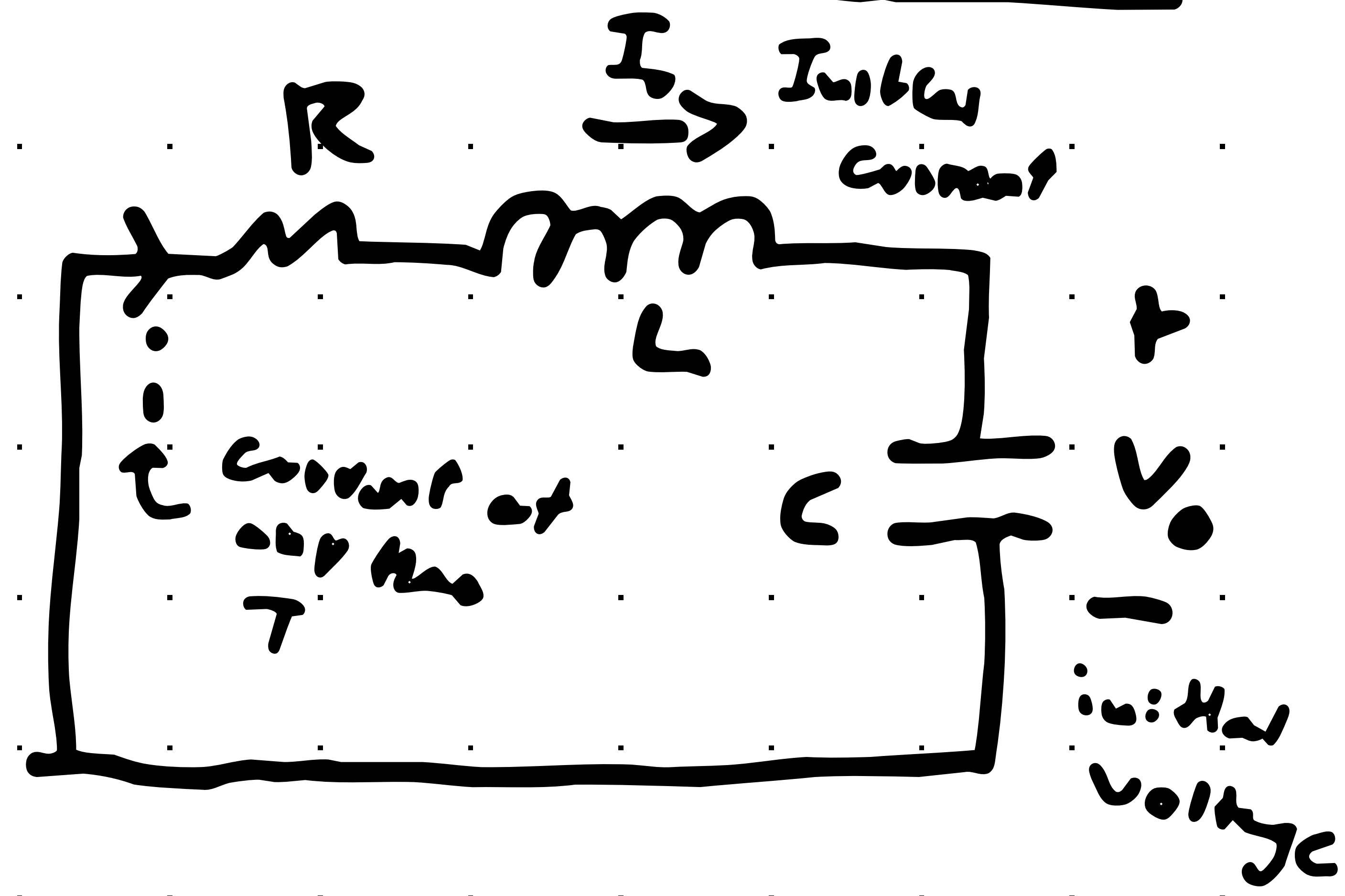
Tutorial 9

The Natural and Step response of a Series RLC

Natural Response, RLC Series



$$\sum_{\text{loop}} V = 0$$

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C} i = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$$

