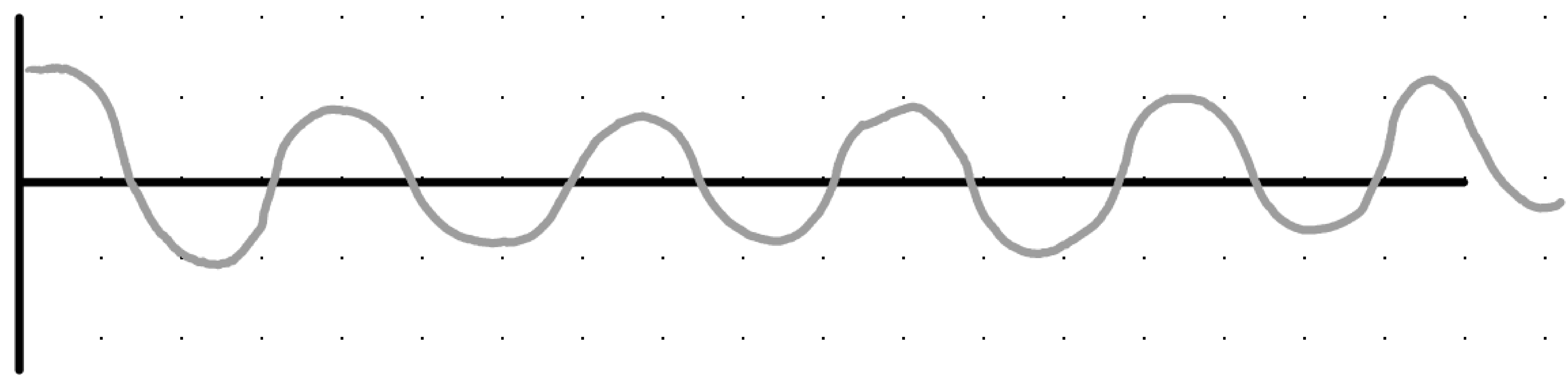
Frequency Response

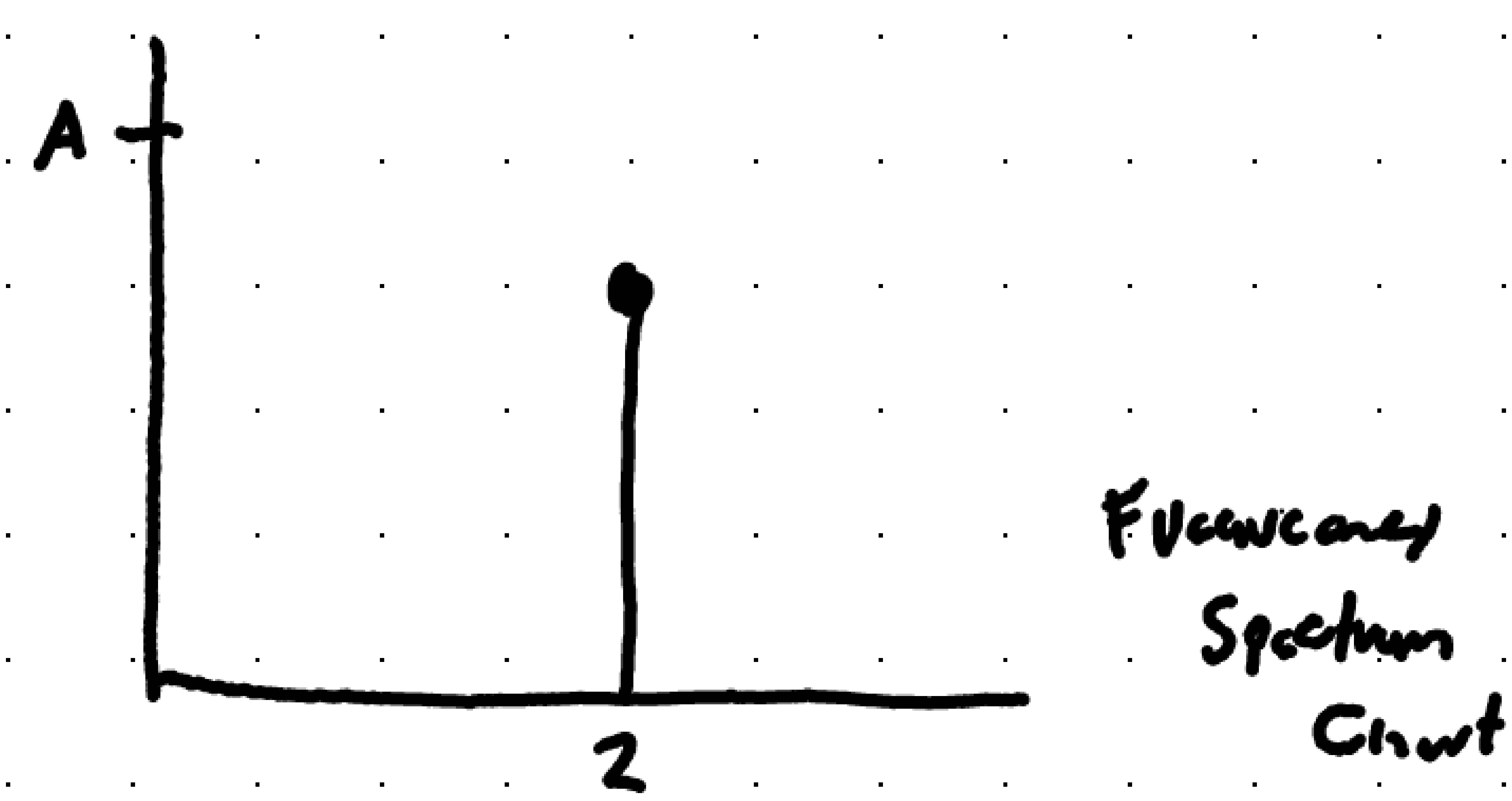
Example of a Sine Wave $\omega = 2\pi f$

