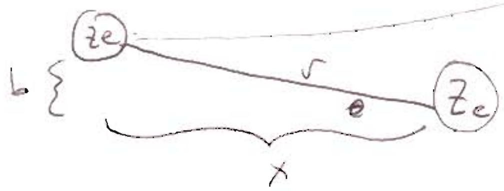
II stopping Power [ANN. Physics vol. 5, 325 (1930)]

a) Bethe Equation (Heavy charged particles)

i.) classical energy loss

Consider the energy lost when a particle of charge (ze) traveling at speed v is scattered by a target of ~~(Ze)~~ charge (Ze).



if gaussian units $k \equiv 1$

coulomb force $= F = k \dfrac{(ze)(Ze)}{r^2}$

Notice : as ze is scattered the horizontal component of "F" flips direction
∴ No net horizontal force

$$F_{vertical} = k \frac{zZe^2}{r^2} \sin\theta = k \frac{zZe^2}{r^2}\left(\frac{b}{r}\right)$$

The momentum change is given by the definition of Impulse

$$\Delta p = \int F\,dt$$

Then the energy transferred to the (ze) electrons is $\dfrac{(\Delta p)^2}{2m_e}$

II stopping power

a) Bethe Equation

1.) classical energy loss

$$\Delta p = \int F\,dt = \int \frac{k z Z e^2 b}{r^3}\,dt \qquad dt = \frac{dx}{v} = \frac{dx}{\beta c}$$

$$r^2 = x^2 + b^2$$

$$= \frac{k z Z e^2 b}{\beta c} \int_{-\infty}^{\infty} \frac{dx}{(x^2+b^2)^{3/2}}$$

$$= \frac{k z Z e^2 b}{\beta c b^3} \int_{-\infty}^{\infty} \frac{dx}{\left[1+\left(\frac{x}{b}\right)^2\right]^{3/2}} = \frac{k z Z e^2}{\beta c b^2} \underbrace{\int_{-\infty}^{\infty} \frac{d\left(\frac{x}{b}\right)}{\left[1+\left(\frac{x}{b}\right)^2\right]^{3/2}}}_{=2}$$

$$= \frac{2 k z Z e^2}{\beta c b}$$

$$r_e \equiv \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = \frac{k e^2}{m_e c^2}$$

$$\Rightarrow \Delta p = \frac{2 z Z r_e m_e c^{\cancel{2}}}{\beta \cancel{c} b}$$

$$\Delta E = \frac{(\Delta p)^2}{2 m_e} = \text{energy transfered to electron} = \text{energy lost by incident particle}$$

$$= 2\left(\frac{r_e m_e}{\beta b}\right)^2 \frac{z^2 \cancel{Z^2}^{\,1}}{m_e} c^2$$

II stopping Power

a) Bethe Equation

1.) Classical energy loss

$-dE = \int_0^\infty P(\Delta E)\, \Delta E\, dx$ = energy lost by incident particle

σ = atomic x-section = $\frac{\#\text{particles scattered}/\Omega}{\#\text{particles in}/A}$ $[cm^2]$

$P(\Delta E)$ = probability of the energy transfer = probability of interaction

$= \left(\frac{N}{A}\right)\sigma$: in units of cm^2/g $\left(\frac{\text{probability}}{\text{target surface density}}\right)$

$P(\Delta E)\,\Delta E = \left(\frac{N}{A}\right)(2\pi\, b\, db)\, Z$ = probability to hit an electron in the area of an annulus of radius $(b+db)$ with an energy transfer of between ΔE & $\Delta E + d(\Delta E)$

$Z \equiv$ # electrons/target atom

$\Rightarrow -dE = \int_0^\infty \left(\frac{N}{A}\right)(2\pi b\, db)\, Z\, \Delta E\, dx$

or $\frac{-dE}{dx} = \int_0^\infty \frac{N}{A}(2\pi b\, db)\, Z\, \Delta E$

$= 2\pi \frac{N}{A} Z \int_0^\infty \Delta E\, b\, db$

$= 2\pi \frac{N}{A} Z \int_0^\infty \left[\frac{2 r_e^2 m_e c^2}{\beta^2 b^2} z^2\right] b\, db$

