Lecture 17

ECED 2□□□

Response of first order RL and RC Circuits

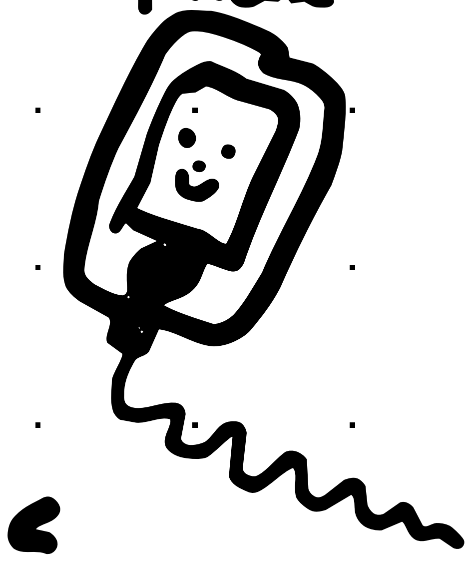
2 The natural response of an RL Circuit

- Only the storage element is connected to the circuit

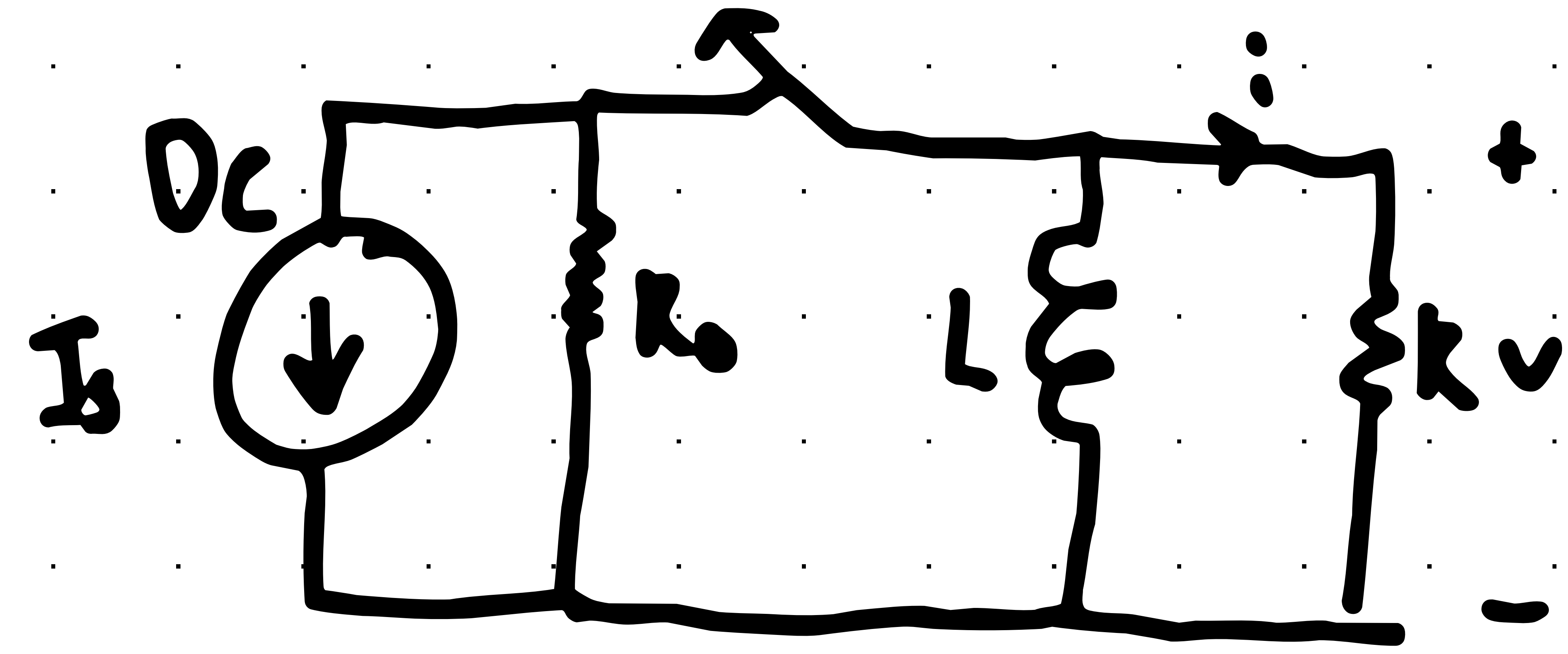
Phone

2 The forced response of an RL Circuit

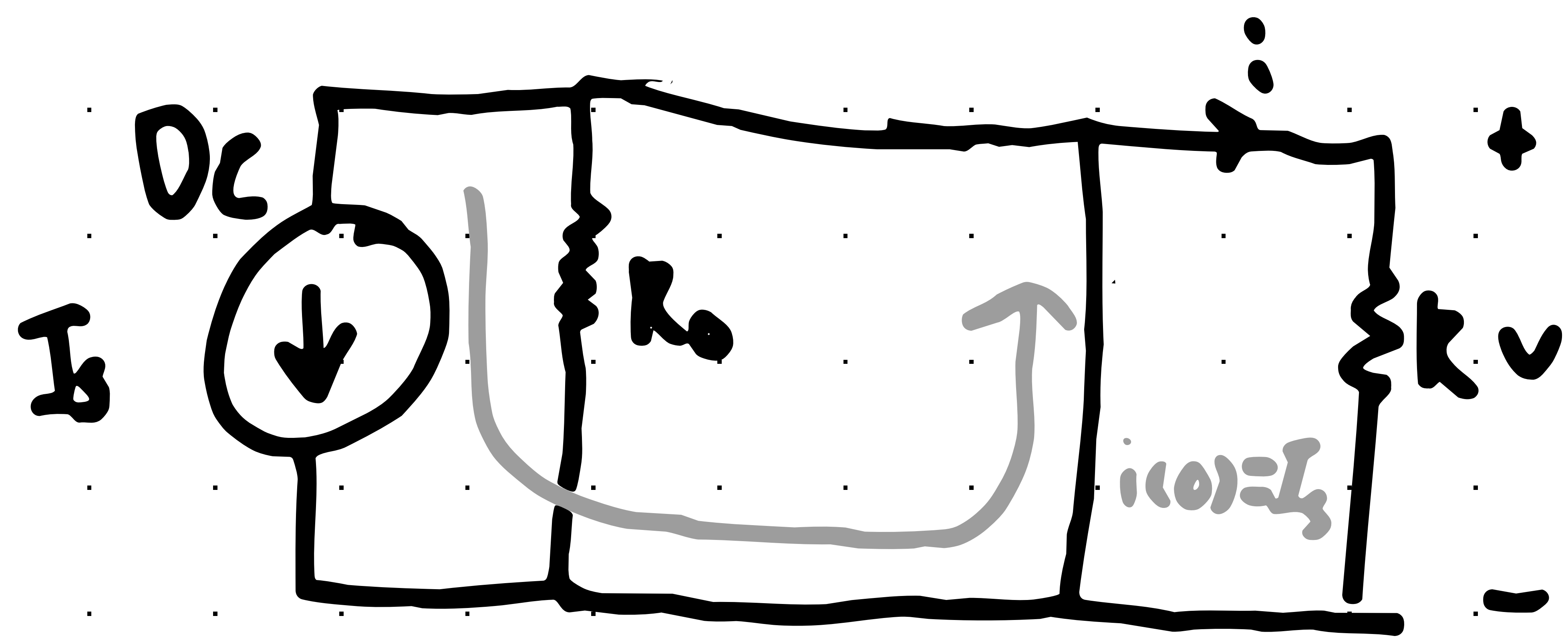
Plugged in Phone



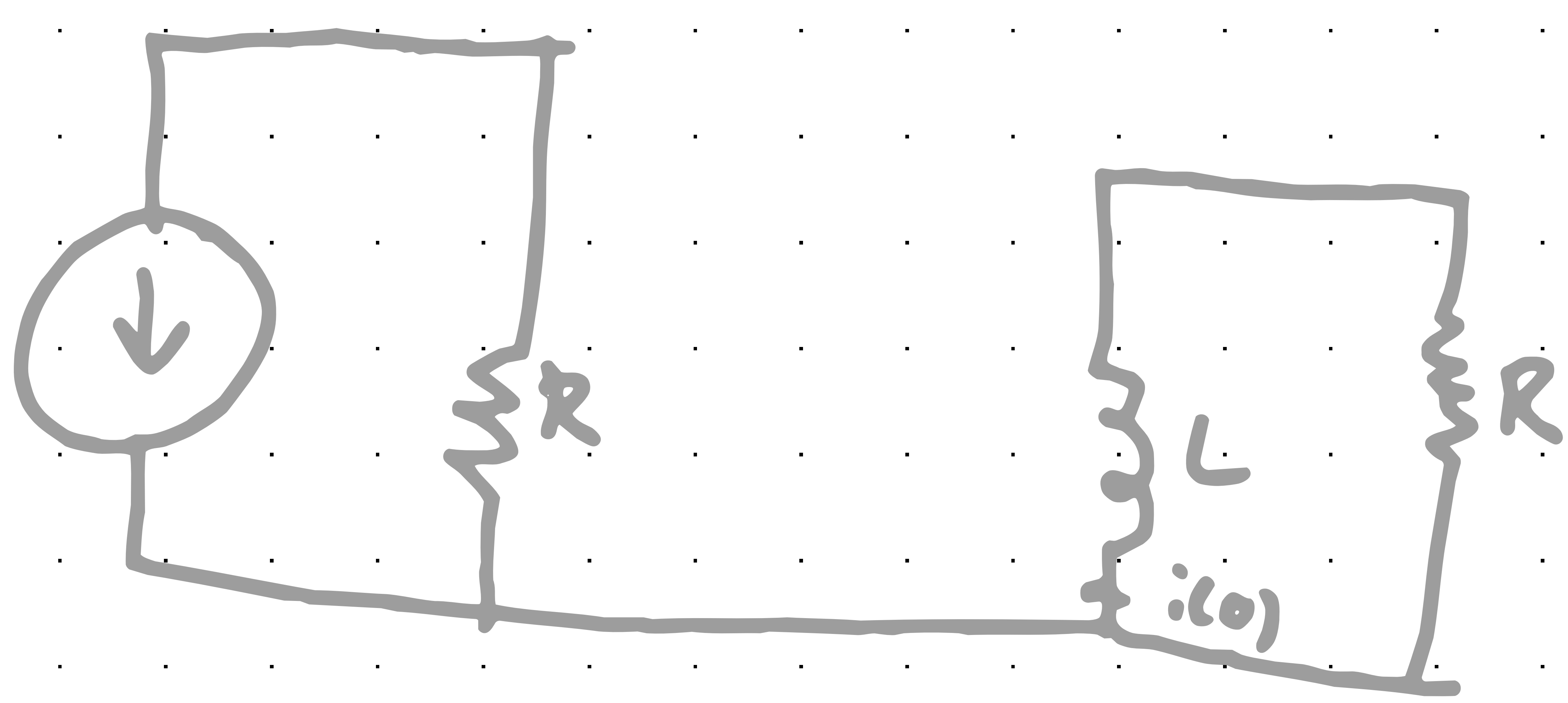
- Step response, the storage element along with a constant DC voltage source



Before the switch is opened, the coil will be short circuited as I_s is constant (for $t<0$)