Solve the above eqn by the same way by assuming $i(t) = Ae^{st}$

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where

$$\alpha = \frac{R}{2L}^{*} \quad \& \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Series

In Series

The Same, series or parallel

There are three different solutions based on

α and ω_0

Case 1 $\alpha^2 > \omega^2$, Overdamped

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Case 2 $\alpha^2 < \omega^2$, Underdamped

$$i(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$$

Case 3 $\alpha^2 = \omega^2$, Critically damped

$$i(t) = D_1 t e^{-\alpha t} + D_2$$

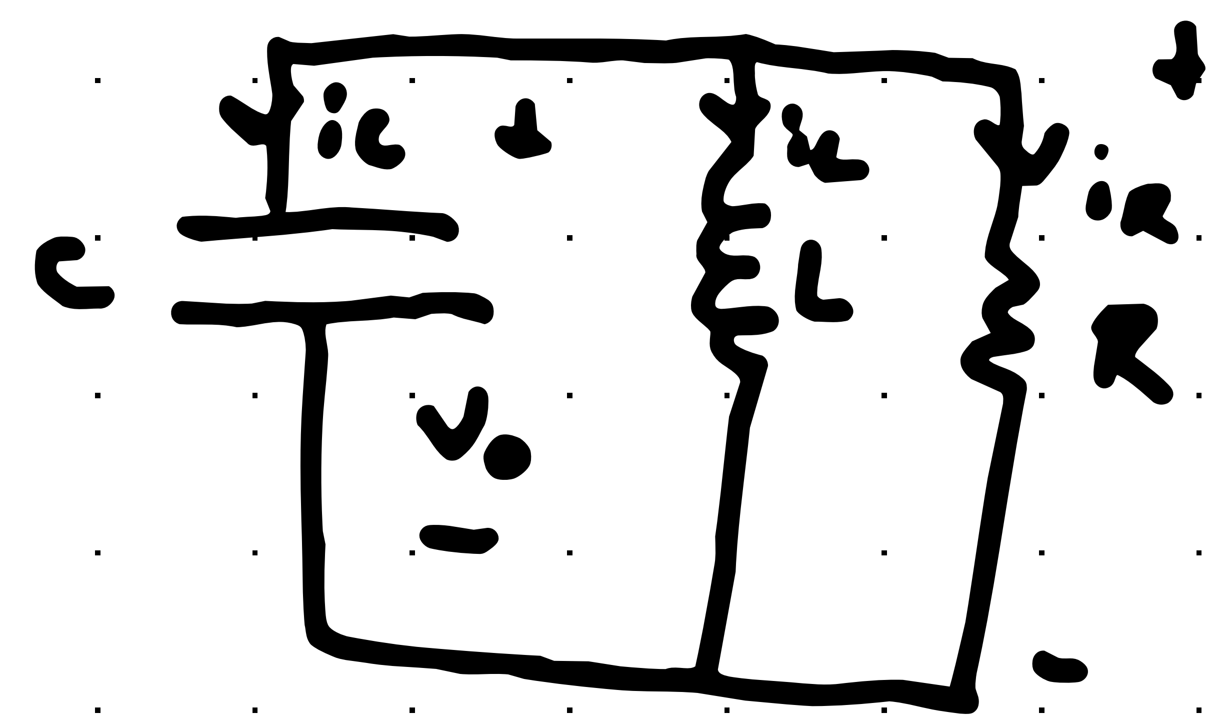
If it is a step response, there are another set of cases.

$V_c(t)$ is over the Capacitor

$i(t)$ is over everything (as it is in series)

Example

Find the roots of the characteristic equation



b) overdamped? underdamped?

c) What value of R causes the response to be critically damped.

