

For RL Step Response

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}$$

Need Voltage Source

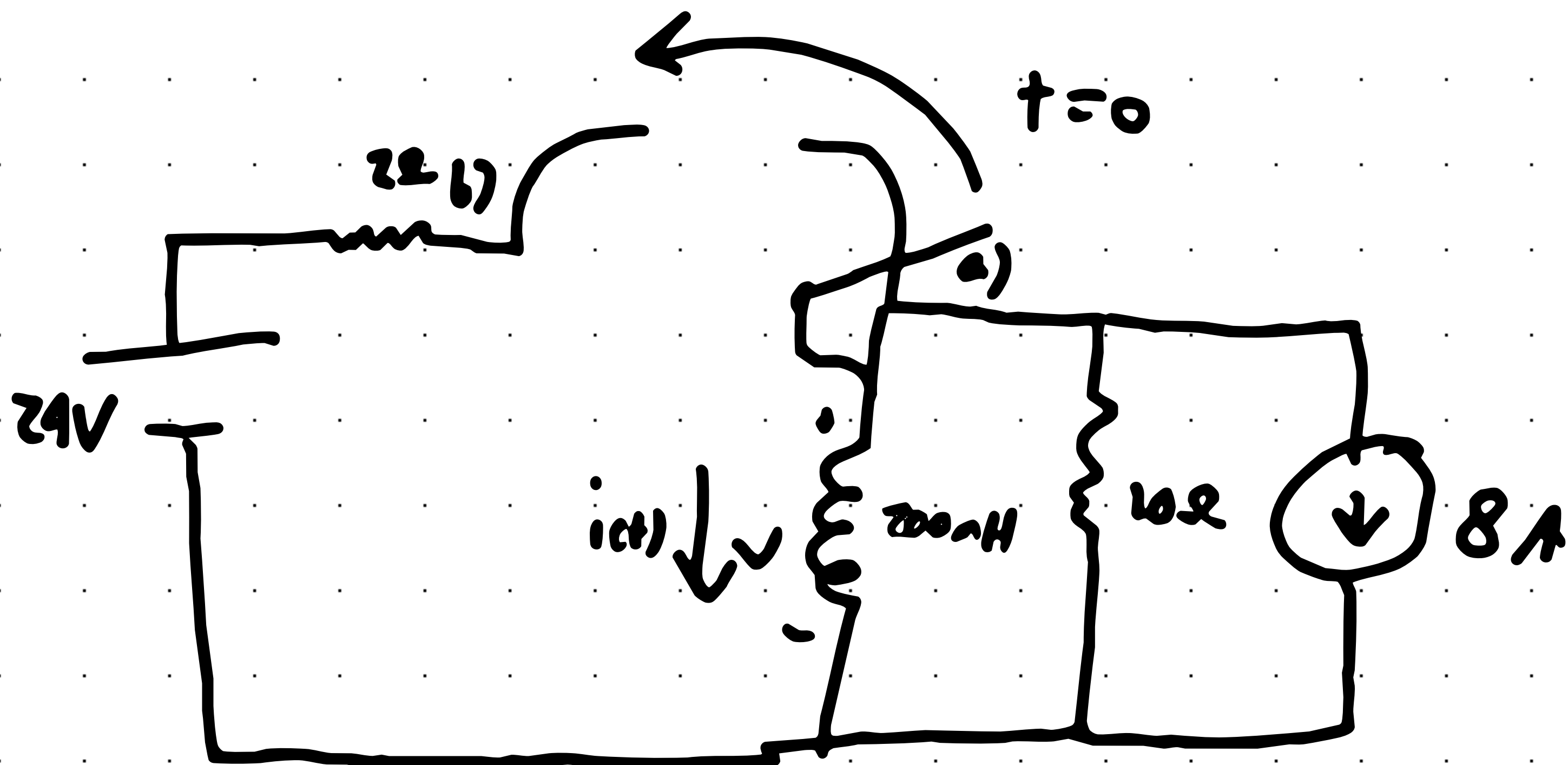
 I_0 = current in inductor at $t=0$ For RC Step Response

$$V_c(t) = I_s R + (V_0 - I_s R) e^{-\frac{t}{RC}}$$

Need Current Source

Example

- Ⓐ Find an Expression for $i(t)$ for $t \geq 0$
- Ⓑ What is the initial voltage across the inductor just after the switch has been moved to Position B?
- Ⓒ How many milliseconds after the switch has been moved does the inductor voltage equal 24V?



