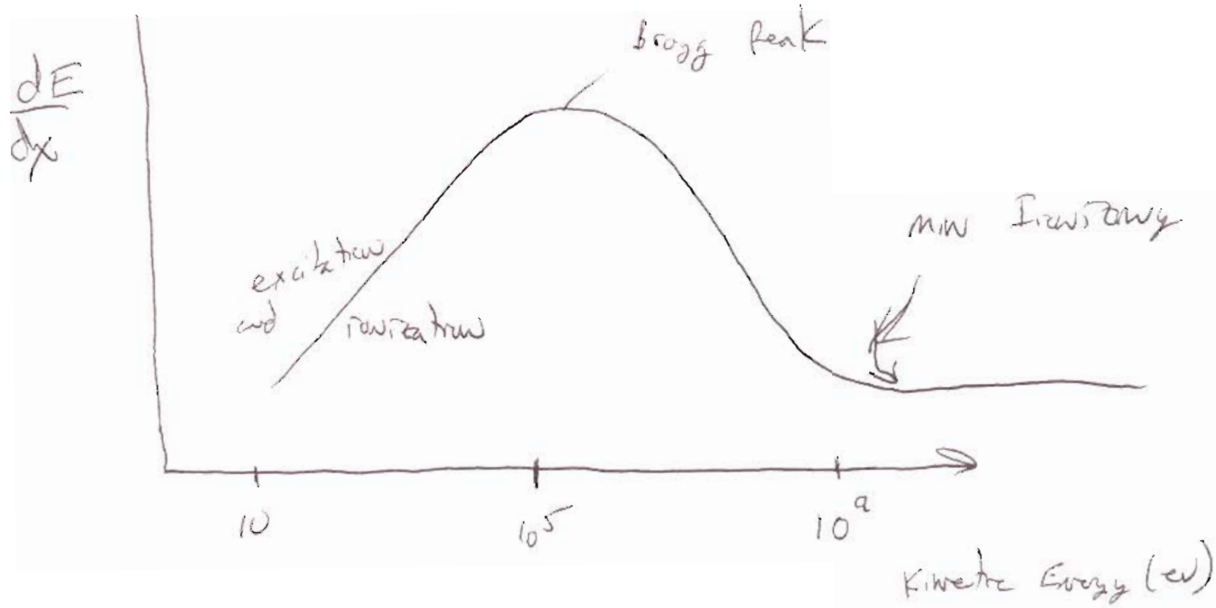
II stopping Power

a.) Bethe Equation

4.) energy dependence



Bethe Bloch breaks down at low energies (below Bragg Peak)

Bethe Bloch is good to better than 10% (accurate to 10%) for

$$10\ \frac{\text{MeV}}{\text{amu}} < E < 2\ \frac{\text{GeV}}{\text{amu}}$$

and

$$z < 26 \quad (\text{Iron})$$

$\frac{1}{\beta^2}$ term dominates between Bragg Peak and Min ionizing

ln term + correction move $\frac{dE}{dx}$ from Min Ionizing region

II stopping power

b.) Energy straggling

while the Bethe - Bloch formula ~~tells~~ gives you ~~the rate of~~ a way to quantify the amount of energy a heavy charged particle looses as a function of distance traveled you should realize that when you calculate the total energy lost

$$\Delta E = \int_{E_i}^{E_f} \left(\frac{dE}{dx}\right) dx$$

you are only determining ~~the~~ the AVERAGE energy loss.

ie: Bethe - Bloch is the A stochastic process

In reality the energy loss process is stochastic because of the statistical fluctuation which occur in the actual number of collisions which take place.

1.) Thick absorber (Gaussian limit)

Thick absorber $\Rightarrow$ Large # of collisions

$\Rightarrow$ Central Limit theorem

says large N random variables $\rightarrow$ Gaussian $(N \to \infty)$

I stopping Power

b) ~~Energy~~ Energy straggling

1) Thick absorber

~~For~~ The variance of the energy loss distribution will be

$$\sigma_0^2 = 4\pi N r_e^2 (m_e c^2)^2 \rho \frac{Z}{A} \cdot X \quad : \text{~~non~~ ~~relativistic~~}$$

the relativistic variance will be

$$\sigma^2 = \frac{(1 - \beta^2/2)}{1 - \beta^2} \sigma_0^2$$

For very thick absorbers see

C. Tshalar, NIM 64, 237 (1968)

61, 141 (1968)

So when simulating energy loss of heavy charged particles the Bethe-Bloch equation may be used to calculate a $\frac{dE}{dx}$ ~~(energy dependent)~~

~~which represents the Average~~

which ~~to~~ can determine the average energy loss at the given kinetic energy of the particle.

