

RLC Resonance

$$Z = R + j\omega L + \frac{1}{j\omega C}$$
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

At the Resonance

$$Z = R$$

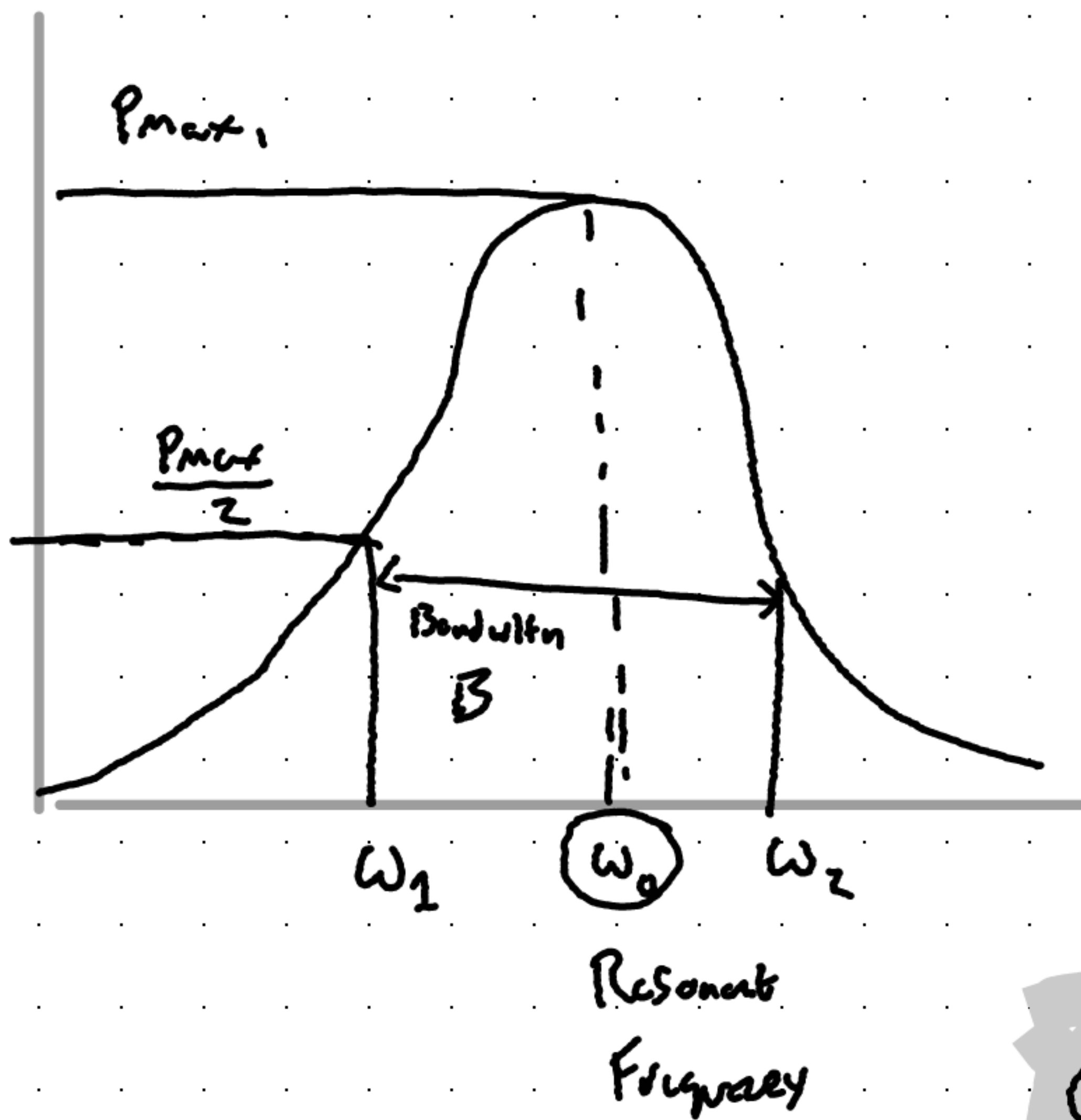
At resonance, $\text{Im}(Z) = 0$

$$\omega_0^2 = \frac{1}{LC} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

rad/s

$$2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$



$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Quality Factor (Q)

