

for RL Step Response

$$i(t) = \frac{V_s}{R} + \left( I_0 - \frac{V_s}{R} \right) e^{-\frac{R}{L}t}$$

Need Voltage Source

$I_0$  = constant in  
Inductor at  $t = 0$

for RC Step Response

$$V_c(t) = I_s R t \left( V_0 - I_s R \right) e^{-\frac{t}{RC}}$$

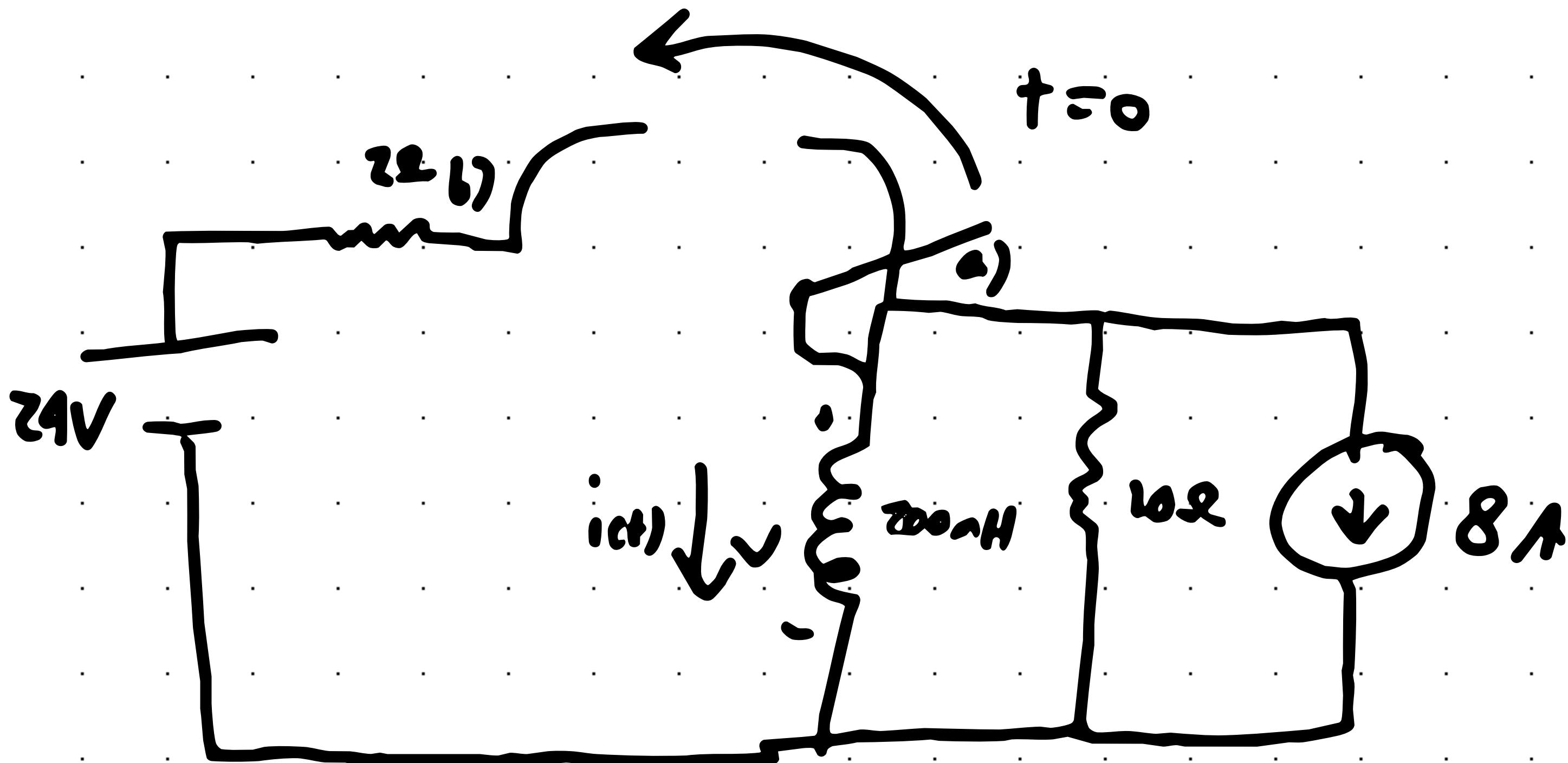
Need Current Source

Example

a) Find an Expression for  $i(t)$  for  $t > 0$

b) What is the initial voltage across the inductor just after the switch has been moved to position B?

c) How many milliseconds after the switch has been moved does the inductor voltage equal 21V?



