

Physical Laws and the Theory of Special Relativity

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The meaning of a physical law is discussed. A distinction is made between specific laws such as: the $1/r^2$ law, the diffusion equation, the law of radioactive decay, Hubble's law, and fundamental laws such as conservation principles. The status of Maxwell's equations is reexamined. It is concluded that: a) physical laws pertain to closed systems, b) physical laws are formulated in inertial reference frames, determined by the center of mass of the closed system, c) there exists a unique, global inertial reference frame, and d) physical laws need not be Lorentz covariant.

1. Introduction

Physics—as a natural science—deals with phenomena observed in nature. The business of physics is the reconstruction of facts in thought, or the abstract quantitative expression of facts (Mach 1960). The “rules” which we form for these reconstructions are the laws of nature. The conviction that such rules are possible lies in the orderliness observed in nature, as well as our belief in causality. The emphasis on the temporal succession (with respect to causality) of events is unnecessary; the concept of causality simply asserts that the phenomena of nature are dependent on one another.

The number of physical laws is surprisingly small and the distinction between “laws” and “principles” is usually tied to their respective “degrees of generality”. For example, Ohm’s law does not deserve at all to be called a “law”, since the linear dependence between current density (\mathbf{j}) and electric field intensity (\mathbf{E}) is valid in the low field approximation only. Every finite conductivity implies energy dissipation and Joule-heating, which—in turn—implies a dependence of the conductivity on the applied electric field. Also, the “one-over-r-square” force law is more general than Kepler’s laws, but less general than the principle of least action. Together with the conservation principles, extremal (or variational) principles are considered to be the most general laws of nature.

In contrast with the above mentioned laws, the so-called “symmetry principles” are tied to the abstract ideas of “form invariance”. While the fascination with the beauty and purity of the symmetry principles has already started to fade away, the space-time (or Lorentz) symmetry of the special theory of relativity (STR) is still believed to play a fundamental role in physics. Although STR is nothing but a hypothetical kinematics, STR has been elevated to the rank of a “super law”—or a “law about laws”—restricting drastically the possible “form” of all existing and hitherto undiscovered laws of physics.

One cannot avoid asking what should be the status of statements like:

a) The diffusion equation:

$$\frac{\partial C}{\partial t} = D \nabla^2 C \quad (1)$$

where C denotes concentration and D the diffusion “constant”;

b) The law of radioactive decay:

$$N = N_0 \cdot \exp\left(-\frac{t}{\tau}\right) \quad (2)$$

where τ is the half-life divided by $\ln 2$,

c) Hubble’s law:

$$v = H \cdot R \quad \text{or} \quad R = R_0 \cdot \exp(Ht) \quad (3)$$

where R , H and v denote distance between two galaxies (or radius of the universe), Hubble’s constant and the relative recession velocity between any two galaxies, respectively.

Are these equations laws, even though they are obviously not Lorentz invariant?

2. Newton’s Principles as Definitions

The status of Newton’s laws—also called principles of dynamics—has often been subjected to dispute. I repeat here my earlier expressed opinion (Galeczki 1990, 1993) that Newton’s “laws” are actually definitions.

The 1st law: “Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by forces impressed upon it.” This famous statement requires the further definitions of “uniform motion” and of “force”. More important, however, it implies the definition of the inertial frame of reference (IFR). The IFR is a non-rotating, non-accelerating frame, in which a free particle (i.e. not acted upon by “forces”) moves with uniform velocity along a straight line. The concept of IFR is of utmost importance in physics, since all presently known laws have been formulated in IFRs.

On further scrutiny it turns out that all practical IFR’s are only approximately so. The “degree of inertiality” of a frame of reference depends on the absence of interactions with other bodies, in other words on its “degree of isolation”. The center of mass (CM) of an ideally isolated system could be thought as an IFR. The question is: What actually constitutes a system? A “free particle” is an uninteresting isolated system, since the lack of internal structure makes the search for laws pertaining to the system superfluous. An isolated hydrogen atom provides a better example, although in the absence of any interaction with the external world, no transitions between its discrete energy levels are possible. A collection of free and non-interacting particles is uninteresting, too, for the simple reason that such an ensemble will expand indefinitely (like a cloud in open space). It is worth mentioning that the so called “constitutive ensemble”—which plays a vital role in one of the typical axiomatic derivations of the Lorentz transformation (LT) (Mittelstaedt 1992)—is precisely such a collection of free and non-interacting point-like particles.

