

Classical Quantum Theory

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From the extensive observations and the ideas of Newton and from classical physical optics the velocity of a quantum particle is given by $\mathbf{w} = \mathbf{S}/E$, where \mathbf{S} is the Poynting vector and E the wave energy density. Integrating $\mathbf{w} = d\mathbf{r}/dt$ yields the quantum particle motion along a discrete trajectory as a function of time and initial conditions. All observables are thus precisely predicted. The wave velocity equals the classical particle velocity, where the de Broglie wavelength and the Planck frequency conditions are $\lambda = \hbar/p$ and $\omega = \mathbf{p} \cdot \mathbf{v}/\hbar$. For bound particle motion, the space part of the wave function $\Psi(\mathbf{r}, t) = \psi(\mathbf{r})T(t)$ satisfies the usual time-independent Schroedinger equation, and so the usual eigenvalues are predicted. The reasons the traditional quantum theory is unable to make precise valid predictions in general and the de Broglie-Bohm interpretation are discussed.

1. Why traditional quantum theory makes no precise valid predictions in general

Apart from a few correct empirical formulas, the traditional quantum theory is generally fundamentally wrong (Wesley 1961, 1965, 1983a, 1984, 1988a, 1991, 1994, 1995). The trouble all started when Schroedinger (1926) based his quantum theory on Hamilton's (1834) geometrical optics approximation instead of basing it on exact physical optics, or classical wave theory. Following extensive experimentation involving the interference and diffraction of light, Newton (1730) concluded that photons were essentially point particles that move such as to yield wave phenomena. Newton's original idea can be implemented with the mathematical apparatus of modern wave theory, or physical optics. The geometrical optics approximation, being valid only for situations where the wavelength may be neglected as small, cannot explain interference and diffraction, where the wavelength plays a vital role and cannot be neglected. However, Schroedinger did recognize the need for exact wave behavior to obtain standing-wave eigenvalues for bound atomic systems, as provided by his time independent Schroedinger equation. As a result, the traditional Schroedinger theory is hopelessly inconsistent. Macroscopically, only the geometrical optics approximation is supposed to be valid, where the wavelength can play no role and observations are supposed to be given by averages over many de Broglie wavelengths, while submicroscopically exact wave behavior is supposed to be true, where the admissible wavelengths determine the desired eigenvalues.

Futile attempts have been made to try to reconcile Schroedinger's hopeless inconsistency. To fit the geometrical optics approximation, Schroedinger (1926) proposed a "wave packet" to represent a quantum particle as a huge object smeared out over thousands of de Broglie wavelengths, and so the individual component wavelengths could be neglected as small. No such "wave packet" has, of course, ever been observed, as it is in principle unobservable (Wesley 1961, 1983b, 1991, 1995). Such a huge "wave packet" cannot fit into a tiny submicroscopic atomic system; nor can it account for interference and diffraction, in agreement with

the limitations of the geometrical optics approximation.

Schroedinger also employed the complex representation of a wave, $\exp[i(kx - \omega t)]$, with an irreplaceable $i = \sqrt{-1}$ which has a 2π ambiguity in phase, as required by the phase independent geometrical optics approximation. Born (1926) introduced the mystical notion that the wave function represents an intrinsic "probability amplitude", such that the precise behavior of an individual quantum particle cannot, even in principle, be either observed or predicted! Heisenberg (1927) introduced the "uncertainty principle", which can be shown (Wesley 1995) to be merely a prescription of the limitation in accuracy required that can make the geometrical optics approximation valid. It is traditionally claimed that observables are given by "expectation values" (Schiff 1949), which are integral averages over all space that eliminate detailed prediction, in agreement with the rough geometrical optics approximation—but, of course, not in agreement with actual detailed observations involving interference and the wavelength.

To try to justify these futile attempts to resolve Schroedinger's original inconsistency, a huge amount of further nonsense has been generated: collapsing wave packets, the nonexistence of nonobserved states, thought experiments in lieu of actual experiments, self interference of a single quantum particle, superposition of wave amplitudes representing the superposition of physical states, the operator approach, the Hilbert-space approach, the von Neumann theorem, wave particle duality, the complementary principle, the claimed violation of Bell's theorem (Wesley 1994), Schroedinger's (1926) time dependent equation with its ambiguous irreplaceable $i = \sqrt{-1}$, etc., etc.