Solution

Before moving the switch

$$I_0 = -8A$$

(current going opposite to source)

Short

Circuit

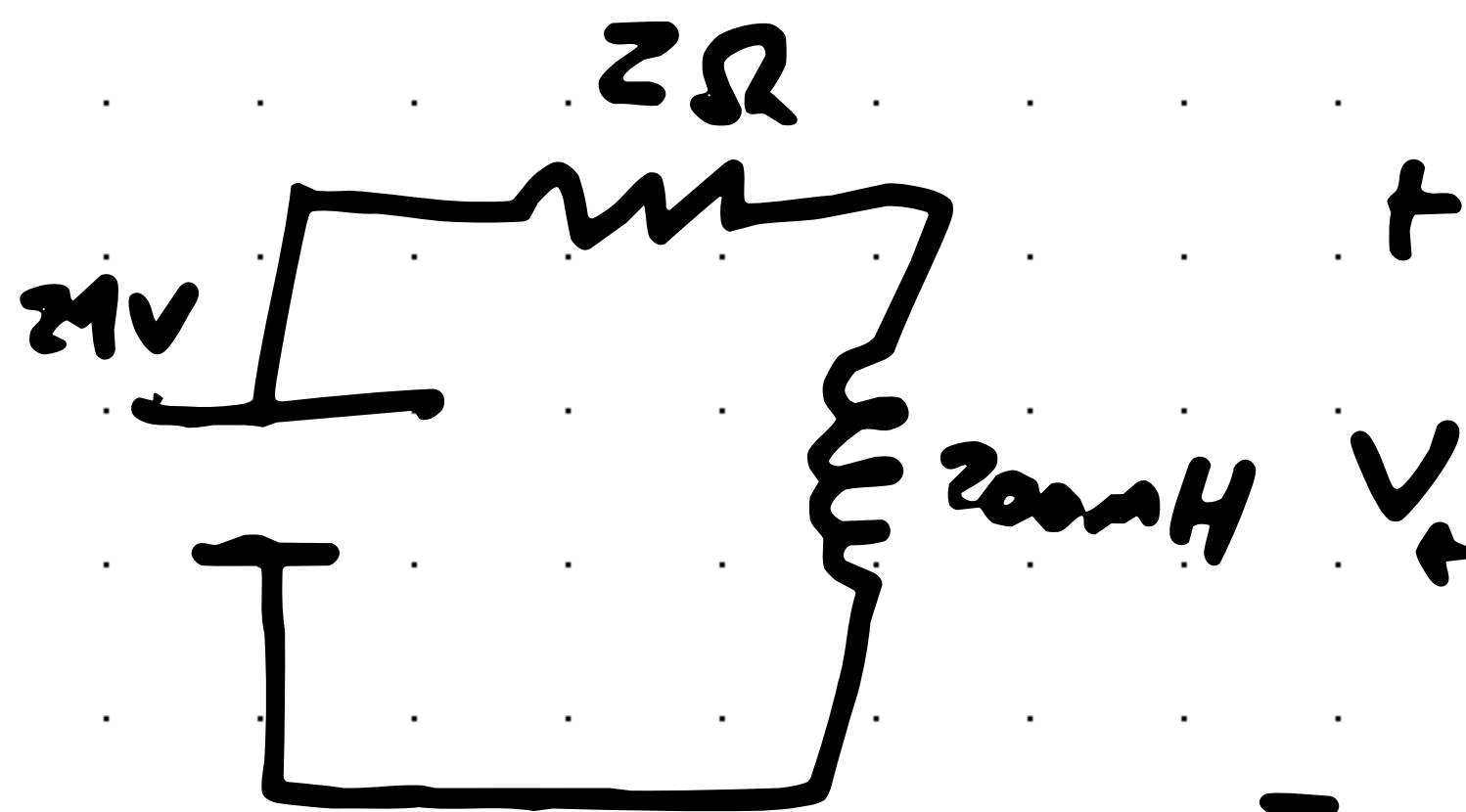


After moving the switch

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}$$

One Resistor ✓

Voltage Source ✓



$$I(t) = \frac{24}{2} + \left(-8 - \frac{24}{2}\right) e^{-\frac{2}{200 \times 10^{-3}} t}$$

