Intro to Quantum Computing

Unclear topics from seminar #1

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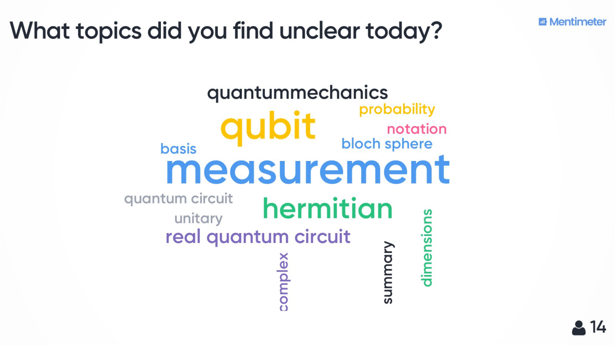
Word cloud

Here is the word cloud generated from the end of last class - thanks for your input!!



I will try and address these as best I can, but some are very broad, and so I will provide some good external resources instead (also, it's always a good idea to use different sources because you might understand them better).

Some notation

Some of the notation I used would be unfamiliar to you if you haven't taken a quantum mechanics class before.

To start, for a complex number

$$\alpha = a + bi, \quad a, b \in \mathbb{R}, \tag{1}$$

we represent its complex conjugate as

$$\alpha^* = a - bi \tag{2}$$

The 'mod-square' of a complex number is itself times its conjugate:

$$|\alpha|^2 = \alpha\alpha^* \tag{3}$$

For complex numbers in polar form $\alpha = re^{i\phi}$, then $|\alpha|^2 = r^2$ is the squared length of α when drawn in the complex plane.

Dirac (bra-ket) notation

Quantum states are often represented using *Dirac notation*.

$|\psi\rangle$ is called a 'ket' and is a column vector, for example:

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{4}$$

where α and β are complex numbers.

$\langle\psi|$ is called a 'bra', and is a row vector that is the conjugate transpose of a ket:

$$\langle\psi| = (|\psi\rangle)^\dagger = \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \tag{5}$$

Dirac (bra-ket) notation

You can take the outer product of a ket and a bra to get a matrix:

$$|\psi\rangle \langle\psi| = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} = \begin{pmatrix} \alpha\alpha^* & \alpha\beta^* \\ \beta\alpha^* & \beta\beta^* \end{pmatrix} \tag{6}$$

You can also take the inner product of a bra and ket to get a number:

$$\langle\psi| \, |\psi\rangle = \langle\psi|\psi\rangle = \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha^*\alpha + \beta^*\beta \tag{7}$$

When I showed measurements in class, that's what $|\langle\phi|\psi\rangle|^2$ meant - it is the mod squared of the inner product of the two vectors $|\phi\rangle$ and $|\psi\rangle$.

Why complex numbers?

Scott Aaronson has (what I think) is a really nice explanation on why quantum mechanical amplitudes have to be complex:

`https://www.scottaaronson.com/democritus/lec9.html`

There is also some discussion on probability here, and the rest of the lectures are in general a great resource for learning quantum computing.

In theory, a qubit is a quantum mechanical system with two distinct states. We usually call these states $|0\rangle$ and $|1\rangle$ and represent them as vectors:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{8}$$

When we manipulate a quantum state, we may end up with a linear combination, or *superposition* of these two states:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{9}$$

where we must have $|\alpha|^2 + |\beta|^2 = 1$.

Bases and dimensions

The space where all the possible qubit states live is called the Hilbert space. It is a 2-dimensional space, meaning it has 2 basis vectors, and has an inner product we can use to measure the length of vectors, and the angles between them.

Basis vectors are linearly independent vectors that, as a set, span the whole space - you can represent any state just by using a combination of the basis vectors. We use bases that are orthonormal, i.e. orthogonal to each other and normalized.

For example, $|0\rangle$ and $|1\rangle$ are an orthonomal basis:

$$\langle 0|0\rangle = 1, \qquad \langle 0|1\rangle = 0 \tag{10}$$

$$\langle 1|0\rangle = 0, \qquad \langle 1|1\rangle = 1 \tag{11}$$

and we can write any qubit state ψ as some combination of $|0\rangle$ and $|1\rangle$.