2. An adequate quantum theory must explain interference

As mentioned above, the Schroedinger (1926) quantum theory, being based upon the geometrical optics approximation of Hamilton, cannot explain interference, where the wavelength plays a vital role and cannot be neglected.

Newton (1730) explained his observations of the interference and diffraction of light in terms of the wavelength (equal

to two of Newton's "fits"). He assumed that photons moved such as to yield this wave behavior. To account for all of Newton's detailed observations, a photon must be regarded as essentially a point particle. The mathematical aspects of the wave behavior of light were developed by Fresnel (1826). This field of study is now called "physical optics". Although the photon aspects of light were largely neglected or even denied in the 1800's, it is now known that Newton was right: light consists of a flux of photons that exhibit wave behavior.

An adequate quantum theory must yield the extremely successful results predicted by physical optics, or classical wave theory, which can be derived from a prescription for the precise behavior of photons, as well as the precise behavior of phonons. Most of the empirical data available that needs to be explained by an adequate quantum theory is provided by the macroscopic observations of classical waves. The denumerable sets of eigenvalues provided by standing waves in bound submicroscopic atomic systems represent a relatively smaller amount of empirical information. In order to fit all of the empirical evidence available, both macroscopic and atomic, the theory presented here is based directly upon exact classical physical optics instead of the rough geometrical optics approximation of the traditional Schroedinger theory.

3. The Wesley wave

An aspect of light unknown to Newton and Fresnel and first discovered by Planck (1901) was its quantization in energy units of

$$E = h\nu = \hbar\omega, \quad (1)$$

where h is Planck's constant, $\hbar = h/2\pi$, and $\omega = 2\pi\nu$ is the angular frequency. The propagation constant k is then also quantized in momentum units given by $p = E/c = \hbar\omega/c = \hbar k/c^2$, or

$$p = \hbar k. \quad (2)$$

de Broglie (1923, 1924) speculated that particles with a nonzero rest mass should also exhibit wave behavior, where the propagation constant (or wavelength $\lambda = 2\pi/k$) is also given by Eq. (2), the so-called de Broglie wavelength condition. His conjecture was confirmed in the laboratory.

Since a classical wave is empirically defined by a flux of quantum particles, the wave velocity u and the particle velocity v must be identical for a free-space wave, or

$$u = v = \frac{\hbar\omega}{\hbar k} = \frac{\omega}{k^2}. \quad (3)$$

From Eq. (2) the angular frequency then becomes, in general, for particles with zero or nonzero rest mass

$$\omega = \frac{\mathbf{p} \cdot \mathbf{v}}{\hbar}. \quad (4)$$

This result (4) is seen to be the Planck frequency condition (1), where for photons $v = c$ and $p = E/c = \hbar\omega/c^2$. The empirically correct free space wave for any quantum particle is then the Wesley (1962, 1965, 1983c) wave,

$$\Psi = \sin \left[\frac{\mathbf{p} \cdot (\mathbf{r} - \mathbf{vt})}{\hbar} \right]. \quad (5)$$

For a slow particle with a nonzero rest mass, the frequency from Eq. (5) or (4) is quantized in units of twice the kinetic energy W ; thus,

$$\omega = \frac{2W}{\hbar}, \quad (6)$$

which may be contrasted with the traditional theory where ω is supposed to be either the total classical energy or, inconsistently, the total energy including the rest energy mc^2 . This empirically correct Wesley wave, Eq. (5), may be compared (Wesley 1965, 1983d) with the physically impossible traditional de Broglie (1923, 1924) wave, where the frequency for a slow particle of nonzero rest mass is supposed to be $\gamma mc^2/\hbar \approx mc^2/\hbar$, and the phase velocity is then supposed to be $u = c^2/v > c$.

