

## Delta functions

$$\ddot{y} + \dot{y} + 2y = \underbrace{e^{-2t} + t^3}_{f(t)}$$

$\delta(t)$

Delta is an instantaneous function!

$$\mathcal{L}\{\delta(t)\} = 1$$

Unit impulse response method (or delta method)

$$\ddot{y} + a_1 \dot{y} + a_0 y = f(t)$$

$$\ddot{h} + a_1 \dot{h} + a_0 h = \delta(t)$$

Turn into h's

$\mathcal{L} \rightarrow \dots$   $H(s) = \text{Something}$

From that, take Laplace inverse  $\mathcal{L}^{-1} h(t)$

Once we get  $h(t)$ , we want to use a Certain formula

$$y(t) = f(t) * g(t) = \int_0^t f(u) g(t-u) du = \int_0^t f(t-u) g(u) du, \quad F(s)G(s)$$

### Example of Delta method

$$\ddot{y} + 5\dot{y} + 6y = e^{-3t}$$

$$\ddot{h} + 5\dot{h} + 6h = f(t)$$

$$\lambda = 1$$

$$s^2 h(s) - s y(0) - \dot{y}(0) + 5(s h(s) - y(0)) + 6h(s) = 1$$

$$h(s)(s^2 + 5s + 6) = 1$$

$$h(s) = \frac{1}{(s^2 + 5s + 6)} = \frac{1}{(s+2)(s+3)}$$

$$\frac{1}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

Cover up:

$$1 = A(s+3) + B(s+2)$$

$$A = 1$$

$$B = -1$$

$$H(s) = \frac{1}{s+2} - \frac{1}{s+3}$$

$$h(t) = e^{-2t} - e^{-3t}$$

Apply Formula

$$y(t) = f(t) * g(t) = \int_0^t f(u) g(t-u) du = \int_0^t f(t-u) g(u) du, \text{ For } f(s)$$

$$y(t) = \int_0^t f(u) h(t-u) du = \int_0^t (e^{-3u})(e^{-2(t-u)} - e^{-3(t-u)}) du$$