$R = 200\Omega$, $L = 50mH$, $C = 0.2\mu F$

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a)

$$\alpha = \frac{1}{2RC} = \frac{1}{2(200)(0.2\times10^{-6})} = 1.25\times10^{4} \text{ rad/sec}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(50\times10^{-3})(0.2\times10^{-6})}} = 10^{4} \text{ rad/sec}$$

α^2 is greater than ω_0

Over damping two real solutions

Use case 1

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -5000$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -20000$$

b)

Over damped

c) For

Critically damped

"when $\alpha^2 > \omega_0^2$"

$$\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC}$$

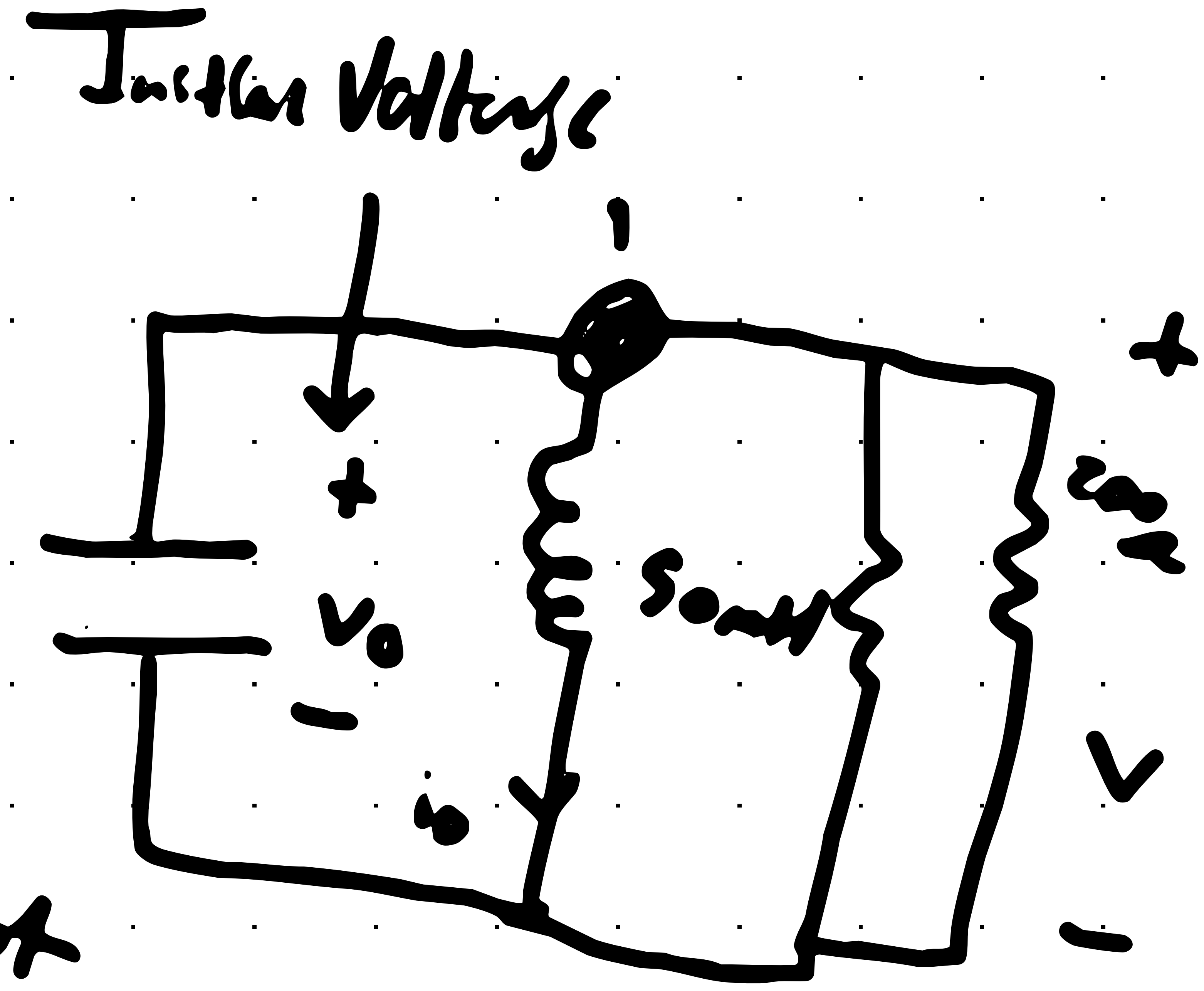
$$\frac{1}{2RC} = \sqrt{\frac{1}{LC}}$$

$$R = \frac{\sqrt{LC}}{2C} = \frac{\sqrt{L}}{2\sqrt{C}} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{2}\sqrt{\frac{50\times10^{-3}}{0.2\times10^{-6}}} = 250\,\Omega$$

Example 2

$V(0^+) = 12V \quad i_L(t) = 30mA$



(a) Calculate the initial current in each branch of the circuit

(b) Find the initial value of $\frac{dv}{dt}$

(c) Find the expression for $V(t)$

(d) Derive the expression for i_R, i_L, i_C at any time.

Solution

a) $i_L(0^+) = 30mA$

