[1] Step One: Find $y_H$

$$\lambda^2 + 2\lambda + \lambda = 0$$

$$(A + Bt)e^{-t}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-2 \pm \sqrt{4 - 4(1)(1)}}{2(1)}$$

*Critically Damped

$$\lambda_{1,2} = -1$$

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[2] Step Two: Find $y_P$

$$\ddot{y} + 2\dot{y} + y = \cos(\omega t)$$

$$\ddot{Y}_p + 2\dot{Y}_p + Y_p = e^{i\omega t}$$

$$Y_p = De^{i\omega t}$$
$$\dot{Y}_p = i\omega De^{i\omega t}$$
$$\ddot{Y}_p = -\omega^2 De^{i\omega t}$$

$$\lambda_{1,2} = -1$$

$$y = RE[Y_P]$$

No dupe

$$De^{i\omega t}(-\omega^2 + 2i\omega + 1) = e^{i\omega t}$$

$$D = \frac{1}{-\omega^2 + 2i\omega + 1}$$

$$Y_p = \left( \frac{1}{-\omega^2 + 2i\omega + 1} \right) e^{i\omega t}$$

$$Y_p = \left( \frac{1}{-\omega^2 + 2i\omega + 1} \right) e^{i\omega t} \cdot \frac{\overbrace{1 - \omega^2}^{a} - \overbrace{2i\omega}^{b}}{\underbrace{1 - \omega^2}_{a} - \underbrace{2i\omega}_{b}}$$

$$= a^2 - b^2$$

$$Y_p = \frac{1 - \omega^2 - 2i\omega}{(1 - \omega^2)^2 + (2\omega)^2} e^{i\omega t}$$

$$Y_p = \frac{2i\omega + \omega^2 - 1}{(1 - \omega^2)^2 + 4\omega^2} e^{i\omega t}$$

$$Y_p = \frac{2i\omega + \omega^2 - 1}{(1 - \omega^2)^2 + 4\omega^2} (\cos \omega t + i \sin \omega t)$$

#Only Want Real Here...

$$y_p = \frac{1}{(1 - \omega^2)^2 + 4\omega^2} \Big( (2i\omega)(i \sin \omega t) + (\omega^2)(\cos \omega t) - (1)(\cos \omega t) \Big)$$

$$y_p = \frac{1}{(1 - \omega^2)^2 + 4\omega^2} \Big( (-2\omega \sin \omega t) + (\omega^2 \cos \omega t) - (\cos \omega t) \Big)$$

$$y_p = \frac{1}{(1 - \omega^2)^2 + 4\omega^2} \Big( (-2\omega \sin \omega t) + (\omega^2 - 1) \cos \omega t \Big)$$

$$y_p = \frac{1}{(1-\omega^2)^2 + 4\omega^2}\left(\underbrace{(-2\omega \sin \omega t)}_{c_1} + \underbrace{(\omega^2 - 1)\cos \omega t}_{c_2}\right)$$

\# prep For Transfer function — Amp Phase Form

$$A = \sqrt{c_1^2 + c_2^2} \quad , \quad A = \sqrt{(-2\omega)^2 + (\omega^2-1)^2}$$