(a) $i(t) = 12 - 20e^{-10t} \quad t \geq 0$

(b) $v(t) = L \frac{di}{dt} = 200 \times 10^3 (0 - 20e^{-10t} (-10))$

$v(t) = 40e^{-10t} \quad t \geq 0$

Initial voltage at $T=0$ is 40V ($40e^{-10 \times 0}$)

(c)

$$24 = 40e^{-10t}$$

$$\ln(24) = \ln(40) + \ln e^{-10t}$$

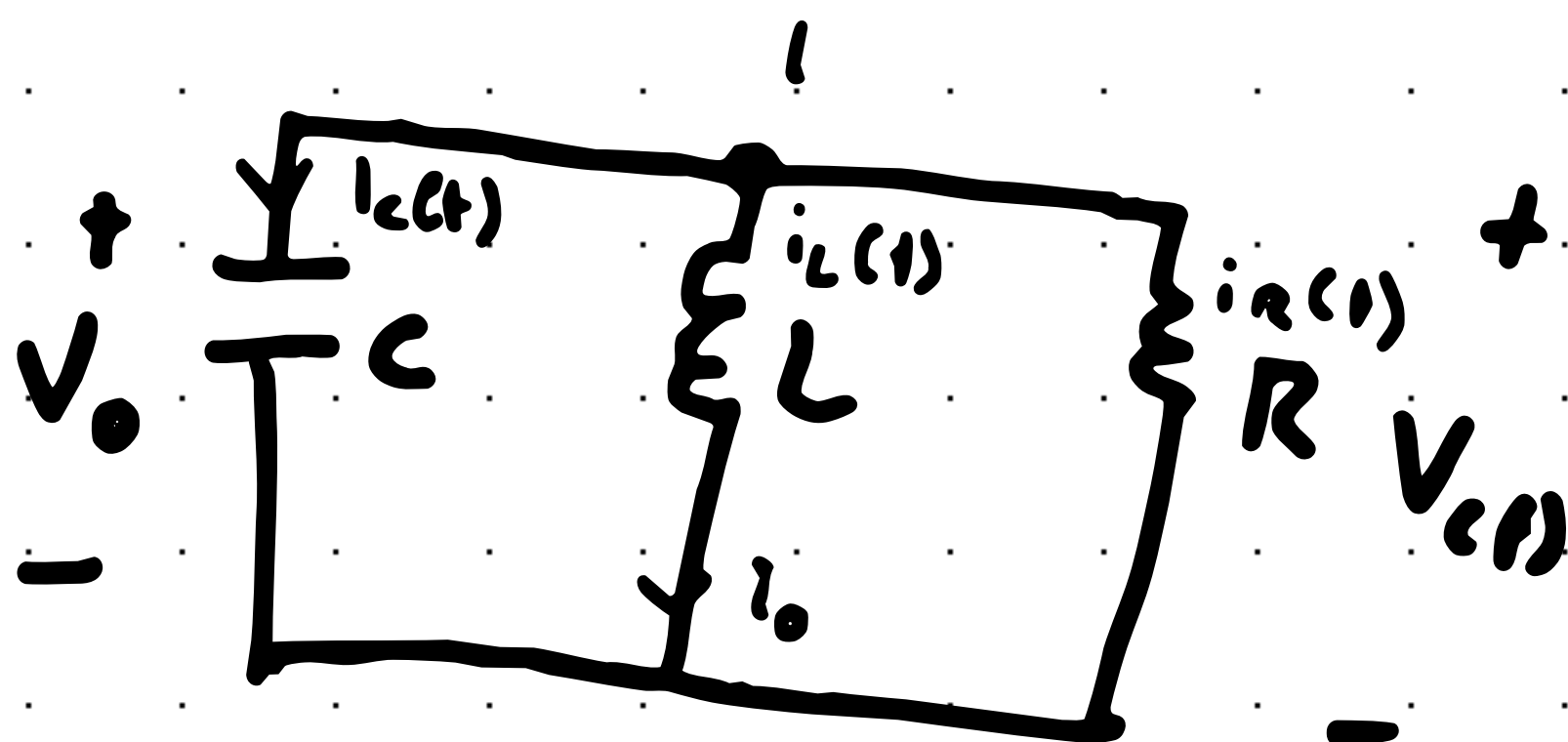
$$\ln 24 - \ln 40 = -10t$$

$t = 51.08 \mu s$

Introduction to the Natural Response of a Parallel RLC Circuit

$$\sum_{\text{node } 1} I = 0$$

CSmall i is
 $i(t)$, not waiting
 to simplify)