$= 4\pi \frac{N}{A} \frac{Z r_e^2 m_e c^2}{\beta^2} z^2 \int_0^\infty \frac{db}{b}$ limits are non physical via b_{min} & b_{max}

$= \cancel{2\pi}\, 4\pi N r_e^2 m_e c^2 \frac{z^2 Z}{A\beta^2} \ln\left(\frac{b_{max}}{b_{min}}\right) \equiv K z^2 \frac{Z}{A}\frac{1}{\beta^2} \ln\left(\frac{b_{max}}{b_{min}}\right)$

II Stopping Power

a) Bethe Equation

1.) classical energy loss

$$-\frac{dE}{dx} = \underbrace{4\pi N r_e^2 m_e c^2}_{K} z^2 \frac{Z}{A} \frac{1}{\beta^2} \int_0^\infty \frac{db}{b}$$

$$= K z^2 \frac{Z}{A} \frac{1}{\beta^2} \int_0^\infty \frac{db}{b}$$

The above classical calculation diverges at the limits because the physics is different at those extremes

∴ introduce a b_{min} & b_{max} with represent the distance regions ~~which~~ through ~~with~~ which the physics is valid.

$$-\frac{dE}{dx} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \int_{b_{min}}^{b_{max}} \frac{db}{b} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

Apply physics: if $b_{min} \to 0$ then $\frac{dE}{dx}$ diverges

and energy transfer $\to \infty$

physically there is a maximum energy that may be transferred before the physics of the problem changes

(ie: to ionization, nuclear excitation, jet production...)

so physically b_{min} corresponds to the maximum energy which may be transferred

II stopping power

a) Bethe Equation

i.) classical energy loss

what is the minimum impact parameter b_{min}?

If the size of the target electron may be approximated in terms of its deBroglie wavelength

the $b_{min} \approx \frac{1}{2}\lambda_{deBroglie}$ = ~~a go~~ a good estimate

$$= \frac{h}{2p} = \frac{h}{2\gamma m_e \beta c}$$

what is b_{max}?

As "b" gets bigger the interaction is "softer" and longer. If the interaction time (τ_i) is so long that it is equivalent to an electron orbit (τ_R) the the atom looks more like it is neutrally charged. The electron orbit is perturbed adiabatically so no orbit change take place and minimal energy is transferred.

$$\tau_i = \frac{b_{max}}{v}\left(\sqrt{1-\beta^2}\right)$$ field at high velocities gets lorentz contracted

$\tau_R = \frac{h}{I}$; $I \equiv$ mean excitation energy of target material

$(E = h\nu) : \left(\frac{h}{\tau}\right)$

II stopping power

a) Bethe Equation

1) classical energy loss

what is b_{max}?

$$\tau_c \cong \tau_R$$

$$\frac{b_{max}}{v}\sqrt{1-\beta^2} = \frac{h}{I}$$

$$b_{max} = \frac{h\,\cancel{\sqrt{1-\beta^2}}\,v}{I\sqrt{1-\beta^2}} = \frac{h\gamma\beta c}{I}$$

$$-\frac{dE}{dx} = K z^2 \frac{Z}{A}\frac{1}{\beta^2}\ln\left(\cancel{\frac{b_{min}}{b_{max}}}\right)\left(\frac{b_{max}}{b_{min}}\right)$$

$$= K z^2 \frac{Z}{A}\frac{1}{\beta^2}\ln\left(\frac{h\gamma\beta c/I}{h/2\gamma m_e\beta c}\right)$$

$$= K z^2 \frac{Z}{A}\frac{1}{\beta^2}\ln\left(\frac{2\gamma^2 m_e\beta^2 c^2}{I}\right)$$

$$\frac{K}{A} \equiv \frac{4\pi N r_e^2 m_e c^2}{A} = 0.307\ \frac{\text{MeV cm}^2}{g} \quad \left(\text{if } A = 1\,\frac{g}{\text{mole}}\right)$$

z^2 = charge of incident particle

$Z = \dfrac{\#\ \text{electrons}}{\text{target atom}}$ = Atomic number of medium

m_e = mass of electron $\qquad I$ = Mean excitation energy

$\beta = \frac{v}{c}$ of projectile $\qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$ of projectile

