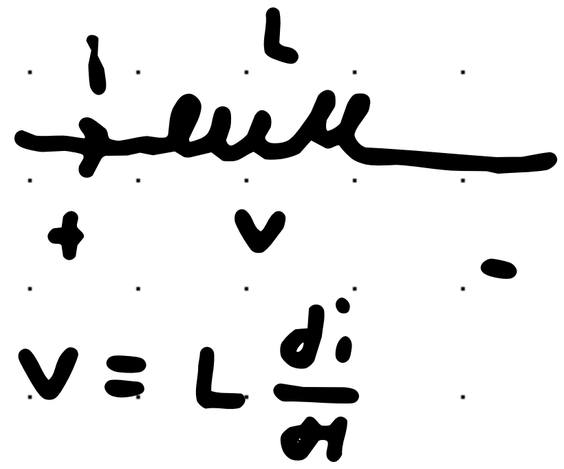
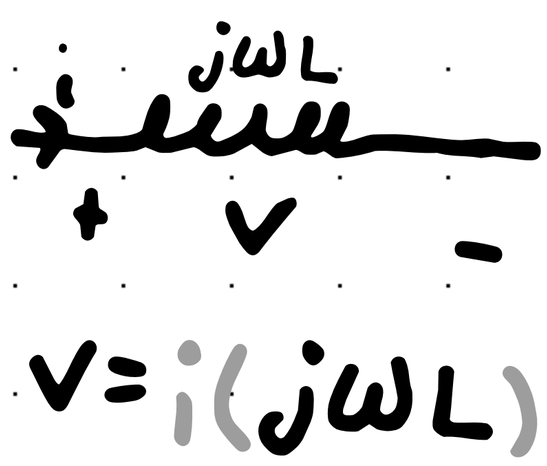
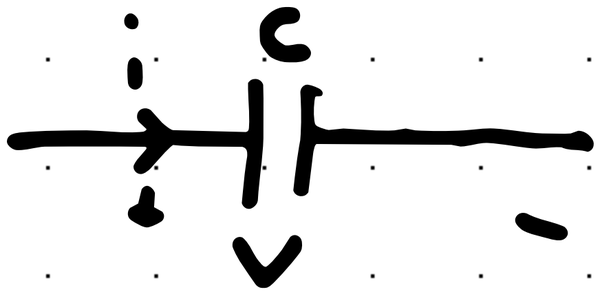