4. Classical quantum particle trajectories

Since quantum particles must carry the energy and momentum, and the classical Poynting vector \mathbf{P} prescribes the energy and momentum flux in a wave, quantum particles must necessarily move along the lines of the Poynting vector to yield wave phenomena. If the wave energy density E is the energy of the quantum particles that produce the wave, then the quantum particle velocity \mathbf{w} is given by

$$\mathbf{w} = \frac{\mathbf{S}}{E}, \quad (7)$$

in agreement with Eq. (5) for a free quantum particle.

A classical scalar wave function Ψ satisfies the wave equation

$$\nabla^2 \Psi - \frac{\partial^2 \Psi / \partial t^2}{u^2} = 0, \quad (8)$$

where u is the phase velocity, subject to appropriate boundary conditions. The classical Poynting vector and energy density for a scalar wave are then given by

$$\mathbf{S} = -K \nabla \Psi \frac{\partial \Psi}{\partial t}, \quad E = K \left[\frac{(\nabla \Psi)^2}{2} + \frac{(\partial \Psi / \partial t)^2}{2u^2} \right], \quad (9)$$

where the appropriate constant K depends upon the amplitude constant chosen for Ψ . If $E/\hbar\omega = P$ is to represent the quantum particle density and $\mathbf{S}/\hbar\omega = \mathbf{G}$ the quantum particle flux, then the constant K must be chosen as

$$K = \frac{\hbar\omega}{k^2}. \quad (10)$$

Light can be treated as a scalar wave. Polarization phenomena can be handled with two such scalar waves (Wesley 1983e). The scalar theory given by Eqs. (7), (8), and (9) may thus be regarded as perfectly general and valid for all quantum particles.

Once the solution Ψ to the wave equation (8) is known as a function of position and time, the motion of a quantum particle along a discrete trajectory as a function of time and the initial conditions are given by integrating Eq. (7), where $\mathbf{w} = d\mathbf{r}/dt$, thus,

$$\mathbf{r}[\mathbf{r}_0, t] = \mathbf{r}_0 + \int_0^t \mathbf{w}(\mathbf{r}, t) dt. \quad (11)$$

Once the precise quantum particle motion is known, all possible observables can be calculated and precisely predicted. For example, the force acting on a slow quantum particle of nonzero rest mass m that gives rise to the motion is given by

$$\mathbf{F} = m \frac{d\mathbf{w}}{dt}, \quad (12)$$

where the velocity \mathbf{w} is given by Eqs. (7) and (9), expressed as a function of time, using Eq. (11). Similarly, the potential that acts in a conservative situation is given by the total energy minus the kinetic energy expressed as a function of position only, using Eq. (11); thus,

$$U = E - \frac{mw^2}{2}. \quad (13)$$

5. Time-average classical quantum particle trajectories

For light, the time variation is harmonic and rapid, a period being of the order of 10^{-15} s; most macroscopic observations of interest are time-average observations. In this case it is of value to define a time-average quantum particle velocity $\bar{\mathbf{w}}$ as

$$\bar{\mathbf{w}} = \frac{\bar{\mathbf{S}}}{\bar{E}}, \quad (14)$$

where $\bar{\mathbf{S}}$ and \bar{E} are given by averaging Eqs. (9) over a cycle. It may be readily shown that the instantaneous trajectories given by Eq. (7) can differ from the time-average approximation, Eq. (14), by at most a quarter wavelength, which is a negligibly small deviation for macroscopic observations (Wesley 1983f).

A specific example is indicated in Fig. 1 for the classical time-average flux of coherent quantum particles passing through Young's double pinholes to produce the famous double pinhole interference pattern (Wesley 1983g, 1984, 1991). This figure was obtained by displaying the slopes of the time-average Poynting vector $\bar{\mathbf{S}}$ on a grid and sketching the trajectories in by eye so that the quantum particles emanate isotropically from each pinhole.

It must be stressed that the approximation $\bar{\mathbf{w}}$, Eq. (14), which is excellent for macroscopic time-average quantum particle motion, is not at all sufficient for submicroscopic atomic systems, where neither the period nor the wavelength can be regarded as small. It cannot be used, for example, to describe the motion of the electron in the hydrogen atom.

