A Non-Lorentzian Force Stronger than the Lorentz Force

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By analyzing a recent Smith-Purcell type experiment (Smith and Purcell 1953, Doucas et al. 1992), we find that the DC force due to spontaneous emission in a periodic electrostatic field is stronger than the oscillating Lorentz force of the electrostatic field that produces spontaneous emission. It is shown that no genuine classical theory has ever explained the net energy transfer from an electromagnetic wave to a medium or a long electron beam whose length is much longer than the wavelength of the electromagnetic wave. Consequently, the force described by the Lorentz force equation as an entity acting on an electron from an electromagnetic wave must be understood to represent only the force due to scattering of the electromagnetic wave by the electron, but not due to any emission or absorption. The force due to the latter, i.e., non-Lorentzian force, may be stronger than the Lorentz force.

Classical electrodynamics was developed at a time when electric and magnetic fields were measured as forces acting on charged objects. Lorentz introduced the force concept into the Lorentz force equation. In Lorentz's lifetime, neither ultra-strong spontaneous emission nor any phenomenon or device of very large net energy transfer from an electromagnetic wave to a medium due to inverse and stimulated bremsstrahlung—such as anomalous absorption in laser fusion, free electron laser (FEL), gyrotron which is an electron cyclotron maser (ECM), laser-plasma acceleration, and so on—was known. Accordingly, it is very natural that, even though the Lorentz force of an electromagnetic (EM) wave is oscillatory, such that any net energy transfer from an EM wave to any medium is most unlikely, no one ever raised any doubts that the Lorentz force is the only macroscopic force acting on a charged particle from an electric or magnetic field or EM field. However, even after two-mode phenomena or devices such as laser fusion, FEL, and ECM were developed, no one ever thought that there must be a non-Lorentzian force until I proposed it (Kim 1984). I explained that the Lorentz force is the force only due to scattering on the quantum mechanical level, and that emission and absorption is not included in the Lorentz force equation (Kim 1984, 1992b). I proposed that the reason why we feel warmth near a stove or why food is cooked in a microwave oven is due to the existence of non-Lorentzian forces (Kim 1986). Further, I have claimed that if we properly harness a non-Lorentzian DC force due to net inverse bremsstrahlung (NIB) of an incident strong electromagnetic wave such as a laser light or a cavity wave, which is called the NIB force, it will be a far stronger alternative force than the present Lorentz force of a cavity TM mode in a radio-frequency (RF) acceleration cavity (Kim 1985, 1988a-c, 1989, 1992b, 1993a). Recently, it has become imperative to find such an alternative force for the following reason: The typical acceleration gradient in the present-day high-energy accelerators (including the scuttled Superconducting Super-Collider) is about 20 MeV/m. With such an acceleration gradient, it is too costly to build a high-energy accelerator of 10 TeV. Thus, in the practical sense, high-energy physics must come to an end unless a new physical mechanism having a higher acceleration gradient is exploited.

In this article, we dismiss the classical concept that there is no non-Lorentzian force, that no force may exceed the Lorentz force, and that energy transfer from an EM wave to a medium is carried out through the Lorentzian force.

Experimental results showing the existence of the non-Lorentzian DC force

There should be a non-Lorentzian force due to net emission or absorption, even if it is not necessarily the NIB force. Extraordinarily strong spontaneous emission can result in a non-Lorentzian force. Now there are many experimental results which show the existence of such a non-Lorentzian force. One of the simplest examples is the so-called Smith-Purcell experiment (Smith and Purcell 1953).

(a) Smith-Purcell experiment

The Smith-Purcell (SP) configuration is shown in Figure 1 of (Kim 1993b). In this an electron loses its kinetic energy as radiation energy. The measured energy loss as an electron travels a grating period d is equal to

$$E_{\text{measured loss}} \approx 10^{14}\,\frac{r_e e^2}{d^2} \tag{1}$$

in the SP configuration, where $r_e \approx 10^{-13}$ cm is the classical electron radius. Thus, the electron experiences a DC force acting in the negative z direction whose magnitude is equal to

$$F_{\text{DC}} = \frac{E_{\text{measured loss}}}{d} \approx 10^{14}\,\frac{r_e e^2}{d^2} = 10^5\,\frac{e^2}{d^2} \tag{2}$$

for $d = 10^{-4}$ cm.