$V(t) = A\cos(\omega t + \theta)$ or say $V(t) = A\cos(2\pi f t + \theta$



Sin and Cos waves have each one frequency, and Amplitude.

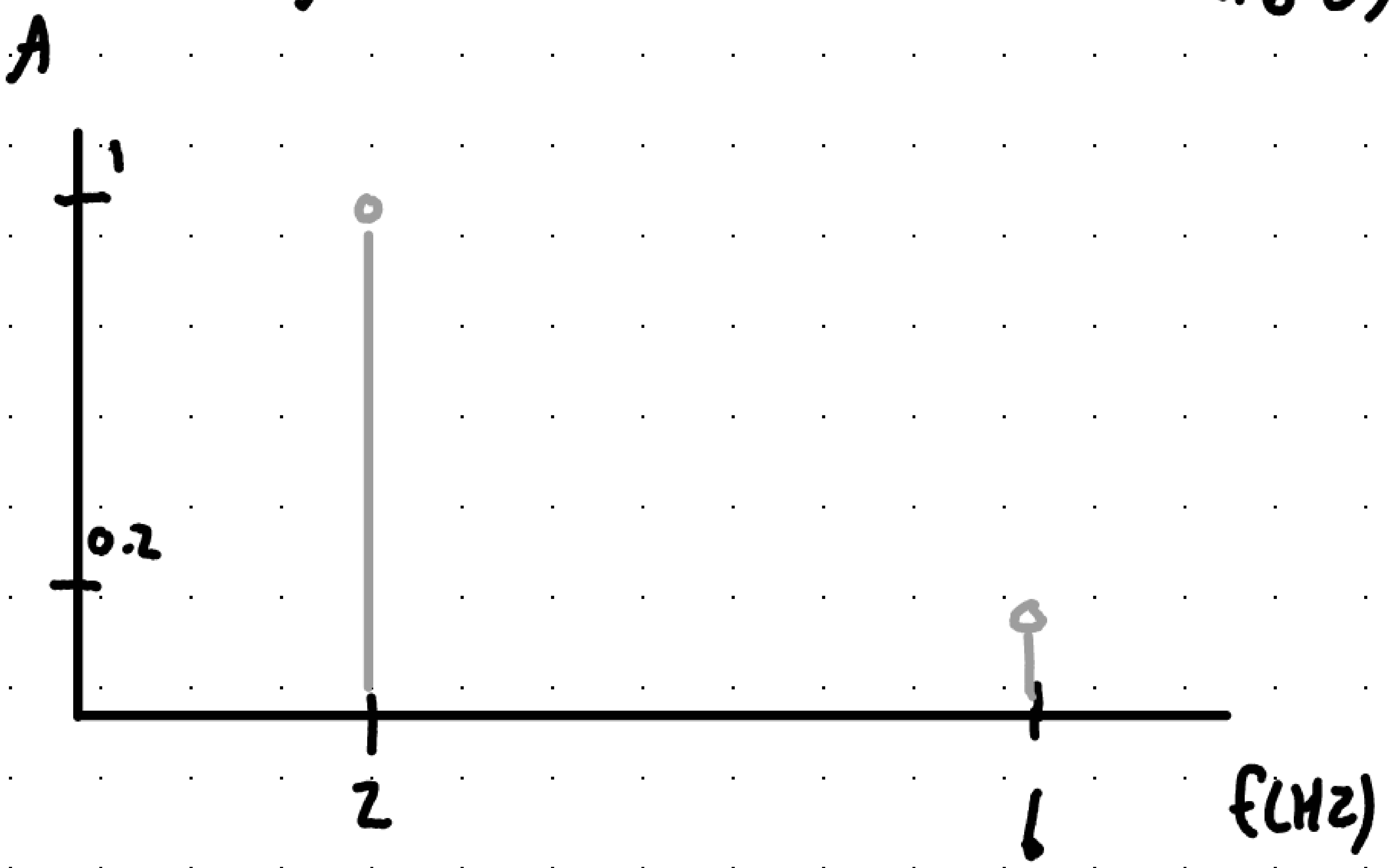
Say we have $x_1 = \sin(2\pi 2 t)$:



