

Homogeneous Systems

\ddot{y} = Acceleration
 \dot{y} = Velocity

$$m\ddot{y} + cy + Ky = 0$$

$$y = e^{\lambda t} \quad | \quad \begin{aligned} \ddot{y} &\rightarrow \lambda^2 \\ \dot{y} &\rightarrow \lambda \\ y &\rightarrow 1 \end{aligned}$$

How to find homogeneous y_h

$$\lambda_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mK}}{2m}$$

There are three cases:

if $c^2 - 4mK = 0$

if $c^2 - 4mK$ is positive (or $c^2 > 4mK$)

if $c^2 - 4mK$ is negative, : imaginary numbers
(or $c^2 < 4mK$) Overdamped

① $C^2 > 4MK$ \longrightarrow over damped

λ_1 and λ_2 would be both real

Door



$$y_h = \underbrace{A e^{\lambda_1 t}}_{\text{First order}} + \underbrace{B e^{\lambda_2 t}}$$

To solve this, we need initial conditions to plug in!

$$\lambda_{1,2} = -\frac{C}{2M}$$

② $C^2 = 4MK$ \longrightarrow Critically damped

$$\lambda_{1,2} = -\frac{C}{2M}$$

$$y_h = A e^{-\frac{Ct}{2M}} + B t e^{-\frac{Ct}{2M}}$$

Proto Caraa

Really Fast Pumping!



Don't want to duplicate!

