical_lines - 1):
    connectivity_table[element_id] = [bottom_indices[i], top_indices[i+1]]
    element_id +=1

# Left diagonal lines (top to bottom)
for i in range(num_vertical_lines - 1):
    connectivity_table[element_id] = [top_indices[i], bottom_indices[i+1]]
    element_id += 1

# plt.plot(coordinates[:, 0], coordinates[:, 1], 'o')
ax = plot_truss_structure_2d(coordinates, connectivity_table,
                        xlabel='x [m]',
                        ylabel='y [m]',
                        ylim=[-2, 8])
```

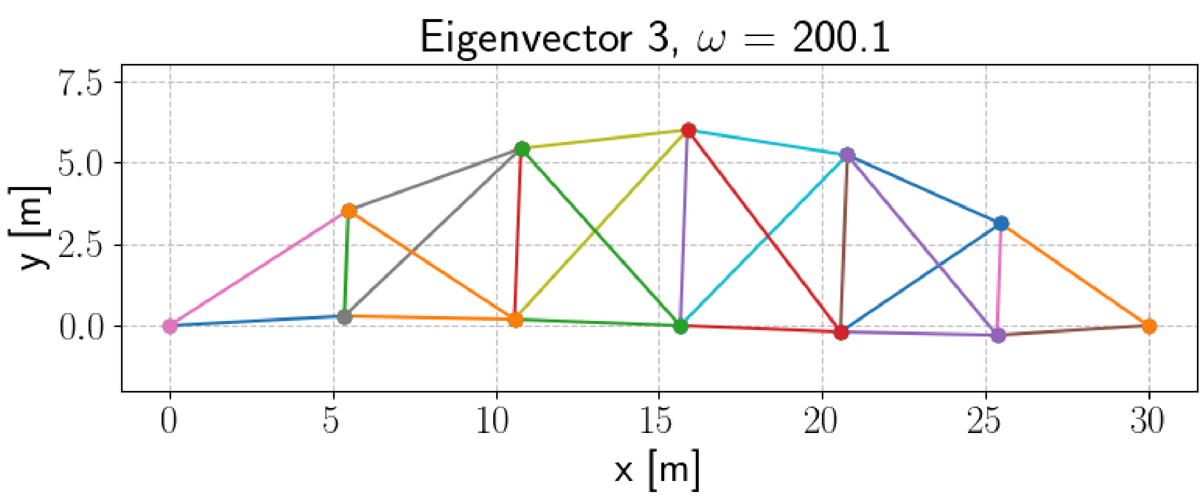


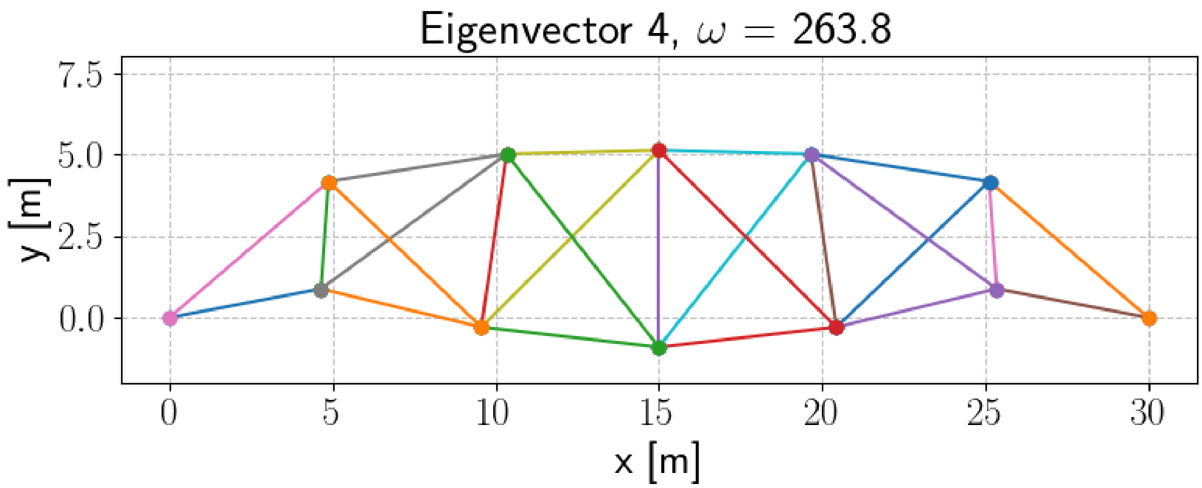
We now solve the modal analysis for the bridge, to find the frequency of vibration and the mode shapes.

In [6]:
```
from src.fem.assemble_matrices import assemble_stiffness_truss_2d, assemble_
from src.fem.boundary_conditions import apply_bcs, restore_data
from src.linear_algebra.solve_eigenproblem import solve_sparse_generalized_e


n_dofs = n_nodes*2


K = assemble_stiffness_truss_2d(coordinates, connectivity_table, EA)
M = assemble_mass_truss_2d(coordinates, connectivity_table, rhoA)

dofs_bcs = [2*left_id, 2*left_id+1, 2*right_id, 2*right_id+1]

