ABSTRACT

("An Efficient Approach to Minor Planet Recovery, Detection, and Characterization")

Advances in the field of planetary science, particularly concerning our own solar system, have been dramatic over the last few decades. These advancements owe largely to developments in observing technology and more comprehensive astronomical surveys across the world. However, with these copious amounts of new data comes a need for more effective methods of analysis. This project offers a solution to the issue by presenting an efficient Python-based approach to aid with the detection, recovery, and characterization of minor planets in the solar system (asteroids, trans-neptunian objects, Kuiper Belt objects, etc.).

The work utilizes data from the DES and DEEP surveys to accomplish the following: 1) recovery/orbit enhancement of known minor planets 2) detection of 22.0-25.0 magnitude objects and 3) object characterization (light curve generation, rotational period analyses).

First, an extensive database of known minor planets is used to obtain orbital elements for all bodies. Orbital elements are evolved using an N-body symplectic integration scheme. Evolved elements are used to predict minor planet positions at the time of the exposure(s). Computed celestial positions and positional uncertainties allow for objects to be matched to single epoch detections.

New object detection is achieved by approximating intra-night trajectories as straight lines. The probabilistic Hough line transform, a modified version of the standard Hough transform (edge detection using random sampling), is used on coadd catalog of object detections as an efficient line detection algorithm.

Finally, detected/recovered objects are characterized through light curve generation, by extracting V-band magnitudes from a SQL database of detections. More complete object characterization requires period determination of produced light curves. For this, an efficient version of the Lomb-Scargle periodogram (*O(N log N)* -- VanDer Plas et al., 2015), is utilized.

Single night analysis results: Recovered ~480 known asteroids across exposures, detection of ~1,000 new objects. Known asteroid recovery rate ~92 percent/CCD. Computation time estimates: ~25 minutes for known object matching, ephemeride generation; 20 seconds/CCD for line detection; 1 minute/object for light curve generation.

An Efficient Approach to Minor Planet Detection and Characterization using N-Body Integration and the Probabilistic Hough Line Transform

Introduction

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This project offers a solution to the issue by presenting an efficient Python-based approach to aid with the detection, recovery, and characterization of minor planets in the solar system (including asteroids, trans-neptunian objects, Kuiper Belt objects, etc.). The work utilizes data from the DES and DEEP surveys to accomplish the following: 1) recovery/orbit enhancement of known minor planets 2) detection of 22.0-25.0 magnitude objects and 3) object characterization (light curve generation, rotational period analyses).

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Model Goals (Per Observing Night):

- ***Number of matches (~100 exposure stack)***
  - ~ 500 known minor planets; 3-5x as many unknown minor planets
  - Per CCD estimates: 20 objects/chip → anticipate decaying number of new matches across consecutive nights (inter night trajectories relatively slow)
- ***Computational time estimates***
  - Initial known asteroid matching (~100 exposure stack) = 20 minutes
  - Ephemerides generation = 20 seconds/match
  - Hough transform = 10 seconds/CCD
  - Line set refining = 1.5 seconds/line
  - Light curve generation = 20 seconds/object

Background: Known Object Recovery

Initial Refining Process (Numerically Solving Kepler's Equations):

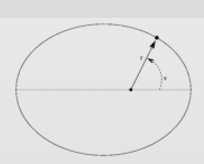
A two-step approach is employed to project known object positions to the time of exposures, assuming asteroid's 6 standard Keplerian elements are known $(e, a, i, \Omega, \omega, v(t_0))$. To eliminate clear mismatches, and propagate position to exposure time(s), Kepler's equation is solved. This is shown in brief detail below:

$$E - e\sin(E) = (t-T)n$$

$$E = M + \sum_{k=1}^{\infty} \frac{2}{k} J_k(ke)\sin(kM)$$

$$r = a(1 - e\cos E)$$

$$\tan\left(\frac{v}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$$



Detailed Orbital Element, Positional Evolution (N-Body Integration):