③ $C^2 < 4MK \longrightarrow$ Underdamped

$$y_h = e^{dt} (A \cos \beta t + B \sin \beta t)$$

$$d = -\frac{c}{2m}$$

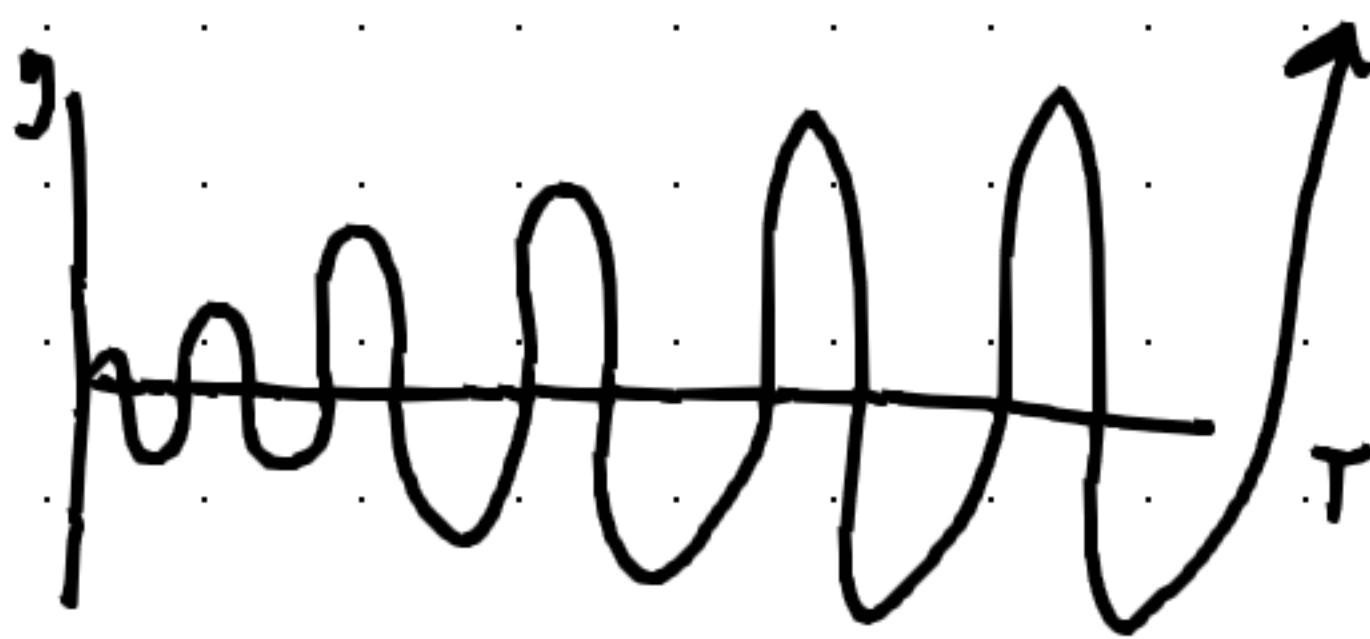
$$\beta = \frac{\sqrt{4MK - C^2}}{2m}$$

$$\frac{\sqrt{-1} \sqrt{C^2 - 4MK}}{2m}$$

$$\lambda_{1,2} = d \pm i\beta$$

C^2 is smaller than
 $4MK$

Stability \longrightarrow Only dependent on the real part of λ (or α)



\leftarrow This is an unstable system!
 $y \rightarrow$ growing
 $t \rightarrow \infty, y \rightarrow \infty$

- { If all λ has negative real part \longrightarrow Stable Decaying to steady constant
 - $y = 2e^{-3t}, \lim_{t \rightarrow \infty} y = 2 \frac{1}{e^{3t}} \cdot \infty \rightarrow 0$ Stable!
- If any λ has positive real part \longrightarrow Unstable Approaches ∞
- If λ has a zero real part \longrightarrow Marginally Stable Some that doesn't grow or shrink.
Usually No damper.

The Stability/Unstability comes ONLY from y_h

TLDR:

Over damped

$$C^2 > 4\mu K$$

Roots

Real, distinct

Response

Slow

Critically Damped

$$C^2 = 4\mu K$$

Real, Repeated

Underdamped

$$C^2 < 4\mu K$$

Complex Conjugates

Oscillatory decay

Fast

Now, we need to take the derivative for the 0

$$\begin{aligned}y_h &= Be^{-4t} + (A + Bt)(-4e^{-4t}) \\&= e^{-4t}(B - 4\end{aligned}$$

Product Rule

Examples:

\rightarrow C "dampener", related to velocity \dot{y}

$$\ddot{y} + 8\dot{y} + 16y = 0$$

$$m=1$$

$$y(0) = 0$$

$$\dot{y}(0) = 6$$

\hookrightarrow K "Spring constant" related to displacement y

$$\ddot{y} = \lambda^2$$

$$\dot{y} = \lambda = \lambda^2 + 8\lambda + 16 = 0$$

$$y = 1$$

$$(\lambda+4)(\lambda+4) = 0$$

$$\lambda_{1,2} = -4$$

Repeating Root,

Critically Damped

$$y_h = Ae^{-4t} + Bte^{-4t}$$



$$y_h = e^{-4t}(A + Bt)$$

$$y(0) = 0 \rightarrow 0 = 1(A + B(0))$$

① $A = 0$

Now, we need to take two derivatives for the other IC

Product Rule: $U'v + uv'$

$$y_h = Be^{-4t} + (A + Bt)(-4e^{-4t})$$
$$= e^{-4t}(B - 4)(A + Bt)$$

$$y'(t) = Be^{-4t}(1 - 4t) \quad \leftarrow \text{Plug in the Already Solved } A = 0$$

Plug in IC

Velocity

$$y'(0) = 6$$

$$y'(0) = Be^0(1 - 4(0)) = 1$$

So,

$$y(t) = 6te^{-4t}$$

$$B = 6$$



Find the time in which the velocity is zero.

or $y'(t) = 0$

$$y'(t) = 6e^{-4t}(1 - 4t)$$

$$0 = 6e^{-4t}(1 - 4t)$$

* Solve for t

You can figure it!

$$1 - 4t = 0$$

$$t = \frac{1}{4}$$

* e can never be zero, even if t is zero!

Solve Question Two

Tucovy

Possibly do Three....