STAT 4160 - Engineering Analysis

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Introduction

Background

Problem: Matrix marks and automated 2d-matrix readers are used in semiconductor factories. Unreadable matrix marks negatively affect factory run rates because it stops the manufacturing process.

Response Variable: Unused Error Correction (UEC), a measure of the unused portion of the redundant information embedded in the 2d-matrix

- UEC of 0 represents the lowest reading that results in a decodable matrix
- UEC of 1 represents the highest reading

Question: Which factor(s) and/or interaction terms seem to have the greatest effect on UEC and warrant further study?

Engineering Experiment

Goals:

- To determine which of the four design factors and their subsequent interactions have the greatest effect on UEC to prevent unreadable marks which negatively affect factory run rates
  - Laser Power (9 and 13 W)
  - Laser Pulse Frequency (4000 and 12000 Hz)
  - Matrix Cell Size (0.07 and 0.12 in)
  - Writing Speed (10 and 20 in./sec)

Experimental Design: A 2^4 factorial experiment that addresses 16 main effects/combinations of interactions

- This type of factorial experiment should address the interactions and/or main effects that have the most impact on UEC for further study

$$\mathbf{Y_{ijklm}} = \mu + \alpha_i + \beta_j + \gamma_k + \delta_l + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\alpha\delta)_{il} + (\beta\gamma)_{jk} + (\beta\delta)_{jl} + (\gamma\delta)_{kl} + (\alpha\beta\gamma)_{ijk} + (\alpha\beta\delta)_{ijl} + (\alpha\gamma\delta)_{ikl} + (\beta\gamma\delta)_{jkl} + (\alpha\beta\gamma\delta)_{ijkl} + \epsilon_{ijklm}$$

$$\text{where } \epsilon_{ijklm} \sim N(0, \sigma^2)$$

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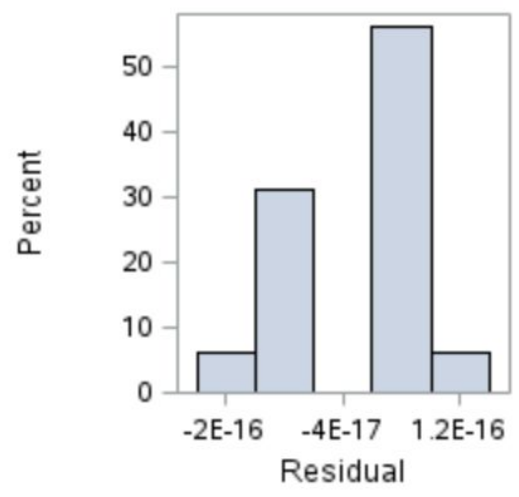
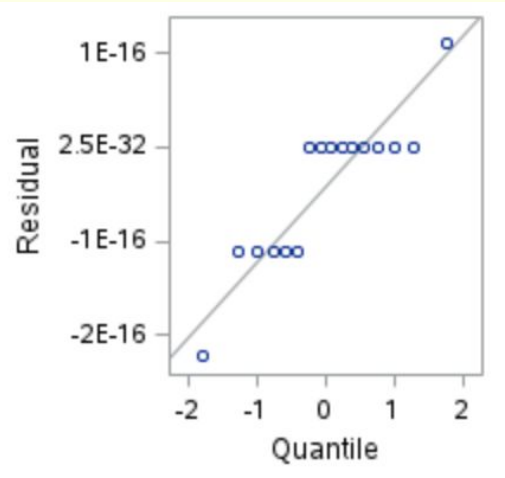
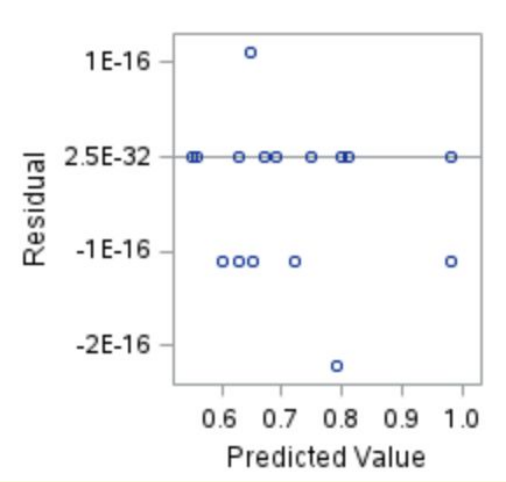
Analysis

Initial Model Assumptions







Initial Model

- The lack of replication in this experiment resulted in an ANOVA table with no F or p-values
- There are insufficient degrees of freedom available to estimate the residual variability → we cannot test the significance of any model effects