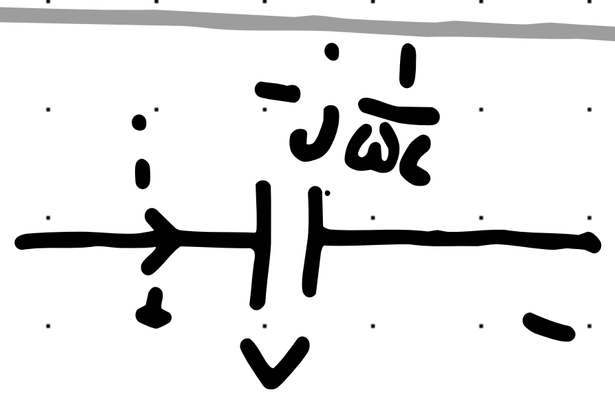
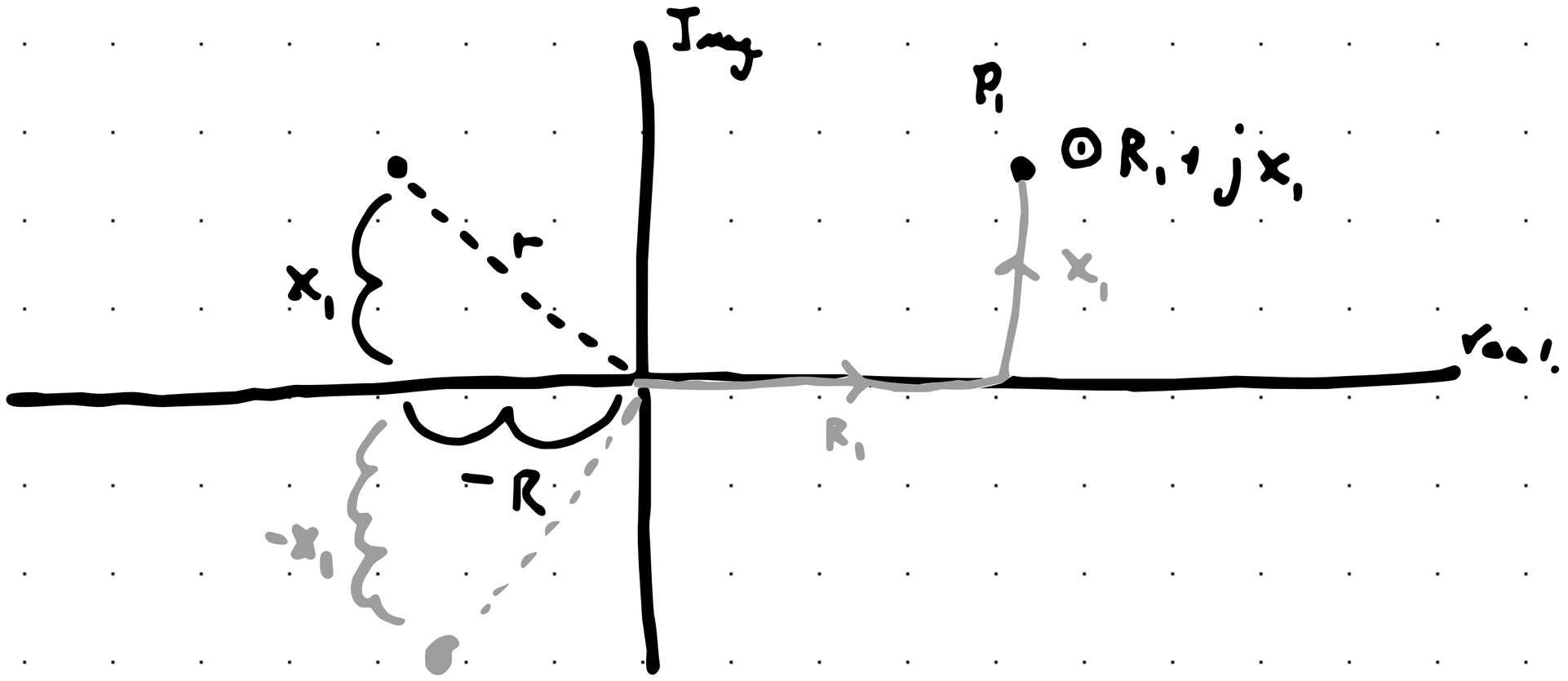
Lecture 20 ECED 2000

