A Fractal Universe with Discrete Spatial Scales: In Memory of Toivo Jaakkola

D.F. Roscoe
School of Mathematics, Sheffield University, Sheffield, S3 7RH, UK.
Email: D.Roscoe@ac.shef.uk

Abstract

The work of this paper is based on work which has been described in a preliminary form elsewhere (Roscoe 1995), and it applies the formalism developed there to the problem of deriving the cosmology for a universe which is in a state of gravitational equilibrium. It predicts that, in such a universe, material is distributed in a fractal fashion with fractal dimension two whilst redshifts necessarily occur in integer multiples of a basic unit and, given a certain model for light propagation, the measured magnitudes of peculiar velocities will increase in direct proportion to cosmological redshift.

The first of these predictions is strongly supported by the results of the most modern pencil-beam and wide-angle surveys, whilst the second conforms with the results of very recent rigorous analyses of accurately measured redshifts of nearby spiral galaxies and the third is in qualitative agreement with the very limited data available. The observational support for these predictions is described in detail in the text.

Toivo Jaakkola was convinced that all the evidence supported the idea of an infinite self-sustaining equilibrium universe. Many of us agreed with some of his arguments, and I was no exception. The following article falls into this pattern: it describes a self-sustaining equilibrium universe; but infinite it is not, and homogeneous it is not.

1. Introduction

The following work describes the application of the gravitation theory described earlier (Roscoe 1995) to the problem of deriving a cosmology. This latter presentation is a preliminary and incomplete development of work now completed, and in preparation for publication elsewhere. Preprints are available on request. The underlying gravitation theory, which is predicated upon the idea of a discrete and finite model universe, is distinguished in the fact that, according to it, concepts of spatial and temporal measurement are undefined in the absence of mass; in this sense, it conforms to the strictest possible interpretation of Mach's Principle.

There is evidence, discussed in §2, to suggest the real Universe is in a state of approximate thermodynamic equilibrium; this possible state is used to justify the cosmological principle that the model universe is in a state of exact gravitational equilibrium. The mass-distribution corresponding to this state is calculated in §3 and §4, and is found to be fractal with a fractal dimension of two. This mass-distribution prediction is very strongly supported by the results of several modern surveys, and this evidence is discussed in §5.

The discrete nature of material in the model universe is considered in §6, and is found to imply a discretization of distance scales which leads, in §7, to the conclusion that redshifts must increase in integer multiples of a basic unit; the evidence supporting this is discussed in §8. The discretization of distance scales occurs in such a way that spatial and temporal measurement scales in remote localities undergo systematic change, discussed in §9, which has implications for kinematics and the nature of light, discussed in §10 and §11 respectively.

The predicted kinematics has implications for the apparent behaviour of the peculiar velocities of galaxies; these are discussed in §12 where it is shown how one consequence of the scale-change phenomenon is that the estimated magnitudes of peculiar velocities will appear to vary linearly with the cosmological redshift. The evidence supporting this conclusion is discussed in §13. The discussion of §3 also leads to the idea of a material vacuum, existing in the model universe, and the implications of this are briefly considered in §14.

The equations of motion, derived for a spherically symmetric distribution of material particles, with an isotropic velocity distribution, are given by

$$\ddot{r} = -\frac{\partial V}{\partial r}\hat{r}, \quad V \equiv -\frac{1}{2}\langle \dot{r}, \dot{r}\rangle = -\frac{r_0 g A}{2} + \frac{B}{2A}\dot{\Phi}^2 \qquad (1)$$

where r is the position vector defined with respect to the global mass-centre, g is the gravitational constant, r_0 is a constant defined below, and A, B are defined by

$$A \equiv \frac{M}{\Phi}, \quad B \equiv -\left[\frac{M}{2\Phi^2} - \frac{M'M'}{2a_0 M}\right], \quad M' \equiv \frac{dM}{d\Phi} \qquad (2)$$