$$i_C + i_R + i_L = 0$$

↓ ↓ ↓

$$C \frac{dv}{dt} + \frac{v}{R} + \frac{1}{L} \int v dt = 0$$

ugly DE

$$C \frac{d^2 v}{dt^2} + \frac{1}{R} \frac{dv}{dt} + \frac{1}{L} (v) = 0 \quad \leftarrow$$

$$\frac{d^2 v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{1}{LC} v = 0$$

Assume $v(t) = Ae^{st}$

$$\frac{dv}{dt} = Ase^{st}$$

$$\frac{d^2 v}{dt^2} = AS^2 e^{st}$$

$$AS^2 e^{st} + \frac{AS}{RC} e^{st} + \frac{A}{LC} e^{st} = 0$$

$$AS^2e^{st} + \frac{AS}{RC}e^{st} + \frac{A}{LC}e^{st} = 0$$

$$(S^2 + \frac{1}{RC}S + \frac{1}{LC}) Ae^{st} = 0$$

$$S^2 + \frac{1}{RC}S + \frac{1}{LC} = 0$$

Note:

$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$S = -\frac{1}{RC} \pm \sqrt{(\frac{1}{RC})^2 - 4(1)(\frac{1}{LC})}$$

$$S_1 = -\frac{1}{2RC} + \sqrt{(\frac{1}{2RC})^2 - (\frac{1}{LC})}$$

$$S_2 = -\frac{1}{2RC} - \sqrt{(\frac{1}{2RC})^2 - (\frac{1}{LC})}$$

These are known
as complex
frequencies

$$\alpha = \frac{1}{2RC}$$

Natural
frequency

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Resonant radian
frequency

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Based on α and ω_0 values, we are expecting three different cases.

Case ① If $\alpha^2 > \omega_0^2$ (Two distinct real solutions)
This case is called (over damped)

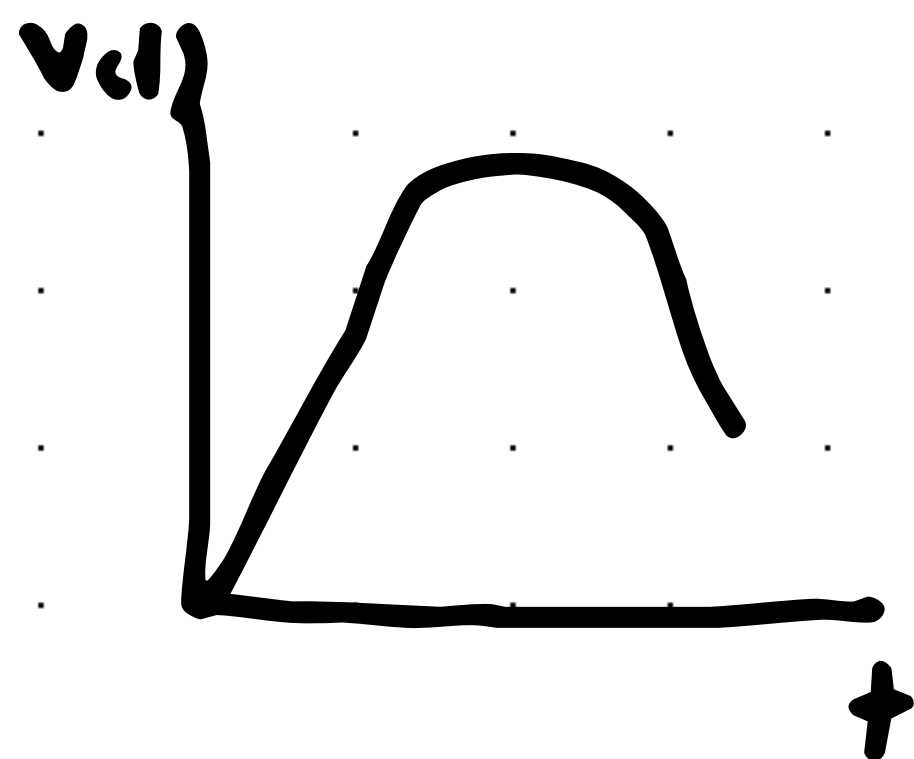
Solution

$$V(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

For A_1 and A_2 calculations, we use initial conditions.