After refining known object candidate list, exposure time position(s) can be more accurately determined using a 15th-order modified Runge-Kutta integration scheme on the Solar System to correctly predict evolution of the objects' orbital elements from initial record to present day. Basic mechanism of IAS15 (Rein and Speigel 2014) is shown below:

$$y'' = F[y', y, t]$$

$$y''[t] \approx y_0'' + a_0 t + a_1 t^2 + \ldots + a_6 t^7.$$
$$y''[h] \approx y_0'' + b_0 h + b_1 h^2 + \ldots + b_6 h^7.$$

$$y''[h] \approx y_0'' + g_1 h + g_2 h(h-h_1) + g_3 h(h-h_1)(h-h_2) + \ldots + g_8 h(h-h_1)\ldots(h-h_7).$$

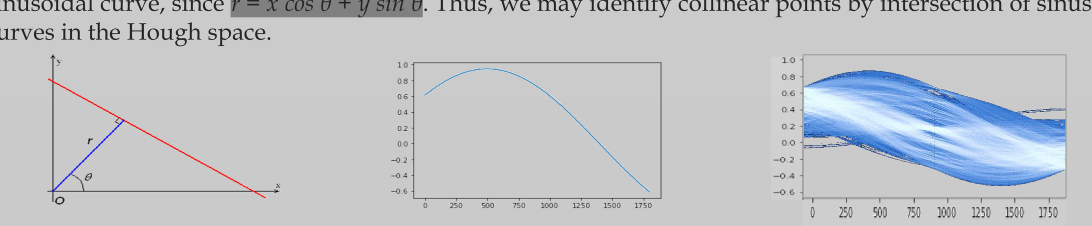
$$y'[h] \approx y_0' + h\,dt\left(y_0'' + \frac{h}{2}\left(b_0 + \frac{2h}{3}\left(b_1 + \ldots\right)\right)\right)$$

$$y[h] \approx y_0 + y_0' h\,dt + \frac{h^2 dt^2}{2}\left(y_0'' + \frac{h}{3}\left(b_0 + \frac{h}{2}\left(b_1 + \ldots\right)\right)\right)$$

Background: Unknown Object Detection

Standard Hough Line Transform Algorithm:

For the short intra-night observational periods being analyzed, it is appropriate to approximate the orbital trajectories as straight lines. Thus, an efficient method of extracting lines from a sparsely scattered set of detections is necessary. First, consider the mechanism of the standard Hough transform, a commonly used line detection algorithm. Note that we can uniquely express every line in the plane of the form $y = mx + b$, in (r, θ)-space (see below). Furthermore, the set of all lines passing through a point can be expressing as a sinusoidal curve, since $r = x\cos\theta + y\sin\theta$. Thus, we may identify collinear points by intersection of sinusoidal curves in the Hough space.



Edge Detection Concerns:

Before performing the Hough transform on the image, one would typically need to apply an edge detection algorithm to identify the relationship between segments. However, for astrophotometric data, which is sparsely scattered, this is not required, as each line feature is independent. Thus, a random point sampling technique is more effective, so the Probabilistic Hough Transform is utilized (see Methodology).

