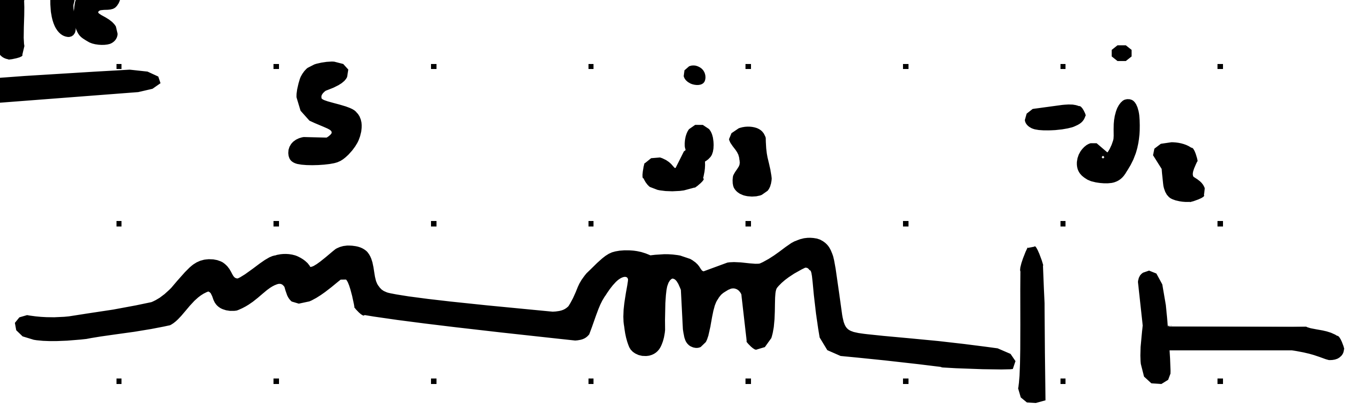
Lecture 21 ECED 2□□□

Impedance : Z (Ω)

Admittance

$$Y = \frac{1}{Z} = \underbrace{G}_{\text{Conductance}} + j\overbrace{B}^{\text{Imaginary}}_{\text{Susceptance}}$$

