$\boxed{1}$ Models IVP's (Initial Value problems)

Laws

Heat: "Newton Cooling"

$$\frac{dT}{dt} \propto T - T_\infty$$

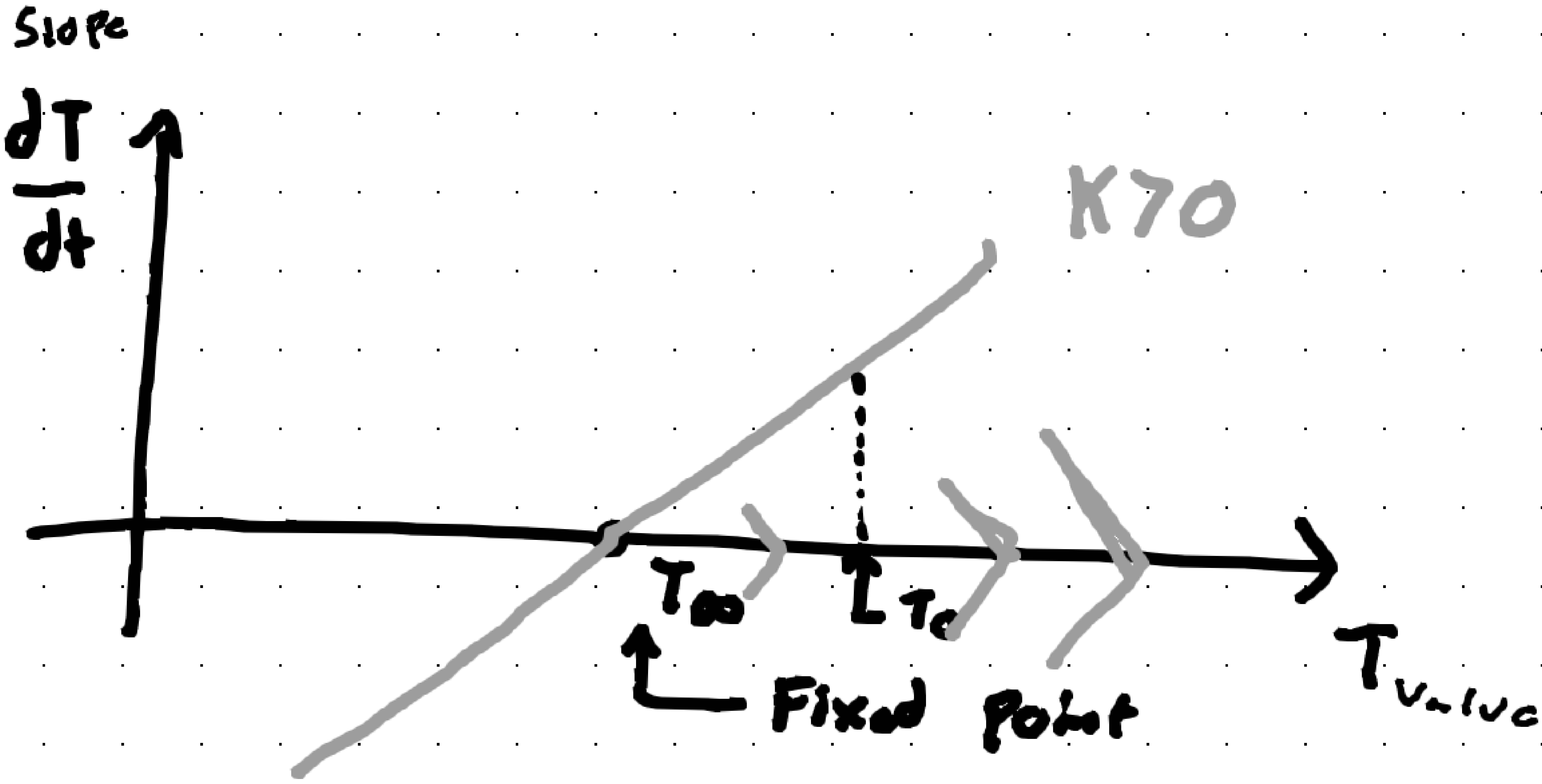
Temp, time, proportional, Ambient

T_∞ "ambient"

