

## Frequency Response

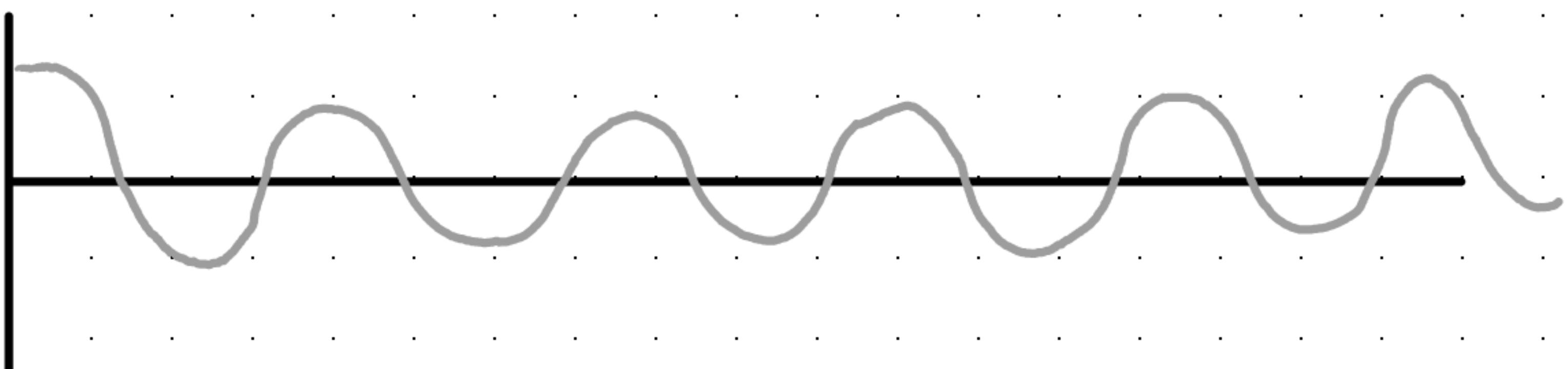
Example of a Sin Wave

$$\omega = 2\pi f$$

$$V(t) = A \cos(\omega t + \phi)$$