After the switch is opened, the circuit looks like



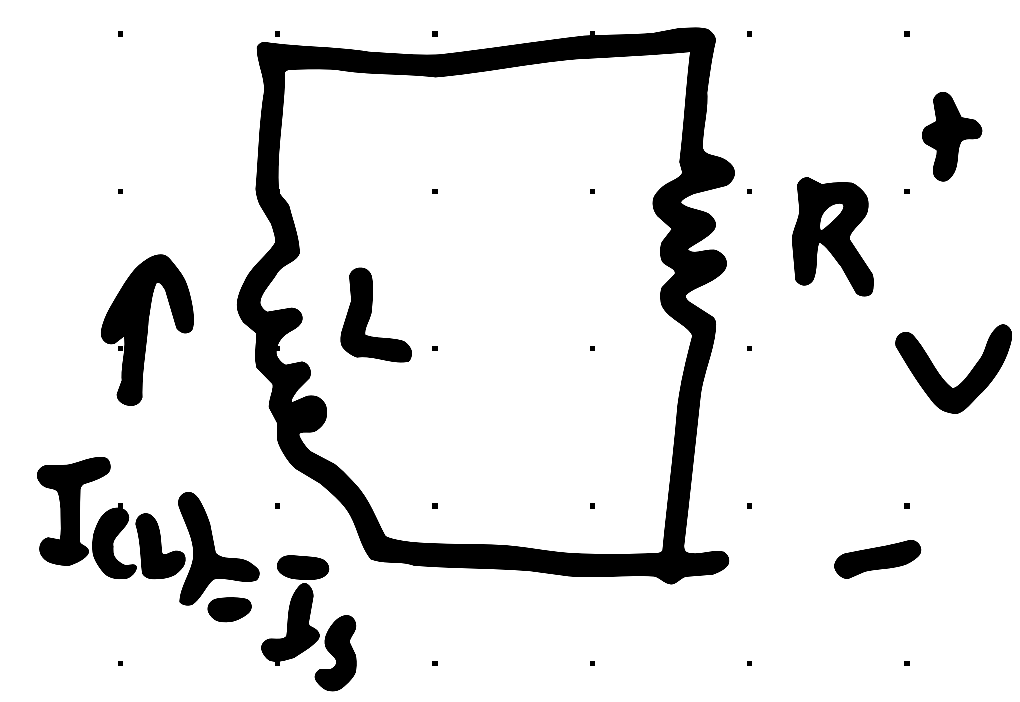
The current in the coil is not changing instantaneously

So

$$\underbrace{i_L(0^-)}_{\text{Immediately before open the switch}} = \underbrace{i_L(0^+)}_{\text{Immediately after closing the switch}} = i_L(0) = I_s$$

The voltage in the coil is changing instantaneously.

We will deal with the following circuit after opening the switch.



$$L\frac{di_L}{dt} + R i_L = 0$$

$$L\frac{di_L}{dt} = -R i_L$$

$$\int_{i_L}^{i_L(t)} \frac{di_L}{dt} = -\int_{t_0}^{t} \frac{R}{L}\,dt = -\frac{R}{L}t\Big|_{t_0}^{t} = -\frac{R}{L}(t-t_0)$$