with $\Phi = \frac{1}{2}\langle r, r\rangle$. The function M is the mass-distribution function, for which a broad admissible class is given by

$$M(r) = m_0\left(\frac{r}{r_0}\right)^g \qquad (3)$$

where m_0 has dimensions of mass, and r_0 is the radius of the volume containing mass m_0. It is to be noted from this expression that M/r^g is a global constant, so that the particular choice of r_0 has no significance for (3). Finally, the defining relationship between time scales and distance scales is given by

$$dt^2 = \frac{\Phi^2}{r_0 g M^2} g_{ij} dx^i dx^j \tag{4}$$

whilst the metric tensor is given by

$$g_{ab} = A\delta_{ab} + Bx^a x^b \tag{5}$$

It follows from (2), (4) and (5) that if $M = 0$ so that there is no mass, then concepts of time and distance are undefined. The foregoing equations of motion can be identified with those given in Roscoe (1995) by making the substitution $M = aU$. It is to be noted that the potential form of the equations is not given in this early development, nor is the interpretation of $M \equiv aU$ as a mass-distribution made there. Preprints of the complete development are available on request.

2. A Simple Cosmological Principle

There is some evidence, briefly discussed below, which suggests the observable universe might be in a state of approximate thermodynamic equilibrium with respect to the various energy sources within it. If this is the case, it would follow that gravitational energy must be included as one of these sources; correspondingly, the most simple realistic cosmological principle applicable to the model universe is the condition that it is in a state of *gravitational equilibrium*.

However, before the consequences of this most simple possible of cosmological principles are worked through, we shall consider some of the evidence supporting the argument that the cosmic ray flux, the cosmic background radiation (both extragalactic sources) and our own galaxy's starlight field are in thermal equilibrium. We are indebted to Assis and Neves (1995) for much of the following discussion.

One of the earliest (if not the earliest) predictions of a background temperature to space, and estimations thereof, is that of Guillaume (1896) who used Stefan's Law (1879) to calculate the equilibrium temperature, arising from stellar radiation, of an inert body placed in the interstellar space of contemporary understanding; this was equivalent to calculating the 'temperature of space', and the figure arrived at was 5.6 K. A similar black-body calculation was given by Eddington in 1926 (reprint 1988), and he arrived at the figure 3.18 K, calling it explicitly the 'temperature of interstellar space'.

It was known by 1928 (Millikan & Cameron) that cosmic rays have an extragalactic origin and, subsequently, Regener (1933 or 1995 for an English translation) calculated the equilibrium temperature of an inert body (having the necessary dimensions to absorb cosmic rays) which is placed in a 'sea' of cosmic radiation, and found this to be 2.8 K. Regener went on to argue that, because of the extragalactic origin of cosmic rays, and because of the (assumed) extreme weakness of starlight in inter-galactic space, then 2.8 K must be the 'temperature of intergalactic space'.

The earliest Hot Big Bang predictions for the existence of the CBR with a black-body spectrum were given by Alpher & Herman (1949) and Gamow (1952, 1953), and these authors estimated the 'temperature of space' variously in the range 5 K to 50 K. After the observations of Penzias & Wilson (1965), we are now aware that the CBR does exist as an additional extragalactic energy field, with a temperature of 2.7 K.

So, there are at least three independent sources of energy - galactic starlight, cosmic rays and the CBR - which have been used to estimate the 'temperature of space', giving answers which suggest that the three sources are in near thermodynamic equilibrium. In addition, Sciama (1971) has pointed out that the turbulent energy density of interstellar gases and the energy density of the interstellar magnetic field is similar to that of the aforementioned sources, and so the net picture is entirely consistent with the idea of a universe which is in an approximate thermodynamic equilibrium.