or  
say

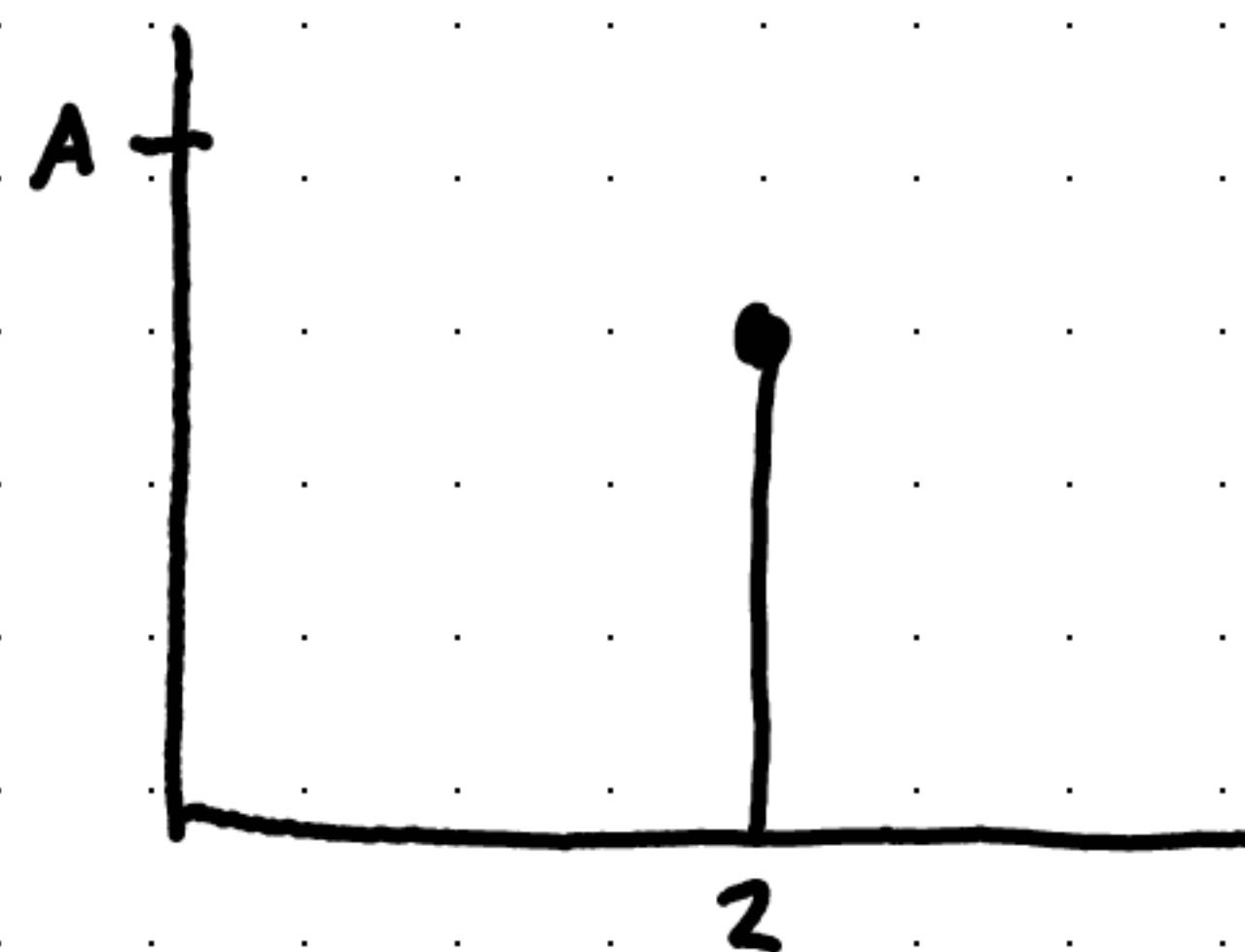
$$V(t) = A \cos(2\pi ft + \phi)$$



Sin and Cos waves have both one frequency,  
and Amplitude.