Methodology and Software Implementation

Methodology: Known Object Detection, Characterization

Fig. 5 Algorithm: Known Object Recovery

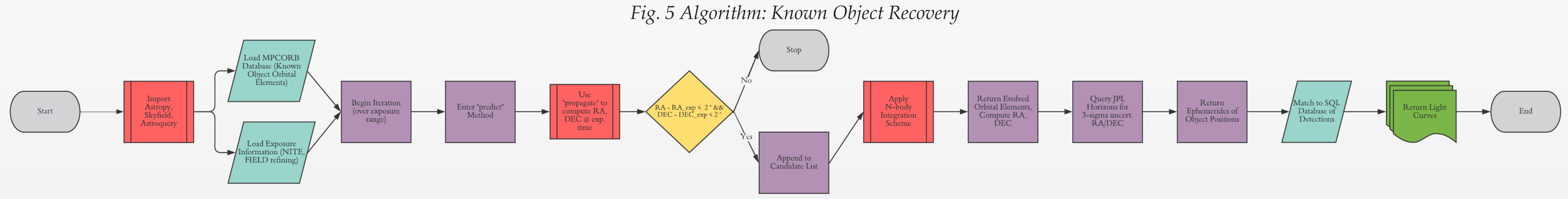


Fig. 6 Implementation of N-Body Integration Scheme

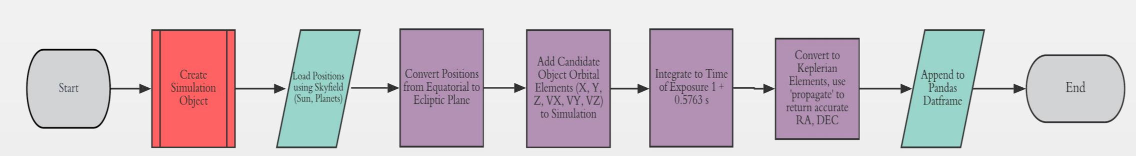
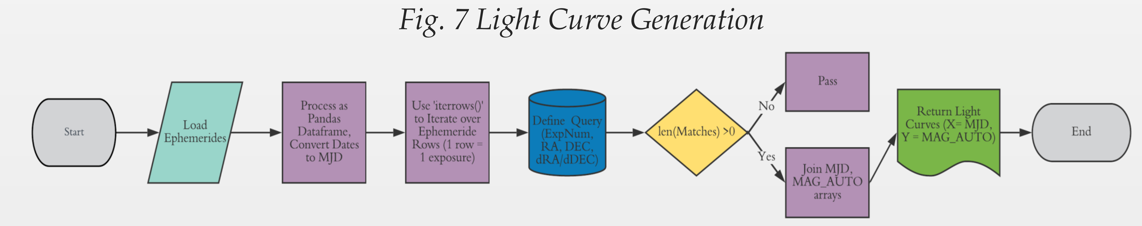
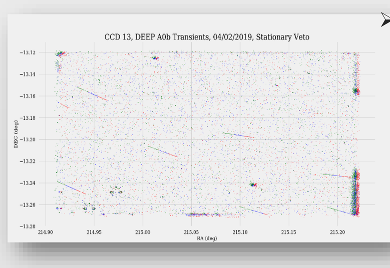


Fig. 7 Light Curve Generation

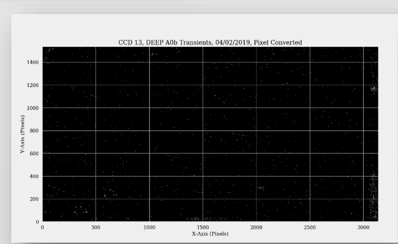


Methodology: Unknown Object Detection, Characterization

Preprocessing

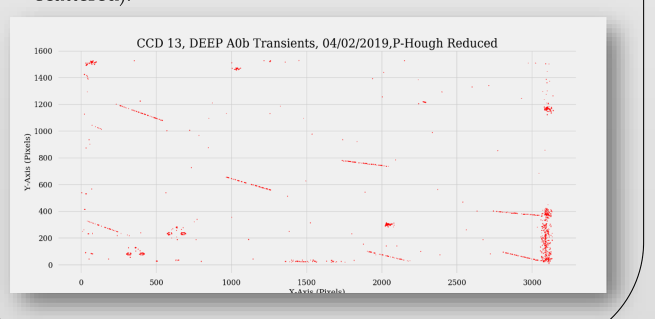


- Stationary objects are removed by referencing to a COADD catalog (stack of exposures, detections within 0.1″ vetoed from set)
- Must convert data from (RA, DEC)-space to pixel space. Hough transform requires image input. Data sparsely scattering → truncation necessary