The 2nd law: “The rate of change of momentum is proportional to the impressed force, and is in the direction in which the force acts”:

$$\frac{d\mathbf{p}}{dt} = \mathbf{F} \quad (4)$$

Several new concepts and/or definitions are encapsulated in this seemingly simple formula:

- a) Linear momentum \mathbf{p} is defined as the product of mass (itself a new dynamical concept) and velocity: $\mathbf{p} = m \cdot \mathbf{v}$.
- b) Velocity \mathbf{v} is defined with respect to the IFR defined by the 1st principle,
- c) The universe is divided between “our system under study” and “the external world”. The system is characterized by the mass m , thought to be concentrated in its CM. The “force” \mathbf{F} represents the action of the external world upon the system, to which a vector applied to the CM is ascribed. The external world reacts with a force defined by the 3rd law of Newton (see below).
- d) Formula (4) defines the relationship between force and linear momentum pertinent to a point-like mass, or to a system seen as concentrated in its CM.

The 3rd law: “To every action there is always opposed an

equal reaction". Taking into consideration that forces are always thought of as acting either upon real, or upon equivalent, point-like CMs, the action of the external world upon the system's CM denoted "b" should be opposite and equal to the reaction of the system to the external world (a):

$$\mathbf{F}_{ab} = -\mathbf{F}_{ba} \quad (5a)$$

or:

$$\mathbf{F}_{ab} + \mathbf{F}_{ba} = 0 \quad (5b)$$

Since nothing exists outside "our system" and "the external world", relation (5b) is—in oversimplified form—the definition of the unique system called a Universe. This is in line with Mach's idea of a Universe closed in itself, with no isolated parts. In such a universe:

- a) no boundary conditions are necessary, and
- b) the mass and/or the inertia of every artificially isolated part is the manifestation of its coupling to the rest of the universe. The possibility of all inertial forces being due to gravitational interaction with other bodies in the universe was recently confirmed by Assis (1989), who extended the velocity dependent electrodynamic potential of Weber to the interaction between masses. The characteristic feature of a closed system, (5b), can be easily generalized for a system of point-like particles interacting by means of two-body forces \mathbf{F}_{ij} obeying (5b). Applying Newton's second law to particle i , one obtains:

$$\frac{d\mathbf{p}_i}{dt} = \sum_{k \neq i} \mathbf{F}_{ki} \quad (6)$$

called the "equation of motion of particle i ", the reference system being Newton's absolute space—the only rigorous IFR, defined by the universe as a whole. In writing down equation (6) we assumed tacitly that all forces are "two-body forces", therefore the summation in (6) is already an approximation:

$$\sum_i \frac{d\mathbf{p}_i}{dt} = 0 \quad (7)$$

since:

$$\sum_i \sum_{k \neq i} \mathbf{F}_{ki} = 0 \quad (8)$$

Equation (7) yields the well-known conservation law of linear momentum.

3. Conservation Laws as Mathematical Identities

Due to the commutativity of summation (over all particles) and time derivative, (7) can be integrated:

$$\sum_i \mathbf{p}_i = \text{constant} \quad (9)$$

the integration constant being unknown. In other words, the total linear momentum of a closed system—rigorously the universe—remains constant in time, provided the two-body forces between the parts of the system obey the principle of equality between action and reaction. This conclusion has to be contrasted with the

so called "Van Dam-Wigner Theorem" (Rindler 1967), according to which "the total momentum and the total energy of a system of particles interacting at-a-distance cannot remain constant in all inertial frames". Since the only rigorously inertial frame is that defined by the Universe as a whole, the validity of the linear momentum conservation in this unique frame is physically much more relevant than its validity in all the fictitious inertial frames which do not belong to this world. The assumption implicit in the Van Dam-Wigner theorem is that without distant simultaneity the total linear momentum cannot remain constant even in one single inertial frame, let alone in all conceivable ones. Without distant simultaneity only the linear momentum of one, isolated, free particle could possibly remain constant in time!