This average is then "smeared" according to a gaussian distribution of variance

$$\sigma^2 = 4\pi N r_e^2 (m_e c^2) \rho \frac{Z}{A} X \frac{(1-\beta^2/2)}{1-\beta^2}$$

II stopping power
b.) energy straggling

a) Thin absorbers:

In thin absorbers the # of collisions is small ∴ Central limit theorem doesn't apply.

~~This is because the large energy transfers that are possible~~

Large energy transfers that are possible give the gaussian a "foot" or high energy tail)

The skewness in the resulting energy loss distribution is quantified as

$$K = \frac{\bar{\Delta}}{W_{max}} = \frac{\text{mean energy loss}}{\text{Max energy transfer in 1 collision}}$$

$\bar{\Delta}$ may be approximated as the lead term in the Bethe-Bloch formula

$$= 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \left(\frac{z}{\beta}\right)^2 x$$

x — thickness of material

ρ — density of absorbing material

II stopping power
b) energy straggling
2.) thin absorbers

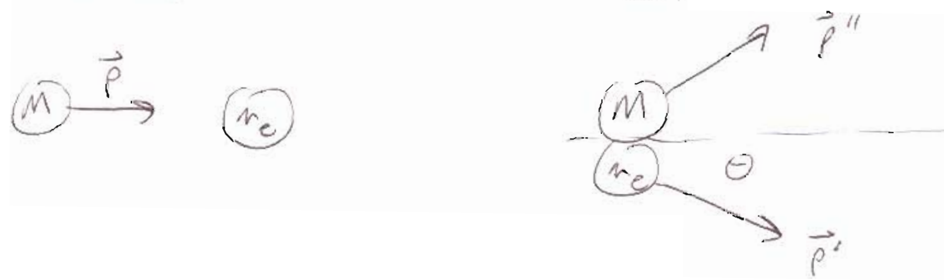
W_{max} = max energy transfered in 1 collision $\Rightarrow$ head on / knock on collision

$$= \frac{(pc)^2}{\frac{1}{2} m_e c^2 + \frac{1}{2}\left(\frac{M^2}{m_e}\right) c^2 + \sqrt{(pc)^2 + (Mc^2)^2}}$$

This comes from relativistic kinematics of an Elastic collision

In Lab Frame

Initial — Final



Cons of p: $\vec{p} = \vec{p}'' + \vec{p}'$

Cons of E:

$$E_{tot} + m_e c^2 = E''_{tot} + E'_{tot}$$

$$\sqrt{(pc)^2 + (Mc^2)^2} + m_e c^2 = \sqrt{(p''c)^2 + (Mc^2)^2} + E_k + m_e c^2$$

$\downarrow$

this ~~???~~ leads to W_{max}

II stopping Power

b) energy straggling

2.) thin absorbers

using cons. of E : p as well as sub. of p'

via: $$E_k + m_e c^2 = \sqrt{(p'c)^2 + (m_e c^2)^2}$$

$$\Rightarrow (p'c)^2 = E_k^2 + 2 E_k m_e c^2$$

will lead to the equation

$$(p''c)^2 = (pc)^2 - 2E_k\sqrt{(pc)^2 + (m_e c^2)^2} + E_k^2 \quad : \text{cons of } E$$

$$= (pc)^2 + E_k^2 + 2E_k m_e c^2 - 2pc\sqrt{E_k^2 + 2E_k m_e c^2}\cos\theta \quad : \text{cons of } p$$

$$\Rightarrow pc\cos\theta\sqrt{1 + \frac{2m_e c^2}{E_k}} = \sqrt{(pc)^2 + (m_e c^2)^2} + m_e c^2$$

solve for $E_k \Rightarrow$

$$E_k = \frac{2m_e c^2 (pc)^2\cos\theta}{\left[\sqrt{(pc)^2 + (m_e c^2)^2} + m_e c^2\right]^2 - (pc)^2\cos^2\theta}$$

$\theta = 0 \Rightarrow$ Head

~~show~~ You will be asked to show the above for Homework V, you need to fill in the algebra.