Example

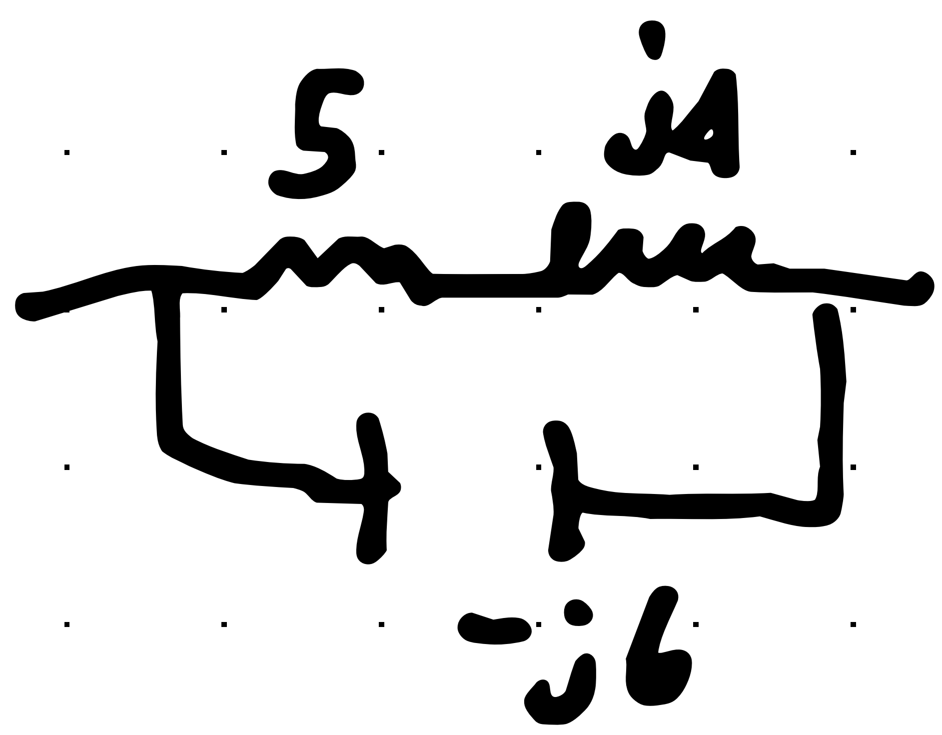


$\Rightarrow Z = 5 + j3 - j2 =$

$Z = 5 + j1\ \Omega$

Example

Find the total Impedance



$$Z = \frac{(5+j4)(-j6)}{5+j4-j6} = \frac{-j30+24}{5-j2}$$

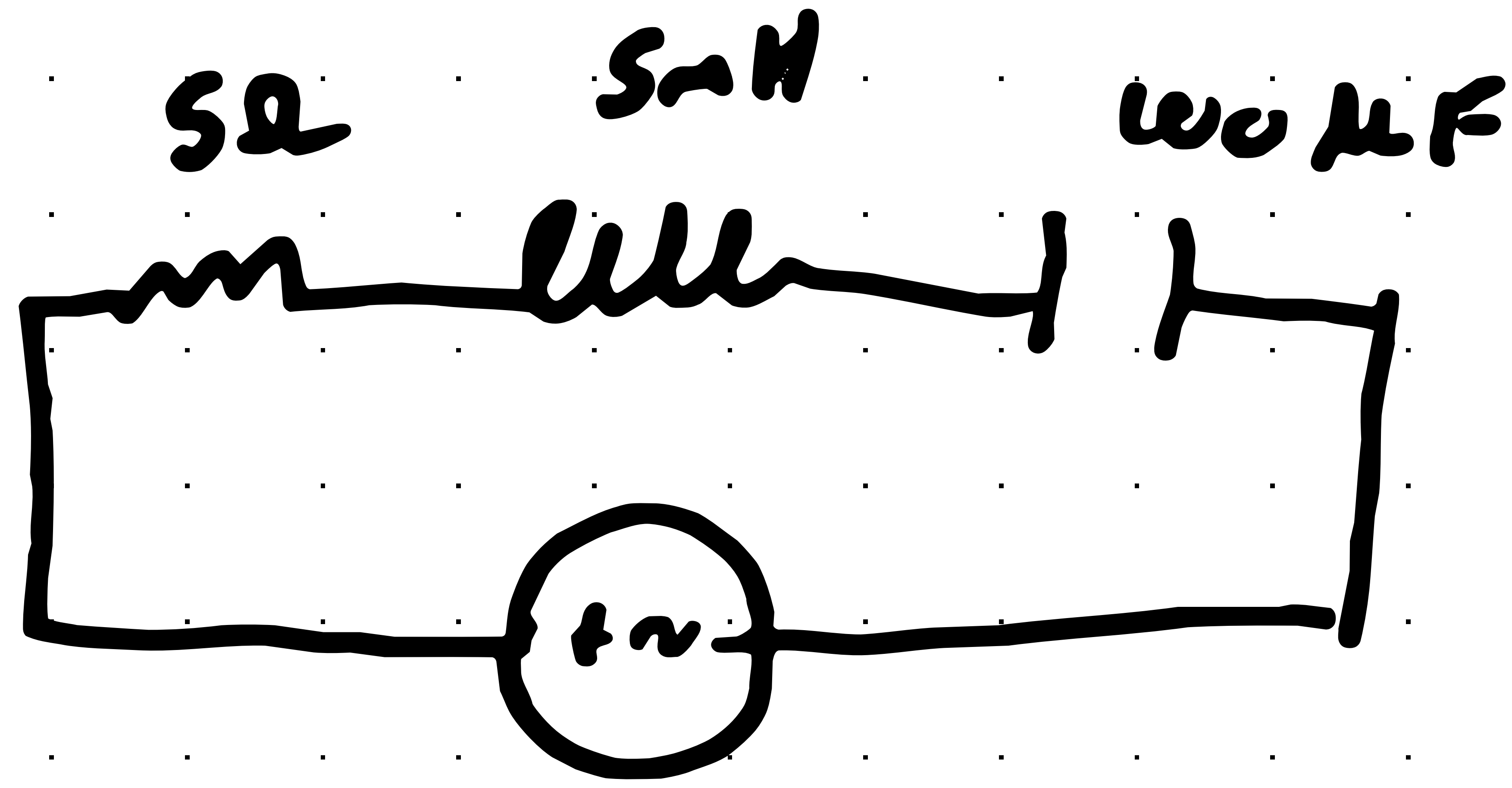
$$= \frac{24-j30}{5-j2}\left(\frac{5+j2}{5+j2}\right)$$ ← Complex conjugate