Say we have

$$x_1 = \sin(2\pi zt)$$



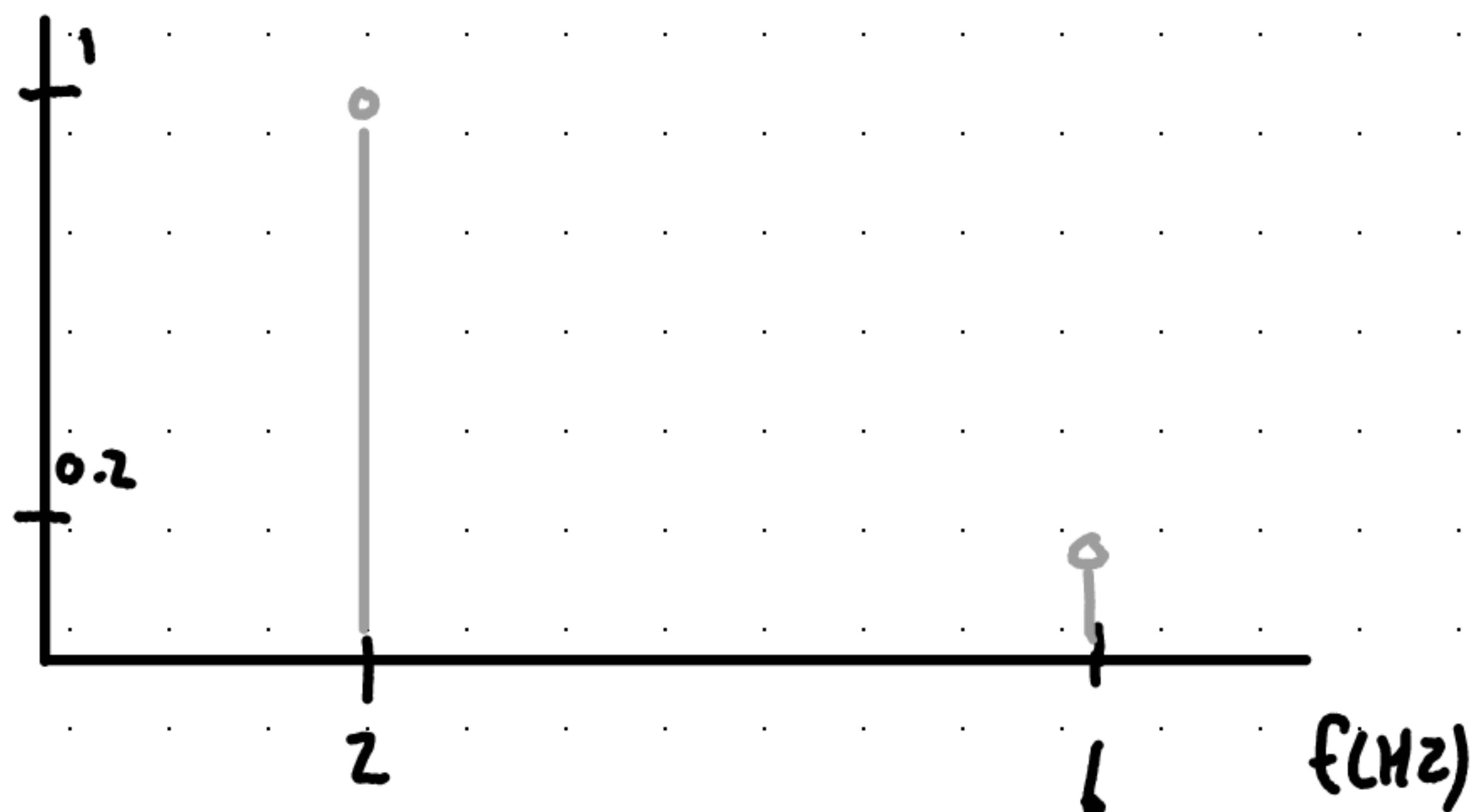
Frequency  
Spectrum  
Chart

We could write a chart, with the one for  
of frequency (in Hz)

Say However,

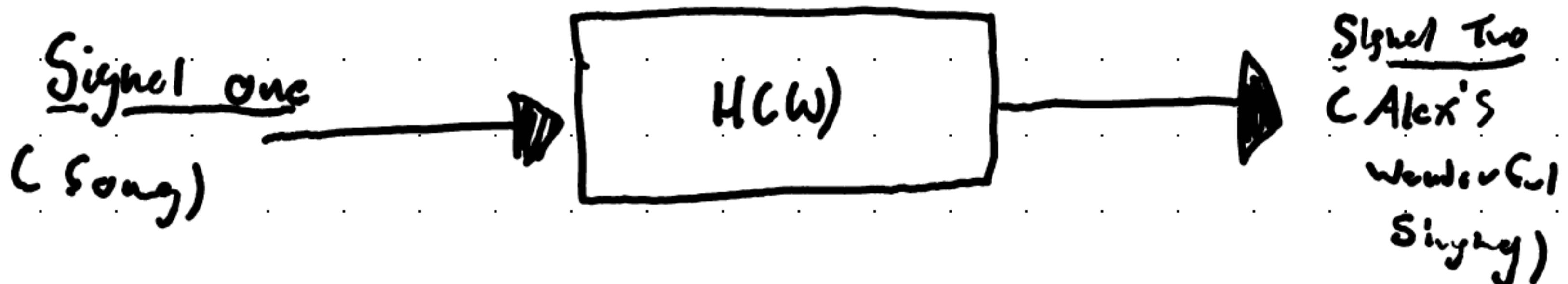
$$X_s = \sin(2\pi 2t) + 0.2 \sin(2\pi 6t)$$