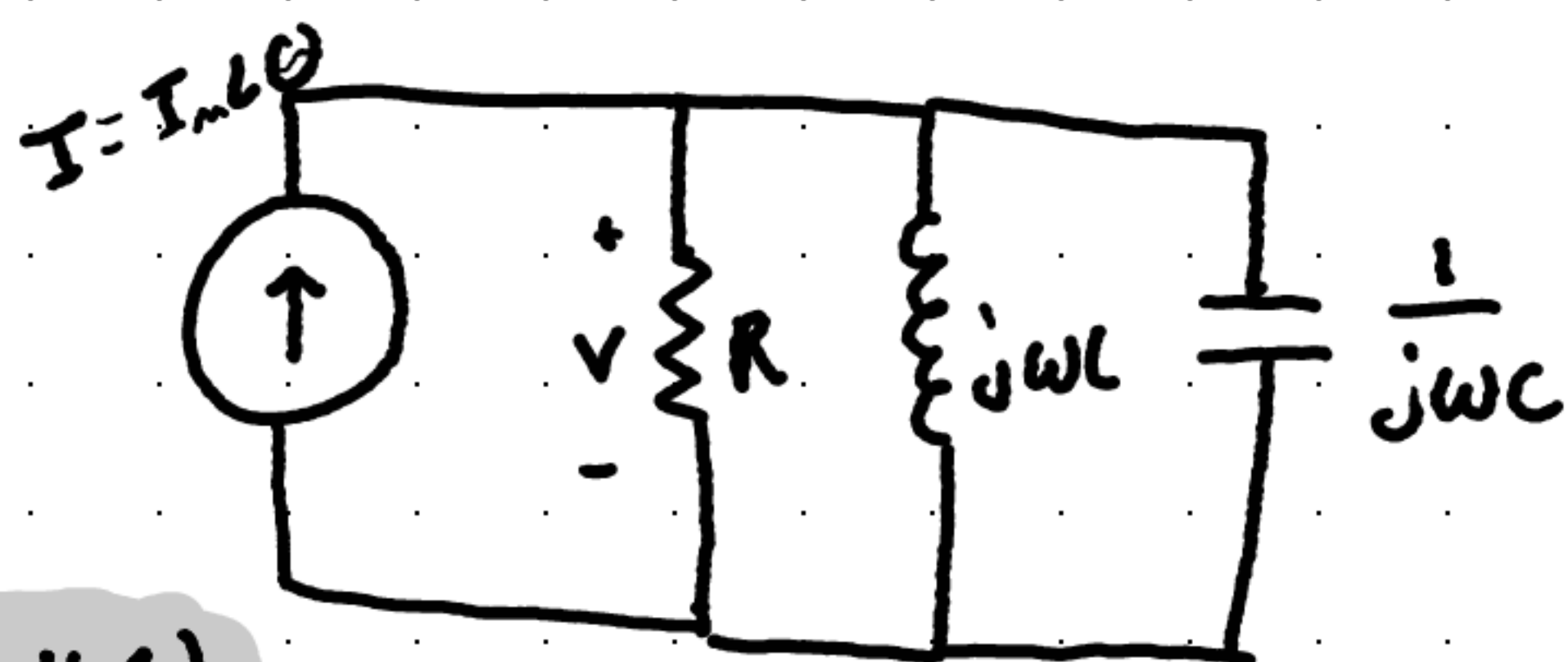
$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

$$Q = \frac{\omega_0}{B}$$

$$B = \text{Bandwidth}$$

$$\frac{P_{max}}{2} = \frac{1}{2} \frac{V_m^2}{R}$$

Parallel Resonance



(Admittance)

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

Resonance occurs when the imaginary part of Y is zero.