The choice of basis is not unique - for qubits there are infinitely many sets of 2 orthonormal vectors. We most commonly use $|0\rangle$ and $|1\rangle$, but another set is[1]

$$
\begin{aligned}
|+\rangle &= \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) && (12)\\
|-\rangle &= \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) && (13)\\
& && (14)
\end{aligned}
$$

You can check that

$$
\begin{aligned}
\langle + | + \rangle = 1, \qquad & \langle + | - \rangle = 0 && (15)\\
\langle - | + \rangle = 0, \qquad & \langle - | - \rangle = 1 && (16)
\end{aligned}
$$

[1] This notation is a bit confusing, but the labels $+$ and - refer to the sign between the $|0\rangle$ and $|1\rangle$.

You can express qubit states in other bases by performing a *change of basis*, i.e. re-expressing the basis vectors in terms of other basis vectors. For example, from the definitions of $|+\rangle$ and $|-\rangle$, you can work out that

$$|0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right), \tag{17}$$

$$|1\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right), \tag{18}$$

$$\tag{19}$$

Then we can rewrite a qubit state in the new basis:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{20}$$

$$= \frac{\alpha}{\sqrt{2}}\left(|+\rangle + |-\rangle\right) + \frac{\beta}{\sqrt{2}}\left(|+\rangle - |-\rangle\right) \tag{21}$$

$$= \left(\frac{\alpha+\beta}{\sqrt{2}}\right)|+\rangle + \left(\frac{\alpha-\beta}{\sqrt{2}}\right)|-\rangle \tag{22}$$

Unitary matrices

A matrix U is *unitary* if

$$UU^\dagger = U^\dagger U = \mathbb{1} \tag{23}$$

Unitary matrices are thus invertible (and reversible).

They also have the nice property that they preserve length. For a state vector $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$,

$$U(|\psi\rangle) = \alpha'|0\rangle + \beta'|1\rangle \tag{24}$$

then

$$|\alpha'|^2 + |\beta'|^2 = |\alpha|^2 + |\beta|^2 = 1 \tag{25}$$

Fun fact: if you have a Hermitian H, then e^{-iHt} is a unitary matrix. This is the idea behind Hamiltonian simulation - you can run the unitaries associated to Hamiltonians on a quantum computer.

Hermitian matrices

A matrix H is *Hermitian* if

$$H = H^\dagger = (H^T)^* \tag{26}$$

Hamiltonians in quantum mechanics are (usually) Hermitian. This is because the allowed energies of quantum system are the eigenvalues of its Hamiltonian, and Hermitian matrices have the nice property that all their eigenvalues (and thus energies) are real numbers.

Measurement

A good video explanation of projective measurements:
https://www.youtube.com/watch?v=HYRTkB0rNII

Another video, part of a larger series that is a great resource:
https://www.youtube.com/watch?v=SMbh0GgCN7I

The Bloch sphere

It's hard to explain this one without pointing at pictures, but here are some resources that explain things pretty clearly:

- http://www.vcpc.univie.ac.at/~ian/hotlist/qc/talks/bloch-sphere.pdf
- https://physics.stackexchange.com/questions/204090/understanding-the-bloch-sphere
- https://medium.com/@quantum_wa/quantum-computation-a-journey-on-the-bloch-sphere-50cc9d73530

And a video explanation:

- https://www.youtube.com/watch?v=vUVkS1XZVCc

Quantum circuits

A video explanation that also discusses measurements, and 2-qubit gates (and also shows the alternative representation of the NOT gate!): `https://www.youtube.com/watch?v=wLv2ORHqlgw`

A methodical explanation from IBM that also discusses qubits, the Bloch sphere, etc.: `https://quantum-computing.ibm.com/support/guides/introduction-to-quantum-circuits?section=5cae613866c1694be21df8cc`

General resources for QC

Great video series which assumes no QM: https://www.youtube.com/playlist?list=PL1826E60FD05B44E4

The 'bible' of QC, by the same author of the above videos: https://en.wikipedia.org/wiki/Quantum_Computation_and_Quantum_Information

A list that I compiled for an earlier lecture series: https://github.com/glassnotes/Intro-QC-TRIUMF/blob/master/Resources.md

General resources on QM

I have fewer suggestions for this, since I took it in undergrad and have not taken (or taught) it since...

I used Griffiths in my undergrad: https://www.cambridge.org/core/books/introduction-to-quantum-mechanics/990799CA07A83FC5312402AF6860311E

I've heard good things about the online courses put up by MIT: https://ocw.mit.edu/courses/physics/8-04-quantum-physics-i-spring-2013/