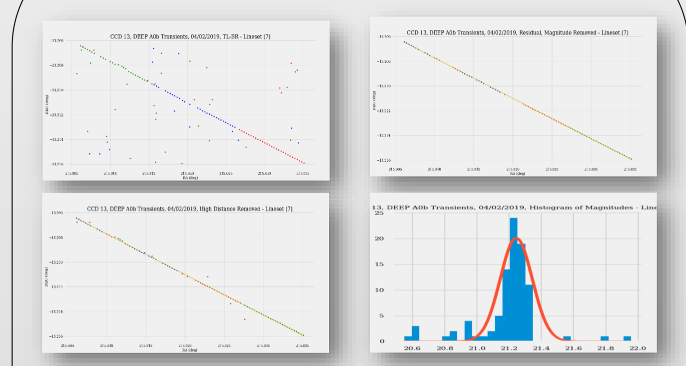


Probabilistic Hough Line Transform

- Employ Probabilistic Hough Line Transform on pixel space data. Accomplished with Skimage module (common image processing library). Probabilistic Hough transform similar to standard Hough Transform (see Background: Unknown Object Recover), except uses random sampling on edges to extract subset of points. Standard Hough transform applied on this subset. Purpose of P-Hough transform is to decrease run time (accuracy not compromised since data is sparsely scattered).



Postprocessing

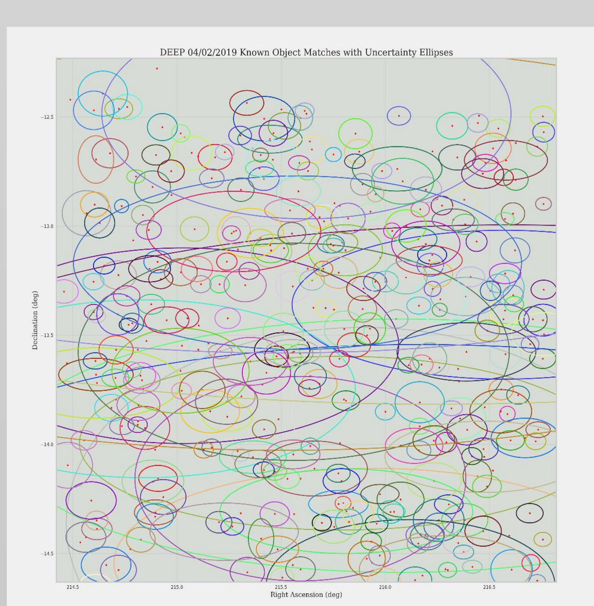


- Postprocessing done by iterating over line sets returned by P-Hough transform. After conversion back to (RA, DEC)-space, need to match back to original detections. First, narrow candidate detections by imposing TL/BR bounds on (RA,DEC)-space. Then, remove points 0.1″ away from all lines in line set. Finally, remove detections using magnitude, residual $(\bar{x} \pm 2\sigma)$ → data approximately normally distributed.

Results: Matched Candidates, Light Curves

Distribution of Known Object Matches:

Below, the distribution of the recovered known objects over the A0b field (04/02/2019) is shown. The predicted position (JPL Horizons/N-body Integration) is shown as a red star, with uncertainty ellipses centered on the predicted position. A total of 394 known objects were recovered, with 250 having low positional uncertainty.



Light Curves:

Positions predicted with ephemerides are cross-referenced against SQL database of detections (catalog-level referencing → detections extracted using SourceExtractor [no differenced imaging]). Predicted V-band magnitude is used to narrow down detection set. Then, plot of MJD (modified Julian date) against VR magnitude is constructed (see Figures 16-18). Finally, a sine regression is performed on each set of detections (useful in estimating rotational periods, along with Lomb-Scargle periodogram).