### Solution

Before moving the switch

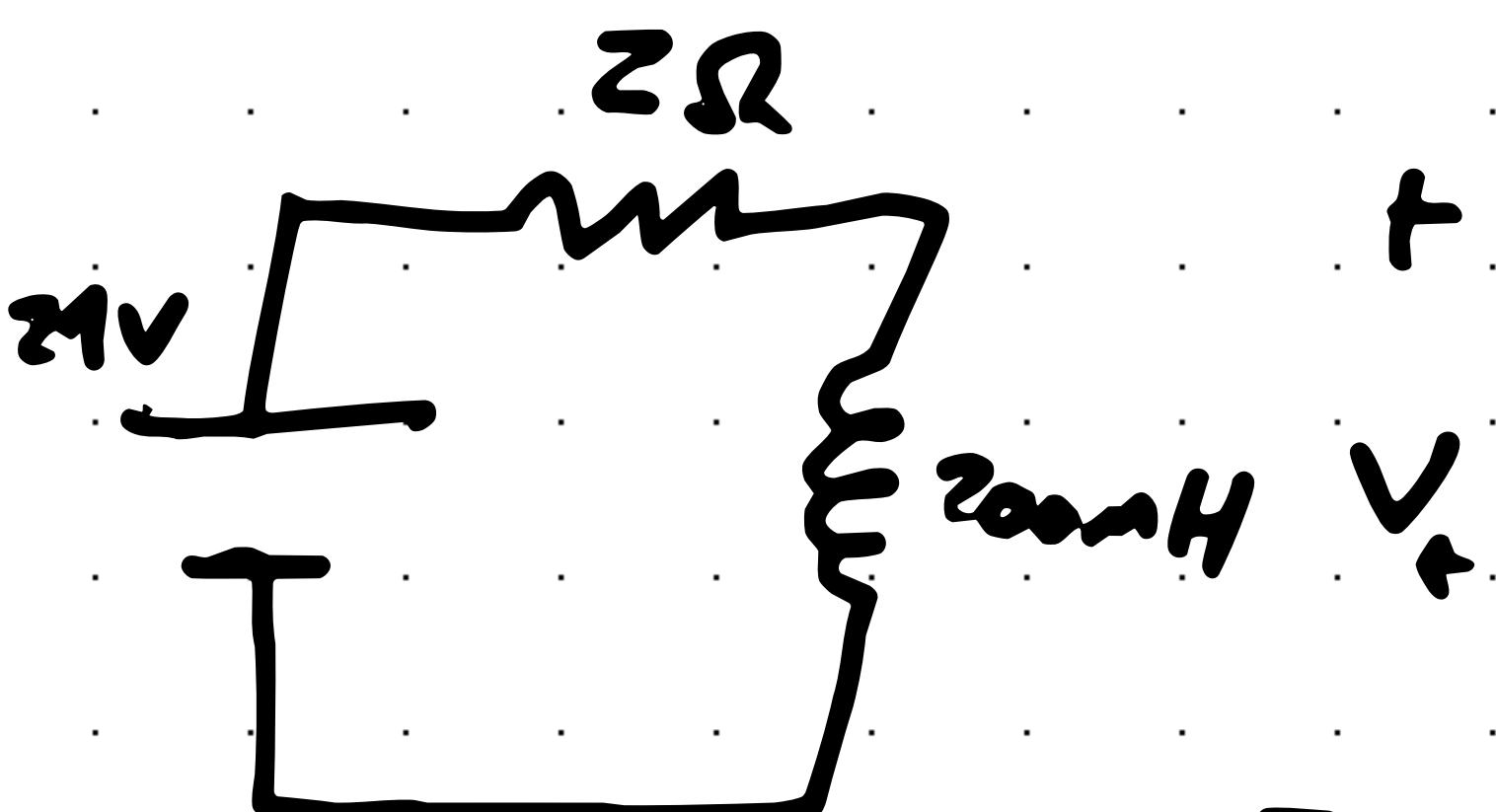
Start circuit

$$I_0 = -8A$$

(current only opposite to sense)