The GLM Procedure

Dependent Variable: UEC

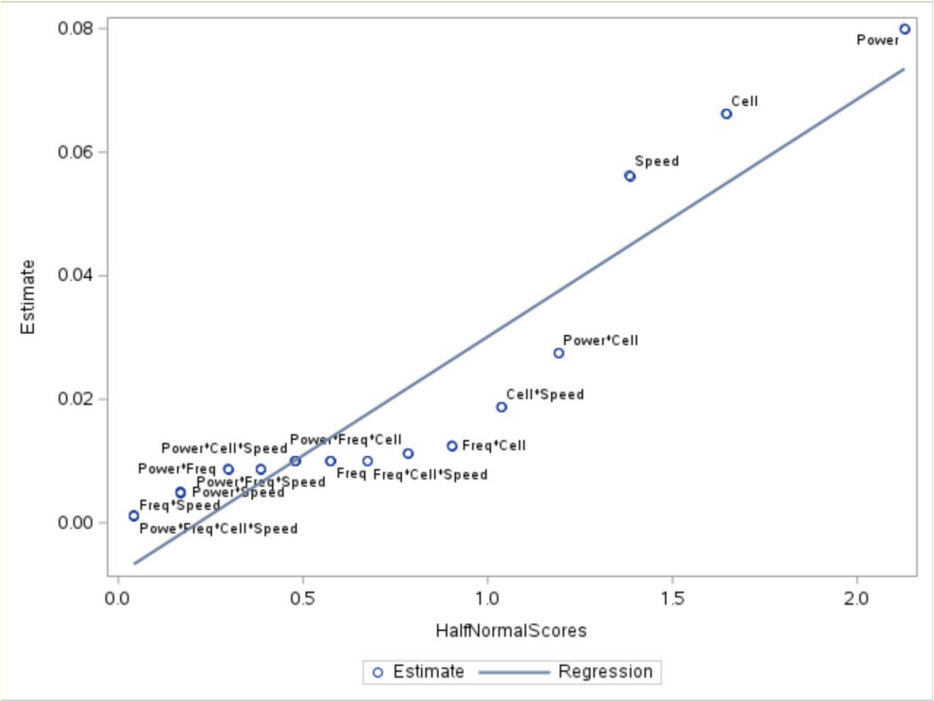
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	15	0.25357500	0.01690500	.	.
Error	0	0.00000000	.		
Corrected Total	15	0.25357500			

R-Square	Coeff Var	Root MSE	UEC Mean
1.000000	.	.	0.716250

Source	DF	Type I SS	Mean Square	F Value	Pr > F
LaserPower	1	0.10240000	0.10240000	.	.
LaserPulseFrequency	1	0.00160000	0.00160000	.	.
LaserPowe*LaserPulse	1	0.00122500	0.00122500	.	.
MatrixCellSize	1	0.07022500	0.07022500	.	.
LaserPowe*MatrixCell	1	0.01210000	0.01210000	.	.
LaserPuls*MatrixCell	1	0.00250000	0.00250000	.	.
LaserP*LaserP*Matrix	1	0.00202500	0.00202500	.	.
WritingSpeed	1	0.05062500	0.05062500	.	.
LaserPowe*WritingSpe	1	0.00040000	0.00040000	.	.
LaserPuls*WritingSpe	1	0.00040000	0.00040000	.	.
LaserP*LaserP*Writin	1	0.00122500	0.00122500	.	.
MatrixCel*WritingSpe	1	0.00562500	0.00562500	.	.
LaserP*Matrix*Writin	1	0.00160000	0.00160000	.	.
LaserP*Matrix*Writin	1	0.00160000	0.00160000	.	.
Lase*Lase*Matr*Writi	1	0.00002500	0.00002500	.	.

Half-Normal Plot

- Looking at the data in this way, allows us to preliminary estimate the significance of the main effects and interactions
- As seen by the non-linearity of many points, there are multiple effects and interactions that appear to be significant



No 4-Way Interaction

- Removed the 4-way interaction because of the Effect Hierarchy Principle
- We now have enough degrees of freedom to accumulate p-values for each remain interaction/main effect
- The Global F-Statistic and p-value indicate a statistically significant result, however, there are still insignificant terms which warrant further removal

The GLM Procedure

Dependent Variable: UEC

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	14	0.25355000	0.01811071	724.43	0.0291
Error	1	0.00002500	0.00002500		
Corrected Total	15	0.25357500			