Here, it should be emphasized that, whatever the reason, Equation (2) simply tells us that the quantities on both sides are equal in magnitude in the SP experiment based on the experimental results obtained by Smith and Purcell (1992). As is seen in Figure 2 of Kim (1993b), the force acting in the z direction is the z component of the Coulomb force from the image induced by the electron. This force alternates between positive and negative values. It is an elementary understanding that the rms value of this force is about e^2/d^2. Thus, the measurement shows that the force due to spontaneous emission is 10^5 times larger than the rms value of the Lorentz force.

In any event, the total force acting on the electron from the surface charge produced by the electron cannot be greater than the vector sum of the forces from the electron to the surface charge. Further, the former cannot be greater than (electron charge)$*$(total surface charge)$/r_{\min}^2 \le e^2/r_{\min}^2$ because the acceleration field from the electron cannot be stronger than the velocity field when $r_{\min} < c(1-\beta^2)/\beta$ (cf. equation (14.14) of Jackson 1975), where $r_{\min}$ is the shortest distance between the electron and its charge center of the surface charge, which is satisfied in the SP configuration. Even if the electron grazes the surface, the force from the surface charge pulls the electron down a distance of order $r_e \approx 10^{-13}$ cm as the electron travels a distance of one grating period $d = 1.67 \times 10^{-4}$ cm (Kim 1993b). Thus, the electron travels practically on a straight path, and the maximum magnitude of the force from the surface charge to the electron is e^2/h^2, where h is the altitude. Therefore, in the classical conception, the amplitude of the force which an average electron experiences is

$$F_o \approx \frac{e^2}{h^2} = \frac{e^2}{r_b^2}\int_{h_m}^{r_b} \frac{r_b^2}{h^2}\,\frac{4\pi h\,dh}{4\pi r_b^2} = \frac{e^2}{r_b^2}\,\frac{r_b - h_m}{h_m}, \tag{3}$$

where $\langle\rangle$ is the average over the beam electrons, $r_b \approx 0.007$ cm is the beam radius, and h_m is the minimum altitude with which the electron can pass over the grating. Here we have made the reasonable assumption that Smith and Purcell measured the SP radiation power on the maximum level, which occurs when the center axis of the beam just grazes the surface, and $r_b \gg h_m$, and an approximation that the cross sectional distribution of the beam electrons is uniform. It is a very conservative assumption that $h_m \approx a_o$, where a_o is the Bohr radius. Thus, $F_o \approx e^2/h^2 \approx 10^{-10} \times 10^5 e^2/a_o^2 \approx 10^{-4} e^2/d^2$. This value is about 10 times smaller than the measured value given by (2).

In the framework of classical electrodynamics, there is no force acting on the electron in the z direction stronger than the

Coulomb force from the image in the SP experiment, apart from the fact that such force is a DC force and its magnitude is larger than the maximum of the Coulomb force from the image. There are many models to explain the SP radiation, including the one by Purcell (Smith and Purcell 1953). The statements that there must be a force other than the Lorentz force, that the force is a DC force, and that its magnitude is about equal to or greater than the amplitude of the Lorentz force, are independent matters from whichever of such models properly describes the mechanism of the SP radiation.

(b) Experiment by Doucas et al.

Unlike the Smith-Purcell experiment, in the experiment done by the Doucas group (Doucas et al. 1992), the beam electrons are bunched as shown in Figure 1. The measured energy loss per unit distance, which is the DC force acting in the negative z direction due to spontaneous emission, is equal to [cf. Equation (4.13) in (Kim 1993b)]

$$F^{\text{meas. val.}}_{\text{net em.}} \approx \frac{\alpha N_b^2 e^2}{\gamma^2 \beta^2 r^2_{\text{min}}}, \quad \text{if } r_{\text{min}} = 1 \text{ mm}, \tag{4}$$

where N_b is the number of electrons in a bunch, $\alpha \approx 1/137$ is the fine-structure constant, and $\beta = v/c$ with v the electron velocity. Equation (4) simply means that, whatever the reason, both sides are equal to each other for the configuration used by the Doucas group. From Figure 1 and the fact that the force acting on the electron from the surface charge cannot be greater than the sum of the force from the electron to the surface charge, and the latter is not greater than $N_b e^2/r^2_{\text{min}}$, the amplitude of the oscillating Lorentz force acting on an average electron from the image of the electron bunch is far less than $N_b e^2/r^2_{\text{min}}$, i.e.,

$$F_{\text{Lorentz}} \ll \frac{N_b e^2}{r^2_{\text{min}}} \tag{5}$$

Since $N_b \approx 10^4$, $\gamma = 8$, $\beta \approx 1$, we find that

$$\left| F^{\text{measured value}}_{\text{net em.}} \right| > 10 \frac{N_b e^2}{r^2_{\text{min}}} \tag{6}$$

or more than 10 times the maximum F_{Lorentz}.