Resonant frequency is still $\omega_0 = \frac{1}{\sqrt{LC}}$

For Parallel Resonance

Half Power (cutoff)
Frequencies

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Bandwidth

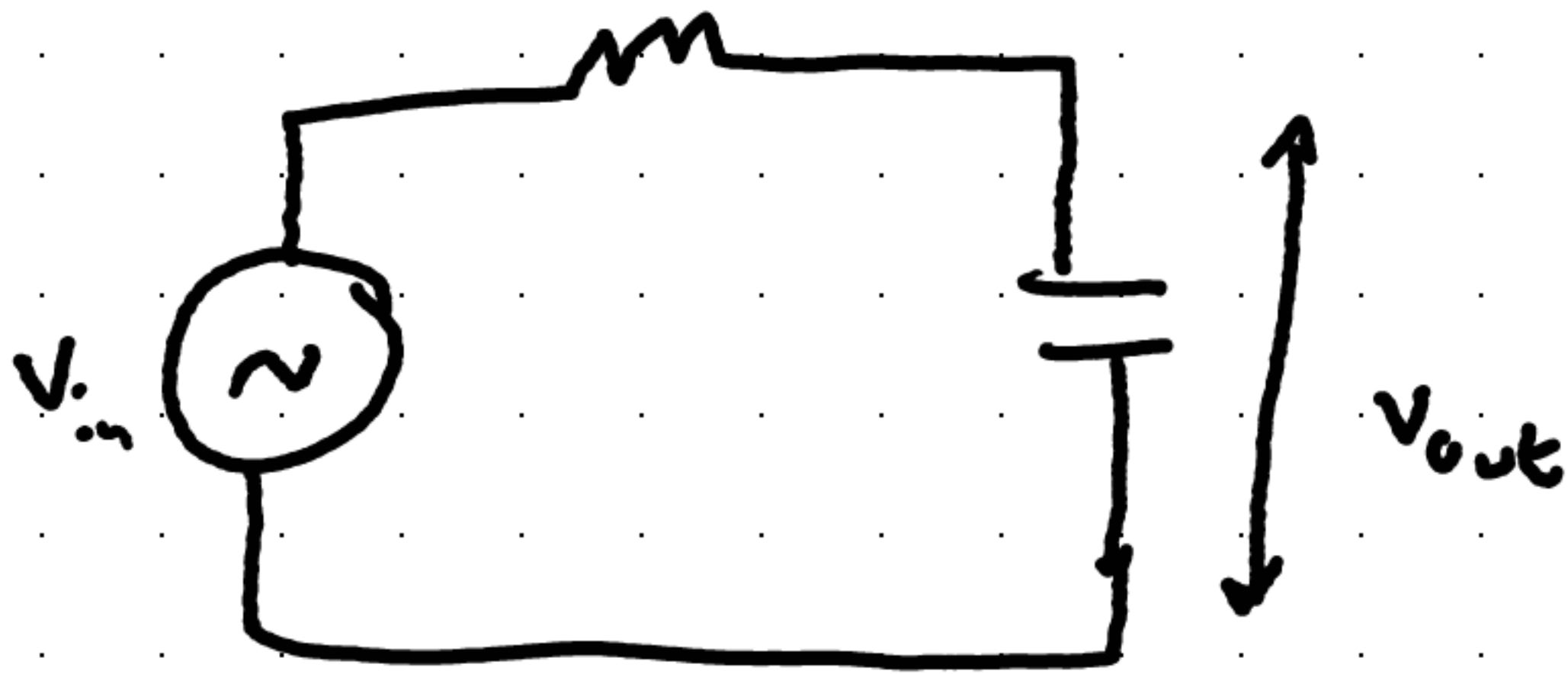
$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