3. The Equilibrium Universe

If the model universe is in gravitational equilibrium, then the net gravitational force at every point within it is necessarily zero, so that $\ddot{r} = 0$ everywhere. Consequently, the potential is constant everywhere so that, by (1),

$$-\frac{r_0 g A}{2} + \frac{B}{2A}\dot{\Phi}^2 = V_0 \tag{6}$$

where V_0 is the value of the constant potential. Using the definitions of A, B given at (2), this equation can be written as

$$\frac{r_0 g M}{r^2} - \frac{v^2}{2}\left[1 - \frac{1}{4a_0}\left(\frac{r}{M}\frac{dM}{dr}\right)^2\right] = V_0 \tag{7}$$

An easy means of solving this equation is arrived at as follows: The equation gives the form of $M(r)$ which is consistent with the constraint $\ddot{r} = 0$ for all motions in the model universe. Of all possible trajectories of this type, there will be a subclass which pass directly through the centre-of-mass, and will therefore have zero angular momentum about this point. These particular trajectories satisfy $\dot{r} =$ constant where, because the speed of the particle concerned is arbitrary, then constant is arbitrary; consequently, these trajectories can be considered specified by $\dot{r}^2 = 2I_1$, for arbitrary positive values of I_1. The above equation for $M(r)$ can be now written

$$\left(\frac{r_0 g M}{r^2} - V_0\right) - I_1\left[1 - \frac{1}{4a_0}\left(\frac{r}{M}\frac{dM}{dr}\right)^2\right] = 0 \tag{8}$$

Since I_1 is simply a measure of an arbitrary constant speed, then this equation must be decomposable into

$$\frac{r_0 g M}{r^2} - V_0 = 0 \quad \text{and} \quad 1 - \frac{1}{4a_0}\left(\frac{r}{M}\frac{dM}{dr}\right)^2 = 0 \tag{9}$$

According to the first of these equations,

$$M(r) = -\frac{V_0 r_0}{g}\left(\frac{r}{r_0}\right)^2 \tag{10}$$

which satisfies the second equation if $a_0 = 1$. This solution is a special case of the more general admissible form (3) so that, finally, the mass-distribution function appropriate to an equilibrium model universe is

$$M(r) = m_0\left(\frac{r}{r_0}\right)^2 \tag{11}$$

where, by comparing the two forms of $M(r)$, the value of the constant potential is found to be given by

$$V_0 = -\frac{gm_0}{r_0} \tag{12}$$

Since m_0 in (8), has dimensions of mass, it must be interpreted as the amount of mass contained in a sphere of arbitrarily chosen radius r_0. It is to be noted that the definitive constant value - lacking all arbitrariness - given to the constant potential in the present equilibrium case can only be interpreted to represent some kind of absolute ground state energy, or vacuum energy, associated with the system.

Finally, if (7) is compared with (6), it can be seen how the second of (7) is equivalent to $B = 0$ so that, with (2), (5) and (8), the metric tensor for the equilibrium universe is given as

$$g_{ab} = \frac{M}{\Phi}\delta_{ab} = \frac{2m_0}{r_0^2}\delta_{ab} \tag{13}$$

4. The Model Fractal Universe

The equilibrium model universe is characterized by $\ddot{r} = 0$ which means that all points in the space are dynamically equivalent. Consequently, there is no dynamical experiment in the space which can distinguish between any pair of points, and hence there is no way of determining the position of a global mass-centre. Since a unique origin for the mass distribution (8) cannot now be defined, then it must be considered true about arbitrarily chosen origins in the space, and this amounts to the statement that mass is distributed in a self-similar, or fractal, fashion with a fractal dimension of two.

A direct corollary of this argument is the fact that, if $M(r)$ has any form, other than (8), then potential gradients must exist, so that $\ddot{r} \neq 0$ necessarily. As a consequence, it becomes possible to determine a unique global-mass centre and so the corresponding $M(r)$ cannot be describing a fractal distribution of mass, since such distributions are necessarily isotropic about all points in the space. So, in conclusion, the only possible fractal distribution of mass in the model universe is the one which has fractal dimension two.