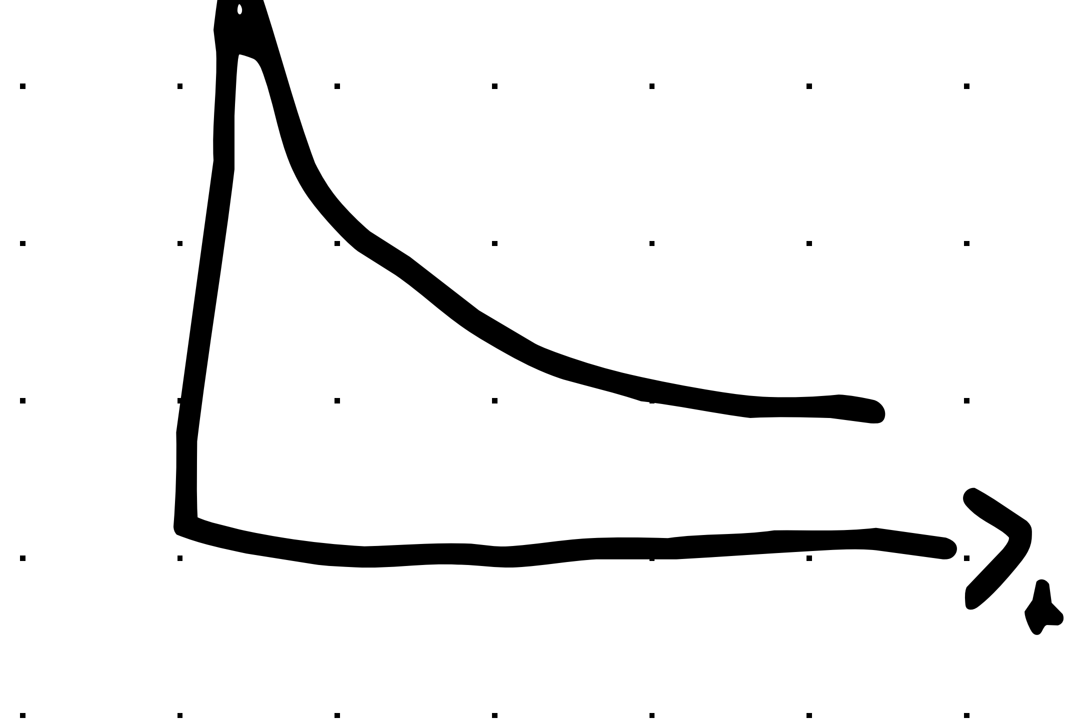
$$\ln\left(\frac{i_L(t)}{i_L(t_0)}\right) = -\frac{R}{L}(t-t_0)$$

$$\frac{i_L(t)}{i_L(t_0)} = e^{-\frac{R}{L}(t-t_0)}$$

$$i_L(t) = i_L(t_0)\,e^{-\frac{R}{L}(t-t_0)}$$

So, LC Discharging Equation

$$\boxed{i_L(t) = i_{L(0)}\, e^{-\frac{R}{L}t}}$$



And there are proofs to say...

Time Constant: $\boxed{\tau = \frac{L}{R}}$ ~63.2% of Change

$$i(t) = i_{L(0)} e^{-t/\tau} \quad t \geq 0$$

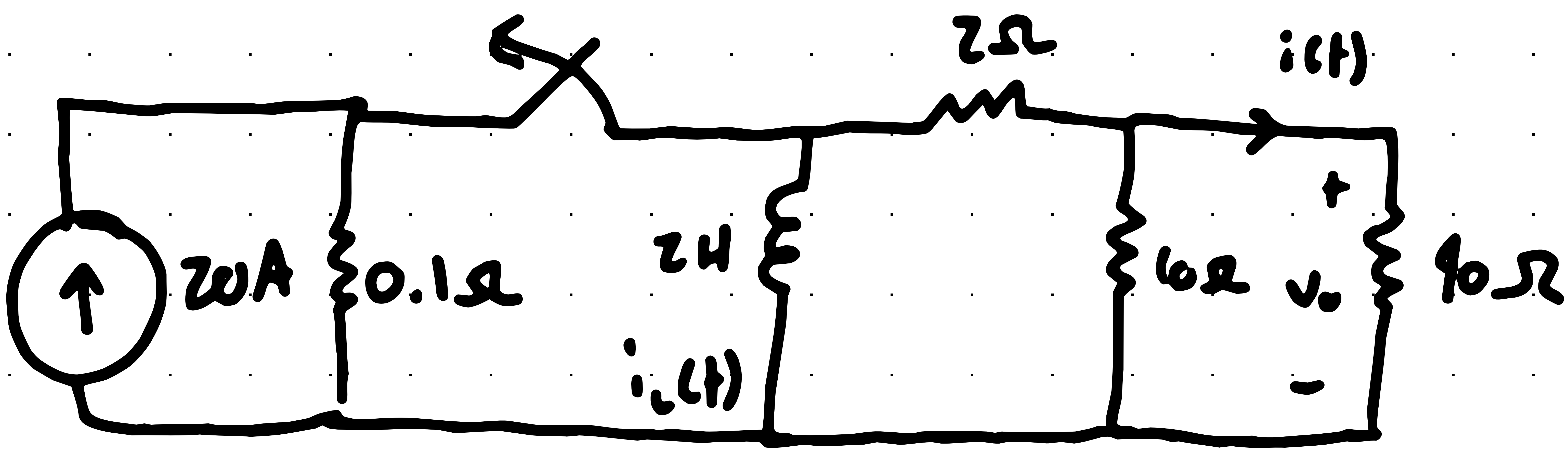
$$v(t) = i_L(0) R e^{-\frac{t}{\tau}} \quad t \geq 0^+$$