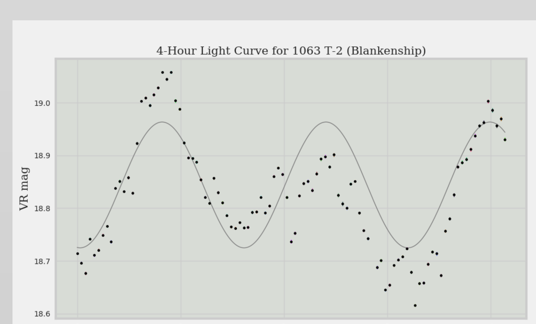


Fig. 16 4-Hour Light Curve for Blankenship, Low Mag. Uncertainty

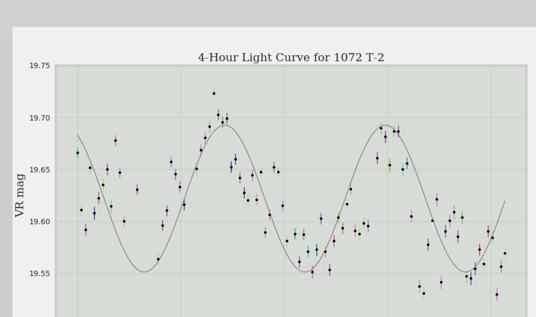


Fig. 17 4-Hour Light Curve for 1072 T-2, Medium Mag. Uncertainty

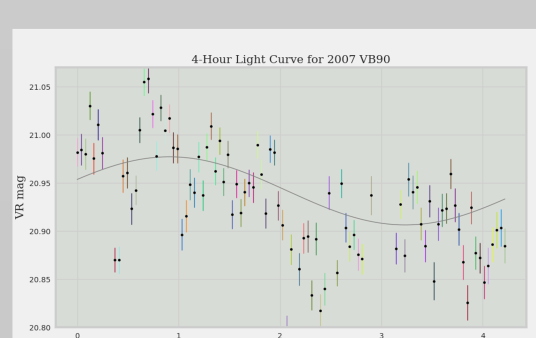


Fig. 18 4-Hour Light Curve for 2007 VB90, High Mag. Uncertainty

Unknown Object Light Curve Construction

In the figures below, 4-hour light curves are plotted for various unknown objects detected in catalog-level data by performing Probabilistic Hough Transform and subsequent post-processing/noise removal on transient catalogs. Specifically, these objects are predicted to be theoretical "fast rotators," with rotational periods too fast to be solely gravitationally bound objects. A more comprehensive analysis of these objects is presented in the "Future Work" section.