A

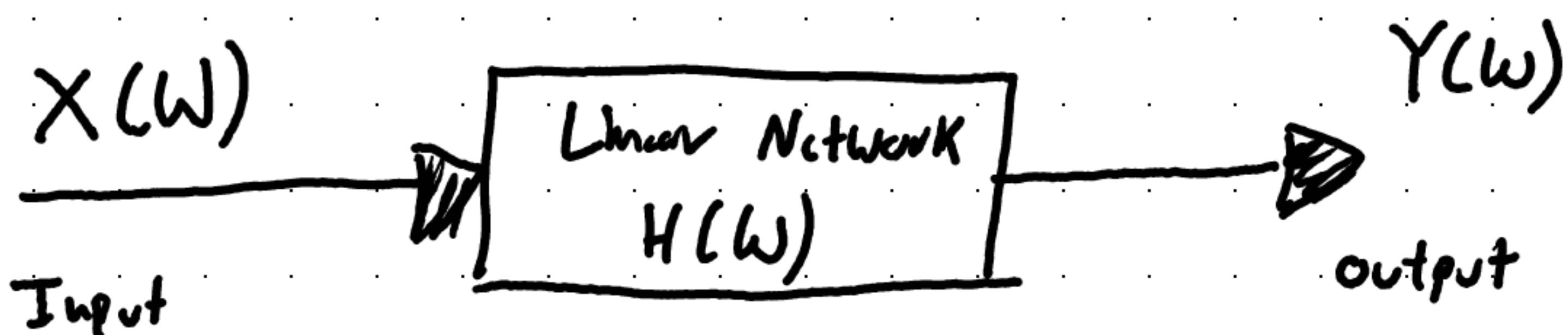


In Signal and Frequency analysis, the Fourier Transform is frequently used to break down complex signals and waves into singular waves.

Think of something like music! We can break it into components.



- The transfer function ( $H(\omega)$ ) is a useful analytical tool for finding the frequency response of a circuit



- In this context,  $X(\omega)$  and  $Y(\omega)$  are denoted as the input and output respectively.
- The transfer function is equal to:

$$H(\omega) = \frac{\text{Output } Y(\omega)}{\text{Input } X(\omega)}$$

And this is known as Gain!

This is also the Voltage gain ( $\frac{V_o(\omega)}{V_i(\omega)}$ ), Current gain ( $\frac{I_o(\omega)}{I_i(\omega)}$ ), Transistor gain ( $\frac{V_o(\omega)}{I_o(\omega)}$ ) and Transfer Admittance ( $\frac{I_o(\omega)}{V_i(\omega)}$ )

## How does one obtain the transfer function?

1. First, convert everything into their frequency domain equivalents
2. Use any circuit technique you'd like to obtain the quantity that you are looking for. ( $V_o$ ,  $V_i$ , etc)
3. Obtain the frequency response by plotting the magnitude and phase of the transfer function  
as frequency Vars.

**TLD R:** Find output and input Vars, and plug em in to the formulas on the previous page

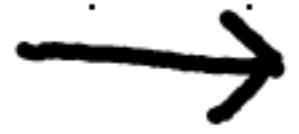
$H(j\omega) \rightarrow$  Plug in  $\rightarrow$  Simplify  $\rightarrow$  take magnitude  $\rightarrow$  Find phi

## RC Circuit

$$H = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$$

$$\phi = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$x(t) = A \cos(\omega t + \phi)$$