$$= \frac{5(24)+(30)(2)+j2(24)-j30(5)}{5^2+2^2}$$

$$= \frac{180}{29} - j\,\frac{102}{29}$$

Example

Calculate the current i in the following circuit



$v = 10\cos(1000t + 20°)$

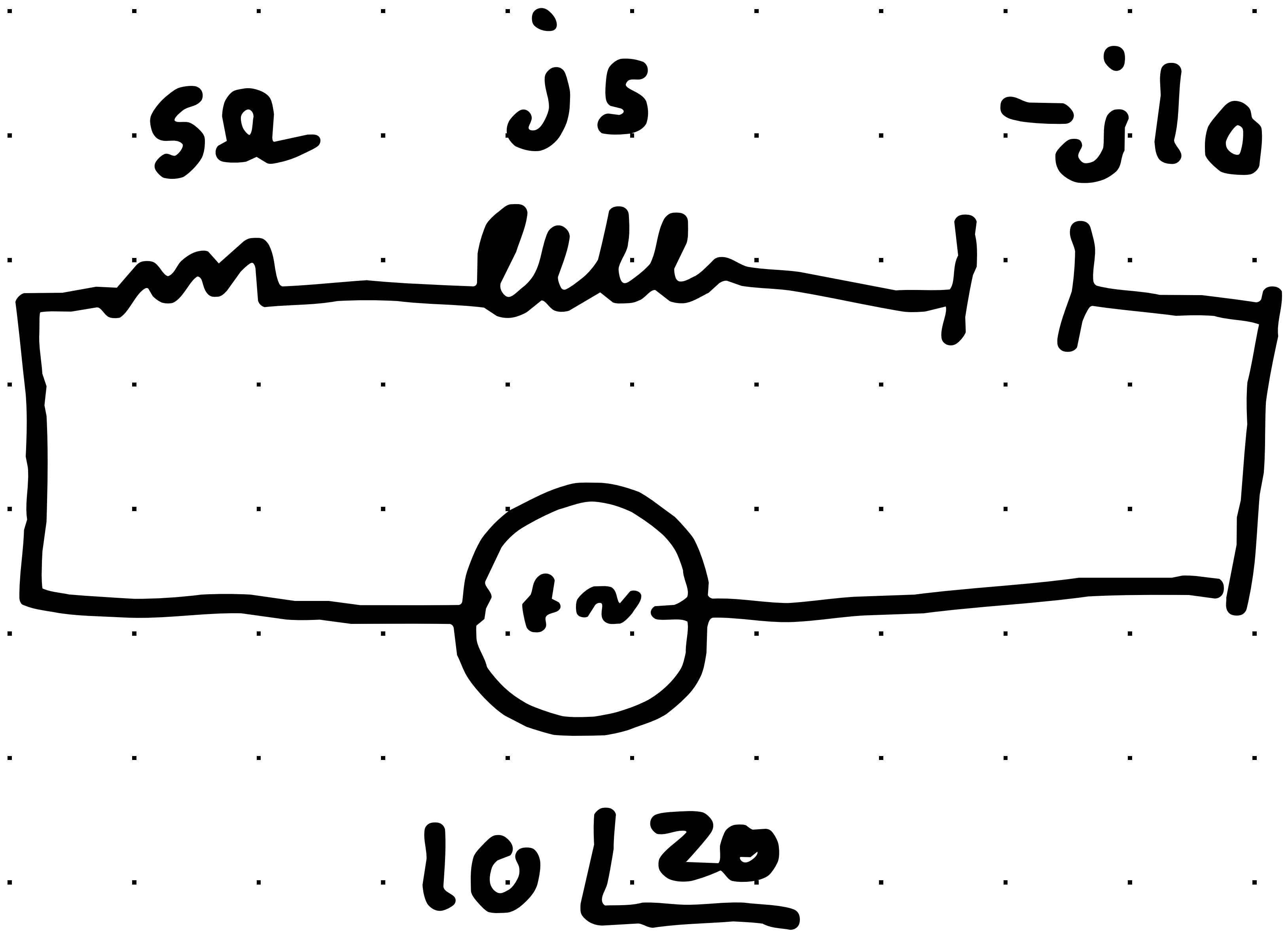
Solution

1 Convert from time domain to Phasor domain.

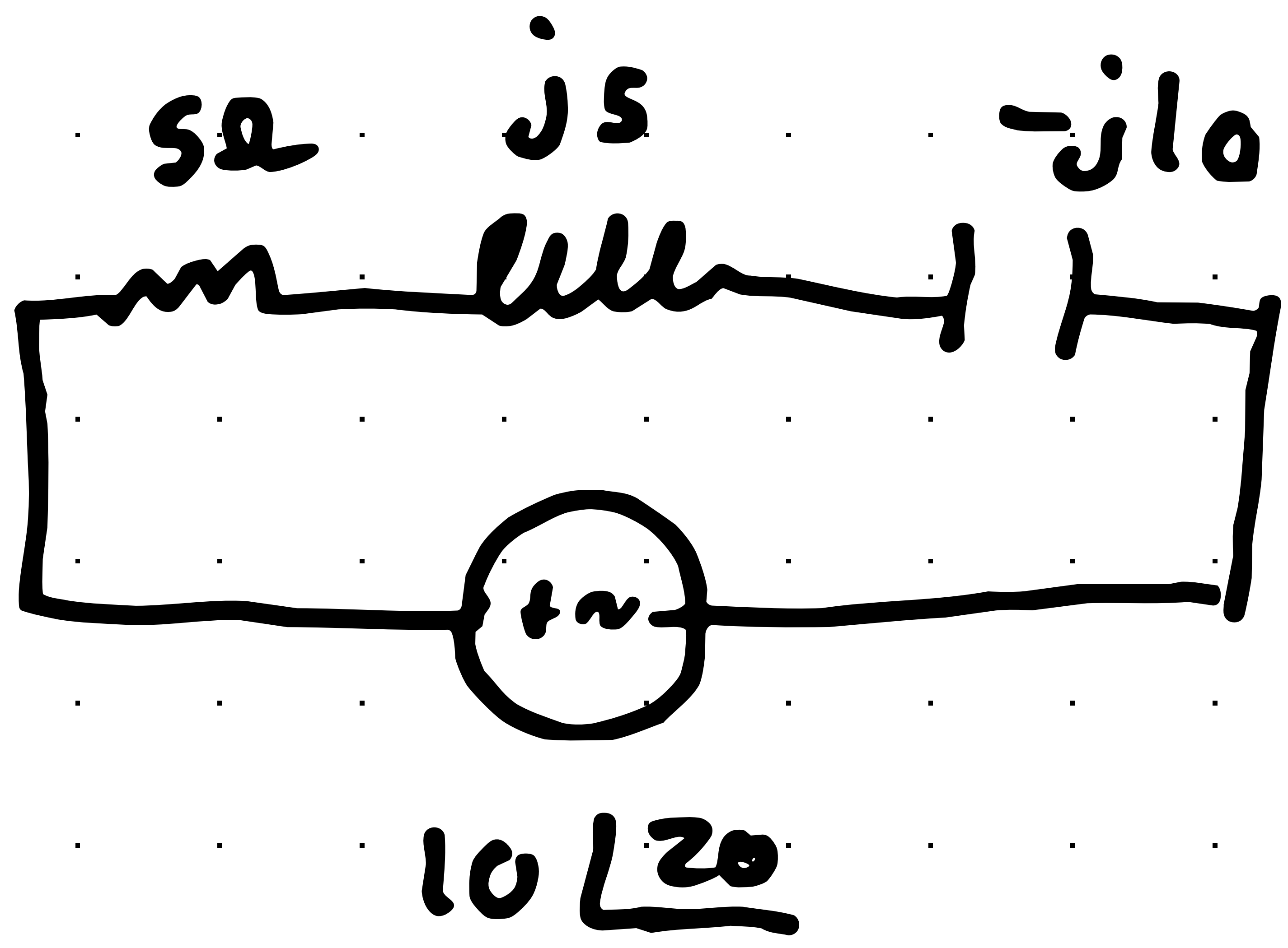
$L \longrightarrow j\omega L \longrightarrow j(1000)(5\times10^{-3}) \rightarrow j5\Omega$

$C \longrightarrow \frac{1}{j\omega C} \longrightarrow j\frac{1}{(1000)(100\times10^{-6})}$ $-j10\Omega$

$V = 10\angle 20$



✓



Here, we look for the total impedance

$$Z = 5 + j5 - j10$$
$$= \underline{5 - j5}$$

$$i = \frac{V}{Z} = \frac{10\angle 20}{5 - j5}$$

Here, either do complex conjugate, OR convert to polar

$$= \frac{10\angle 20}{\sqrt{5^2 + 5^2}\ \angle -\tan^{-1}\left(\frac{5}{5}\right)}$$

$$= \frac{10\angle 20}{5\sqrt{2}\ \angle -45} = \frac{10}{5\sqrt{2}} \angle 20-(-45)$$