Clearly, the mathematically trivial operation of integrating (7) assumes tacitly the existence of a parameter t valid for the entire system: the universe. This parameter has to be independent of the positions and the velocities of the particles—all conceivable "observers" included—belonging to the system. More explicitly, the very possibility of writing down equations (4) and (9) requires a universal (or cosmic) time t , as well as distant simultaneity. The "special" relativistic jargon expresses this fact in the following manner: "the energy-momentum conservation law is formulated in a hyperplane $t = \text{constant}$ ", the hyperplane belonging to the Minkowskian space-time world. In general relativity, where only global conservation laws exist, another formulation is preferred: "In general relativity conservation laws are locally valid, since a curved Riemannian space is locally Minkowskian (i.e. 'plane')".

I would prefer to avoid unnecessary linguistic acrobatics and put it in plain language: The conservation law (9) is formulated in principle for the whole universe, in absolute space and universal time. Its validity for smaller systems depends on the degree to which the system could be considered as isolated. Even when isolating a system from the whole, we must always keep in mind that it is just a part of the whole and that the mass of every piece of matter is a manifestation of its coupling to the whole system. The particle velocities are defined in absolute sense with respect to the whole system, and all possible velocity dependent effects have to be absolute in nature. Relative velocity, implying relative motion, is the difference between two absolute velocities:

$$\mathbf{v}_{ij} = \mathbf{v}_j - \mathbf{v}_i = -(\mathbf{v}_i - \mathbf{v}_j) = -\mathbf{v}_{ji} \quad (10)$$

Since the velocities \mathbf{v}_i and \mathbf{v}_j are defined at the same (universal) instant " t ", it should be clear that the very possibility of relative motion implies distant simultaneity and absolute, universal time. The reciprocity of relative velocities is a natural feature of their definition as algebraic differences of absolute velocities.

The status of the law of conservation of angular momentum is similar to that of the linear momentum. There is, however, one point which deserves special attention, namely the undisputed absolute character of rotational motion. Very often, the absolute character of accelerated motion in general and of rotational motion in particular, as compared to linear, uniform motion, is seen as similar to the absolute difference between any curve and a straight line. Nevertheless, the straight line and the linear, uniform motion are only idealized concepts extrapolated

from experience, with the corollary that a sound physical theory should predict a gradual increase of the effects induced by gradual deviations from uniform, linear motion. The most unusual (i.e. non-physical) exception to this natural expectation is, again, Einstein's STR, which introduces a sharp demarcation between unaccelerated, linear motion and accelerated motion, no matter how small the acceleration and how short its duration. This strange behaviour is best revealed in the (in)famous clock "paradox" of STR, where the claimed time-lag between the initially synchronized twin clocks is tied exclusively to the second clock being temporarily accelerated, no matter how weak the acceleration, or how short its duration was.

In the same vein, the null-effect in the Michelson-Morley (MM) experiment and the positive, first-order effect in the Michelson-Gale (Michelson and Gale 1925) experiment are ascribed to the total absence, or to the manifest presence of a small acceleration, respectively. Both claims are questionable since:

- (a) on one hand, the orbital motion of the Earth around the Sun (expected to cause the fringe-shift in the MM experiment) is not uniform, and
- (b) on the other hand, during 10–15 seconds (the characteristic time of the optical experiment) the tangential velocity of Earth's rotation around its own axis could well be seen as linear and uniform.