After moving the switch



$$i(t) = \frac{V_s}{R} + \left( I_0 - \frac{V_s}{R} \right) e^{-\frac{R}{L}t}$$

One Resistor ✓

Voltage Source ✓

$$I_{(C)} = \frac{24}{2} + \left(-8 - \frac{24}{2}\right) e^{-\frac{t}{200 \times 10^3}} +$$

a)  $i(t) = 12 - 20e^{-10t}$   $t \geq 0$

b)  $V_{(C)} = L \frac{di}{dt} = 200 \times 10^3 (0 - 20e^{-10t} (-10))$

$V(t) = 40e^{-10t}$   $t \geq 0$

Initial voltage at  $T=0$  is 40V ( $40e^{-10 \times 0}$ )

c)  $24 = 40e^{-10t}$

$$\ln(24) = \ln(40) + \ln e^{-10t}$$

$$\ln 24 - \ln 40 = -10t$$

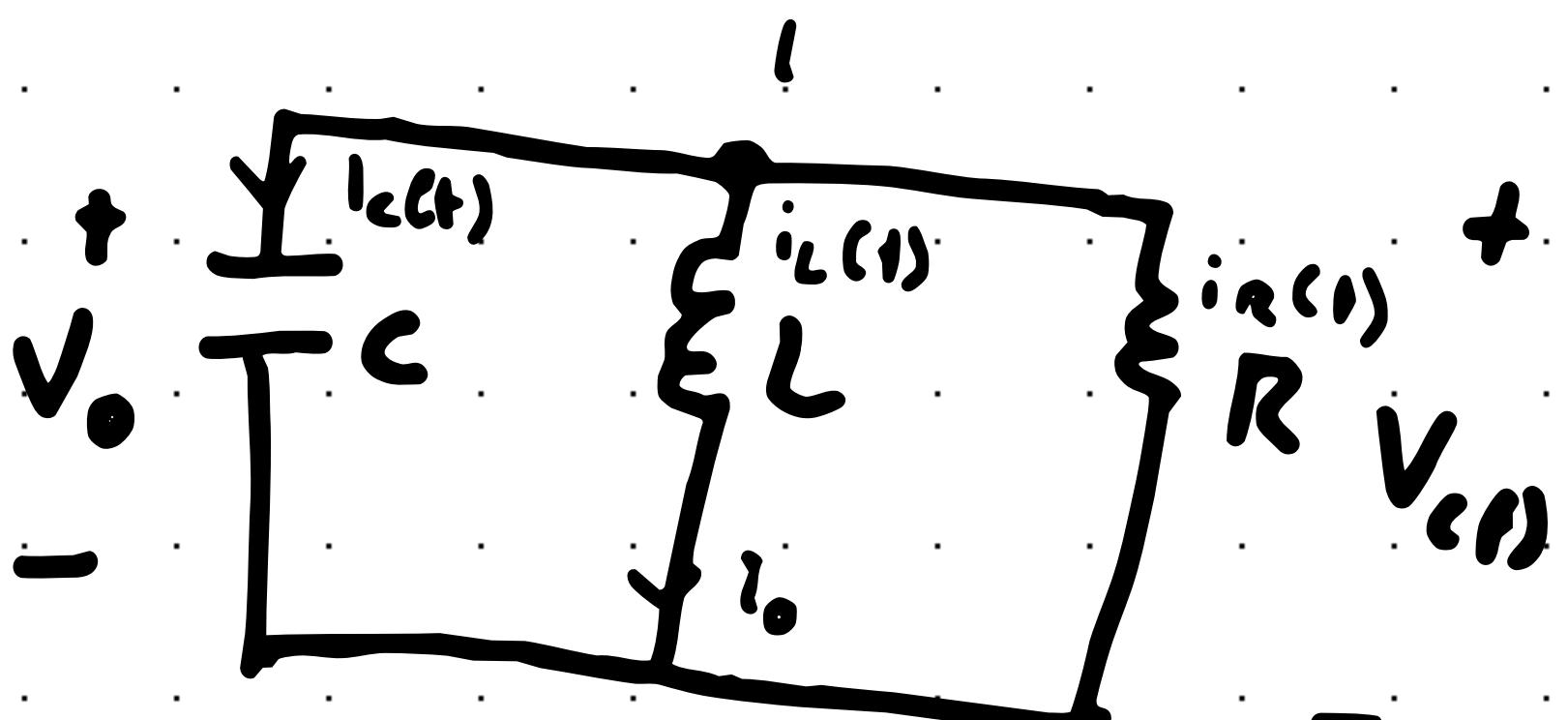
$$t = 51.08 \text{ ms}$$

# Introduction to the Natural Response of a Parallel RLC Circuit

$$\sum I = 0$$

make

$C_{\text{small}}$  is  
 $i_c(t)$ , not worthy  
 to simplify.)



$$i_C + i_R + i_L = 0$$

$\downarrow \quad \downarrow \quad \downarrow$

$$C \frac{dv}{dt} + \frac{v}{R} + \frac{1}{L} \int v dt = 0$$

ugly DE

$$C \frac{d^2v}{dt^2} + \frac{1}{R} \frac{dv}{dt} + \frac{1}{L} (v) = 0 \quad \leftarrow$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{1}{LC} v = 0$$

$$\text{Assume } v(t) = A e^{st}$$

$$\frac{dv}{dt} = A s e^{st}$$

$$\frac{d^2v}{dt^2} = A s^2 e^{st}$$