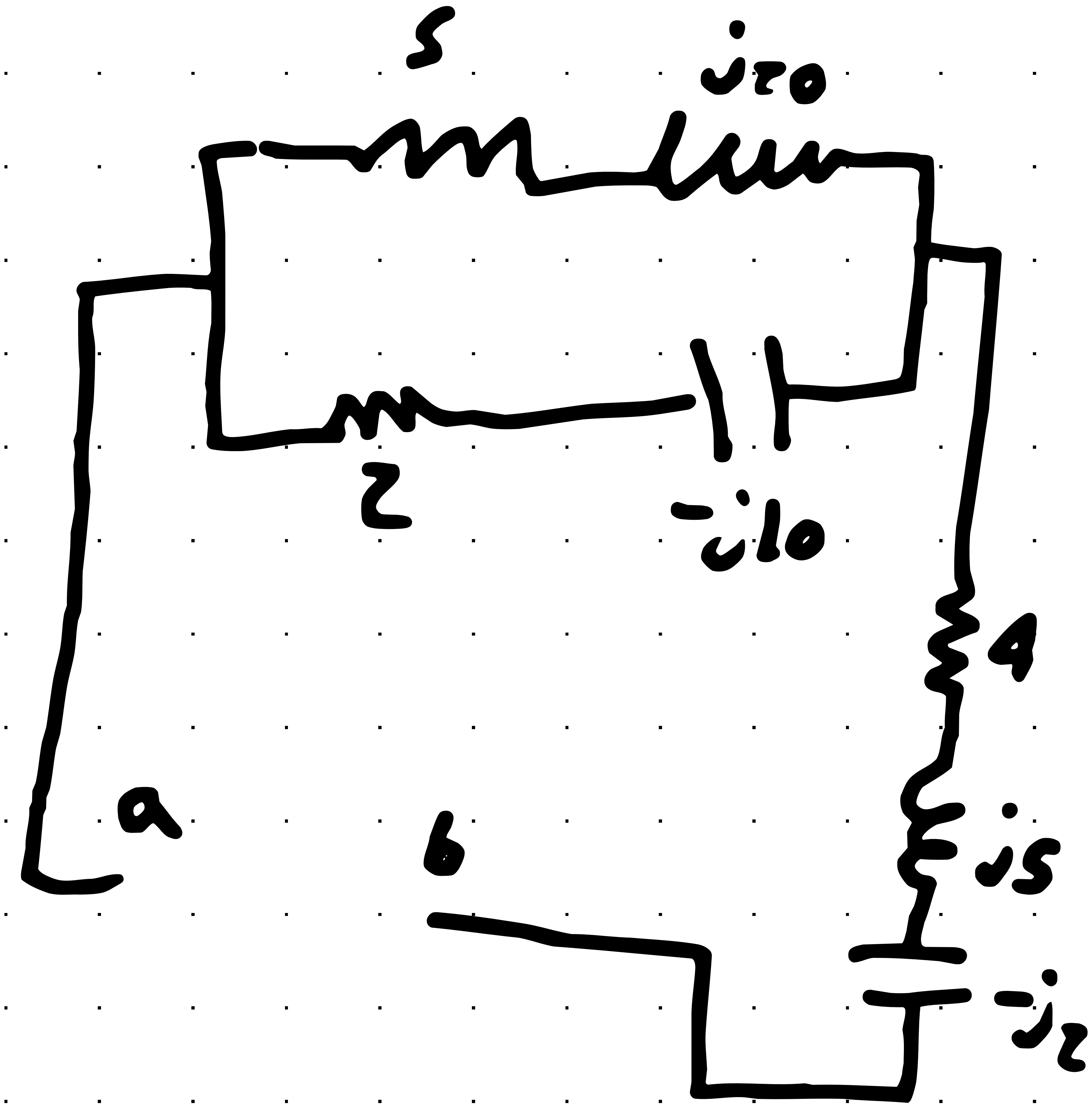
$$= \frac{2}{\sqrt{2}} \angle 65$$ Phasor

Convert back to time domain

$$i(t) = \sqrt{2}\cos(1000t + 65)\ A$$

Example

Calculate the total impedance between a and b



Solution

$(5+j20)//(2-j10)$ (Series, parallel up top)