These SP (1953) and Doucas et al. (1992) examples demonstrate that the classical concept is basically flawed [The mystery of the SP radiation was recently cleared up by identifying the SP radiation as the so-called free-electron two-quantum Stark (FETQS) emission, which dismisses the correspondence principle as a groundless faith (Kim 1993b-e)].

Fallacy of the classical force concept

In the framework of classical electrodynamics, the force due to spontaneous emission is the Abraham-Lorentz radiative reaction force (Jackson 1975) whose magnitude is too small to be measured with any man-made equipment. However, we have shown that the DC force due to spontaneous emission is much stronger than the Lorentz force producing spontaneous emission in SP type configurations, and, moreover, in the SP configuration the former is on the order of 100 MV/m. In the presence of a sufficiently strong radiation field, induced emission and absorption

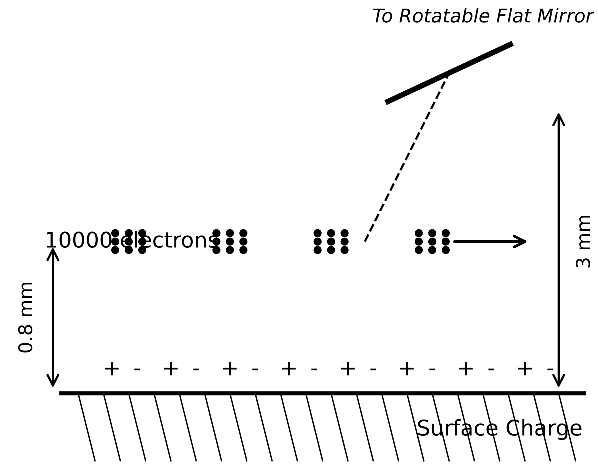

Figure 1: Schematic of the Doucas *et al.* configuration.

prevails over spontaneous emission. Analysis of the results of a laser-plasma acceleration experiment (Martin et al. 1986) has revealed that the force due to NIB of a laser in a plasma wave is about 1 GeV/m (Kim and Chen 1986).

Putting aside the above special cases, we can see, in general, DC forces due to net emission or absorption in various devices which can be measured macroscopically, as shown in Table 1.

The net energy transfer from the beam electrons to a laser field is possible only when there is a DC force which cannot be a Lorentz force. However, the classical FEL theory claims to explain the net energy transfer in an FEL. In the following, we will explain why the classical theory is flawed.

The magnetic wiggler cannot work here, since it is a magnetic field. However, it changes some of the axial kinetic energy to transversely wiggling kinetic energy. The frequency of the transverse wiggling can be the same as the frequency of the electric field of an EM wave as seen by the electron [the frequency of the so-called ponderomotive potential in the laboratory frame (Kim 1992a,b)]. Since the relative phase between the electric field in the transverse direction and the transverse motion is locked (i.e., constant in time), there will be a net energy transfer between the electron and the EM wave. So far there is nothing wrong here.

The net energy transfer depends on the relative phase between the electric field of the laser wave as seen by the electron and the electron transverse velocity. Consequently, one can raise the following question: "If we inject a long electron beam, the phases will be distributed uniformly from 0 to 2π, so that there will be no net energy transfer between all the beam electrons and the EM wave". The classical FEL theory answers this question as follows. If we calculate the phase distribution for a given laser wave and a long mono-energetic electron beam, we find that the initial uniform phase distribution always changes to an asymmetrical distribution (such that net energy flows from the electrons to the laser wave) in the beginning, and later to an opposite asymmetrical distribution (such that net energy flows from the

Table 1: Forces in various configurations

	E_i MeV	Length m	Efficiency	DC force	DC force/ Lorentz force	Comment
First FEL	24	5.2		30 kV/m	0.03	st. em.*
ELF	3.5	1.4	6%	100 kV/m	$\ll 1$	ASFETQS**
gyrotron	0.05	0.3	30%	50 kV/m	0.1	ASFETQS**
SP	0.3			100 MV/m	≥ 10	FETQS
Doucas	3			100 V/m	$\gg 10$	FETQS
Laser accel.				>1 GeV/m	≥ 0.1	NIB