K_red = apply_bcs(K, dofs_bcs)
M_red = apply_bcs(M, dofs_bcs)

omega_squared, modes_red = solve_sparse_generalized_eigenproblem(K_red, M_re
omega_vec = np.sqrt(np.real(omega_squared))

eigenvectors = restore_data(modes_red, dofs_bcs)

n_modes = 4
for ii in range(n_modes):
    coordinates_eigenmodes = coordinates + eigenvectors[:, ii].reshape(-1, 2
    ax = plot_truss_structure_2d(coordinates_eigenmodes, connectivity_table,
                    xlabel='x [m]',
                    ylabel='y [m]',
                    ylim=[-2, 8],
                    show_element_labels=False,
                    show_node_labels=False,
                    title=f'Eigenvector {ii+1}, $\omega$ = {omega_vec[ii
```


Eigenvector 1, $\omega = 72.2$
y [m]
x [m]
7.5
5.0
2.5
0.0
0
5
10
15
20
25
30


Eigenvector 2, $\omega = 121.4$
y [m]
x [m]
7.5
5.0
2.5
0.0
0
5
10
15
20
25
30


Eigenvector 3, $\omega = 200.1$
y [m]
x [m]
7.5
5.0
2.5
0.0
0
5
10
15
20
25
30



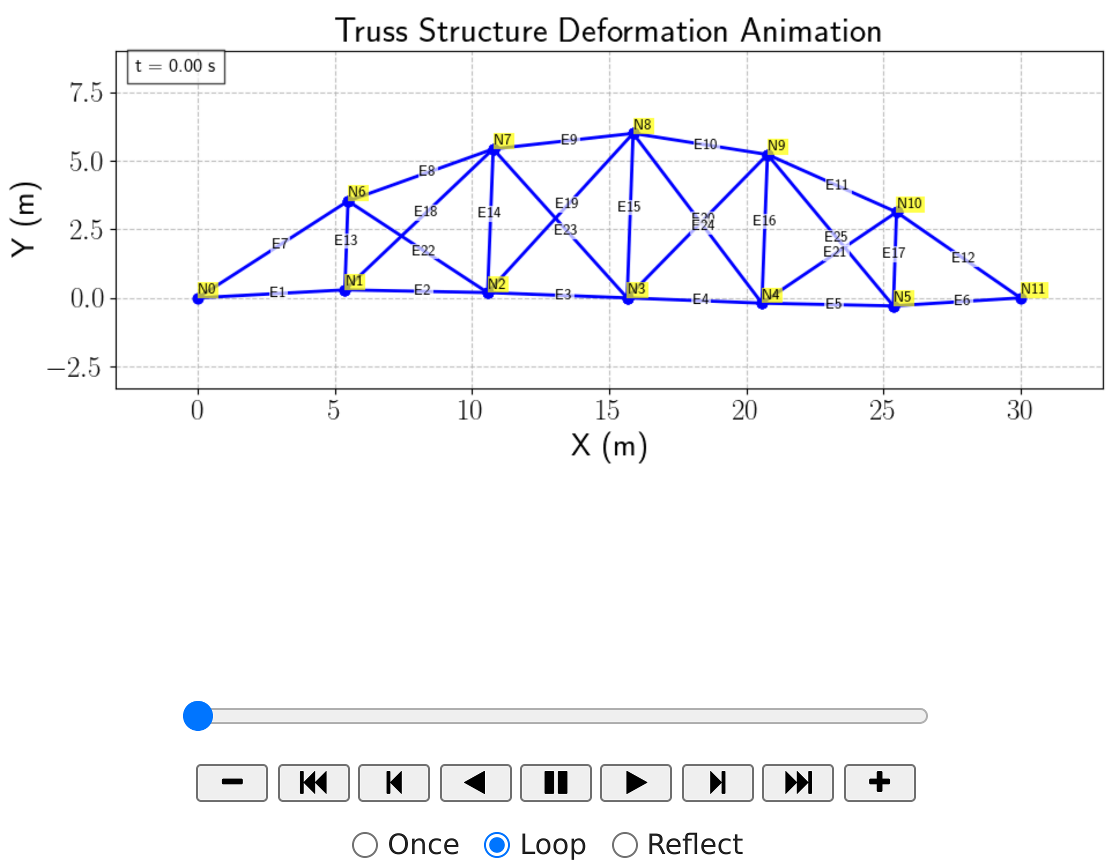
Now an initial condition for the velocity field is considered. In particular, we consider an initial velocity given by the first eigenmode.

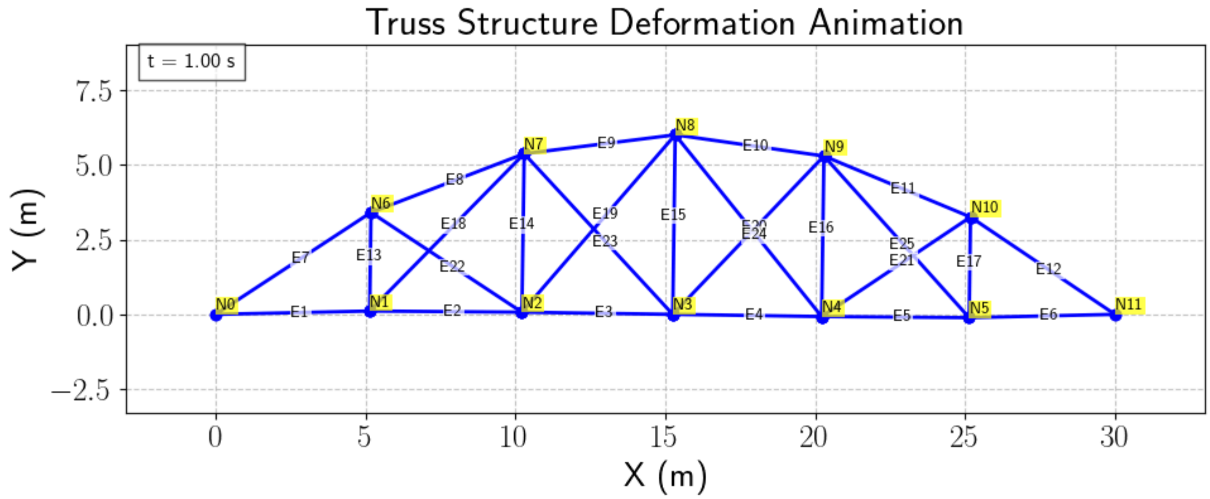
In [4]:
```
from src.linear_algebra.time_integration import newmark
from src.postprocessing.animate_mesh import animate_truss_structure
from IPython.display import HTML
import os