The status of the energy conservation principle is similar to that of linear and angular momentum, respectively. Due to the fact that, neither at some "initial moment", nor at any other moment " t " are the velocities and positions of all the particles of a system known, the equality between "energy" at two distinct moments (or for two distinct "states" of the system) represents a mathematical identity. There is, however, a considerable difficulty—as compared with linear and/or angular momentum—stemming from the need to properly define the energy of the system in question. Kinetic energy is the only form of energy which has a straightforward definition. So strong is our belief in the general validity of an energy conservation principle, that every time a seeming violation occurs, a new form of (hitherto unknown) energy is introduced. In its most general form, the conservation of energy is known as the first principle of thermodynamics. Potential energy, usually a function of the relative positions of the particles composing a system, is the prototype of action-at-a-distance and implies the possibility of defining absolute, universal time, as well as distant simultaneity. In spite of its ubiquitous occurrence in books which pay lip-service to STR, potential energy (or any force which depends on relative positions, only) should have had no place in theories obeying the basic ideas of STR.

Before closing this section, I shall point out an important difference between discrete systems composed of point-like particles and continuous media. As already discussed, the conservation principles hold rigorously for the unique closed system—the Universe—and every artificial isolation of a sub-system is necessarily an approximation. These non-local, or global conservation laws, valid for discrete systems, have to be contrasted with the local conservation laws valid for continuous, fluid-like media. The expression of the conservation law is in this case the

equality between the time variation of the conserved quantity (for example, momentum, energy, charge, etc.) within an imaginary volume of the medium, and the total flux through the surface confining that volume. This equality is usually written in the form of a local "equation of continuity". For a fluid, which has to be confined to a container of volume V under hydrostatic pressure p , the role of the energy is played by the enthalpy $E + pV$. The above discussion reveals the genuine local or point-event character of STR, which has to be contrasted with the non-local character of both the Machian version of Newton's mechanics and quantum mechanics. The point-event character of STR can be traced back to the unfortunate redefinition of the concept of distant simultaneity. Before 1905, simultaneity was a dyadic relationship: two distant events were (or were not) considered to be simultaneous in an absolute sense. After 1905 simultaneity was redefined as a triadic relationship: two distant events are (or are not) simultaneous—according to STR—with respect to a third point where an "observer" is present. The two kinds of simultaneity imply two distinct kinematics, Galilean and Lorentzian, respectively. One is compatible with global conservation laws (having nothing to do with the spirit of GTR), the other with local conservation laws, valid for isolated point-like particles, for fictive mathematical points in a continuous medium or in a "field".

4. Two Approaches to Maxwell's Equations

Although the physical content of Maxwell's equations is best revealed in integral form, it is their local, "microscopic" form, given by Lorentz in 1892, which is generally known as Maxwell's equations (ME). In the microscopic view the world is reduced to point charges and vacuum, the ME providing a link between the particle variables ρ , \mathbf{j} and the field variables \mathbf{E} and \mathbf{B} . The field could be determined if the sources ρ and \mathbf{j} are given (as functions of \mathbf{r} and t), or vice-versa. The equation of continuity, expressing local charge conservation, relates the sources of the field:

$$\frac{\partial \rho}{\partial t} + \text{div} \mathbf{j} = 0 \quad (11)$$

—remarkable, for the Giorgi system ME can be brought into a form which is free of any constants or parameters reminiscent of the properties of any one particular medium:

$$\begin{aligned} \text{curl} \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}; & \text{div} \mathbf{B} &= 0 \\ \text{curl} \mathbf{H} &= -\frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}; & \text{div} \mathbf{D} &= \rho \end{aligned} \quad (12)$$

These equations display natural invariance in the sense that they are metric independent and retain their form in non-inertial frames of reference as well. The metric dependence is relegated to the constitutive relations:

$$D_i = \epsilon_{ik} E_k; \quad B_i = \mu_{ik} H_k \quad (13)$$

A free-space inertial situation is defined by the explicit relations:

$$\mathbf{D} = \epsilon_0 \mathbf{E}; \quad \mathbf{B} = \mu_0 \mathbf{H}; \quad \epsilon_0 \mu_0 = c^{-2} \quad (14)$$

which are invariant under the Lorentz transformations and under the conformal group. The wave equation in vacuum:

$$\square^2 \mathbf{E} \equiv \left(\nabla^2 - \frac{1}{c^2} \cdot \frac{\partial^2}{\partial t^2} \right) \mathbf{E} = 0 \quad (15)$$

is obviously Lorentz invariant, too, but the wave equation, even in a linear and isotropic medium:

$$\left(\nabla^2 - \epsilon \mu \cdot \frac{\partial^2}{\partial t^2} \right) \mathbf{E} = 0 \quad (16)$$

is not. For this reason only, it seems to me that the requirement of Lorentz invariance—or at least of covariance—of all physical laws is a farfetched extrapolation.