$$= \frac{(5+j20)(2-j10)}{5+j20+2-j10} = \frac{(10+200)+j(40-50)}{7+j10}$$

$$= \frac{210+j10}{7+j10} = \frac{210+j10}{7+j10}\,\frac{(7-j10)}{(7-j10)}$$

complex conjugate

$(j \times j) = -1$

$$= \frac{7(210)-100-j(2100+70}{7^2+10^2}$$

$$= \frac{1370-j2170}{149} = 9.19-j14.56\,\Omega$$

$(9.19 - j14.56)$ Series $(4 + j5 - j2)$

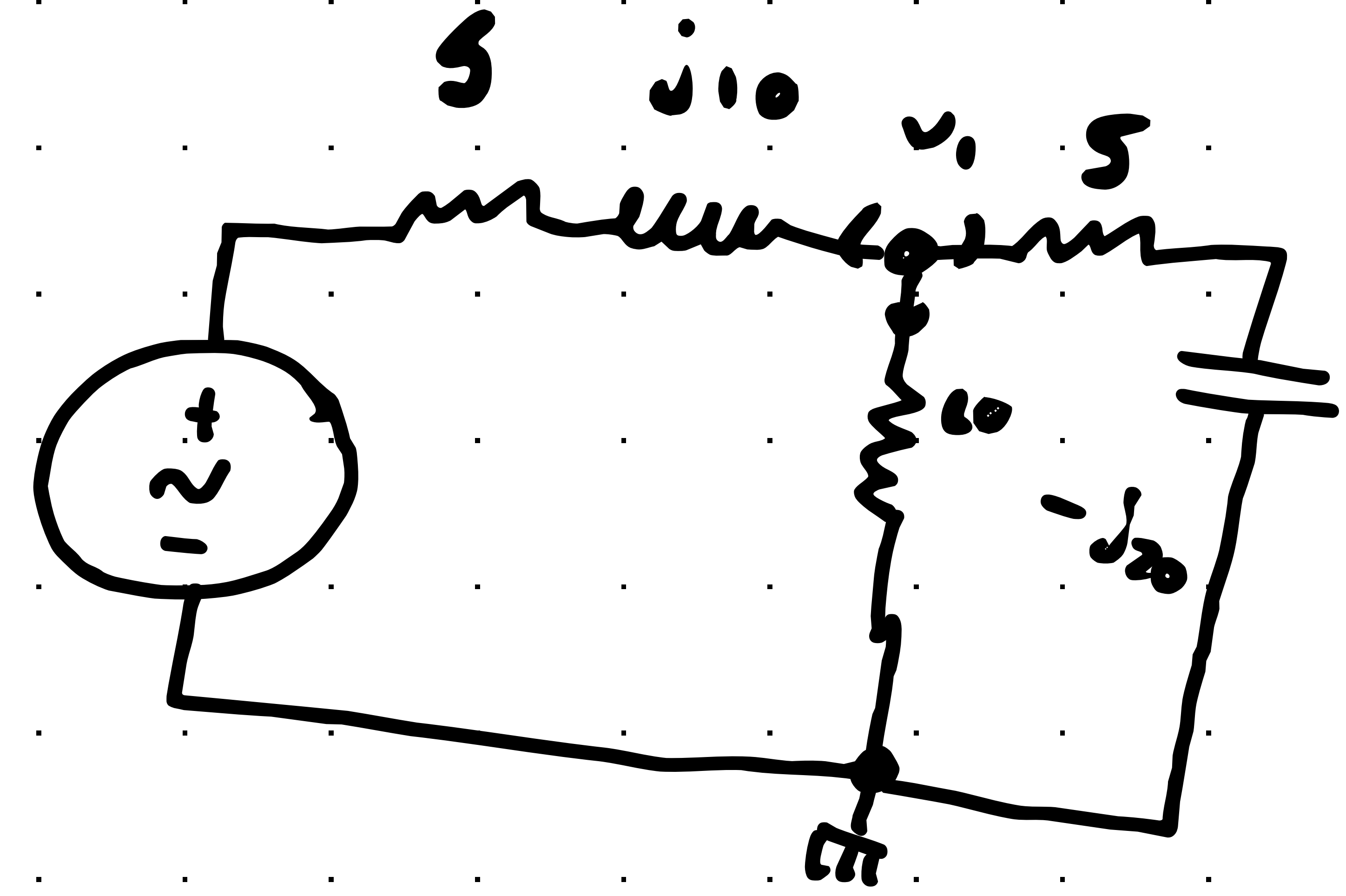
$$Z_{total} = 9.19 - j14.56 + 4 + j5 - j2$$

$$= 13.19 - j11.56\ \Omega$$

* Add Real to Real, and Imaginary to Imaginary

Example

Solve the following problem to calculate V_1



$\sum I = 0$ Node 1

$$\frac{V_1 - 100\angle 0^\circ}{5 + j10} + \frac{V_1}{10} + \frac{V_1}{5 - j20}$$