$$p(t) = i_{L(0)}^2 R e^{-\frac{2t}{\tau}} \quad t \geq 0^+$$

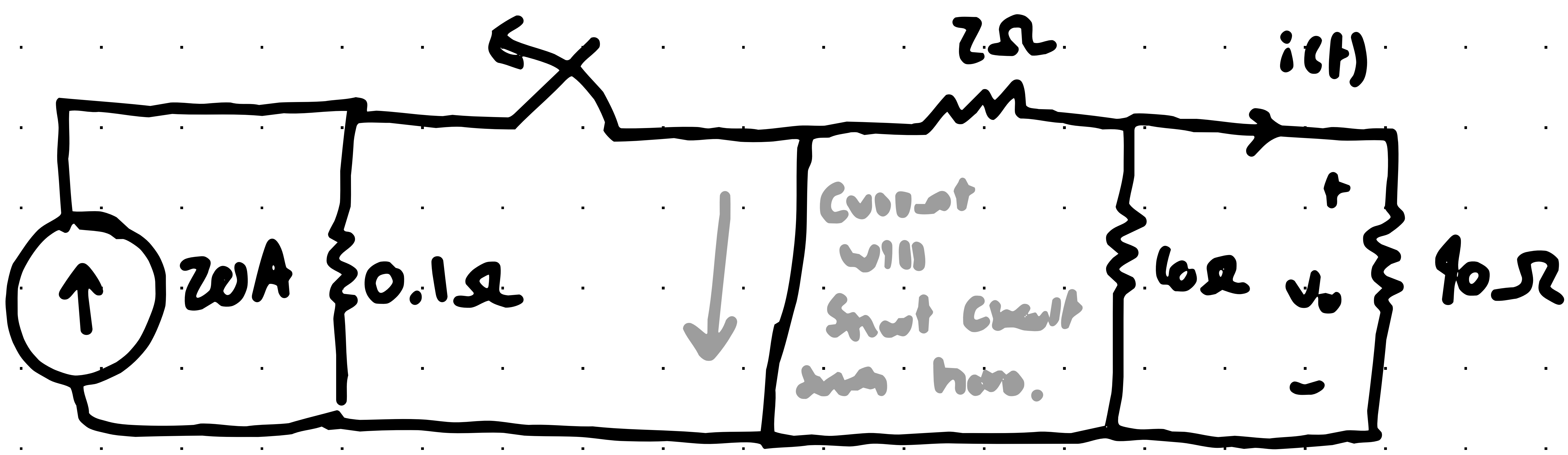
$$w(t) = \frac{1}{2} L i_{L(0)}^2 \left(1 - e^{\frac{-2t}{\tau}}\right) \quad t \geq 0$$