Sinusoidal Steady State

Element	Time domain	Phasor domain
① Resistor	 $v = IR$	 $v = IR$
② Inductor	 $v = L\frac{di}{dt}$	 $V = i(j\omega L)$
③ Capacitor	 $v = \frac{1}{C}\int i\,dt + V_{(t_0)}$	 $V = i(-j\frac{1}{\omega C})$

Complex Numbers



Complex plane

P_1 is represented by either:

Rectangular Form $R_1 + jx_1$

Polar Form $r\angle\theta$

$$r\angle\theta = R_1 + jx_1 = r\cos\theta + jr\sin\theta,$$

$$R_1 jx_1 = \sqrt{R_1^2 + x_1^2}\ \angle \tan^{-1}\left(\frac{x}{R}\right) = r\angle\theta$$

For the Second Quadrant

$$-R_1 + jx_1 = \sqrt{R_1^2 + x_1^2}\ \angle 180 - \tan^{-1}\frac{x_1}{R_1}$$

For the Third Quadrant

$$-R_1 - jX_1 = \sqrt{R_1^2 + X_1^2} \angle 180 + \tan^{-1}\frac{X_1}{R_1}$$

For the fourth Quadrant

$$-R_1 - jX_1 = \sqrt{R_1^2 + X_1^2} \angle -\tan^{-1}\frac{X_1}{R_1}$$

As well $j = \sqrt{-1}$, $j^2 = 1$

Adding or Subtracting two Complex Numbers

$$\Rightarrow (R_1 + jX_1) \pm (R_2 + jX_2) =$$

$$= (R_1 \pm R_2) + j(X_1 \pm X_2)$$

$$\Rightarrow r_1\angle\theta + r_2\angle\theta_2 =$$

$$= r_1\cos\theta_1 + jr_1\sin\theta_1 + r_2\cos\theta_2 + jr_2\sin\theta_2 =$$

$$= (r_1\cos\theta_1 + r_2\cos\theta) + j(r_1\sin\theta_1 + r_2\sin\theta_2)$$

Multiplying or dividing two Complex numbers

$$(R_1 + jX_1)(R_2 + jX_2) = R_1R_2 + jR_1X_2 + jR_2X_1 - X_1X_2$$

$$= (R_1R_2 - X_1X_2) + j(R_1X_2 + R_2X_1)$$

Multiplication

$$\left(\frac{R_1 + jX_1}{R_2 + jX_2}\right)\left(\frac{R_2 - jX_2}{R_2 - jX_2}\right)$$

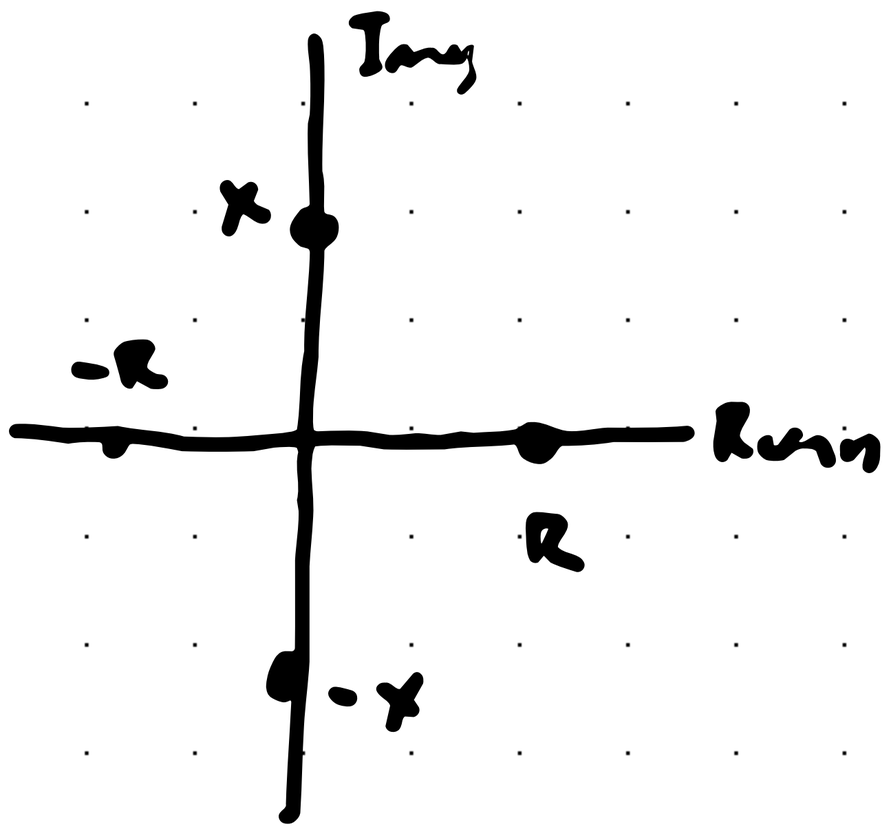
Complex Conjugate needed

Division

Polar

$$\frac{r_1 \angle \theta_1}{r_2 \angle \theta_2} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2,$$