→ Convert to polar

$$\frac{V_1 - 100\angle 0^\circ}{\sqrt{5^2 + 10^2}\,\angle \tan^{-1}\left(\frac{10}{5}\right)} + \frac{V_1}{10} + \frac{V_1}{\sqrt{5^2 + 20^2}\,\angle -\tan^{-1}\left(\frac{20}{5}\right)} = 0$$

$$\frac{V_1 - 100}{11.2\angle 63.4} + \frac{V_1}{10} + \frac{V_1}{20.6\angle -76} = 0$$

$$\frac{1\angle -63.4}{11.2} + \frac{V_1}{10} + \frac{V_1}{20.6\angle -76} = 0$$

$$0 = \frac{1\angle -63.4}{11.2} V_1 - \frac{100}{11.2}\angle -63.4 + \frac{1}{10} V_1 + \frac{1\angle 76}{20.6} V_1$$

$$0.089\angle -63.4\, V_1 - 8.93\angle -63.4 + 0.1 V_1 + 0.05\angle 76\, V_1$$

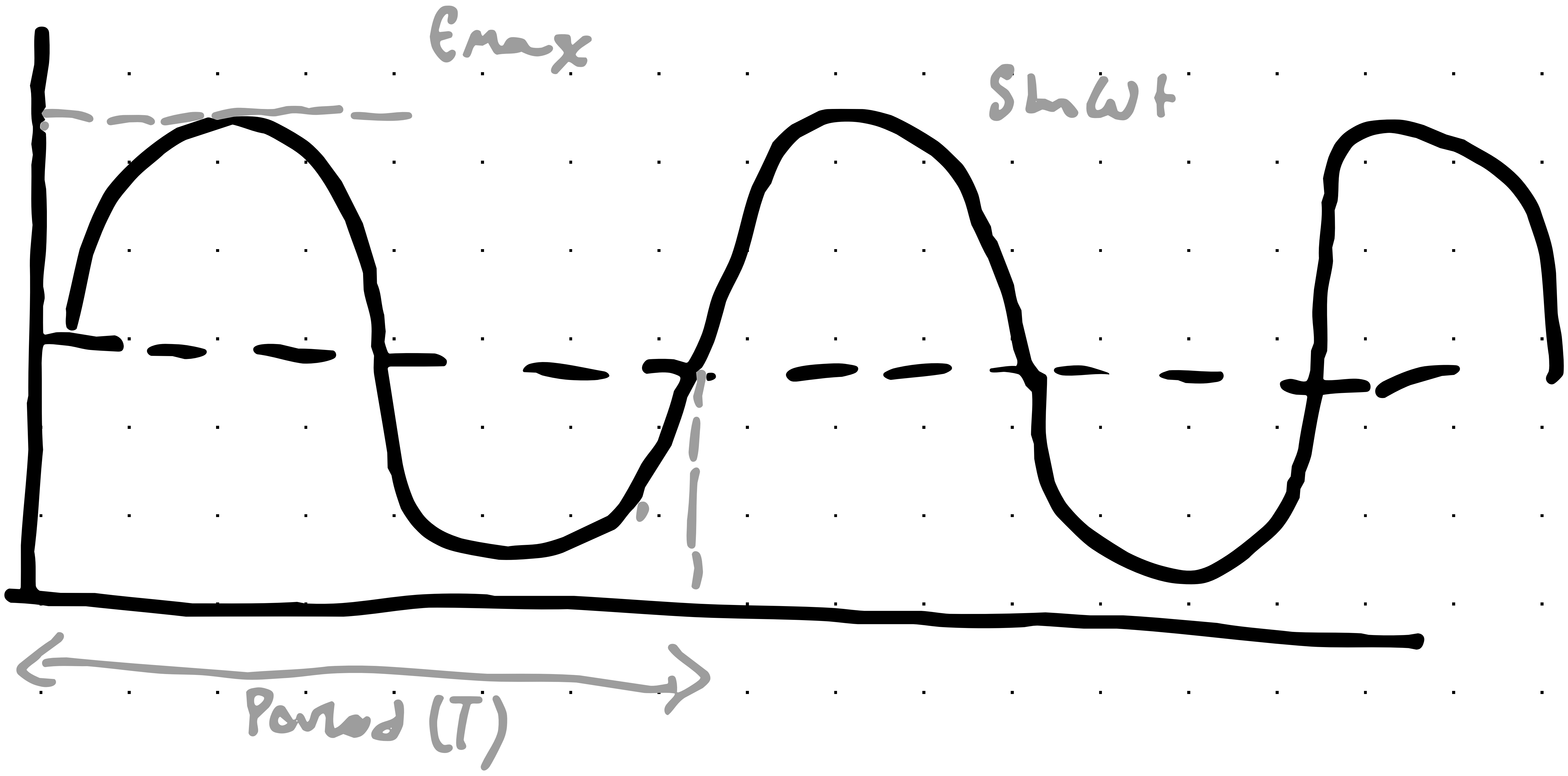
$$V_1 (0.089\angle -63.4 + 0.1 + 0.05\angle 76) = 8.93\angle -63.4$$

head back to non polar domain to add...