5. A Fractal Universe, The Evidence

A basic assumption of the Standard Model is that, on some scale, the universe is homogeneous; however, in early responses to suspicions that the accruing data was more consistent with Charlier's conceptions of an hierarchical universe (Charlier 1908, 1922, 1924) than with the requirements of the Standard Model, de Vaucouleurs (1970) showed that, within wide limits, the available data satisfied a mass distribution law $M(r) \approx r^{1.3}$ whilst Peebles (1980) found $M(r) \approx r^{1.23}$. The situation, from the point of view of the Standard Model, has continued to deteriorate with the growth of the data-base to the point that (Baryshev et al. 1995):

> ...the scale of the largest inhomogeneities (discovered to date) is comparable with the extent of the surveys, so that the largest known structures are limited by the boundaries of the survey in which they are detected.

For example, several recent redshift surveys, such as those performed by Huchra et al. (1983), Giovanelli and Haynes (1986), De Lapparent et al. (1988), Broadhurst et al. (1990), Da Costa et al. (1994) and Vettolani et al. (1994) etc. have discovered massive structures such as sheets, filaments, superclusters and voids, and show that large structures are common features of the observable universe; the most significant conclusion to be drawn from all of these surveys is that the scale of the largest inhomogeneities observed is comparable with the spatial extent of the surveys themselves. So, to date, evidence that the assumption of homogeneity in the universe is realistic does not exist.

By contrast, evidence for the fractal nature of the matter distribution is becoming increasingly strong; for example, Coleman et al. (1988) analysed the CfA1 redshift survey of Huchra et al. (1983), and found $M(r) \propto r^{1.4}$ for this sample; subsequently, the CfA2 survey of Da Costa et al. (1994), which is an extension of the CfA1 survey out to about twice the depth, has been analysed by Pietronero and Sylos Labini (1995) to reveal $M(r) \propto r^{1.9}$. The pencil beam survey data accumulated in ESO Slice Project (Vettolani et al. 1994), which reaches out to 800 Mpc, has been similarly analysed (Pietronero and Sylos Labini (1994)) to conclude that, within this data, the distribution of galaxies conforms to the fractal law $M(r) \propto r^2$ up to the sample limits and, according to Baryshev et al. (1995), this same result of fractal distribution of dimension ≈ 2 has been found in the analysis of other deep redshift surveys such as those of Guzzo et al. (1992) and Moore et al. (1994).

To summarize, for more than twenty years evidence has been accumulating that material in the universe appears to be distributed in an hierarchical, or fractal way in direct opposition to the requirements of the Standard Model and the results of the most modern deep and wide angle surveys are consistent in suggesting the distribution law $M(r) \propto r^2$, valid about arbitrarily chosen centres. This empirical law, which describes a self-similar mass distribution of fractal dimension two, is in direct conformity with the mass distribution law derived, for a universe in gravitational equilibrium, in this paper.

6. Discrete Mass Implies Discretized Distance Scales

The model universe was defined, in the first instance, to consist of a finite amount of discrete material, and it was the finite quality which allowed the definition of the global mass-centre, and hence enabled the theory to be developed as it has been; in the following, the discrete quality of mass in the model universe is considered, and shown to imply the discretization of distance scales. At first sight, this seems to be a surprising conclusion but, when it is remembered that, according to the theory, concepts of space and time cannot be formulated in the absence of mass, then it appears reasonable to expect that a discrete matter distribution must imply discrete space.

We begin by considering the mass distribution function which, according to (3), is given by

$$M(r) = m_0 \left(\frac{r}{r_0}\right)^g \tag{14}$$

When g is real then $M(r)$ varies continuously through real values with r, and so the discrete quality of the model universe cannot be made manifest in this case. However, the analysis which gave rise to the foregoing expression for $M(r)$ does not exclude the possibility of g assuming complex values so that, with g written as explicitly complex, the most general expression of $M(r)$ is

$$M(r) = m_0 \left(\frac{r}{r_0}\right)^{g_1+ig_2} \tag{15}$$

for real g_1 and g_2. The function $M(r)$ now only takes real positive values at the set of discrete points

$$r_k = r_0 \exp\left\{\frac{2k\pi}{g_2}\right\}, \quad k = 0, \pm 1, \pm 2, ... \tag{16}$$

and so, from point to point, $M(r)$ varies discretely over real values as required for the model universe. It follows that, for perfect rigour, the whole analysis to this point should be recast from a continuum form into a discrete form, where r is discretized according to (10). However, for the sake of brevity and convenience, the discrete analysis will only be applied from (7) onwards.