We could create a chart, with the one point of frequency (in Hz)

Say however,

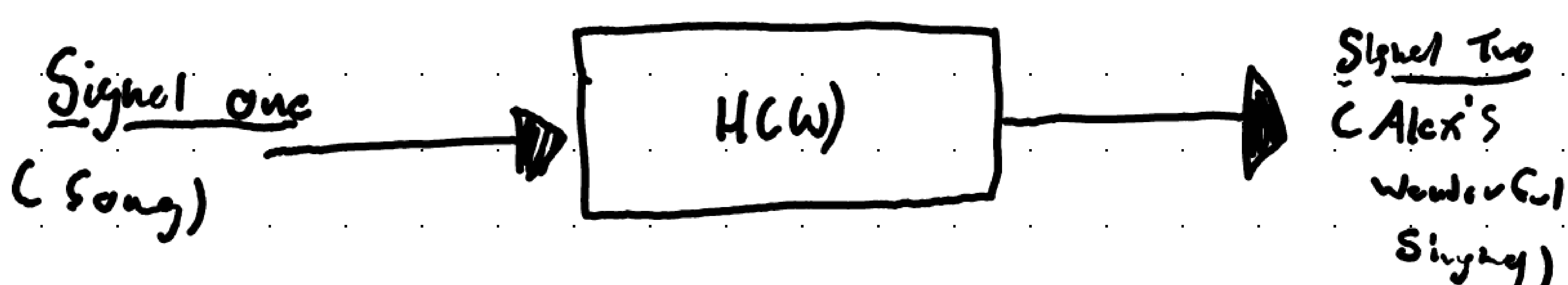
$$x_s = \sin(2\pi 2t) + 0.2\sin(2\pi 6t)$$



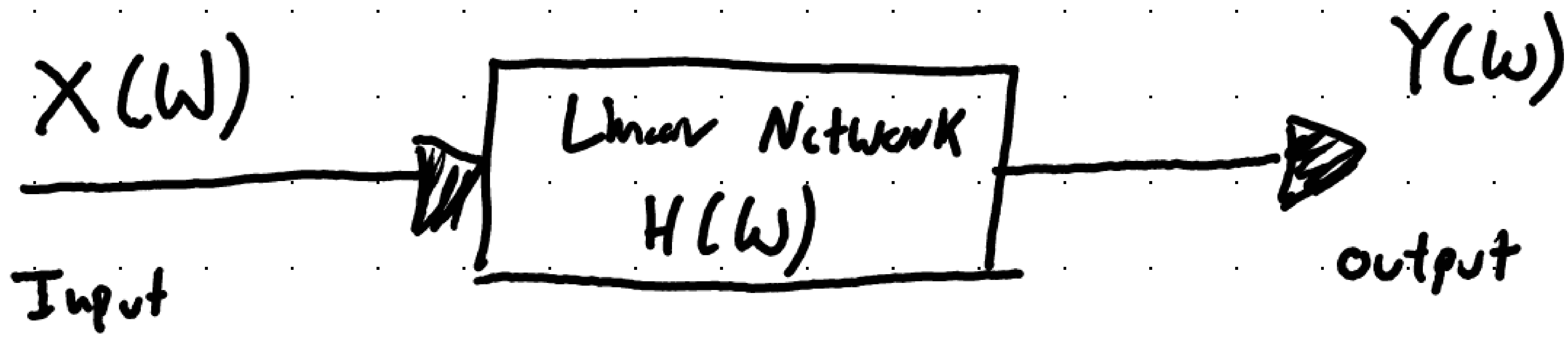
We would have something that looks more like this!