6. Arbitrary flux and density of the traditional theory

In contrast to the present classical quantum theory, which is based directly upon the classical empirical evidence provided by Newton and classical physical optics, the particle flux and density of the Schroedinger (1926) theory is based upon theoretical speculation only. As a consequence it is difficult to discover any empirical merit for his arbitrary specification. In terms of his unfortunate improper use of complex notation involving an irreplaceable $i = \sqrt{-1}$, Schroedinger's (1926) "relativistic", particle flux and density (where for photons

and phonons $mc^2 = \hbar\omega$) are given by

$$\mathbf{G}' = \frac{\hbar}{2im} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*), \quad P' = \frac{i}{2\omega} \left(\Psi \frac{\partial \Psi^*}{\partial t} - \Psi^* \frac{\partial \Psi}{\partial t} \right) = \Psi \Psi^*, \quad (15)$$

where the time variation is always prescribed by the time harmonic factor $\exp(-i\omega t)$, in which ω is a constant.

Clearly Schroedinger's arbitrary prescription (15) cannot agree with the correct empirically observed instantaneous flux and density given by the classical expressions (9) (for the appropriate choice of the constant K). The traditional prescription (15) is not a function of the time; it can, at best, only represent a steady state flow for time harmonic waves averaged over a period.

Classically, only pure real wave functions are admissible, and if complex notation is used, they must be reducible to a pure real result. A pure real time harmonic function f can be expressed as the real part of a complex function $\Psi = \psi(\mathbf{r}) \exp(-i\omega t)$; thus,

$$f = A(\mathbf{r}) \cos \omega t + B(\mathbf{r}) \sin \omega t = \text{Re}[\psi(\mathbf{r}) \exp(-i\omega t)] = \frac{\Psi + \Psi^*}{2}, \quad (16)$$

where $\psi(\mathbf{r}) = A(\mathbf{r}) + iB(\mathbf{r})$. The time average of a product of two real time harmonic functions f_1 and f_2 of the same angular frequency is then given by

$$\overline{f_1 f_2} = \frac{\Psi_1 \Psi_2^* + \Psi_1^* \Psi_2}{4}. \quad (17)$$

From Eqs. (16), (17), (9), and (10) the empirically correct classical time-average quantum particle flux and density in correct complex notation are given by

$$\mathbf{G} = \frac{\hbar}{2im} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*), \quad P = \frac{\nabla \Psi^* \nabla \Psi + k^2 \Psi \Psi^*}{2k^2}, \quad (18)$$

which may be compared with the arbitrary traditional flux and density, Eq. (15). The Schroedinger flux \mathbf{G}' is seen to agree with the empirically observed classical time-average flux \mathbf{G} for time harmonic waves. But the Schroedinger density P' does not agree with the observed classical time-average density P , except for the trivial case of a free particle, where $\Psi = \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$.

The fact that the Schroedinger flux happens to agree with the correct classical time-average Poynting vector for time harmonic waves does not mean the traditional theory is thereby rescued, because: 1) The traditional flux, \mathbf{G}' , the first of Eqs. (15), is supposed to be the instantaneous flux, which does not agree with the correct instantaneous flux given by the first of Eqs. (9) for real Ψ functions. 2) Since the Schroedinger specification (15) assumes only time harmonic waves; it cannot yield the quantum particle flow for transient waves. In contrast, the correct classical specification (9) yields particle flow for transient waves, as well as for time harmonic waves. 3) The traditional specification (15) is assumed to be valid for submicroscopic atomic systems, which led Schroedinger to make the impossible claim that bound systems are devoid of any motion what-so-ever. 4) In

the Schroedinger theory \mathbf{G}' , Eq. (15), is supposed to be exact without any approximation being involved; but the time-average of the correct classical flux \mathbf{G} , Eq. (9) (that equals \mathbf{G}'), for time harmonic waves is an approximation that cannot agree with the exact instantaneous flux. 5) The traditional Copenhagen interpretation claims that the flow lines defined by Schroedinger's flux, \mathbf{G}' have no physical meaning; because quantum particles are not supposed to follow precise (or even approximate) trajectories.

