Homework V

1.) ~~Find~~ show that the maximum energy transfered in a relativistic head on collision ~~a~~ elastic collision

$$T_{max} = \frac{p^2c^2}{\frac{1}{2}m_ec^2 + \frac{1}{2}\left(\frac{M^2}{m_e}\right)c^2 + \sqrt{p^2c^2 + M^2c^4}}$$

where

p = incident momentum of mass M
m_e = mass of target electron which is at rest initially.

In LAB Frame

Initial | Final

cons of p : $\vec{p} = \vec{p}'' + \vec{p}'$

cons of E :

$$E_{tot} + m_ec^2 = E''_{tot} + E'_{tot}$$

$$\sqrt{(pc)^2 + (Mc^2)^2} + m_ec^2 = \sqrt{(p''c)^2 + (Mc^2)^2} + \sqrt{(p'c)^2 + (m_ec^2)^2}$$

Homework V

1.)

cons. of Momentum:

$$\vec{p}'' = \vec{p} - \vec{p}'$$

$$(p'')^2 = (p)^2 + (p')^2 - 2pp'\cos\theta$$

$$(p''c)^2 = (pc)^2 + (p'c)^2 - 2pp'c^2\cos\theta$$

Find equation for p':

$$E_k \equiv \text{K.E. of scattered particle}$$
$$= \text{Energy transfered}$$

$$E'_{tot} = E_k + m_ec^2 = \sqrt{(p'c)^2 + (m_ec^2)^2} = \gamma m_e c^2$$

$$\Rightarrow \left(E_k + m_ec^2\right)^2 = (p'c)^2 + (m_ec^2)^2$$

$$(p'c)^2 = E_k^2 + 2E_k m_e c^2$$

sub. into cons. of Momentum

$$(p''c)^2 = (pc)^2 + \left(E_k^2 + 2E_k m_e c^2\right)$$
$$- 2pc\sqrt{E_k^2 + 2E_k m_e c^2}\cos\theta$$

cons. of E: $E_{tot} + m_ec^2 = E''_{tot} + E'_{tot}$

$$\sqrt{(pc)^2 + (Mc^2)^2} + m_ec^2 = \sqrt{(p''c)^2 + (Mc^2)^2} + E_k + m_ec^2$$

Homework V

1.)

cons. of E:

$$\sqrt{(pc)^2 + (m_e c^2)^2} = \sqrt{(p''c)^2 + (m_e c^2)^2} + E_h$$

$$(p''c)^2 + (mc^2)^2 = \left[\sqrt{(pc)^2 + (mc^2)^2} - E_h\right]^2$$

$$= (pc)^2 + (mc^2)^2 - 2E_h\sqrt{(pc)^2 + (mc^2)^2} + E_h^2$$

$$(p''c)^2 = (pc)^2 - 2E_h\sqrt{(pc)^2 + (mc^2)^2} + E_h^2$$

$$= (pc)^2 + E_h^2 + 2E_h m_e c^2 \cancel{-2pc}$$

$$- 2pc\sqrt{E_h^2 + 2E_h m_e c^2}\cos\theta$$

cons. of p

$$\Rightarrow -2E_h\sqrt{(pc)^2 + (mc^2)^2} = 2E_h m_e c^2 - 2pc\sqrt{E_h^2 + 2E_h m_e c^2}\cos\theta$$

$$\Rightarrow pc\cos\theta\sqrt{E_h^2 + 2E_h m_e c^2} = E_h\sqrt{(pc)^2 + (mc^2)^2} + E_h m_e c^2$$

$$pc\cos\theta\sqrt{1 + \frac{2m_e c^2}{E_h}} = \sqrt{(pc)^2 + (mc^2)^2} + m_e c^2$$

solve for E_h:

$$\sqrt{1 + \frac{2m_e c^2}{E_h}} = \frac{\sqrt{(pc)^2 + (mc^2)^2} + m_e c^2}{pc\cos\theta}$$

Homework V

1.)