II stopping power
a) Bethe Equation
1.) classical energy loss

Example: Find dE/dx for a 10 MeV proton hitting an LH_2 target

$\rho_{LH2} = .07 \frac{g}{cm^3}$ $\qquad \beta_{proton} = ?$ $\qquad K.E = 10\ \text{MeV} = (\gamma - 1) mc^2$

$\Rightarrow \gamma = \frac{K.E}{mc^2} + 1 \simeq 1$

$\therefore$ proton is not relativistic

$v^2 = \frac{2KE}{m} = \frac{2(10\ \text{MeV})}{938\ \text{MeV}} = 2 \times 10^{-2} c^2 \quad ; \quad c^2 \equiv 1$

$\therefore \beta^2 = \frac{v^2}{c^2} = 2 \times 10^{-2}$

$$\frac{dE}{dx} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \ln\left(\frac{2\gamma^2 m_e \beta^2 c^2}{I}\right)$$

$I = 21.6$ eV ; see solid data point in Fig 23.4 of PDG pg 145

$$\frac{dE}{dx} = \left(0.307 \frac{\text{MeV} \cdot \text{cm}^2}{g}\right)(1)^2 (1) \frac{1}{(2\times10^{-2})} \ln\left(\frac{2\gamma^2 (.511\ \text{MeV})(2\times10^{-2})}{21.6\ \text{eV}} \cdot \frac{10^6\ \text{eV}}{\text{MeV}}\right)$$

$= 105 \frac{\text{MeV cm}^2}{g}$ $\qquad$ (Triumf kinematics handbook $\Rightarrow 100 \frac{\text{MeV cm}^2}{g}$)

How much energy is lost after 0.3 cm

$\Delta E = \left(105 \frac{\text{MeV} \cdot \text{cm}^2}{g}\right)\left(.07 \frac{g}{cm^3}\right)(.3\ \text{cm}) = 2.2$ MeV

From: "Triumf Kinematic handbook", 2nd edition, sept. 1987, L.G. Greeniaus

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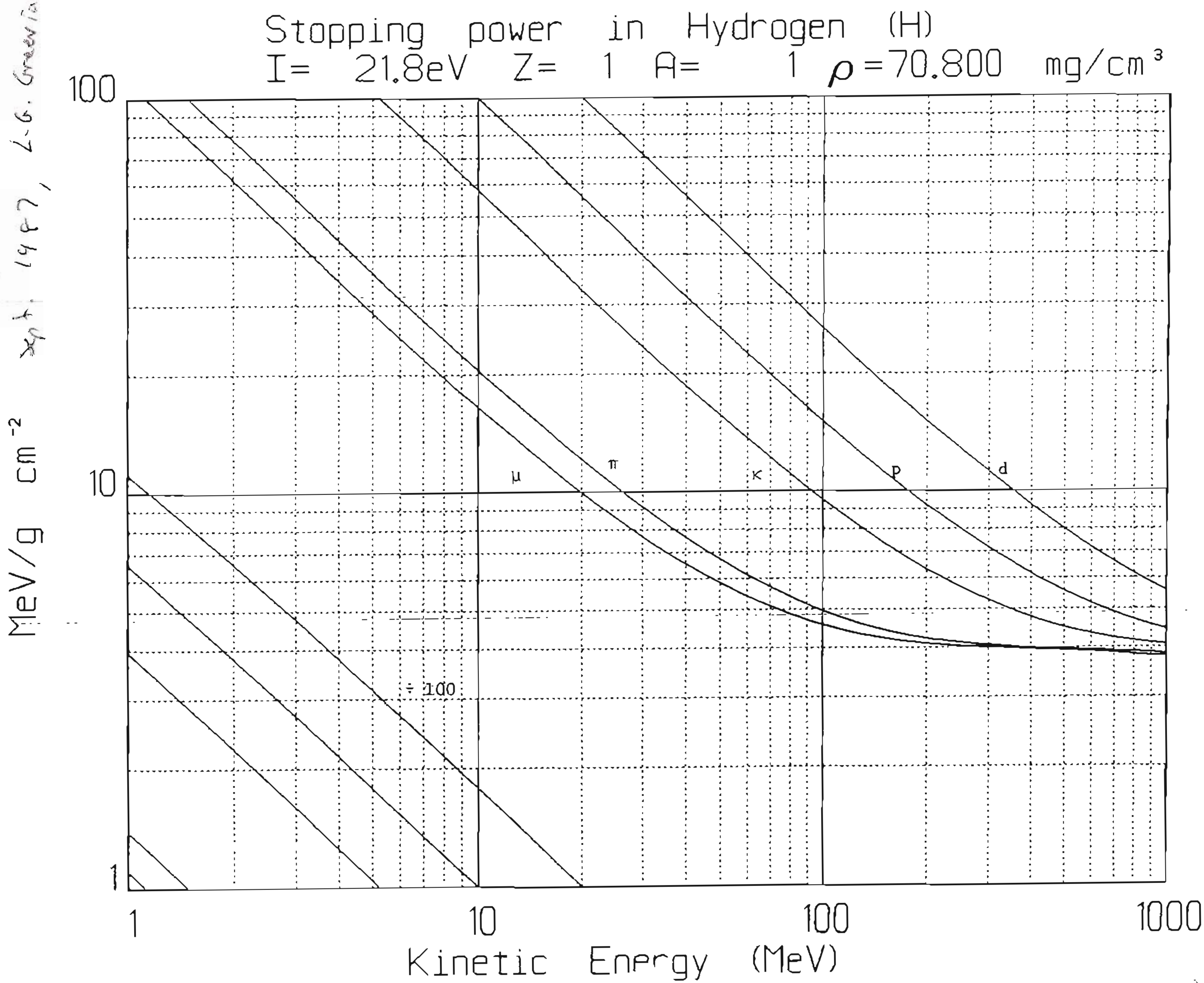