II Stopping power
  b) energy straggling
    2) thin absorbers

i.) $\kappa \le 0.01$ (Landau theory)

Landau assumed

1) $W_{max} = \infty$ is max energy transfer possible

2) electrons are free (energy transfer is so large you can neglect binding)

3) incident particle maintains velocity
   ie: momentum transfer ⇒ large v to ~~...~~ small mass electron

L. Landau, "One the Energy loss of Fast particles by ionization", J. Phys. vol 8 pg 201 (1944)

instead of the gaussian distribution

$$f(x,\Delta) \propto e^{-(\Delta-\bar{\Delta})^2/2\sigma^2}$$

$$\cancel{f(x)}\ f(x,\Delta) = \frac{1}{\xi\pi}\int_0^\infty e^{(-u\ln u - u\lambda)} \sin \pi u\, du$$

where $\lambda = \frac{1}{\xi}\left[\Delta - \xi(\ln\xi - \ln\epsilon + 1 - C)\right]$

$$\xi = 2\pi N_a r_e^2 m_e c^2 \rho \left(\frac{Z}{A}\right)\left(\frac{z}{\beta}\right)^2 x$$

$C = 0.577$

II stopping Power

b) energy straggling

1) thin absorbers

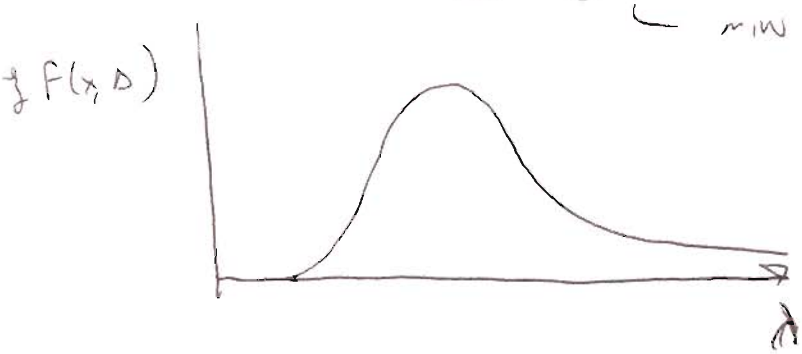
i) $\kappa \leq 0.01$ Landau theory

$$f(x,\Delta) = \frac{1}{\xi\pi}\int_0^\infty e^{(-u\ln u - u\lambda)} \sin(\pi u)\, du$$

$$\lambda = \frac{1}{\xi}\left[\Delta - \xi\ln\xi - \ln\varepsilon + 1 - C)\right]$$

$$\xi = 2\pi N_a r_e^2 m_e c^2 \rho \left(\frac{Z}{A}\right)\left(\frac{z}{\beta}\right)^2 x$$

$$\ln\varepsilon = \ln\left[\underbrace{\frac{(1-\beta^2)\, I^2}{2mc^2\beta^2} + \beta^2}_{\text{min energy transferred}}\right] \qquad C = 0.577$$



II stopping power

b) energy straggling

2) thin absorbers

(ii) $.01 < K < \infty$

↳ ~~tells~~ really the Gaussian limit

Vavilov's Theory: P. V. Vavilov, "Ionization losses of High-Energy Heavy Particles", Soviet physics JETP, vol 5, pg 749 (1957)

$$f(x,\lambda) = \frac{1}{\xi\pi} \, \kappa e^{\kappa(1+\beta^2 C)} \int_0^\infty e^{\kappa f_1} \cos(y\lambda_1 + \kappa f_2)\, dy$$

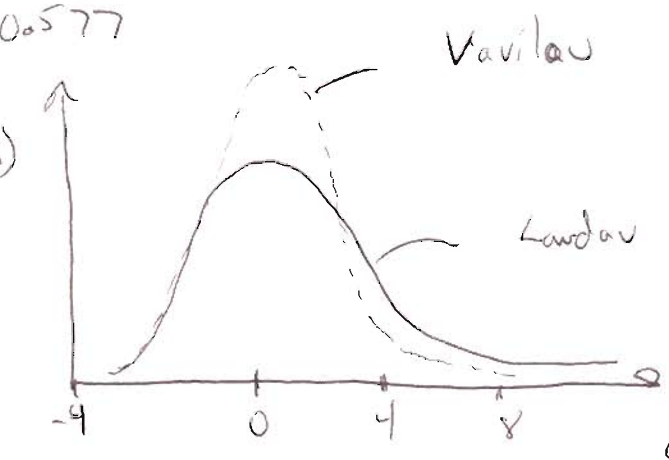
$$f_1 = \beta^2 \left[\ln(y) - C_i(y)\right] - \cos(y) - y\, S_i(y)$$

$$f_2 = y\left[\ln(y) - C_i(y)\right] + \sin(y) + \beta^2 S_i(y)$$

$$C_i(y) \equiv -\int_y^\infty \frac{\cos(t)}{t}\, dt$$

$$S_i(y) \equiv \int_0^y \frac{\sin(t)}{t}\, dt$$

$C = 0.577$



II. stopping Power
b) energy straggling
3) GEANT 4's implementation

↳ Urbana, NIM A362, 416 (1995)