$$V_1 [0.089\cos(-63.4) + j0.089\sin(-63.4) + 0.1 + 0.05\cos 76 + j0.05\sin 76]$$

$$= 8.93\angle -63.4$$

Root Mean Square



Frequency $= \frac{1}{T}$

Root Mean Square $= \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt}$

For Any Sinusoidal Function

Root Mean Square $= \frac{\text{True Amplitude (max)}}{\sqrt{2}}$

$= \frac{f_{max}}{\sqrt{2}}$

R.M.S Examples

Example

$$V_1(t) = 50\cos(\omega t) \rightarrow V_{1\,rms} = \frac{50}{\sqrt{2}}\angle 0$$

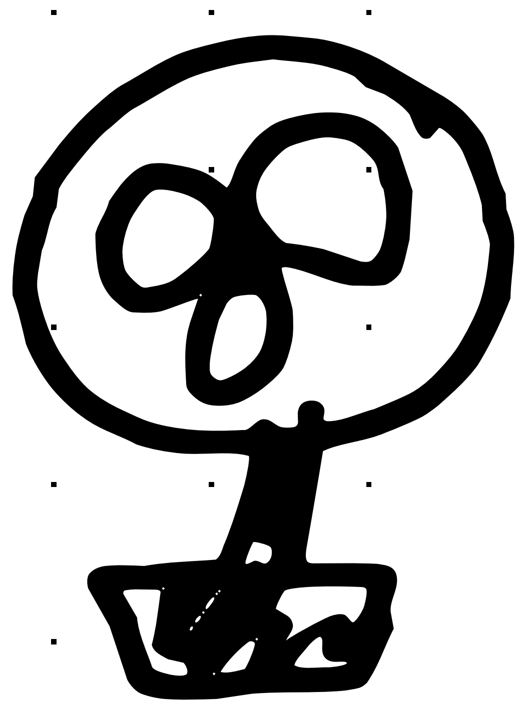
$$V_2(t) = 100\cos(\omega t + 20) \rightarrow V_{2\,rms}\ \frac{100}{\sqrt{2}}\angle 20$$

Complex Apparent Power S

Real Power VS. Imaginary Power

Without Imaginary power, we would not be able to use the real power.

For example, a fan.



- A fan requires electrical (Real) power to provide current.
- Inside the fan, there is a motor, that contains a magnetic.
- The magnetic force does not spin the fan by itself, it needs current.
- But the opposite is also true. The real electrical power is used, but not possible without the magnet.

$$S = V_{rms}\, i_{rms} = P + jQ$$ (VA) (Volt-Amperes)

Active Power (P)

P

$$P = \sum |i_{rms}^2| R$$ or

$$= |i_{rms}|\, |V_{rms}| \cos(\phi_v - \phi_i)$$

Watts

Reactive Power (Q)

$$Q = \sum |i_{rms}^2| X_L - \sum |i_{rms}^2| X_C$$

or

$$Q = |V_{rms}|\, |i_{rms}| \sin(\phi_v - \phi_i)$$

VAR

Volt-Amp-Reactive

$$\text{Power Factor} = \cos(\phi_v - \phi_i)$$

Ex

$$V(t) = 100\cos(1000t + 20)$$

$$i(t) = 5\cos(1000t - 40)$$

* Note, both i(t) and V(t) must be the same. Otherwise, add 90° and convert.

Calculate the active power, reactive power complex power and the impedance

$$V_{rms} = \frac{100}{\sqrt{2}}\angle 20 \qquad i_{rms} = \frac{5}{\sqrt{2}}\angle -40$$

Real power

$$P = |V_{rms}||i_{rms}|\cos(\phi_v - \phi_i) =$$

$$= \frac{100}{\sqrt{2}}\left(\frac{5}{\sqrt{5}}\right)\cos(20-(-40)) = 125 \text{ watts}$$

Reactive power

$$Q = |V_{rms}||i_{rms}|\sin(\phi_v - \phi_i) =$$

216.5 VAR

Complex power

$$S = P + jQ$$
$$= 125 + j216.5$$

Impedance

$$Z = \frac{V}{I} = \frac{\frac{100}{\sqrt{2}}\angle 20}{\frac{5}{\sqrt{2}}\angle -40} = 20\angle 60$$
$$= 20\cos 60 + j20\sin 60$$
$$= 10 + j17.32\,\Omega$$