$$A s^2 e^{st} + \frac{A s}{RC} e^{st} + \frac{1}{LC} e^{st} = 0$$

$$AS^2c^{st} + \frac{AS}{RC} c^{st} + \frac{A}{LC} c^{st} = 0$$

$$(S^2 + \frac{1}{RC}S + \frac{1}{LC}) Ac^{st} = 0$$

$$S^2 + \frac{1}{RC}S + \frac{1}{LC} = 0$$

Note:

$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$S = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - 4(1)\left(\frac{1}{LC}\right)}$$

$$S_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{LC}\right)}$$

$$S_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{LC}\right)}$$

These are known  
as complex  
frequencies

$$\alpha = \frac{1}{2RC}$$

Natural  
frequency

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Resonant natural  
frequency

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2},$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Based on  $\alpha$  and  $\omega_0$  values, we are expecting three different cases.

Case ①

$$\text{If } \alpha^2 > \omega_0^2$$

(Two distinct real solutions)

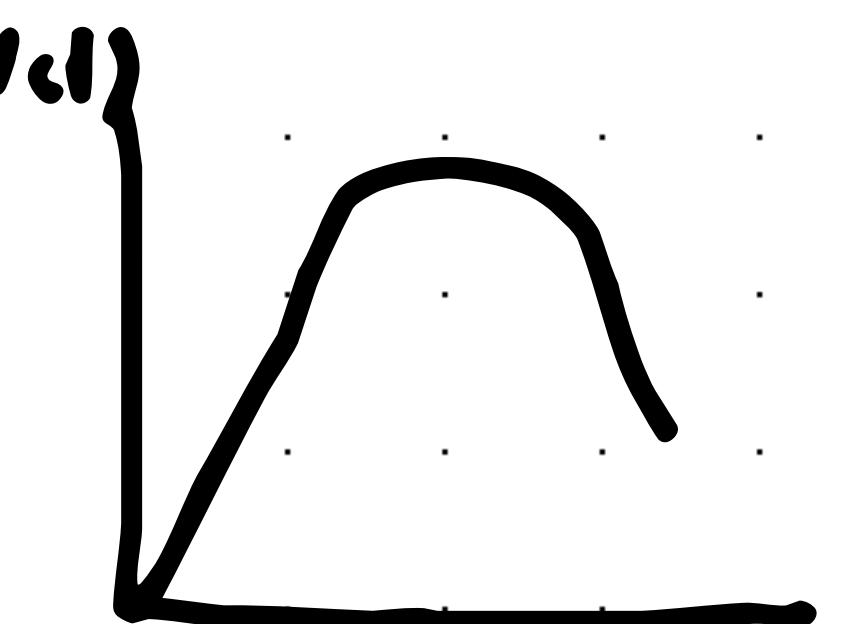
Solution

$$V(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

For  $A_1$  and  $A_2$  calculations, we use initial conditions.

$$V(0)^+ = A_1 + A_2 ;$$

$$\frac{dV(0)^+}{dt} = S_1 A_1 + S_2 A_2 = \frac{i_C(0)}{C}$$



- Step one: solve for  $\alpha$  and  $\omega$
- Step two: compare  $\alpha$  and  $\omega$
- Step three: if  $\alpha^2$  is less than  $\omega_0^2$ , use case ①

Case ②

$$\text{If } \omega^2 < \omega_0^2$$

(Under damped case)

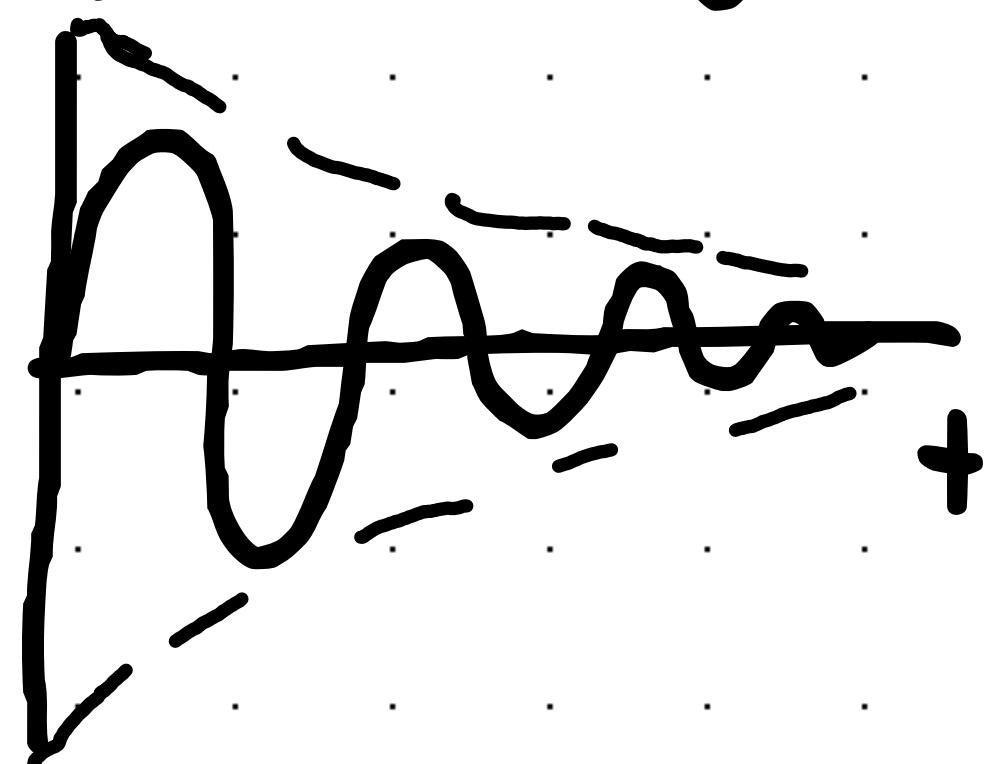
Two complex

$$V(t) = \beta_1 e^{-\alpha t} \cos \omega_d t + \beta_2 e^{-\alpha t} \sin \omega_d t$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$V(t)$$

conjugates



For finding  $\beta_1$  and  $\beta_2$ , use initial conditions

$$V(0)^+ = \beta_1$$

$$\frac{dV(0)}{dt} = -\alpha \beta_1 + \omega_d \beta_2$$

Case ③

$$\text{if } \omega^2 = \alpha^2$$

(Critically damped)

(Repeated Roots)