In signal and frequency analysis, the Fourier Transform is frequently used to break down complex signals and waves into singular waves.

Think of something like music! We can break it into components.



- The transfer function ($H(W)$) is a useful analytical tool for finding the frequency response of a circuit



- In this context, $X(W)$ and $Y(W)$ are denoted as the input and output respectively.

- The transfer function is equal to:

$$H(W) = \frac{\text{Output } Y(W)}{\text{Input } X(W)}$$

And this is known as Gain!

This is also true for voltage Gain $\left(\frac{V_o(W)}{V_i(W)}\right)$, Current gain $\left(\frac{I_o(W)}{I_i(W)}\right)$, Transfer gain $\left(\frac{V_o(W)}{I_o(W)}\right)$ and Transfer Admittance $\left(\frac{I_o(W)}{V_i(W)}\right)$

How does one obtain the transfer function?

1. First, convert everything into their frequency domain equivalents

2. Use any circuit technique you'd like to obtain the quantity that you are looking for. (V_i, V_o, etc)

3. Obtain the frequency response by plotting the magnitude and phase of the transfer function as frequency varies.

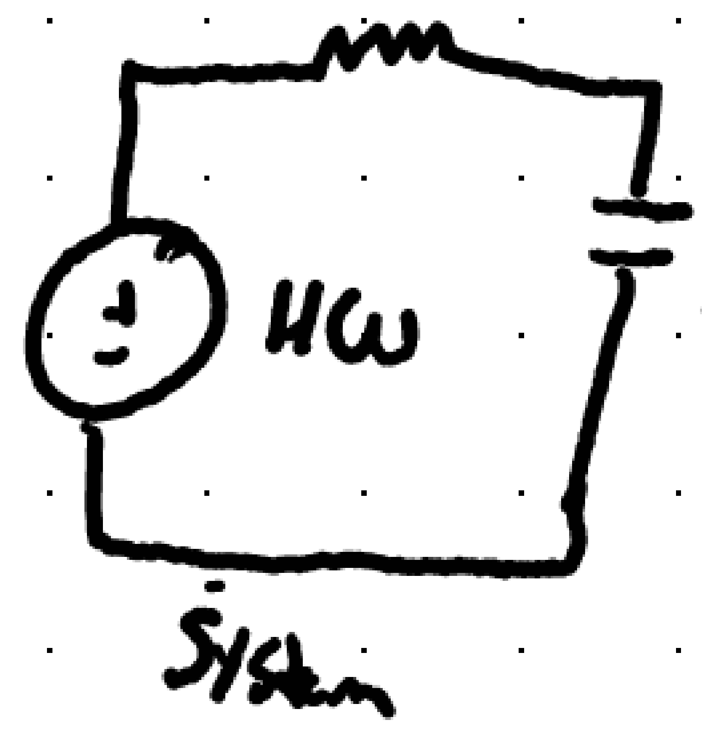
TLDR: Find output and input values, and plug em' in to the formulas on the previous page

$H(\omega)$ → plug in → simplify → take magnitude → find ph

Rc Circuit

$$H = \frac{1}{\sqrt{1+(W/W_0)^2}} \qquad \Phi = -\tan^{-1}\left(\frac{W}{W_0}\right)$$