7. Bohm's interpretation of the traditional theory

Madelung (1926), de Broglie (1927) and Bohm (1952, Bohm and Hiley 1993) tried to resolve Schroedinger's original inconsistency by proposing that quantum particles follow discrete trajectories given by integrating a particle velocity \mathbf{w}' defined by

$$\mathbf{p} = m\mathbf{w}' = -i\hbar\nabla \ln \Psi, \quad (19)$$

where Ψ is the usual traditional complex wave function. Since the particle velocity must be pure real, the complex conjugate of the right side of Eq. (19) must yield the same particle velocity. Hence, Eq. (19) can be written in the symmetrical form

$$\mathbf{w}' = \frac{\hbar}{2im} (\nabla \ln \Psi - \nabla \ln \Psi^*) = \frac{\hbar}{2im} \cdot \frac{\Psi^* \nabla \Psi - \Psi \nabla \Psi^*}{\Psi \Psi^*}. \quad (20)$$

Making the replacement $mc^2 = \hbar\omega$, Eq. (20) is seen to be simply the particle velocity expected from Schroedinger's flux and density, Eq. (15), for time harmonic solutions. In particular,

$$\mathbf{w}' = \frac{\mathbf{G}'}{P'}. \quad (21)$$

Thus, the de Broglie-Bohm theory offers nothing new. It merely says that Schroedinger's arbitrary flow lines, \mathbf{G}' , should be taken seriously and should be accepted as actual quantum particle trajectories.

Although Eq. (19) or (21) yields the correct time-average approximate trajectories for macroscopic observations for time harmonic waves; it does not give the correct magnitude of the particle velocity along these trajectories; because the correct particle density P , the second of Eqs. (18), does not equal $P' = \Psi\Psi^*$. Moreover, the particle velocity as prescribed by Eq. (19) or (21) cannot, contrary to Bohm's claims, yield the correct trajectories, nor the correct velocities for submicroscopic atomic systems where the time-average approximation is not admissible. Like Schroedinger, Bohm must make the impossible claim that bound atomic systems are necessarily completely motionless.

The Bohm interpretation of the traditional quantum theory, thus, merely serves to make Schroedinger's original inconsistency painfully obvious without, however, resolving it. The present classical quantum theory exhibits no such inconsistency.

8. Classical quantum theory for bound systems

To satisfy boundary conditions, a standing wave solution $\Psi(\mathbf{r}, t)$ to the wave equation (8) appropriate for bound quan-

tum particle motion must be separable as a product of a function of position only $\psi(\mathbf{r})$ times a function of the time only $T(t)$; thus,

$$\Psi(\mathbf{r}, t) = \psi(\mathbf{r})T(t). \quad (22)$$

Substituting Eq. (22) into (8) then yields two separated equations

$$\nabla^2\psi + k^2\psi = 0, \quad \frac{\partial^2 T}{\partial t^2} + \omega^2 T = 0, \quad (23)$$

where the phase velocity squared is given by

$$u^2 = \frac{\omega^2}{k^2}. \quad (24)$$

To solve these Eqs. (23), k^2 must be expressed as a function of position only and ω^2 must be expressed as a function of time only. Since for classical macroscopic observations a free-traveling wave travels with the quantum particle and the phase velocity equals the particle velocity, the phase velocity for bound particle motion should also be taken as the classical particle velocity (ignoring possible quantum wave effects). Because energy is conserved for bound particle motion, the phase velocity squared that occurs in the wave equation can be expressed in terms of the energy. For a slow particle of nonzero rest mass, using the de Broglie wavelength condition (2), the propagation constant squared k^2 to be used in the first of Eqs. (23) becomes

$$k^2 = \frac{p^2}{\hbar^2} = \frac{2m[E - V(\mathbf{r})]}{\hbar^2}, \quad (25)$$

the kinetic energy being $p^2/2m$, and where E is the total constant energy. Here, k^2 , Eq. (25), is seen to be expressed as a function of the position only, as desired.