Equations

Example Find $i_o(t),\ t \geq 0$, $i_o(t),\ t \geq 0^+$, $V_o(t)\ t \geq 0^+$

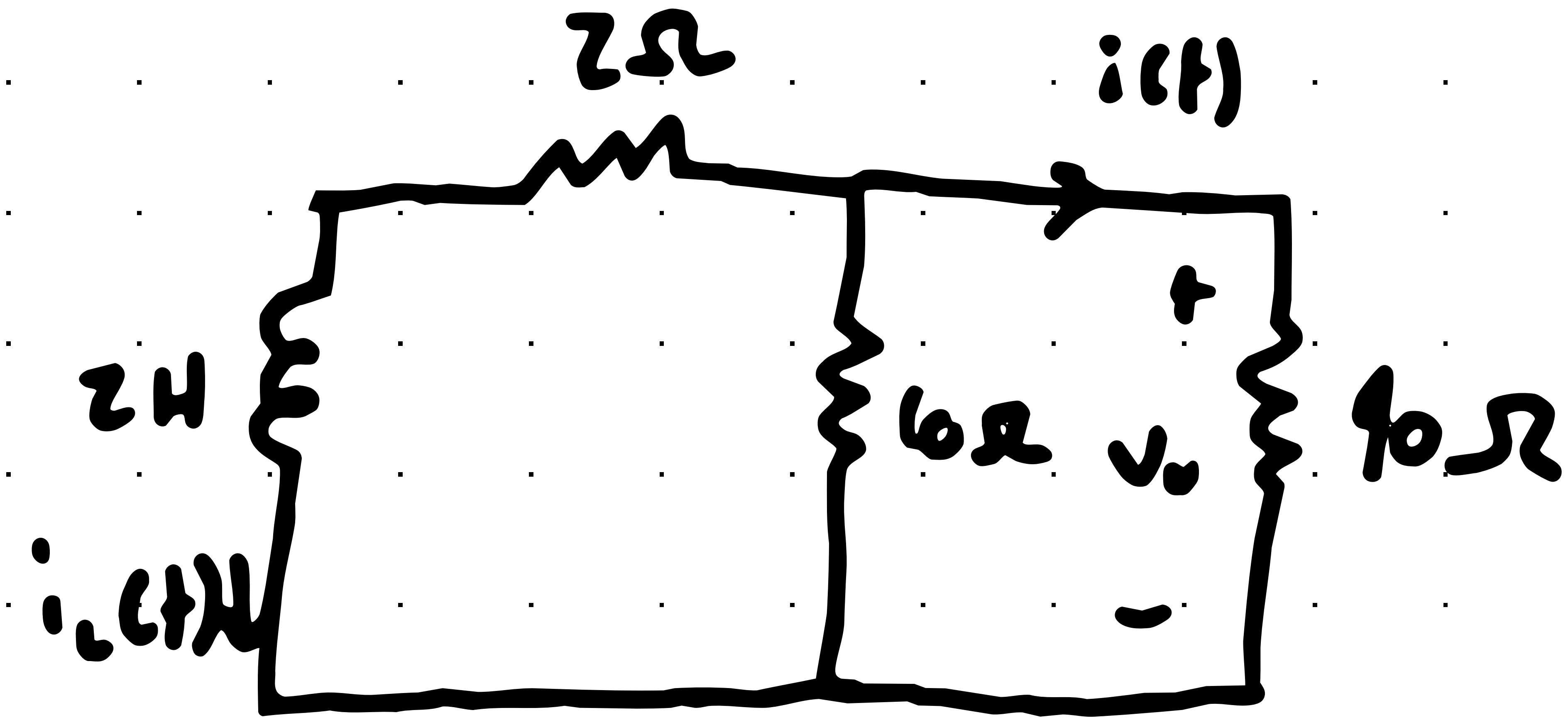


At $t=0$ the coil looks like a SC



$i_L(0) = 20A$

After opening the switch...



$i_L(t) = i_L(0)e^{-t/\tau}$

$\tau = \frac{L}{R}$ ← One L
← Many R's
Find R_{eq}

$$R_{eq} = \frac{10(40)}{10+40} + 2 = 10\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{10} = \underline{0.2 \text{ Seconds.}}$$

a)