$$V(0)^+ = A_1 + A_2;$$

$$\frac{dV(t)^+}{dt} = S_1 A_1 + S_2 A_2 = \frac{i_c(0)}{C}$$



- Step one: Solve for α and ω
- Step two: Compare α and ω
- Step three: If α^2 is less than ω , use case ①

Case ② If $\alpha^2 < \omega_0^2$

(Under damped case)
Two complex
conjugates

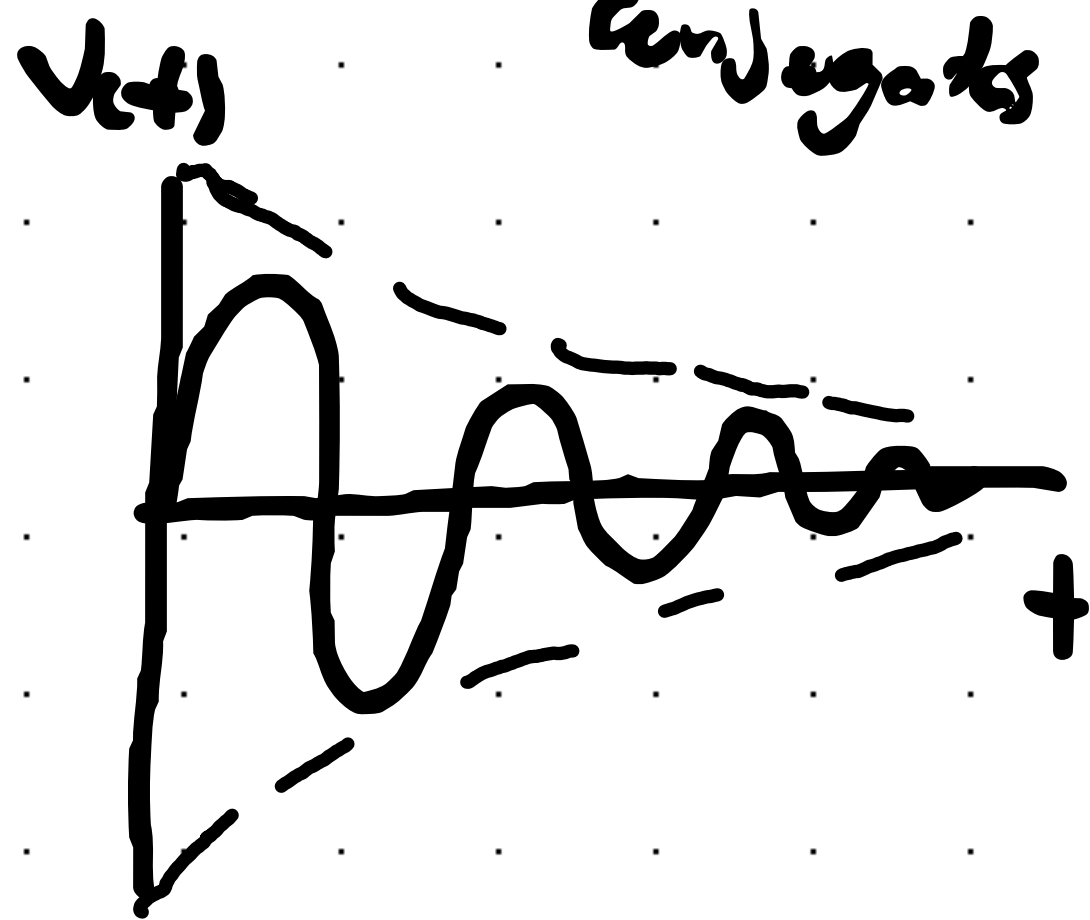
$$v(t) = \beta_1 e^{-\alpha t} \cos \omega_d t + \beta_2 e^{-\alpha t} \sin \omega_d t$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