$$1 + \frac{2m_e c^2}{E_h} = \frac{\left[\sqrt{(pc)^2 + (mc^2)^2} + m_e c^2\right]^2}{(pc)^2 \cos^2\theta}$$

$$\frac{2m_e c^2}{E_h} = \frac{\left[\sqrt{(pc)^2 + (mc^2)^2} + m_e c^2\right]^2 - (pc)^2\cos^2\theta}{(pc)^2 \cos^2\theta}$$

$$E_h = \frac{2m_e c^2 (pc)^2 \cos^2\theta}{\left[\sqrt{(pc)^2 + (mc^2)^2} + m_e c^2\right]^2 - (pc)^2\cos^2\theta}$$

for Max energy transfer $\theta = 0 \Rightarrow$ head on collision.

$$E_h = \frac{2m_e c^2 (pc)^2}{\cancel{(pc)^2} + (mc^2)^2 + 2m_e c^2\sqrt{(pc)^2 + (mc^2)^2} + (m_e c^2)^2 - \cancel{(pc)^2}}$$

$$= \frac{(pc)^2}{\frac{1}{2}m_e c^2 + \frac{1}{2}\left(\frac{M^2}{m_e}\right)c^2 + \sqrt{(pc)^2 + (Mc^2)^2}}$$

given; initial momentum $\cancel{P}$ (p) of mass M

you can find Max K.E transfered to electron

Note: $E_{tot} = \gamma mc^2 = \sqrt{p^2c^2 + M^2c^4}$

Homework V

2.) Find the maximum K.E. transfer for a 10 GeV proton hitting an atomic electron.

$$E_{tot} = 10\text{ GeV} = \gamma M c^2 \Rightarrow \gamma = \frac{10\text{ GeV}}{.938\text{ GeV}} \approx 10$$

$$= \sqrt{(pc)^2 + (Mc^2)^2} = E_k + Mc^2$$

$$\gamma^2 (Mc^2)^2 = (pc)^2 + (Mc^2)^2 \Rightarrow (pc)^2 = (\gamma^2 - 1)(Mc^2)^2$$

$$= \left(\frac{1}{1-\beta^2} - 1\right)(Mc^2)^2$$

$$= \gamma^2 \beta^2 (Mc^2)^2$$

$$E_k = \frac{(pc)^2}{\frac{m_e c^2}{2} + \left(\frac{M^2}{m_e}\right)\frac{c^2}{2} + \sqrt{(pc)^2 + (Mc)^2}}$$

$$= \frac{2 m_e c^2 (pc)^2}{(Mc^2)^2 + (m_e c^2)^2 + 2 m_e c^2 \sqrt{(pc)^2 + (Mc^2)^2}}$$

$$= \frac{2 m_e c^2 (pc)^2}{(Mc^2)^2 + (m_e c^2)^2 + 2 m_e c^2 \gamma M c^2}$$

$$= \frac{2 m_e c^2 (\gamma^2 \beta^2)(Mc^2)^2}{(Mc^2)^2 + (m_e c^2)^2 + 2 m_e c^2 \gamma (Mc^2)}$$

$$= \frac{2 m_e c^2 \gamma^2 \beta^2}{1 + \left(\frac{m_e}{M}\right)^2 + 2\gamma \frac{m_e}{M}}$$

Homework V

2) T_{max} for 10 GeV protons

$$T_{max} = \frac{2 m_e c^2 \gamma^2 \beta^2}{1 + \left(\frac{m_e}{m}\right)^2 + 2\gamma \frac{m_e}{m}}$$

$\gamma = 10 = \frac{1}{\sqrt{1-\beta^2}}$ $\quad \beta = \frac{v}{c} = 1 - \frac{1}{\gamma^2} = 1 - \frac{1}{10} = \frac{.9}{10}$

$$T_{max} = \frac{2(.511\ \text{MeV})(10)^2\left(\frac{9}{10}\right)^2}{1 + \left(\frac{.511}{938}\right)^2 + 2(10)\left(\frac{.511}{938}\right)}$$

$= 82$ MeV

Note: if $p \gg \frac{M^2 c}{m_e}$ then

$T_{max} \approx pc \approx E_i$