~~if $K \equiv \frac{\bar{\Delta}}{W_{max}} = 10$ then we have energy straggling~~

Geant 4 sets the skewness parameter

$$K \equiv \frac{\bar{\Delta}}{W_{max}} = 10$$

If $K > 10$ ~~then~~ and we have a thick absorber (step size) then the energy straggling function chosen is a Gaussian just like in II.b.1.)

ie. ΔE is calculated via $\int_{E_i}^{E_f} \frac{dE}{dx}\, dx$
then the actual ΔE to use is
chosen from a gaussian distribution with the above ΔE average which has a width

$$\sigma^2 = 2\pi r_c^2 m_e c^2 N_{el} \left(\frac{Z_h^2}{\beta^2}\right) T_c\, s \left(1 - \frac{\beta^2}{2}\right)$$

where N_{el} = electron density of the medium
Z_h = charge of incident particle

T_c = cut off ~~energy for~~ kinetic energy for δ-electrons
s = step size

II stopping Power

b) energy straggling

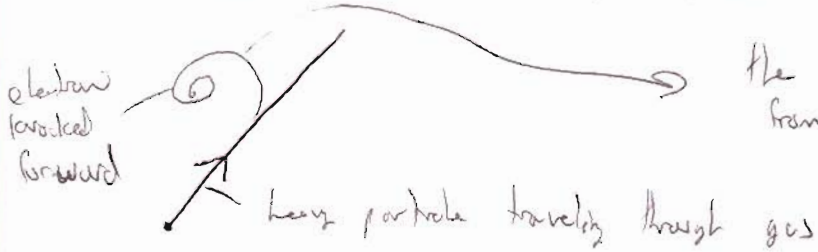
3) GEANT 4's implementation.

what is a δ-electron?

δ-electrons also called knock-on electrons
a.k.a. ~~delta~~ delta rays (δ).

as heavy particles traverse medium they can ~~ionized~~ ionize electrons from atoms

in cloud chamber it looks like



the high energy ~~the~~ electron from ionization by the heavy particle is given enough energy to ionize the gas in the cloud chamber

$T_{cut} > 1$ keV in Geant 4 (Note: B.E. of electron in H is 13.6 eV)

B.E. Ar - 15.7 eV → 3.2 keV ; Gas detector Excitation : Ionization energies ~ 5-20 eV

If $K < \frac{\Delta E}{T_c}$; $T_c \geq \frac{T_{max}}{2}$

Then GEANT 4 uses a "fluctuations model" to ~~determ~~ determine the energy loss

II stopping Power
b.) energy straggling
3) GEANT 4's implementation.

Fluctuations model.

- the atom is assumed to have only 2 energy levels E_1 ; E_2

- you can excite the atom and loose either E_1 or E_2 amount of energy

  OR

  You can ionize the atom and loose energy according to a $1/E^2$ function

The total energy loss in a step will be $\Delta E = \Delta E_{exc} + \Delta E_{ion}$

where
$$\Delta E_{exc} = n_1 E_1 + n_2 E_2$$

$$\Delta E_{ion} = \sum_{j=1}^{n_3} \frac{I}{1 - u_j \frac{T_{up} - I}{T_{up}}}$$

where $n_1, n_2, ; n_3$ are the # of collisions which are sampled from a poisson distribution (this is how the straggling is inserted into the calculation

II stopping Power
b) energy straggling
3) GEANT 4's implementation

Fluctuations model:

$$u_j = \int_I^{E_j} \frac{I\, T_{up}}{T_{up} - I} \frac{dX}{X^2} , \quad E_j = \frac{I}{1 - (\text{rnd}) \frac{T_{up} - I}{T_{up}}}$$

random # between 0, 1

T_{up} = threshold energy for δ-ray production ($\sim$ keV) or T_{max} if it is smaller

I = mean ionization energy

$$E_2 \approx (10\text{ eV})\, Z^2$$

$$\ln E_1 = \frac{\ln(I) - f_2 \ln(E_2)}{f_1} \text{ where } f_1 + f_2 = 1 \;\&\; f_2 = \begin{cases} 0 & Z=1 \\ \frac{2}{Z} & Z \geq 2 \end{cases}$$

The "fluctuations model" was compared to the data in

K. Lassila-Perini and L. Urbán, NIM, A362, pg 416 (1995)

The X-sections for excitation & ionization may be found in

H. Bichsel, Rev. Mod. Phys., Vol 60, pg 663 (1988)