* A CO_2 laser with $I = 10^5$ W/cm^2 laser is externally injected to produce stimulated emission 10^9 times stronger than spontaneous emission (Elias et al. 1976). **ASFETQS = Amplified Spontaneous FETQS. E_i = initial kinetic energy. *st. em. = stimulated emission.

laser wave to the electrons) if the beam energy is tuned to a certain energy. If we design the length of the laser cavity such that the electrons emerge from the wiggler cavity before the distribution becomes the latter type, we can always make the beam electrons lase in the cavity. Depending on the initial energy, the opposite can occur (inverse free-electron laser). The claim is correct if there is a laser wave. The reason is that the emission system is not symmetrical about the reflection of the z coordinate, since both the electron beam and the laser wave travel in the positive z direction. Now the problem is: “There is no laser wave at the inlet in a single-pass laser (this logic applies to any feedback laser since there is no laser wave in the beginning). So how can such a classical theory be applied to the single-pass laser”? To this question, the classical FEL theory replies: “Indeed, there is no laser wave at the inlet. However, the laser wave is produced by lasing. Thus, we can apply the classical FEL theory at some distance from the inlet”.

However, we still cannot be satisfied with the classical FEL theory, for the following reason. From the classical viewpoint, the laser wave cannot be idle, that is, it must force the electron to emit. Thus, when a laser wave exists in a cavity, only stimulated emission can occur in the classical concept. In stimulated emission, the electron emits an EM wave whose frequency is the same as the that of the incident EM wave inducing the emission. Therefore, in the quantum mechanical concept, the electron must experience at least one full cycle of the incident EM wave. Since the electron must see that the phase of the incident EM wave varies continuously over at least a full range from 0 to 2π, it cannot say which phase in the range from 0 to 2π is the phase of the incident EM wave, that is, the electron cannot know the phase of the incident EM wave. Since the electron does not know the phase of the incident EM wave, it cannot emit the EM wave whose phase is the same as that of the incident EM wave, and must give off EM waves of random phase.

While the reasoning is different, the same result holds in the framework of the so-called classical FEL concept, as follows: The lasing electrons think that the ponderomotive force forcing them to lase is a constant field (Kim 1992a,b). Since the electrons do not know the phase with respect to the ponderomotive force, they cannot emit electromagnetic waves whose phases are identical with that of the incident ponderomotive force. Thus, the laser wave must have random phase with respect to an electron, and accordingly no net energy transfer is possible between the electrons and the laser wave.

This is the final blow to the classical theory (Kim 1993b). The classical FEL theory still insists on a groundless claim (Marshal 1975). The classical FEL theory says that the above claim is wrong because such a quantum mechanical concept does not apply to the phenomenon exhibited by the classical electrons. It insists that the laser wave preserves its coherence, because the phase of the incident EM wave must be the same as the phase of the induced emission. However, the real ultimate blow to the classical theory was delivered by the present writer by showing that stimulated emission is inherently phase incoherent (Kim 1992a), even by purely classical reasoning. Further, I showed that the preservation of the laser coherence an inherent quantum mechanical phenomenon.

The above paragraph reveals that the classical FEL theory's claim that net energy between the electrons and a laser wave can be explained based on the Lorentz force concept is based on defective logic.

Conclusions

We have found that the magnitude of the DC force due to spontaneous emission induced by the periodic Lorentz force in SP type configurations (Smith and Purcell 1953; Doucas et al. 1992) is much stronger than the amplitude of the periodic Lorentz force which is the only force causing emission in the classical conception. In the classical view, such a force is nothing but the Abraham-Lorentz radiative reaction force which is too weak to be measured with any man-made equipment (Ch. 17 of Jackson 1975).

Since the Lorentz force of an EM wave is oscillatory, net energy transfer between an EM wave and a medium or a long electron beam through the Lorentz force does not exist, even in the general sense. The classical theory has claimed that the net energy transfer can be explained by the Lorentz force concept. However, we have revealed that the classical theory is nothing but a deceptive faith. Thus, we must admit that the heating of food in a microwave oven or the reason why we feel warmth near a stove is due the existence of a force that is not included in the Lorentz force equation.

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