

Homogeneous Systems

\ddot{y} = Acceleration

\dot{y} = Velocity

$$m\ddot{y} + cy + ky = 0$$

$$y = e^{\lambda t} \quad \left| \begin{array}{l} \ddot{y} \rightarrow \lambda^2 \\ \dot{y} \rightarrow \lambda \\ y \rightarrow 1 \end{array} \right.$$

How to find homogeneous y_h

$$\lambda_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mK}}{2m}$$

There are three cases:

if $c^2 - 4mK = 0$

if $c^2 - 4mK$ is positive

if $c^2 - 4mK$ is negative,

(or $c^2 > 4mK$) Overdamped

imaginary numbers

(or $c^2 < 4mK$)

① $c^2 > 4mK \longrightarrow$ overdamped ——— •

λ_1 and λ_2 would be both real

Door



$$y_h = \underbrace{Ae^{\lambda_1 t}}_{\text{First order}} + Be^{\lambda_2 t}$$

To solve this, we need initial conditions to plug in!

$$\lambda_{1,2} = -\frac{c}{2m}$$

② $c^2 = 4mK \longrightarrow$ Critically damped ——— •

$$\lambda_{1,2} = -\frac{c}{2m}$$

Photo Camera

Really Fast Ramping!

$$y_h = Ae^{-\frac{c}{2m}t} + Bte^{-\frac{c}{2m}t}$$

⤴ Don't want to duplicate!