$X(t) = A\cos(\omega t + \phi) \longrightarrow$

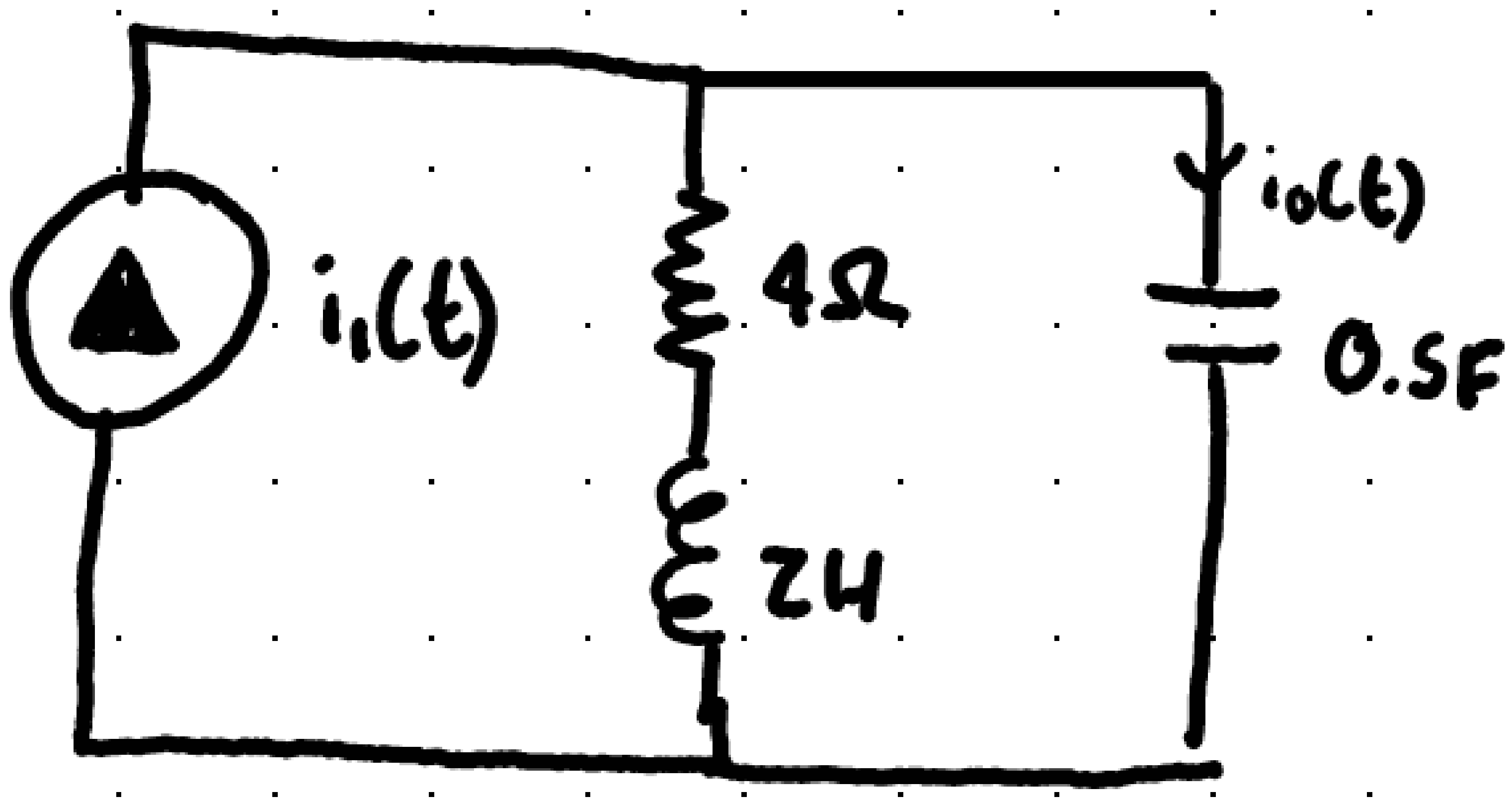


$\longrightarrow y(t) =$

$A|H(\omega)|\cos(\omega t + \phi + \arg(H(\omega))$

Example

Calculate the gain $I_o(\omega)/I_i(\omega)$, and its poles and zeros



Current Divider

$$i_o(\omega) = \frac{4 + j2\omega}{4 + j2\omega + \frac{1}{j0.5\omega}}$$

$$H = \frac{\text{Output}}{\text{Input}} = \frac{I_o(\omega)}{I_i(\omega)} = \frac{2j\omega + (j\omega)^2}{1 + 2j\omega + (j\omega)^2}$$

Let $s = j\omega$

$$H = \frac{2s + s^2}{1 + 2s + s^2} = \frac{s(s+2)}{(s+1)^2}$$

Zeros at $z_1 = 0, \; z_2 = -2$