Defining the derivative in (7) in terms of differences, according to

$$\frac{dM}{dr} \equiv \frac{M_k - M_{k-1}}{r_k - r_{k-1}} \tag{17}$$

and using (10), the first of (7) is found to be satisfied by

$$M_k \equiv M(r_k) = -\frac{V_0 r_k}{g}\left(\frac{r_k}{r_0}\right)^2 \tag{18}$$

whilst the second of (7) is found to be satisfied only when

$$4a_0 = \frac{R}{1 + \exp\left\{-\frac{2\pi}{g_2}\right\}} \tag{19}$$

Notice that, according to (10), there is no such thing as an origin $r = 0$; it then becomes natural to interpret r_0 as a form of 'reference surface' from which displacements are calculated. In this case, (10) gives, for the value of non-negative displacements,

$$\Delta_k \equiv r_k - r_0 = r_0\left[\exp\left(\frac{2k\pi}{g_2}\right) - 1\right] \tag{20}$$

[Publisher's Note: The full text of the original document beyond this point was not fully recoverable. The references listed below were reconstructed from the available metadata and snippets.]

References

- Alpher, R.A., Herman, R.C. 1949. Phys. Rev. 75, 1089.
- Assis, A.K.T., Neves, M.C.D. 1995. Apeiron, 2, 79.
- Baryshev, Y., Sylos Labini, F., Montuori, M., Pietronero, L. 1995. Vistas in Astronomy 38, 419.
- Broadhurst, T.J., Ellis, R.S., Koo, D.C., Szalay, A.S. 1990. Nature 343, 726.
- Charlier, C.V.L. 1908. Arkiv. Math. Astr. Fys. 4, 24.
- Charlier, C.V.L. 1922. Arkiv. Math. Astr. Fys. 16, 22.
- Coleman, P.H., Pietronero, L., Sanders, R.H. 1988. Astron. Astrophys. 200, L32.
- Da Costa, L.N. et al. 1994. Astrophys. J. 424, L1.
- De Lapparent, V., Geller, M.J., Huchra, J.P. 1988. Astrophys. J. 332, 44.
- de Vaucouleurs, G. 1970. Science 167, 1203.
- Eddington, A.S. 1926 (1988). The Internal Constitution of the Stars, CUP.
- Gamow, G. 1952. Phys. Rev. 86, 251.
- Giovanelli, R., Haynes, M.P. 1986. Astrophys. J. 303, L55.
- Guillaume, C.E. 1896. La Nature 24, series 2, 234.
- Huchra, J.P., Davis, M., Latham, D., Tonry, J. 1983. Astrophys. J. Suppl. 52, 89.
- Peebles, P.J.E. 1980. The Large Scale Structure of the Universe, Princeton Univ. Press.
- Penzias, A.A., Wilson, R.W. 1965. Astrophys. J. 142, 419.
- Pietronero, L., Sylos Labini, F. 1995. In Birth of the Universe and Fundamental Physics, ed. Occhionero, F., Springer Verlag.
- Regener, E. 1933. Z. Phys. 80, 666. (English trans. 1995, Apeiron 2, 85).
- Roscoe, D.F. 1995. Physics Essays 8, 79.
- Sciama, D.W. 1971. Modern Cosmology, CUP.
- Vettolani, G. et al. 1994. In Cosmology and Large Scale Structure in the Universe, ASP Conf. Series.