# Initial conditions corresponding to first mode
num_mode = 2

q0 = np.zeros(n_dofs)
v0 = np.zeros(n_dofs)

q0[::2] = eigenvectors[0::2, num_mode]
q0[1::2] = eigenvectors[1::2, num_mode]

q0_red = np.delete(q0, dofs_bcs)
v0_red = np.delete(v0, dofs_bcs)

# Solve dynamic response
T_end = 1  # Total simulation time
dt = 2*np.pi/omega_vec[num_mode]/10  # Time step
n_times = int(np.ceil(T_end/dt))
q_array_red, v_array_red = newmark(q0_red, v0_red, M_red, K_red, dt, n_times

q_array = restore_data(q_array_red, dofs_bcs)
# Post-processing
coordinates_deformed = np.zeros((n_nodes, 2, n_times))
for t in range(n_times):
    coordinates_deformed[:, :, t] = coordinates + q_array[:, t].reshape(-1,

folder_res = './PC1/truss_example_3/results/'
if not os.path.exists(folder_res):
    os.makedirs(folder_res)

# np.save(folder_res + 'coordinates_deformed.npy', coordinates_deformed)
animation = animate_truss_structure(
        coordinates_deformed,
```

```
        connectivity_table,
        title="Truss Structure Deformation Animation",
        xlabel="X (m)",
        ylabel="Y (m)",
        interval=50,  # milliseconds between frames
        show_element_labels=True,
        show_node_labels=True,
        show_time=True,
        time_values=np.linspace(0, T_end, n_times+1),
        time_format="t = {:.2f} s",
        save_path=folder_res + "truss_animation.mp4"  # Uncomment to save ar
    )

HTML(animation.to_jshtml())
```

```
Animation size has reached 20993945 bytes, exceeding the limit of 20971520.
0. If you're sure you want a larger animation embedded, set the animation.em
bed_limit rc parameter to a larger value (in MB). This and further frames wi
ll be dropped.
```

Out[4]:





Truss Structure Deformation Animation
t = 1.00 s
N0
N1
N2
N3
N4
N5
N6
N7
N8
N9
N10
N11
E1
E2
E3
E4
E5
E6
E7
E8
E9
E10
E11
E12
E13
E14
E15
E16
E17
E18
E19
E20
E21
E22
E23
E24
E25
7.5
5.0
2.5
0.0
−2.5
0
5
10
15
20
25
30
X (m)
Y (m)

In [ ]: