Y_p

Amp Phase Form

Original Form

$$y = A\sin\Theta + B\cos\Theta$$

Sin

$$y = A\sin(\Theta + \Phi)$$

Cos

$$y = A\cos(\Theta - \Phi)$$

Natural Frequency

$$y(t) = C_1 \cos\omega t + C_2 \sin\omega t$$

☆ Convert to Amp Phase.

Sin	Cos
$y = A\sin(\omega t + \boxed{\phi})$	$y = A\cos(\omega t \boxed{-\psi})$
$A = \sqrt{C_1^2 + C_2^2}$	$A = \sqrt{C_1^2 + C_2^2}$
$\tan\phi = \dfrac{C_1}{C_2}$	$\tan\psi = \dfrac{C_2}{C_1}$

C_1 & C_2 are the coefficients on sin and cos

Complex Numbers

$$Z = a + ib$$

$$D = \frac{1}{Z} = \frac{1}{a + iB}$$

$$\frac{1}{a+iB} \times \frac{a-iB}{a-iB} = \frac{a-ib}{(a+ib)(a-ib)}$$

$$= \frac{a-ib}{a^2 + i^2 b^2}$$

☆ Remember!

$i^2 = -1$

$$= \frac{a-ib}{a^2 - b^2}$$

$$D = \underbrace{D_{real}}_{\text{Real}} + \underbrace{iD_{Im}}_{\text{Imaginary}}$$

* We want to get rid of the complex number in the denominator!

We use the complex conjugate.

Example

Convert to Amp Phase (either cos or sin)

$$\dot{y} + 2y = 4e^{-3t}\cos(5t)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Euler's Formula

$$e^{i5t} = \underbrace{\cos 5t}_{\text{Real}} + \underbrace{i\sin 5t}_{\text{Imag}}$$

$$\cos 5t = \mathrm{Re}[e^{i5t}]$$

$$\dot{y} + 2y = 4e^{-3t}\,\mathrm{Re}(e^{i5t})$$

$$= 4\,\mathrm{Re}(e^{-3t}e^{i5t})$$

$$= 4\,\mathrm{Re}(e^{(-3+i5)t})$$

☆ IF SIN WAS GIVEN

$y(0)=0$

Find the IMAGINARY component instead of the real.

We have to make guesses for y_p

But Kember wants us to do it weirdly... He wants us to express sin and cos in terms of eulers notation

(The complex approach)

Again, we ONLY use this for oscillators

$$y_p = \mathrm{Re}(Y_p)$$

* A Guess

$$\dot{Y}(t) + 2Y(t) = 4e^{(-3+i5)t}$$

And now, we can make our guesses!

$$Y_p = De^{(-3+i5)t}$$

$$\dot{Y}_p = D(-3+i5)e^{(-3+i5)t}$$

This is Kembers Auxillary Problem

We now have Big Y_p

This is weird, but helpful

So, lets plug back in for Y_p!

$$D(-3+i5)e^{(-3+i5)t} + 2De^{(-3+i5)t} = \text{RHS Stuff}$$

$$D[(-3+i5)+2]e^{(-3+i5)t} = \text{RHS Stuff}$$

$$D(-1+i5)e^{(-3+i5)t} = 4e^{(-3+i5)t}$$

$$D(-1+i5) = 4$$

$$D = \frac{4}{-1+i5}$$

And we can plug D back in...

$$Y_p = \left(\frac{4}{-1+i5}\right) e^{(-3+i5)t}$$

Now, we need to find the Real part of Y_p to get back to y_p.

Usually, that is easiest in this form.

$$\underbrace{\qquad\quad}_{Re} + i\underbrace{\qquad\quad}_{Im}$$

Typically, you'll run into two problems doing this!

① Most commonly, having **i** in the denominator.
(like in our case)

$$Y_p\left(\frac{4}{-1+i5}\right)e^{(-3+i5)t}$$

↳ $$\frac{4}{-1+i5} \times \frac{-1-i5}{-1-i5} = \frac{4(-1-i5)}{(-1)^2-\underbrace{(i)^2}_{-1}(5)^2} = \boxed{\frac{4(-1-i5)}{26}}$$

Complex Conjugate

② Having **i** in the exponential (Also a problem)

$$Y_p\left(\frac{4(-1-i5)}{26}\right)e^{(-3+i5)t}$$

↳ First, lets split that e back up from earlier!
(we want to use euler again)

↳ $e^{-3t}e^{i5t}$

↳ $e^{i5t} = \cos 5t + i\sin 5t$

↳ $e^{-3t}(\cos 5t + i\sin 5t)$

$$Y_p = \frac{-2}{13}(1+i5)e^{-3t}(\cos 5t + i\sin 5t)$$

We Can actually Simplify this a bit!

$$Y_p = -\frac{2}{13}e^{-3t}(1 + i5)(\cos 5t + i\sin 5t)$$

And we're going to have to expand this to find the reals and Imaginaries.

$$(1 + i5)(\cos 5t + i\sin 5t) =$$

$$\underbrace{1\cos 5t}_{Re} + \underbrace{1 i\sin 5t}_{Im} + \underbrace{i5\cos 5t}_{Im} + \underbrace{\cancel{i^2}^{-1}\, 5\sin 5t}_{Re}$$

$$Y_p = -\frac{2}{13}e^{-3t}(\underbrace{\cos 5t - 5\sin 5t}_{Real} + \underbrace{i(\sin 5t + 5\cos 5t)}_{Imag})$$

$$y_p = RE(Y_p) = -\frac{2}{13}e^{-3t}(\cos 5t - 5\sin 5t)$$

Little y_p Now!

Big Y_p

Real Component!

Done!

Well, Almost. We need to get that Solution into Amp-phase form like we talked about.

$$y_p = -\frac{2}{13}e^{-3t}(\cos 5t - 5\sin 5t)$$

We're going to bring it into sin. Let's recap.

$$y = A\sin(\omega t + \phi)$$

$$A = \sqrt{c_1^2 + c_2^2}$$

$$\tan\phi = \frac{c_1}{c_2}$$

$$A = \sqrt{1^2 + (-5)^2} = \sqrt{26}$$

$$= A\sin(5t + \phi)$$

$$\tan\phi = \frac{1}{-5}$$

$$= \sqrt{25}\,\sin(5t + \phi)$$

So, let's solve for y!

$$y = y_h + y_p$$

$$y = Ce^{-2t} + \sqrt{25}\sin(5t + \phi)$$

y_h from previous notes.

We can solve for c!

$y(0) = 0$

$$\boxed{C = \frac{2}{13}}$$

$$y = \frac{2}{13}e^{-2t} + \sqrt{25}\sin(5t + \phi)$$