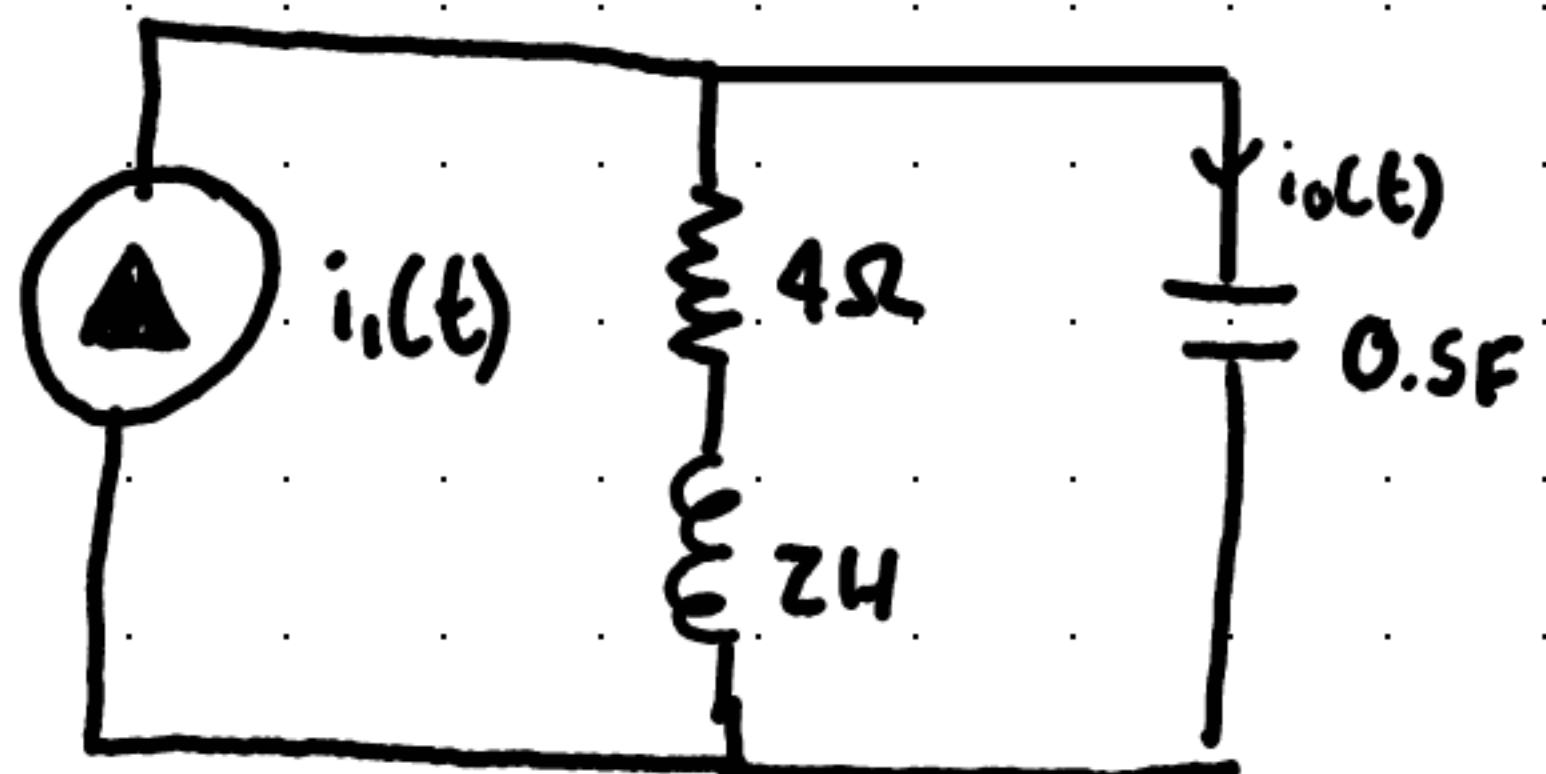


$$y(t) =$$

$$A |H(\omega)| \cos(\omega t + \phi + \arg(H\omega))$$

## Example

Calculate the gain  $I_o(\omega)/I_i(\omega)$ , and its poles and Zeros



Current Divider

$$i_o(\omega) = \frac{4 + j2\omega}{4 + j2\omega + j0.5\omega}$$

$$H = \frac{\text{Output}}{\text{Input}} = \frac{I_o(\omega)}{I_i(\omega)} = \frac{2j\omega + (j\omega)^2}{1 + 2j\omega + (j\omega)^2}$$

$$\text{Let } s = j\omega$$

$$H = \frac{2s + s^2}{1 + 2s + s^2} = \frac{s(s+2)}{(s+1)^2}$$

$$\text{Zeros at } z_1=0, z_2=-2$$