Quality factor

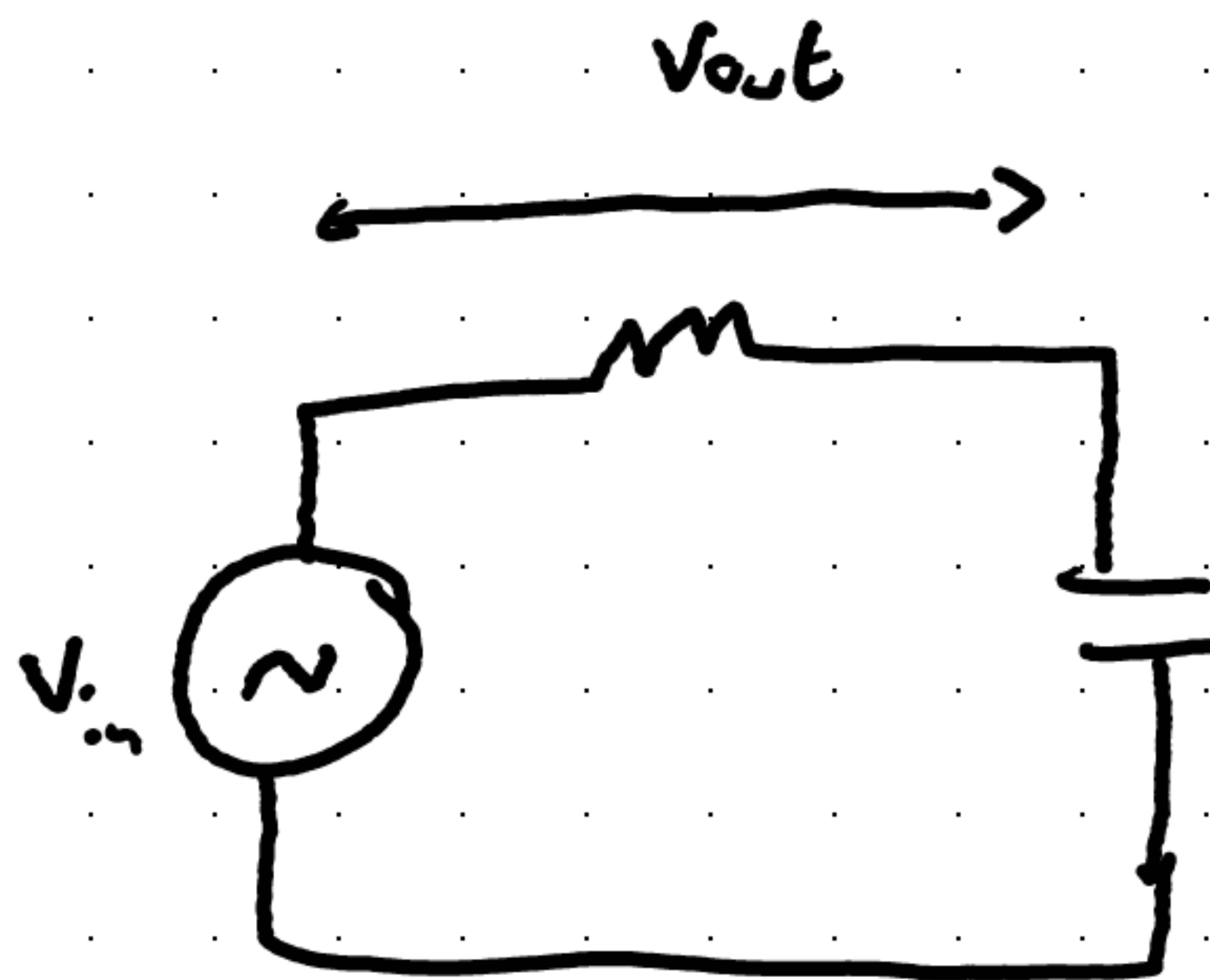
$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