(Real)

$$V(t) = D_1 + D_2 e^{-\alpha t} + D_3 t e^{-\alpha t}$$

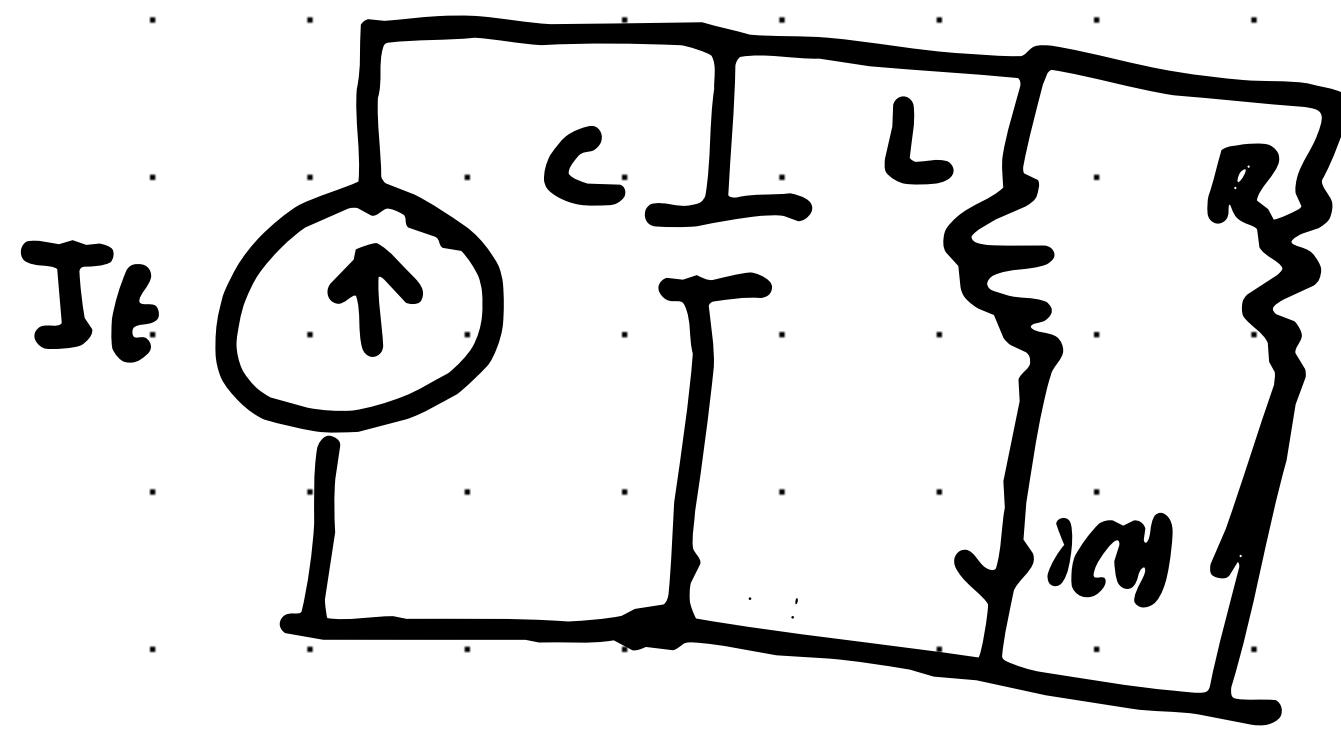
$\rightarrow$  for  $D_1$  and  $D_2$ , use Initial conditions

$$V(0) = D_1$$

$$\frac{dV(0)}{dt} = D_2 - \alpha D_1$$

In Case of Step Response RLC

Parallel



Case ①

If  $\alpha^2 > \omega_0^2$ , then

$$i_L(t) = I_E + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Case ②

If  $\alpha^2 < \omega_0^2$ , then

$$i_L(t) = I_E + \beta_1 e^{-\alpha t} \cos \omega_0 t + \beta_2 e^{-\alpha t} \sin \omega_0 t$$

Case ③

If  $\alpha^2 = \omega_0^2$ , then

$$i(t) = I_E + D_1 t e^{-\alpha t} + D_2 t^2 e^{-\alpha t}$$

$$\alpha = \frac{1}{2RC}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