$$= \sqrt{\omega^4 + 2\omega^2 + 1}$$

$$\phi = \tan^{-1}\left(\frac{c_2}{c_1}\right)$$

cos form

$c_1$ and $c_2$ flipped for Sin

$$\tan^{-1}\left(\frac{\omega^2-1}{-2\omega}\right)$$

$$y_p = \frac{1}{(1-\omega^2)^2 + 4\omega^2}\left(\sqrt{\omega^4 + 2\omega^2 + 1}\,\cos\left(\omega t - \tan^{-1}\left(\frac{\omega^2-1}{-2\omega}\right)\right)\right)$$
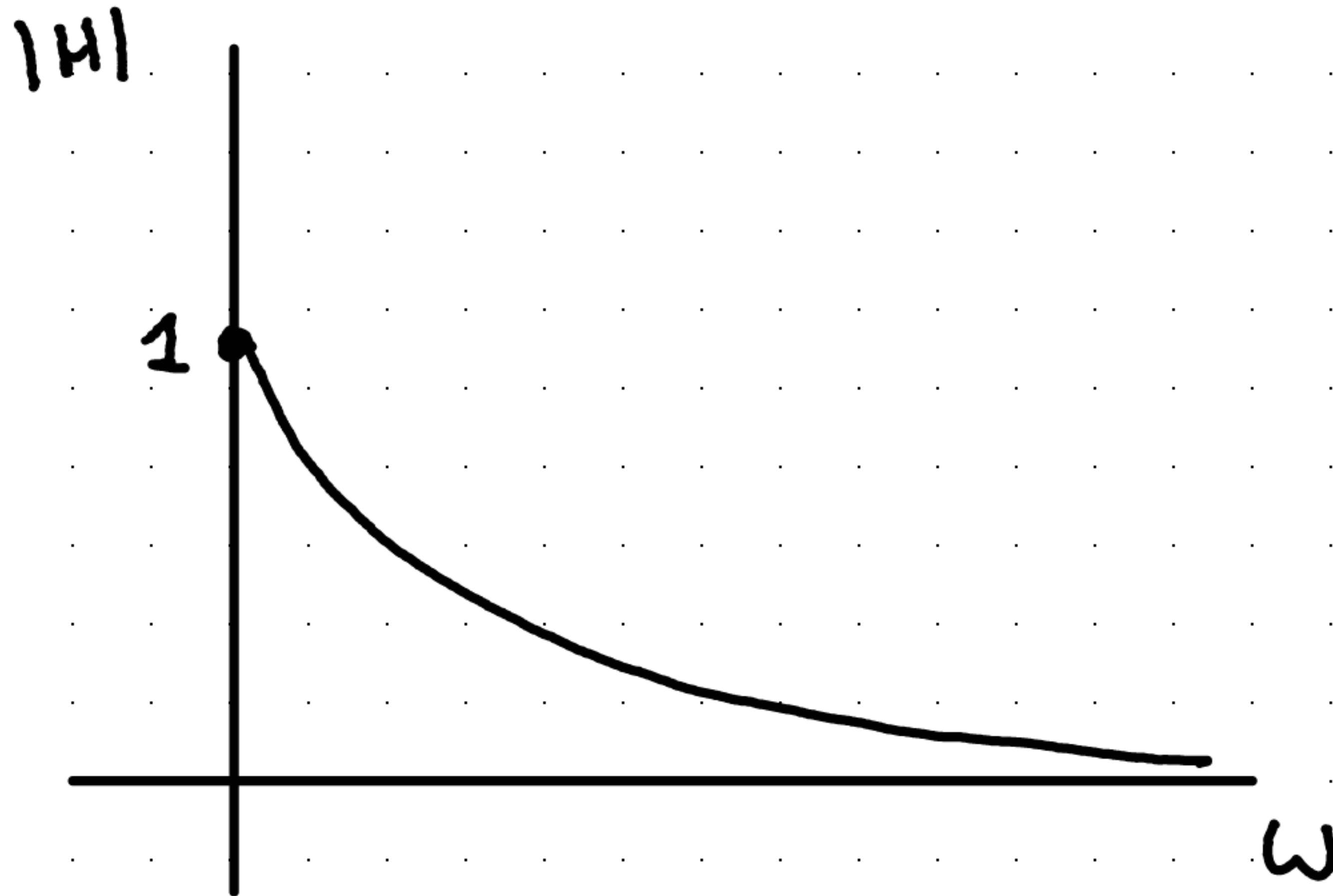
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Ready for Transfer Function

$$H(\omega) = \frac{\text{Amp of Response}}{\text{Amp of forcing}} = \frac{\sqrt{\omega^4 + 2\omega^2 + 1}}{\frac{(\omega^4 + 2\omega + 1)}{1}}$$

$$\omega^4 - 2\omega^2 + 1 + 4\omega^2$$
$$\omega^4 + 2\omega^2 + 1$$

$$= \frac{1}{\sqrt{\omega^4 + 2\omega^2 + 1}}$$

Graphing Transfer function:

$$\frac{1}{\sqrt{\omega^4 + 2\omega^2 + 1}}$$

|H|

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Big Time goes to Zero.