Resonant Frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



Low
Pass
Filter

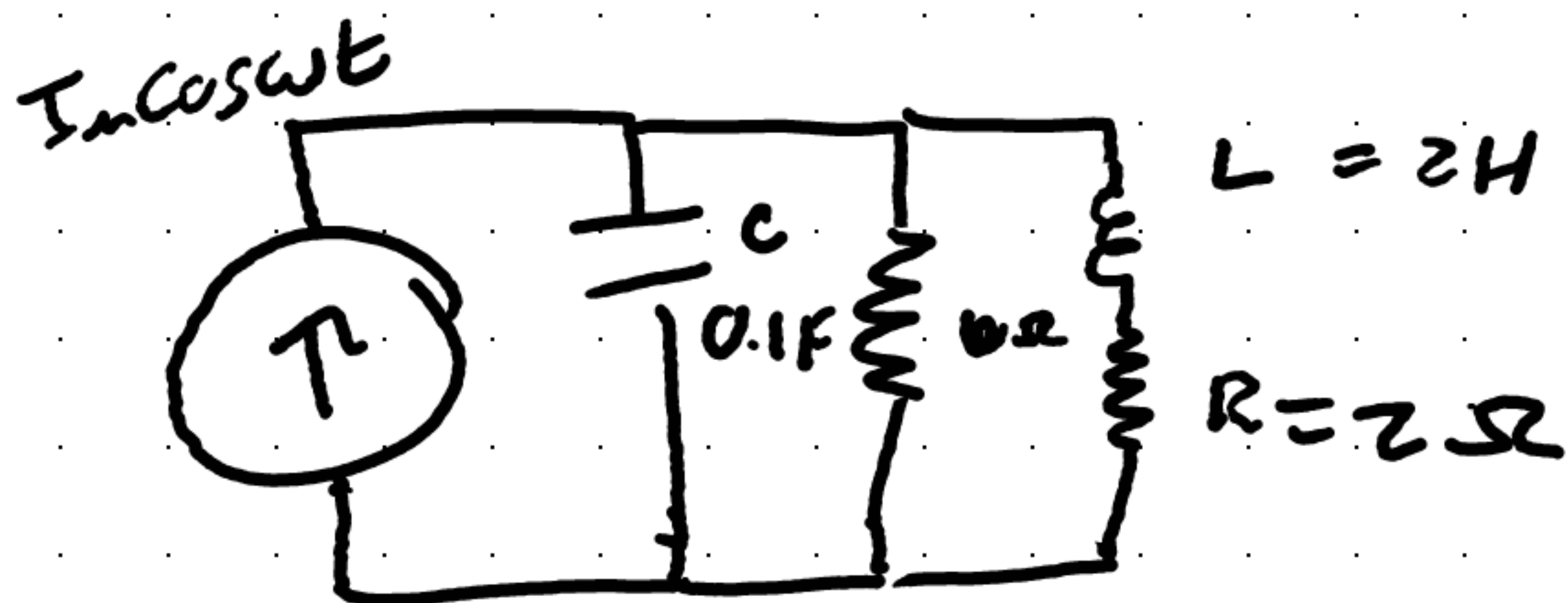


High
Pass
Filter

Source Transformation Formula

$$I = \frac{V}{R}$$





$$Y = \frac{1}{10} + j\omega(0.1) + \frac{1}{2 + 2j\omega}$$

$$Y = \frac{1}{10} + j\omega(0.1) + \frac{2 + 2j\omega}{4 + 4\omega^2}$$

$$Y = \underbrace{\left(0.1 + \frac{2}{4 + 4\omega^2}\right)}_{\text{RE}} + j \underbrace{\left(0.1\omega - \frac{2}{4 + 4\omega^2}\right)}_{\text{Im}}$$

The resonance occurs when $\text{Im}(Y) = 0$

$$0.1\omega_0 - \frac{2\omega_0}{4 + 4\omega_0^2} = 0$$

$$4 + 4\omega_0^2 = 20$$

$$\omega_0 = 2 \text{ rad/s}$$