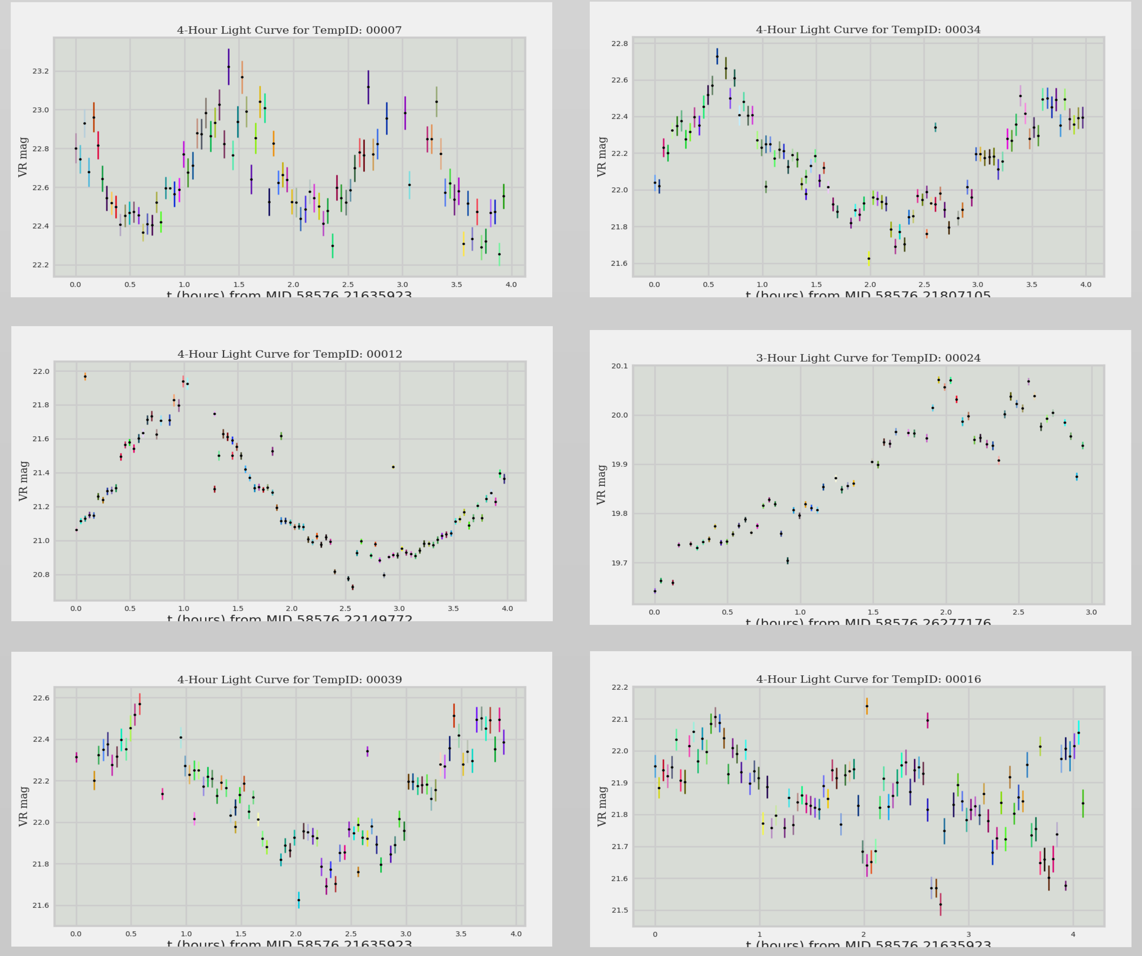


Fig. 19-24 4-Hour Light Curves for Unknown Predicted Fast Rotators

Results: Period Analysis

Lomb-Scargle Periodogram:

In order to conduct further analysis on the recovered/newly detected objects, rotational periods of the light curves constructed must be extracted. To do this, the Lomb-Scargle (LS) Periodogram is utilized. The LS-periodogram is an efficient algorithm to detect and characterize periodicity in unevenly sampled data, as is the case here. The LS-periodogram is similar in method to the standard technique of Fourier analysis (and associated Power Spectrum), as well as a least-squares method Employing the LS-periodogram for unevenly sampled data allows for the probing of frequencies larger than the classical Nyquist frequency in the limit of evenly spaced data. Here , an efficient version of the LS-periodogram is used (*O (N log N)*). Below, light curves and associated LS-periodograms are shown for 2 objects with relatively large VR-mag uncertainties.