For finding β_1 and β_2 , use initial
conditions

$$v(0)^+ = \beta_1$$

$$\frac{dv(0)^+}{dt} = -\alpha \beta_1 + \omega_d \beta_2$$



Case ③ if $\omega^2 = \alpha^2$

(Critically damped)
(Repeated Roots)
(Real)

$$v(t) = D_1 + e^{-\alpha t} + D_2 e^{-\alpha t}$$

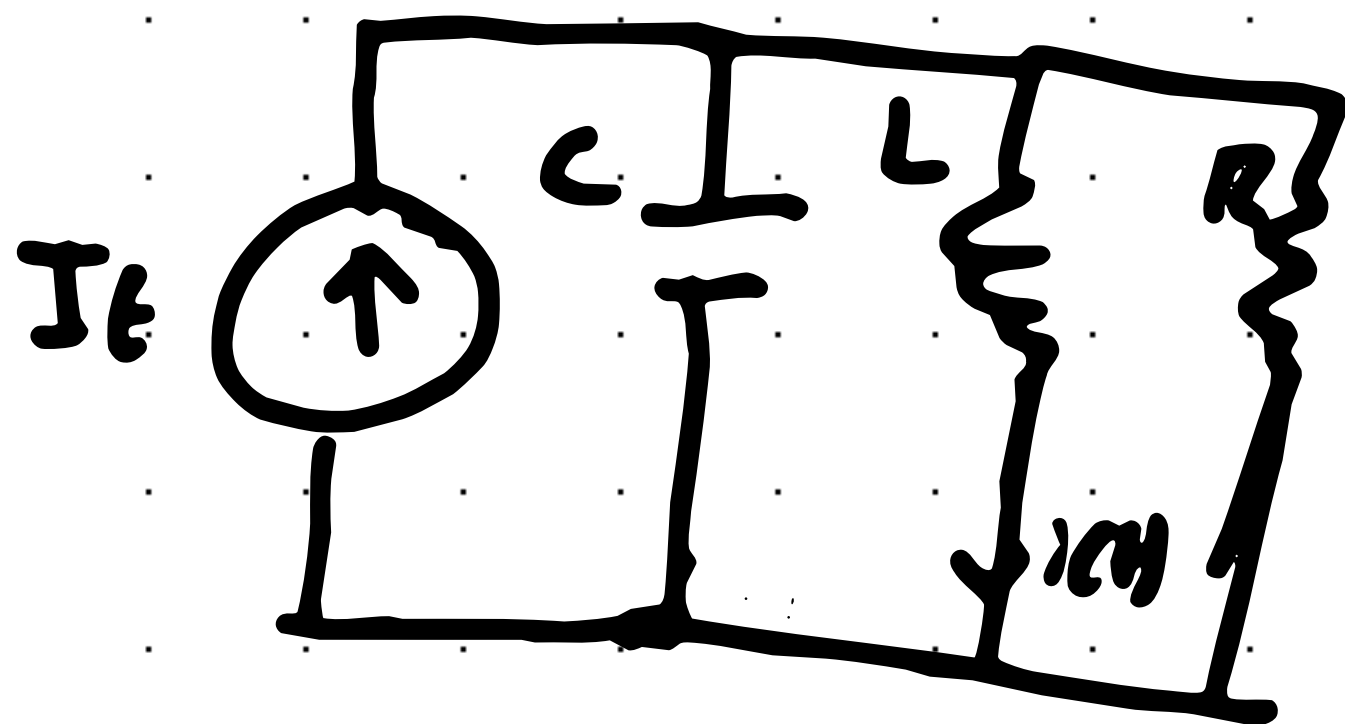
→ for D_1 and D_2 , use Initial conditions

$$v(0) = D_1$$

$$\frac{dv(0)^+}{dt} = D_1 - \alpha D_1$$

In Case of Step Response RLC

Parallel



Case ①

If $\alpha^2 > \omega_0^2$, then

$$i_L(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Case ②

If $\alpha^2 < \omega_0^2$, then

$$i_L(t) = I_s + \beta_1 e^{-\alpha t} \cos \omega_d t + \beta_2 e^{-\alpha t} \sin \omega_d t$$

Case ③

If $\alpha^2 = \omega_0^2$, then

$$i(t) = I_s + D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

$$\alpha = \frac{1}{2RC}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