③ $c^2 < 4mK \longrightarrow$ Underdamped

$$y_h = e^{\alpha t} (A \cos \beta t + B \sin \beta t)$$

$$\alpha = -\frac{c}{2m}$$

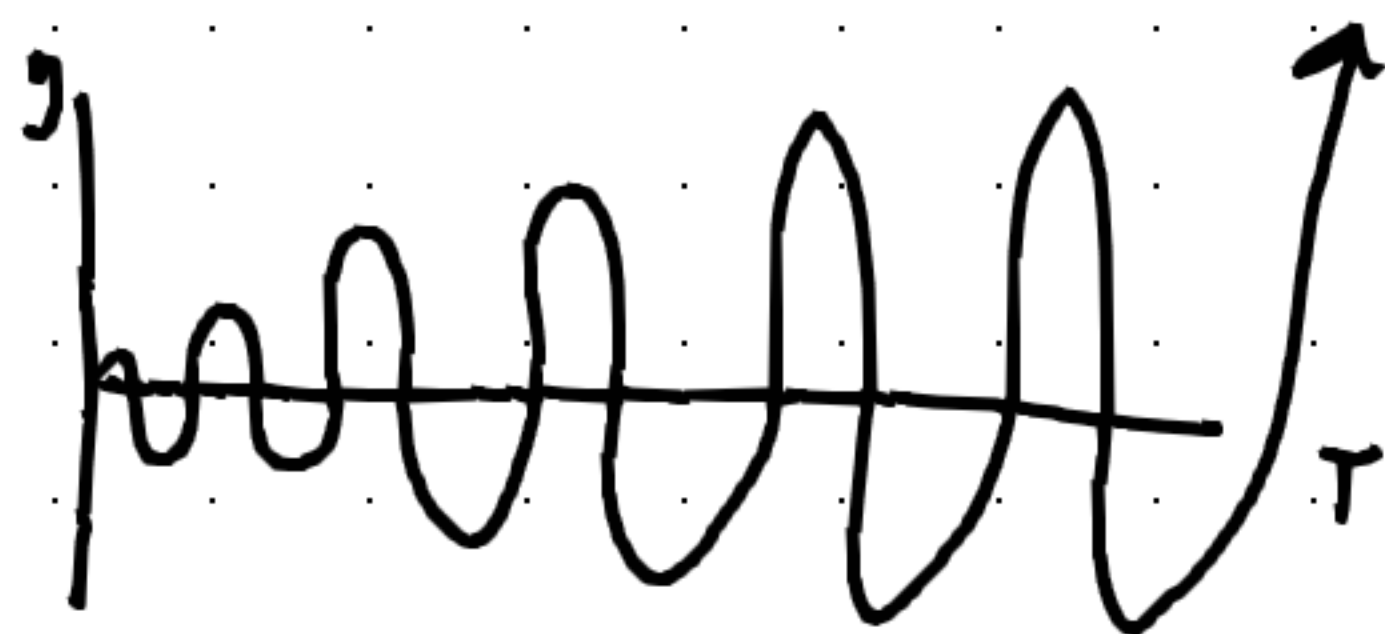
$$\beta = \frac{\sqrt{4mK - c^2}}{2m}$$

$$\frac{\sqrt{-1} \sqrt{c^2 - 4mK}}{2m}$$

\swarrow
 \nwarrow
 c^2 is smaller than $4mK$

$$\lambda_{1,2} = \alpha \pm i\beta$$

Stability \longrightarrow Only dependent on the Real part of λ (or α)



\leftarrow This is an unstable system!
 y is growing
 $t \rightarrow \infty, y \rightarrow \infty$

- If all λ has negative real part \longrightarrow **Stable** Decaying to Steady constant

$y = 2e^{-3t}, \lim_{t \rightarrow \infty} y = 2 \frac{1}{e^{3t}} \rightarrow 0$ **Stable!**
 - If **any** λ has positive real part \longrightarrow **Unstable** Approaches ∞
 - If λ has a zero real part \longrightarrow **Marginally Stable** Some that doesn't grow or Shrink. (Think \cos or \sin)

\uparrow Usually No damper.
- The Stability/Unstability Comes **ONLY** from y_h

TLDR:

		<u>Roots</u>	<u>Response</u>
Overdamped	$C^2 > 4mK$	Real, distinct	Slow
Critically Damped	$C^2 = 4mK$	Real, Repeated	Fast
Underdamped	$C^2 < 4mK$	Complex Conjugates	Oscillatory decay.

Now, we need to take the derivative for the 0

$$\begin{aligned} y' &= B e^{-4t} + (A + Bt)(-4e^{-4t}) \\ &= e^{-4t}(B - 4A - 4Bt) \end{aligned}$$

Product Rule

Examples:

\rightarrow C "dumper", related to velocity \dot{y}

$$\ddot{y} + 8\dot{y} + 16y = 0$$

\rightarrow K "Spring constant" related to

related to

displacement y

$$y(0) = 0$$

$$y'(0) = 6$$

$$n=1$$

$$\ddot{y} = \lambda^2$$

$$\dot{y} = \lambda$$

$$y = 1$$

$$= \lambda^2 + 8\lambda + 16 = 0$$

$$(\lambda + 4)(\lambda + 4) = 0$$

$$\lambda_{1,2} = -4$$

Repeated Root,

Critically Damped

$$y_h = Ae^{-4t} + Bte^{-4t}$$

\downarrow

$$y_h = e^{-4t}(A + Bt)$$

$$y(0) = 0 \rightarrow 0 = 1(A + B(0))$$

$$\textcircled{1} A = 0$$

Now, we need to take the derivative for the other IC

Product Rule: $u'v + uv'$

$$y_h = Be^{-4t} + (A+Bt)(-4e^{-4t})$$
$$= e^{-4t}(B-4)(A+Bt)$$

$$y'(t) = Be^{-4t}(1-4t) \quad \leftarrow \text{Plug in the Already Solved } A=0$$

Plug in IC

Velocity

$$y'(0) = 6$$

$$y'(0) = Be^0(1-4(0)) = 6$$

$$B = 6$$

So,

$$y(t) = 6te^{-4t}$$

Find the time in which the velocity is zero.

$$\text{or } y'(t) = 0$$

$$y'(t) = 6e^{-4t}(1-4t)$$

$$0 = 6e^{-4t}(1-4t)$$

$$1-4t = 0$$

$$t = \frac{1}{4}$$

Solve for t

You can Ignore e !

e can never be zero, even if t is zero!

Solve Question Two

Theory

Possibly do Three.....