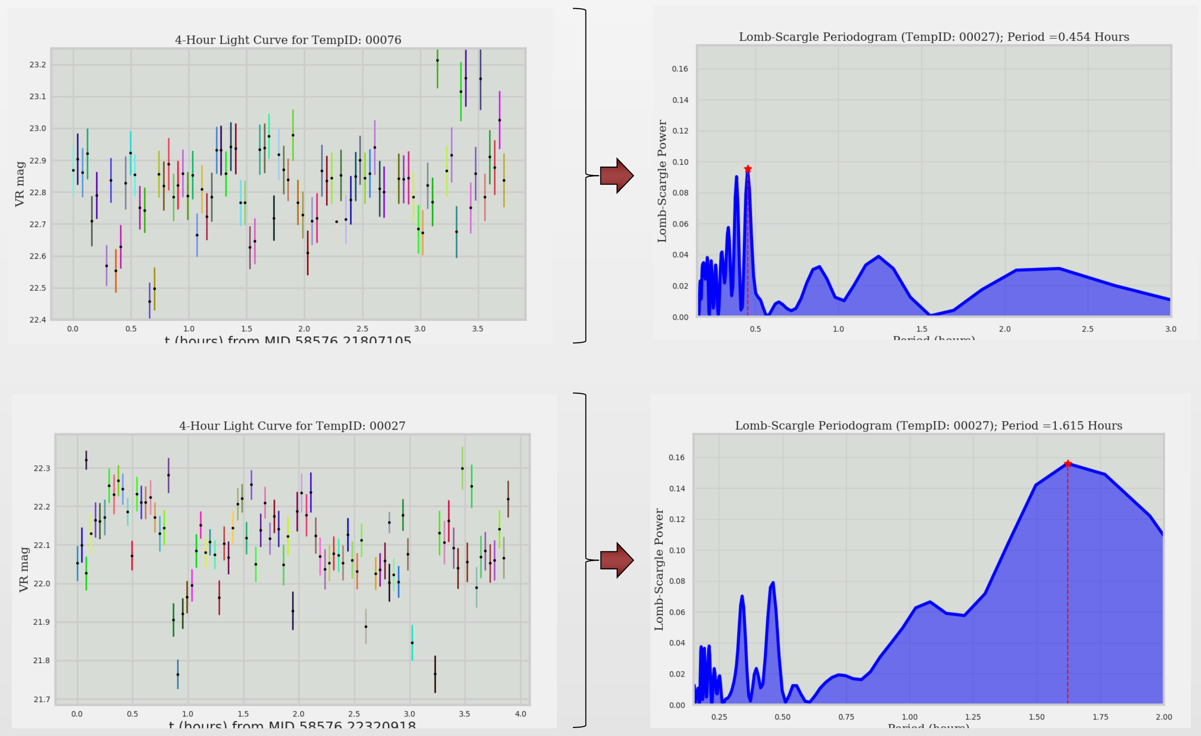


Fig. 19-24 4-Hour Light Curves for Unknown Predicted Fast Rotators

Conclusions

Known Object Recovery:

In this work, an efficient approach to known minor planet recovery was presented and demonstrated using data from the DES and DEEP surveys. Known minor planets are recovered in the datasets by utilizing a 3 step approach. First, object positions are roughly estimated by solving Kepler's equation. Then, orbital elements are evolved using an N-body integrator. Finally, positional uncertainties are returned by querying JPL Horizon's ephemeris system. *Per night results: 389 candidate matches, Runtime: 00: 16: 06.*

Unknown Object Detection:

Unknown minor planets (lost/new objects) are detected in the datasets by utilizing the probabilistic Hough line transform, an efficient line detection algorithm. Post-processing is performed on the lines recovered to isolate objects, and match to real detection. This includes: magnitude-based eliminations, temporal elimination, high residual cuts. See "Methodology." *Per night results: 12 objects/CCD (average), Runtime: 8 seconds/CCD (total = 64 CCDs).*

Object Characterization:

Recovered/detected objects are characterized by constructing light curves in VR-band. Rotational periods are extracted from constructed light curves using the Lomb-Scargle periodogram, an analog of the Fourier Power Spectrum for unevenly sampled data. Future work proposes identification of fast/slow rotators. See "Results: Period Analysis." *Runtime: 0.5 seconds/object*

Future Work

Cross-Referencing Known Object Databases, Join Inter-Night Trajectories

Future work proposes matching the positions of recovered known objects/detected unknown object to preexisting known object databases, like the Minor Planet Center. Generated period data may be compared against any existing light curves to minimize uncertainties in predictions. Furthermore, unknown objects found across multiple nights may be linked by again employing an N-body integration scheme (requires true velocity determination, see below).

Average Diameter Estimation, Further Rotational Period Analyses:

The size of the newly detected minor planets, particularly main-belt asteroids/Jupiter Trojans, may be estimated by determining distance and employing the following empirical equation that relates the geometric albedo, V-band magnitude, and mean diameter of the object:

$$D = \frac{1329}{\sqrt{p_G}} 10^{-0.2 \cdot H_v}$$

Mean diameter estimate, along with Lomb-Scargle period extraction, can be used to identify fast and slow rotators, where rotational periods are not in alignment with periods predicted by a gravitationally bound system. Moreover, analysis of peak width(s) in light curve can inform us about the shape and topography of the detected object (spherical, rod-shaped, contact-binary etc.).

Employing Shift-and-Add/Image Stacking:

Adopting a shift-and-stack scheme to further denoise the catalog-level data set allows for true speed determination, and detection of minor planets below the magnitude threshold of the observing apparatus (with higher signal to noise ratios). This facilitates more matches in outer Solar System (Neptune trojans, TNOs, etc.).