Combining the first of Eqs. (23) and (25) yields the fruitful time independent Schroedinger equation,

$$\hbar^2 \nabla^2 \psi + 2m[E - V(\mathbf{r})]\psi = 0. \quad (26)$$

From the Planck frequency condition (4), ω^2 to be used in the second of Eqs. (23) becomes

$$\omega^2 = \frac{(\mathbf{p} \cdot \mathbf{v})^2}{\hbar^2} = \frac{4W^2(t)}{\hbar^2}, \quad (27)$$

where the kinetic energy W is to be expressed as a function of the time only. The second of Eqs. (23) becomes

$$\frac{d^2 T}{dt^2} + \frac{4W^2(t)}{\hbar^2} T = 0. \quad (28)$$

Consistent with the differential equation for the space part, the Schroedinger equation (26), $W(t)$ is to be taken as the classical kinetic energy expressed as a function of the time only.

For fast particles, where the momentum \mathbf{p} and kinetic energy W are given by

$$\mathbf{p} = m\gamma\mathbf{v}, \quad W = E^* - V = mc^2\gamma, \quad (29)$$

where E^* is the total constant energy including the rest energy, the appropriate Eqs. (23) from (2) and (4) become

$$\hbar^2 c^2 \nabla^2 \psi + [(E^* - V)^2 - m^2 c^4] \psi = 0, \quad \frac{\partial^2 T}{\partial t^2} + \frac{W^2 - m^2 c^4}{\hbar^2} T = 0 \quad (30)$$

where W in the second of Eqs. (30) is to be expressed as a function of the time only. The first of these Eqs. (30) is Schroedinger's "relativistic" wave equation.

To derive the motion for a bound quantum particle, the classical problem, independent of possible quantum wave effects, must first be solved. Second, the resulting classical expressions for the kinetic energy $W = E - V$ (or else $E^* - V$) expressed as a function of position and also as a function of time only are then to be substituted into the appropriate differential equations (26) and (28) (or else Eqs. (30)). Third, after these differential equations are solved, $\Psi = \psi(\mathbf{r})T(t)$ is substituted into Eqs. (9) and (7) for the quantum particle velocity \mathbf{w} , which includes both classical and quantum effects. Fourth, Eq. (7) is integrated to yield the desired quantum particle motion along discrete trajectories as a function of time and initial conditions.

The approximate time average velocity $\bar{\mathbf{w}}$, Eq. (14), and the approximate trajectories obtained by integration, which are appropriate for macroscopic observations of time harmonic waves, cannot be used for bound particle motion, because the wavelength involved cannot be regarded as small.

In the traditional Schroedinger theory the variation with time is assumed to be nonexistent for bound particles. The function $T(t)$ must be assumed traditionally to be a constant. This peculiar idea results from Schroedinger's improper use of complex notation to try to represent pure real physical phenomena.

Some explicit examples of bound particle motion prescribed by the present classical quantum theory may be found elsewhere (Wesley 1983h, 1984, 1991).

9. Discussion

The classical quantum theory presented here is able to successfully account for a vast amount of empirical evidence that cannot be explained by the traditional Schroedinger quantum theory. The classical quantum theory readily reveals the precise motion of quantum particles along discrete trajectories that yield interference and transient wave phenomena. It indicates, in terms of the initial positions of photons in the incident beam, which photons are transmitted and which are reflected at a dielectric interface (Wesley 1988), no intrinsic probabilities being involved. The idea of inherent unpredictability, which the traditional quantum theory needs to excuse its inability to make precise valid predictions, is not a feature of the precise classical quantum theory.

Despite the extreme superiority of the classical quantum theory presented here, it seems to lack a truly fundamental character. To a first approximation, the classical motion without quantum wave effects is derived. To a second approximation, quantum wave effects are generated from the first approximation. It then appears that quantum (or wave)

behavior may be merely a superficial effect superposed upon the classical motion. It would seem that perhaps a third approximation might be needed, or, of course, a truly fundamental theory.

The arrogant ideas of Heisenberg, Born, Dirac, Bohr, and others that the Copenhagen quantum theory represents a new fundamental science or "quantum mechanics" is hardly justified. The observation of quantum phenomena is not new. The general ideas of how quantum particles move to produce interference and diffraction were already investigated by Newton 300 years ago.

The underlying cause of the wave behavior exhibited by quantum particles remains obscure. The classical quantum theory presented here is able to show how quantum particles move to exhibit interference and wave phenomena; but it does not say why quantum particles should move in this fashion.

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