R-Square	Coeff Var	Root MSE	UEC Mean
0.999901	0.698080	0.005000	0.716250

Source	DF	Type I SS	Mean Square	F Value	Pr > F
LaserPower	1	0.10240000	0.10240000	4096.00	0.0099
LaserPulseFrequency	1	0.00160000	0.00160000	64.00	0.0792
LaserPowe*LaserPulse	1	0.00122500	0.00122500	49.00	0.0903
MatrixCellSize	1	0.07022500	0.07022500	2809.00	0.0120
LaserPowe*MatrixCell	1	0.01210000	0.01210000	484.00	0.0289
LaserPuls*MatrixCell	1	0.00250000	0.00250000	100.00	0.0635
LaserP*LaserP*Matrix	1	0.00202500	0.00202500	81.00	0.0704
WritingSpeed	1	0.05062500	0.05062500	2025.00	0.0141
LaserPowe*WritingSpe	1	0.00040000	0.00040000	16.00	0.1560
LaserPuls*WritingSpe	1	0.00040000	0.00040000	16.00	0.1560
LaserP*LaserP*Writin	1	0.00122500	0.00122500	49.00	0.0903
MatrixCel*WritingSpe	1	0.00562500	0.00562500	225.00	0.0424
LaserP*Matrix*Writin	1	0.00160000	0.00160000	64.00	0.0792
LaserP*Matrix*Writin	1	0.00160000	0.00160000	64.00	0.0792

No LaserFrequency Effect

- We are able to remove the Laser Frequency interactions and main effects because the Effect Heredity Principle does not apply because of the insignificance of LaserFrequency as a main effect
- The global p-value has decreased, but there are still insignificant terms → further removal

The GLM Procedure

Dependent Variable: UEC

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	0.24297500	0.03471071	26.20	<.0001
Error	8	0.01060000	0.00132500		
Corrected Total	15	0.25357500			

R-Square	Coeff Var	Root MSE	UEC Mean
0.958198	5.082101	0.036401	0.716250

Source	DF	Type I SS	Mean Square	F Value	Pr > F
LaserPower	1	0.10240000	0.10240000	77.28	<.0001
WritingSpeed	1	0.05062500	0.05062500	38.21	0.0003
LaserPowe*WritingSpe	1	0.00040000	0.00040000	0.30	0.5977
MatrixCellSize	1	0.07022500	0.07022500	53.00	<.0001
LaserPowe*MatrixCell	1	0.01210000	0.01210000	9.13	0.0165
WritingSp*MatrixCell	1	0.00562500	0.00562500	4.25	0.0733
LaserP*Writin*Matrix	1	0.00160000	0.00160000	1.21	0.3038

Final Model

- Best model so far, with a statistically significant Global p-value and all significant main effects/interaction terms

The GLM Procedure

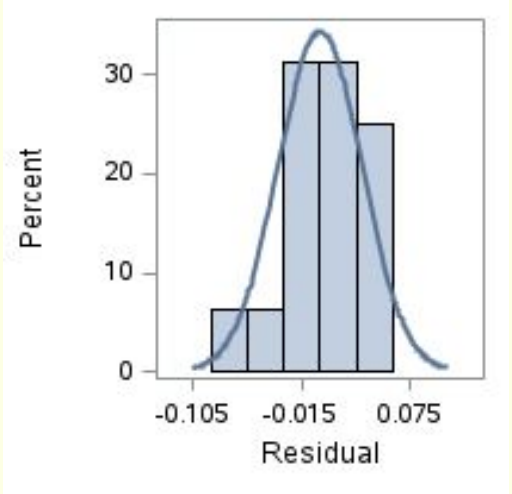
Dependent Variable: UEC

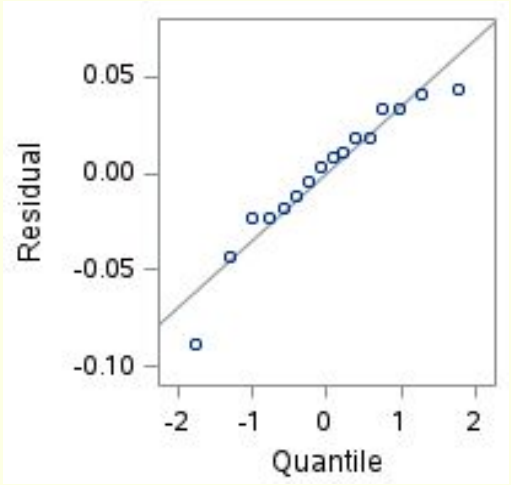
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	0.23535000	0.05883750	35.51	<.0001
Error	11	0.01822500	0.00165682		
Corrected Total	15	0.25357500			