$$i_L(t) = i_L(0)\, e^{-t/\tau} = 20e^{\frac{-t}{0.2}} = 20e^{-5t}$$

$$t \geq 0$$

b) $$i_{(o)}(t) = -\,i_L(t)\,\frac{10}{10+40}$$

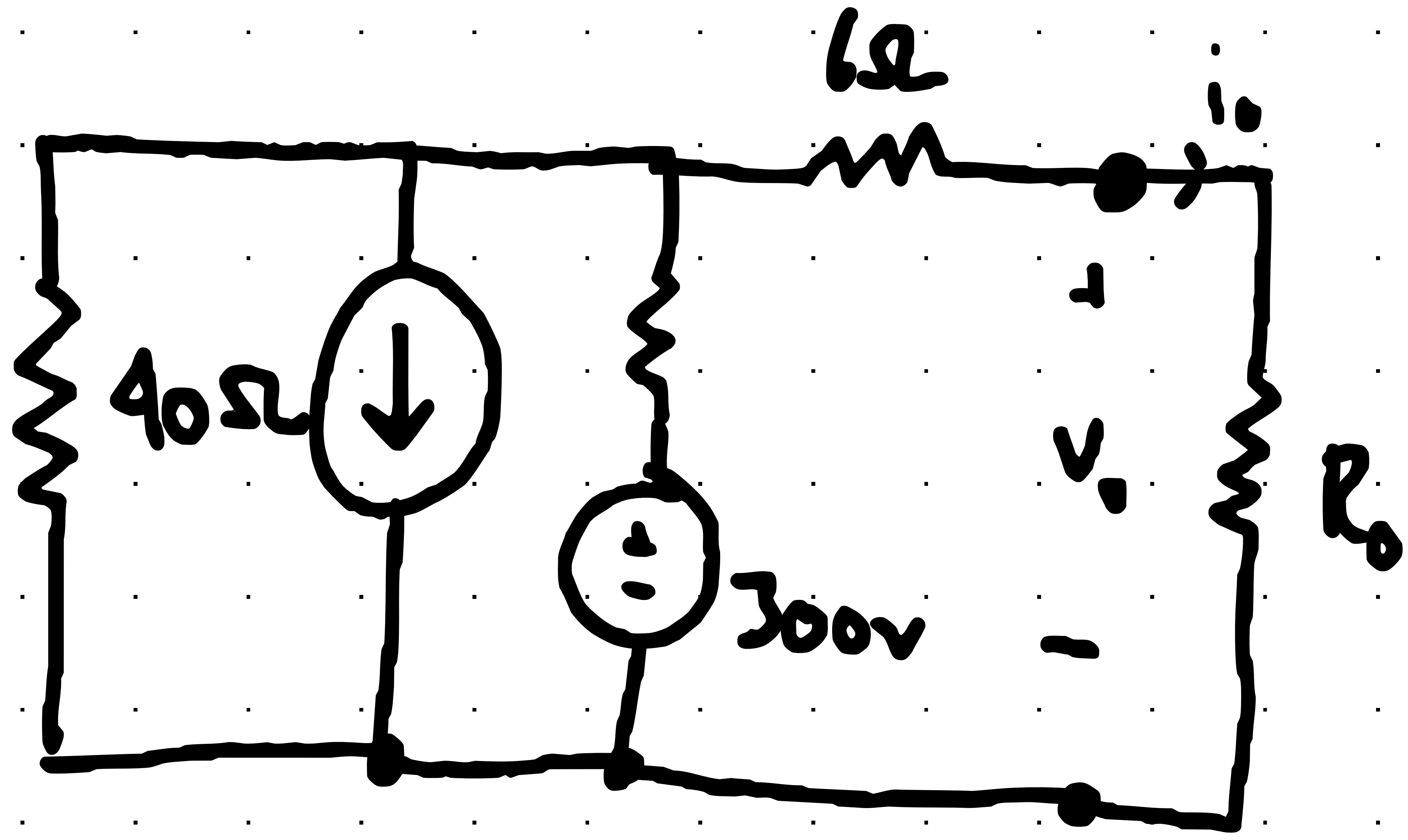
Based on current divider.

Because of opposite current direction

$$i_o(t) = -4e^{-5t} A \qquad t \geq 0^+$$

c)

$$v_o(t) = i_o(t) R = -160\, e^{-5t} \qquad t \geq 0^+$$



6Ω
i_o
40Ω
+
v_o
−
R_o
300v