$$i_R(0^+) = \frac{V_C(0^+)}{R} = \frac{12}{200} = 60mA$$

$\sum I = 0$

Node 1 $\quad I_R + I_L + I_C = 0$

$$I_C(0^+) = -i_L(0^+) - I_R(0^+) = -90mA$$

b) $i_c = C\frac{dv}{dt}$

$$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{-90\times10^{-3}}{0.2\times10^{-6}} = -450 \text{ KV/sec}$$

c)

$$\alpha_{\text{Parallel}} = \frac{1}{2RC} = 1.25\times10^4 \quad \omega_0 = \frac{1}{\sqrt{LC}} = 1\times10^4$$

$$\alpha^2 > \omega_0^2 \quad \text{(overdamped)}$$

$$V(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$S_1 = -\alpha + \sqrt{(\alpha)^2 + \omega_0^2} = -5000$$

$$S_2 \; -\alpha - \sqrt{(\alpha)^2 + \omega_0^2} = -20000$$

$$V(t) = A_1 e^{-5000t} + A_2 e^{-20000t}$$

* Missing A_1 and A_2

So, must solve...

For Calculating A_1 and A_2, use Initial Conditions.

$$V_{(0)} = A_1 + A_2 = 12 \quad \text{Eqn } 1$$

$$\frac{dv_{(0)}}{dt} = -450 = -5000A_1 - 20000A_2 \quad \text{Eqn } 2$$

Solve for A_1 and A_2

$A_1 = -14$
$A_2 = 26$

So,

$$\boxed{V(t) = -14e^{-5000t} + 26e^{-20000t}}$$

d) derive the expression

$$V = IR$$

$$I_R(t) = \frac{V(t)}{R_{(200)}} = -70e^{-5000t} + 130e^{-20000t} \text{ mA}$$

$$I_C(t) = C\frac{dv}{dt} = 0.2\times10^{-6}\left[-14(-5000)e^{-5000t} + 26(-20{,}000)e^{-20000t}\right.$$