There is a long-standing tradition, started with an article by Leigh Page in 1912 “A Derivation of the Fundamental Relations of Electrodynamics from those of Electrostatics” (Page 1912), which claims that:

(Maxwell’s Equations) = (Coulomb’s Law) + (Lorentz Transformations)

Page’s derivation presupposes a transformation rule for “the force experienced by a particle in IFR (S’), to calculate the force as determined by an observer in (S) which moves with velocity v_o relative to (S’). Next, the derivation includes a definition of the field \mathbf{H} :

$$\mathbf{H} \equiv (\mathbf{v}_o \times \mathbf{D}) \quad (17)$$

However, the most problematic point in Page’s derivation is its reliance upon Coulomb’s law as written in (S’). Here I must confess that I have never understood, why on the one hand Newton’s $1/r^2$ -law is seen as action-at-a-distance, while on the other hand Coulomb’s $1/r^2$ -law is supposed to coexist peacefully with Maxwell’s contiguous action theory?

An approach initiated by Planck (Planck 1949), that contrasts with Page’s derivation of ME, derives ME and Lorentz’s force law in vacuum as theorems, from the conservation principles of energy and momentum (Imai 1991). The basic assumptions of this approach are:

- a) Electric “field lines” connect positive and negative charges
- b) Magnetic charges do not exist
- c) Electric charge is conserved
- d) Superposition principle is valid
- e) Energy and momentum are stored in the electromagnetic field; they are per unit volume given by:

$$W \equiv 0.5(\mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B}); \quad \mathbf{g} \equiv \mathbf{D} \times \mathbf{B} \quad (18)$$

- f) Across any surface in the electromagnetic field there are fluxes of electromagnetic energy and momentum:

$$\mathbf{S} \equiv \mathbf{E} \times \mathbf{H}; \quad \mathbf{T} \equiv \mathbf{E}\mathbf{D}_n + \mathbf{H}\mathbf{B}_n - W\mathbf{n} \quad (19)$$

where \mathbf{n} denotes the unit vector normal to the surface, \mathbf{S} is the Poynting vector and \mathbf{T}_n is related to Maxwell’s tensor $T_i = T_{ij}n_j$. It should be noted that ‘force’ does not appear in the above statements. In contrast to energy and momentum, this approach does

not consider force as a fundamental physical quantity. It could be shown that the dissipation of electromagnetic momentum per unit volume and per unit time is:

$$\mathbf{f} = \rho \mathbf{E} + \mathbf{j} \times \mathbf{B} \quad (20)$$

Only for the extreme situation of a point charge q moving with the uniform velocity \mathbf{v} one obtains the Lorentz force:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (21)$$

Like Newton’s second and third laws, Maxwell’s equations hold rigorously for the closed universe only. The validity of ME for smaller systems is conditioned by the weakness of the coupling of these systems to the “rest of the universe”. Even if many systems, from atoms to galaxies, are electrically neutral, endowing electromagnetic fields with individuality means, actually, that they cannot be completely screened. (This problem was part of a U.S. Department of Defense project devoted to protecting (super)computers from the strong electromagnetic radiation following nuclear explosions.) As a matter of fact, with the exception of some cosmic rays and neutrinos, all the information about the universe external to our solar system reaches us in the form of electromagnetic radiation.