II stopping Power

a.) Bethe Equation

2.) Bethe - Bloch equation

while the classical equation above works in a limited kinematic regime, the Bethe-Bloch equation includes the corrections needed to cover most kinematic regimes for heavy particle ~~A~~ energy loss

$$\frac{dE}{dx} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln\left(\frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} \right) - \beta^2 - \frac{\delta}{2} \right]$$

: from ~~TBD~~ PDG "passage of particles through matter" section 23.1 equation 23.1

$-\beta^2$: electron spin and very distant collisions which only deform electron atomic orbits each reduce dE/dx by $\beta^2/2$

$\delta/2$: density correction term: in the classical ~~φ~~ derivation the material is treated as just a system of "N" atoms uniformly distributed in space. These atoms, however, give the material a polarizability which can reduce the electric field (dielectric)

II stopping Power

a) Bethe equation

3.) GEANT 4 version

The GEANT 4 file:

sources/processes/electromagnetic/standard/src/G4BetheBlochModel.cc

is used to ~~to~~ calculate hadron energy loss.

line 132 =)

$$\frac{dE}{dx} = \log\left[\frac{2m_e c^2 \left[\tau(\tau+2)\right] E_{min}}{I^2}\right] - \left(1+\frac{E_{min}}{E_{max}}\right)\beta^2$$

where $\tau = \frac{K.E.}{m}$

line 143 =) $\underset{\substack{\text{means to} \\ \text{subtract R.H.S. from current value} \\ \text{of } dE/dx}}{\frac{dE}{dx}-} = \underbrace{\log\left[\tau(\tau+2)\right] - cden}_{\delta/2}$: Density correction

line 148 =) $\frac{dE}{dx} - = \frac{2C}{Z_{target}}$: Shell correction

this corrects the classical assumption that the atomic electron velocity is initially zero (or that the incident particle's velocity is far greater ~~that~~ than the atomic electron's)

not 4! because of $\frac{1}{2}\ln(\ldots)$

line 154 =) $\frac{dE}{dx} * = \frac{2\pi m_e c^2 r_e^2 z^2}{\beta^2} \rho_e$ =) $\frac{NZ}{A}$: constant term