$$I_C(t) = 14e^{-5000t} - 104e^{-20000t} \text{ mA}$$

$$I_L(t) = -i_c(t) - i_R(t) = 56e^{-5000t} - 26e^{-20000} \text{ mA}$$

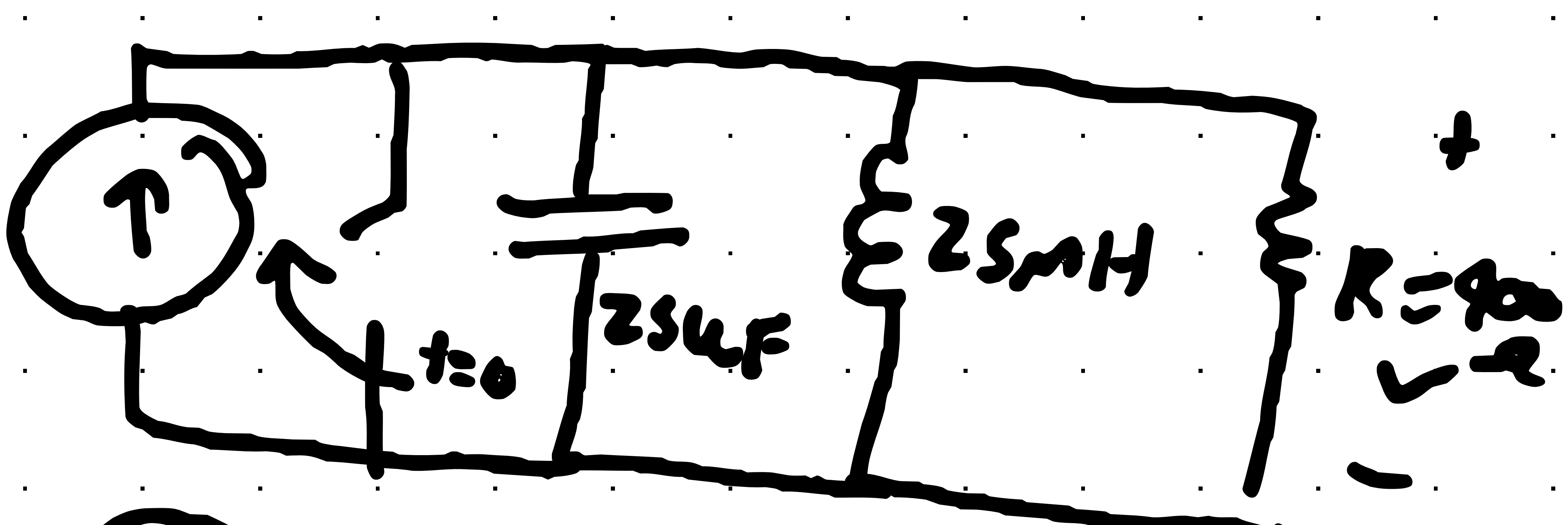
Example 3

The initial energy stored in the circuit is zero

At $t=0$ a dc current is applied to the circuit

The value of the resistor is $400\,\Omega$

(a) What is the initial value of i_L?
(b) What is the initial value of di/dt
(c) What are the roots of the characteristic Eq
(d) What is the numerical expression of $I_L(t)$ when $t \geq 0$?



Solution

(a)

Initial energy stored is zero

Capacitor $V_C(0) = 0$

Inductor $I_L(0) = 0$

(b)

$$V_L = L\frac{di}{dt} \rightarrow \frac{di_L(0)}{dt} = \frac{0}{L} = 0$$

(c)

$$\alpha_{Par} = \frac{1}{2RC}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\alpha = 25\times10^4$$

$$\omega_0 = 4\times10^4$$

$$\alpha^2 > \omega_0^2 \quad \text{(over damped)}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega^2} = -20000 \text{ rad/sec}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega^2} = -80000 \text{ rad/sec}$$

$$i_L(t) = I_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$I_L(t) = 24 + A_1 e^{-20000t} + A_2 e^{-80000t}$$

For Finding A_1 and A_2, use I.C

$$I_L(0) = 0 = A_1 + A_2 + 24\times10^{-3} \quad (1)$$

$$\frac{di_{L(0)}}{dt} = 0 = 0 + A_1(-20{,}000) + A_2(-80{,}000) \quad (2)$$

$A_1 = -32 mA$

$A_2 = 8 mA$

$$\underline{I_L(t) = 24 + (-32)e^{-20000t} + 8e^{-80000t} \ mA}$$