5. The Status of the Preferred Frame of Reference

Even if the expression “the fractal structure of the universe” is lip-service paid to a trendy theory of our times, the hierarchy from nucleons to clusters of galaxies is one of the characteristic features of the universe. Although every system has a center of mass (CM) and an associated “proper” frame of reference, the universe as a whole provides a unique, privileged and preferred frame of reference. Foucault’s pendulum, the Bradley aberration, the Sagnac-type effects and the Kennard-Müller-type unipolar induction experiments (Kennard 1917) provide (for some of us) convincing evidence for the unique, preferred frame of reference. Nevertheless, typical reactions to the enquiry of Yaes (1993)—namely the violation of special relativity due to the existence of a global, cosmically preferred reference frame, in which the background microwave radiation is isotropic—were:

- a) “it is my (T.P. Krisher) understanding that the principle of relativity is not necessarily violated by the mere existence of a universal frame of reference”.
 - b) “if there existed some interaction that violated Lorentz invariance, then velocity-dependent effects could become locally apparent to a moving observer (T.P. Krisher).
 - c) “the relativity principle is in fact a statement about the invariance properties of the laws governing the behavior of physical systems and not about the invariance properties of particular states of those systems” (J. Anderson).
 - d) “The reference frame in which the distribution (of the cosmic microwave background radiation, G.G.) is isotropic can be said to constitute a preferred frame, but only in respect to the particular way in which the expansion began...” (A.C. Dottson).
- To these reactions one may add the reply of Combourieu and Vigier (1993) to the claim that realistic interpretations of quantum mechanics are in trouble with the Lorentz invariance:

The aim of the present Letter is to show that this result is incorrect if one follows Lorentz's interpretation of the relativistic formalism (based on the idea that rods contract and clocks slow down) when moving relative to a privileged inertial frame similar to the rest frame R_o, \dots, R_o is the 2.7 °K background microwave radiation isotropic at all frequencies.

A similar difficulty arises with respect to Hubble's law (3), where R is taken to be the distance to a star (or a galaxy) that is moving out in the "exploding universe", relative to the origin of the "big bang" and H is Hubble's constant. The law (3) is obviously not covariant and uses the Newtonian concept of absolute time—where the time measure is the same from all (comoving) reference frames. Both the isotropic microwave background radiation and Hubble's law clearly violate the Einsteinian relativity principle. One may wonder what is left of Einstein's theory if the sound evidence for the existence of a privileged and preferred reference frame is there, and further it is assumed that rods contract and clocks slow down when moving relative to this frame!! Why then the vehement protests against Sir Edmund (Whittaker) who dared to name the relevant chapter of his monumental History (Whittaker 1953): "The Relativity Theory of Lorentz and Poincaré"?

If true, the Einsteinian revolution would have been a counter-revolution against the Copernican one! Galileo's famous "Eppur si muove" makes sense only if the motion of the earth has an absolute meaning. The CM of any (approximately) closed system is a privileged frame of reference, and only in such a frame was the discovery of the $1/r^2$ -law possible. As to the one-way velocity of electromagnetic energy propagation, it has to be c with respect to the cosmically preferred frame of reference only, otherwise the Bradley aberration would not reveal the orbital motion of the Earth. The same orbital velocity could be obtained locally from a careful analysis of a Michelson-Gale type experiment, too. Moreover, the velocity of Earth with respect to the cosmically preferred frame of reference was determined in a closed laboratory (i.e. locally) in the ingenious coupled-mirror experiment of Marinov (1980). Last but not least, the microwave background radiation is not necessarily a remnant of an imaginary "big bang"; therefore the preferred frame of reference need not be a particular state characterized by a general, isotropic expansion.

It is one of the peculiarities of scientific thought that Newton, whose dynamics ascribed no physical effects to absolute velocities, nevertheless, held a firm belief in a preferred frame of reference, while Einstein, who fathered the "paradox" of an absolute effect ascribed to relative motion (for example, the asymmetric aging of twins), rejected the very idea of absolute velocities referred to a privileged frame of reference.

Acknowledgment

I am grateful to Dr. Peter Marquardt for his patience, his critical spirit and his thoroughness, which were of great help in bringing this paper to its present form. Special thanks are due to him for bringing the theorem of Van Dam-Wigner to my attention.

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