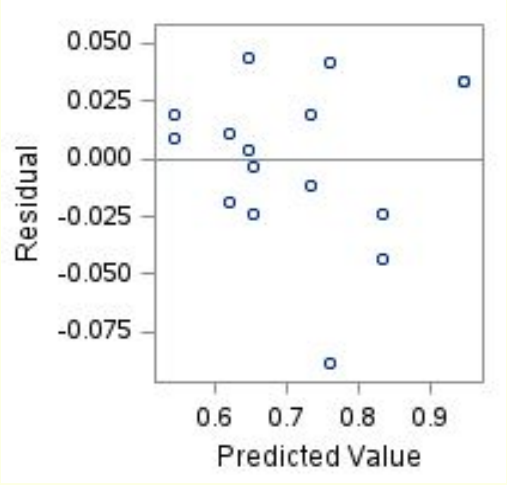
R-Square	Coeff Var	Root MSE	UEC Mean
0.928128	5.682936	0.040704	0.716250

Source	DF	Type I SS	Mean Square	F Value	Pr > F
LaserPower	1	0.10240000	0.10240000	61.81	<.0001
MatrixCellSize	1	0.07022500	0.07022500	42.39	<.0001
LaserPowe*MatrixCell	1	0.01210000	0.01210000	7.30	0.0206
WritingSpeed	1	0.05062500	0.05062500	30.56	0.0002

Final Model Assumptions







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Conclusions

Main Effect Pairwise Testing

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Tukey

LaserPower	UEC LSMEAN	H0:LSMean1=LSMean2	
		t Value	Pr > |t|
-1	0.63625000	-7.86	<.0001
1	0.79625000		

LaserPower

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Tukey

MatrixCellSize	UEC LSMEAN	H0:LSMean1=LSMean2	
		t Value	Pr > |t|
-1	0.78250000	6.51	<.0001
1	0.65000000		

MatrixCellSize

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Tukey

WritingSpeed	UEC LSMEAN	H0:LSMean1=LSMean2	
		t Value	Pr > |t|
-1	0.77250000	5.53	0.0002
1	0.66000000		

WritingSpeed

Interaction Pairwise Testing

- LSMeans is the ONLY groups where every pairwise interaction has a p-value <0.002
  - LaserPower is 13 Watts (1) and MatrixCellSize is 0.07 inches (-1)
- LSMean 1 & 4 are the only combinations that are very likely to occur if the null hypothesis is true
  - LSMean (1) = LSMean (4)

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Tukey

LaserPower	MatrixCellSize	UEC LSMEAN	LSMEAN Number
-1	-1	0.67500000	1
-1	1	0.59750000	2
1	-1	0.89000000	3
1	1	0.70250000	4

Least Squares Means for Effect LaserPowe*MatrixCell t for H0: LSMean(i)=LSMean(j) / Pr > \|t\|

Dependent Variable: UEC

i/j	1	2	3	4
1		2.692646 0.0843	-7.46992 <.0001	-0.95546 0.7765
2	-2.69265 0.0843		-10.1626 <.0001	-3.6481 0.0172
3	7.469921 <.0001	10.16257 <.0001		6.514466 0.0002
4	0.955455 0.7765	3.648101 0.0172	-6.51447 0.0002	

Factors Affecting UEC

- Very strong evidence to suggest that these main effects/interactions effect UEC:
  - LaserPower
  - MatrixCellSize
  - WritingSpeed
  - LaserPower*MatrixCellSize

- The results from our analysis indicate that these factors warrant further study to fully understand the significance and effects they have on UEC

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Limitations and Future Work

A Different Approach

- Considering limitations of the original experiment, we believe that conducting a 2^(k-1) experiment may be more advantageous than the original 2^4 approach
    - This would result in a reduction in time, money, and resources spent
    - It would also give the flexibility to screen for multiple levels of different effects

- This approach could introduces the risk of confounding factors - it is important to proceed with caution

Our Own Changes

- We excluded several interaction terms and even a main effect in our final model
    - This introduces the potential loss of important information that could be used in our analysis
- Considering this - we could have adjusted our analysis alpha level to 0.1 instead of 0.05 which would allow us to include the interaction terms that were on the cusp of significance in previous models

Larger Follow-Up Experiment

- Introduce Replication
  - This would improve the reliability of experiment and ensure that our estimates of the effects/significance are more precise

- Balanced Design
  - This would ensure that all combination of factor levels are represented equally and would address the potential for confounding factors if we were to complete a 2^(k-1) fractional experiment