$$(r_1 \angle \theta_1)(r_2 \angle \theta_2) = r_1 r_2 \angle \theta_1 + \theta_2$$



$$R = R + j0 = R\angle 0$$

(All real, so angle is zero)

$$jx = x\angle 90^\circ$$

(All Imaginary, so angle is 90°)

$$-R = R\angle 180^\circ$$

$$-jx = x\angle 270^\circ$$

Resistor

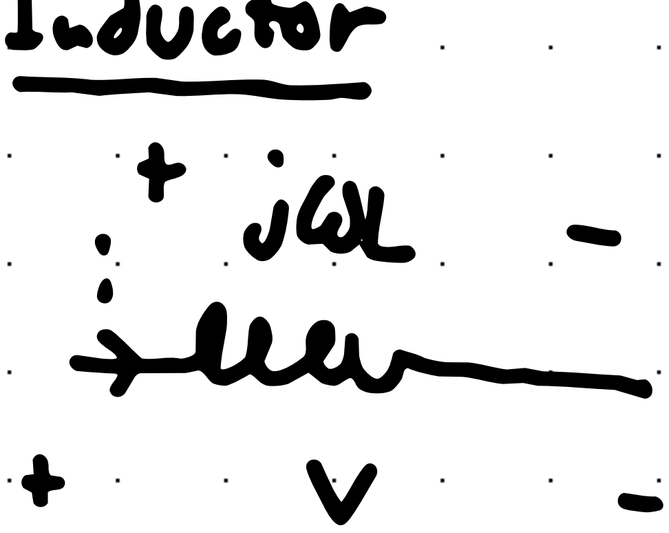
$$V = IR$$

Angle of voltage, and Angle of current is the same

No phase shift

$$5\angle 90 = 5_i$$

Inductor



$$jX_L = j\omega L = j2\pi fL$$

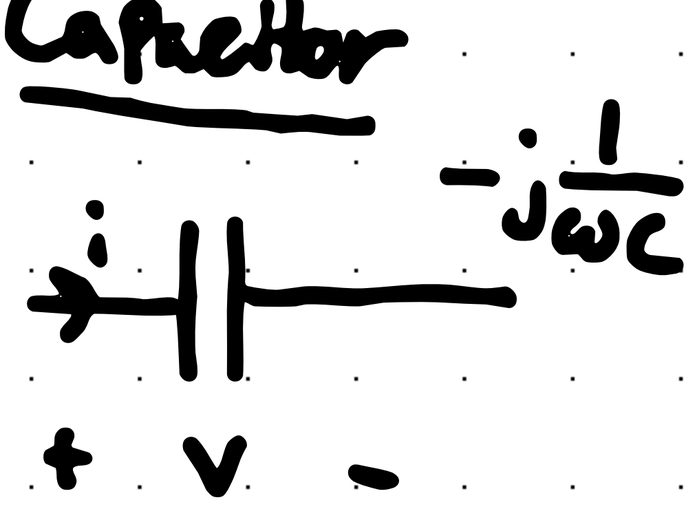
☆ Reactance (Ω)

$$V = i(j\omega L)$$

$$= i\omega L \angle 90^\circ$$

Voltage leads current by 90°. Phase Shift is 90°

Capacitor



$$-j\left(\frac{1}{\omega C}\right) = -j\frac{1}{2\pi fC}$$

$$V = i\left(-j\frac{1}{\omega C}\right) = i(-jX_C)$$

Voltage in Phasor domain

- $v(t) = V_m \cos(\omega t + \phi)$ ← Phi

  Time Domain

  ⇓

  Phasor

$$V = V_m \angle \phi$$

Current in Phasor domain

- $i(t) = I_m \cos(\omega t + \theta)$

  Time Domain

  ⇓

  Phasor

$$i(t) = I_m \angle \theta$$