Starting value T_0

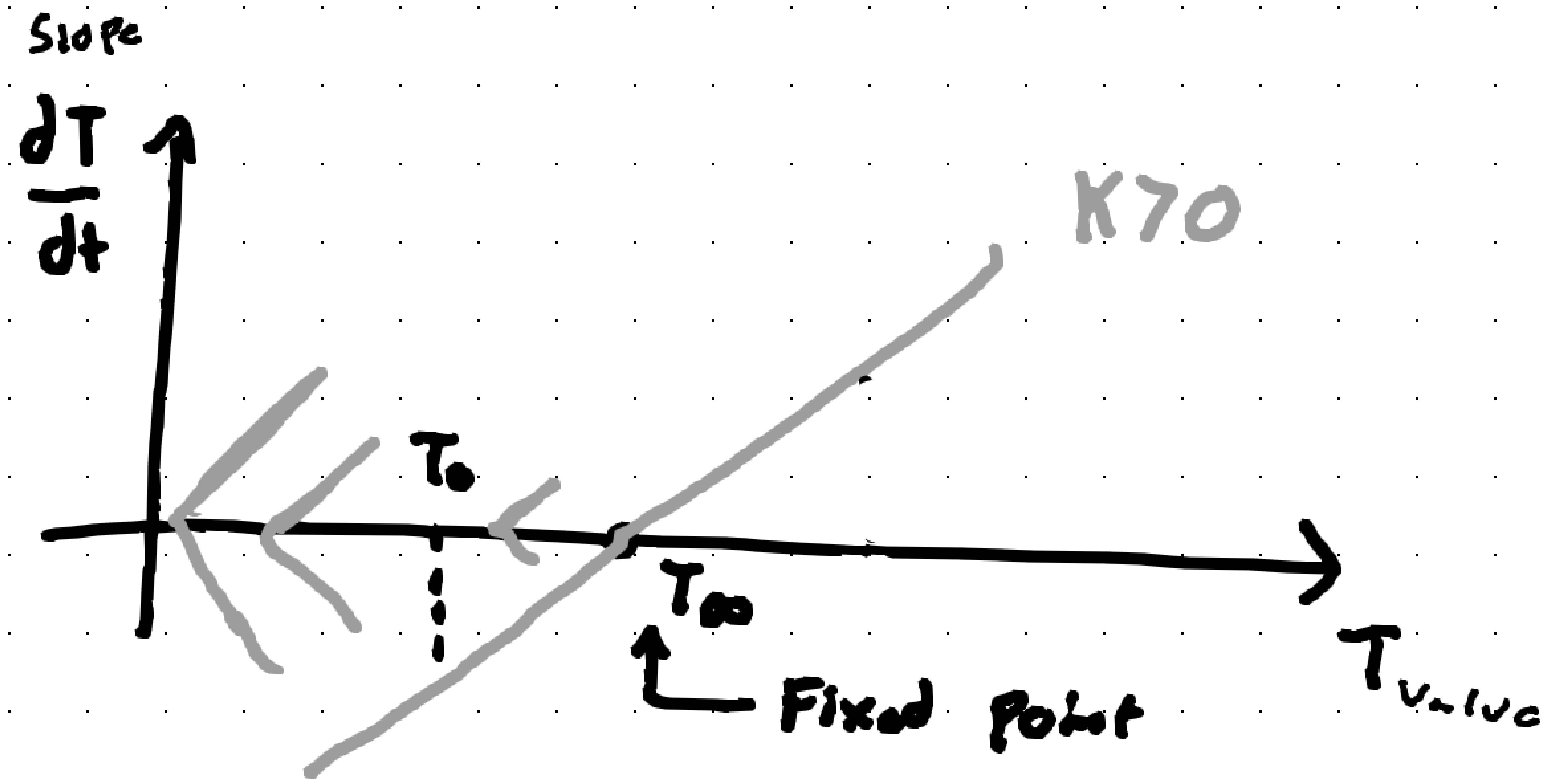
$$\frac{dT}{dt} = K(T - T_\infty)$$

We want $K>0$?



$K>0$. Because of this, and T_0 is warmer than the ambient, we know if we continue, we'll keep going up!

Because $\frac{dT}{dt}$ is expanding, we know that this system is actually accelerating away from the ambient!



If we start cold and go down, it's going to start accelerating downwards!

This tells us that the system is

!! Unstable !!

(Accelerating away from the Fixed point)

The Axiom → something you don't need to prove.

Here

$$T \longrightarrow T_\infty$$

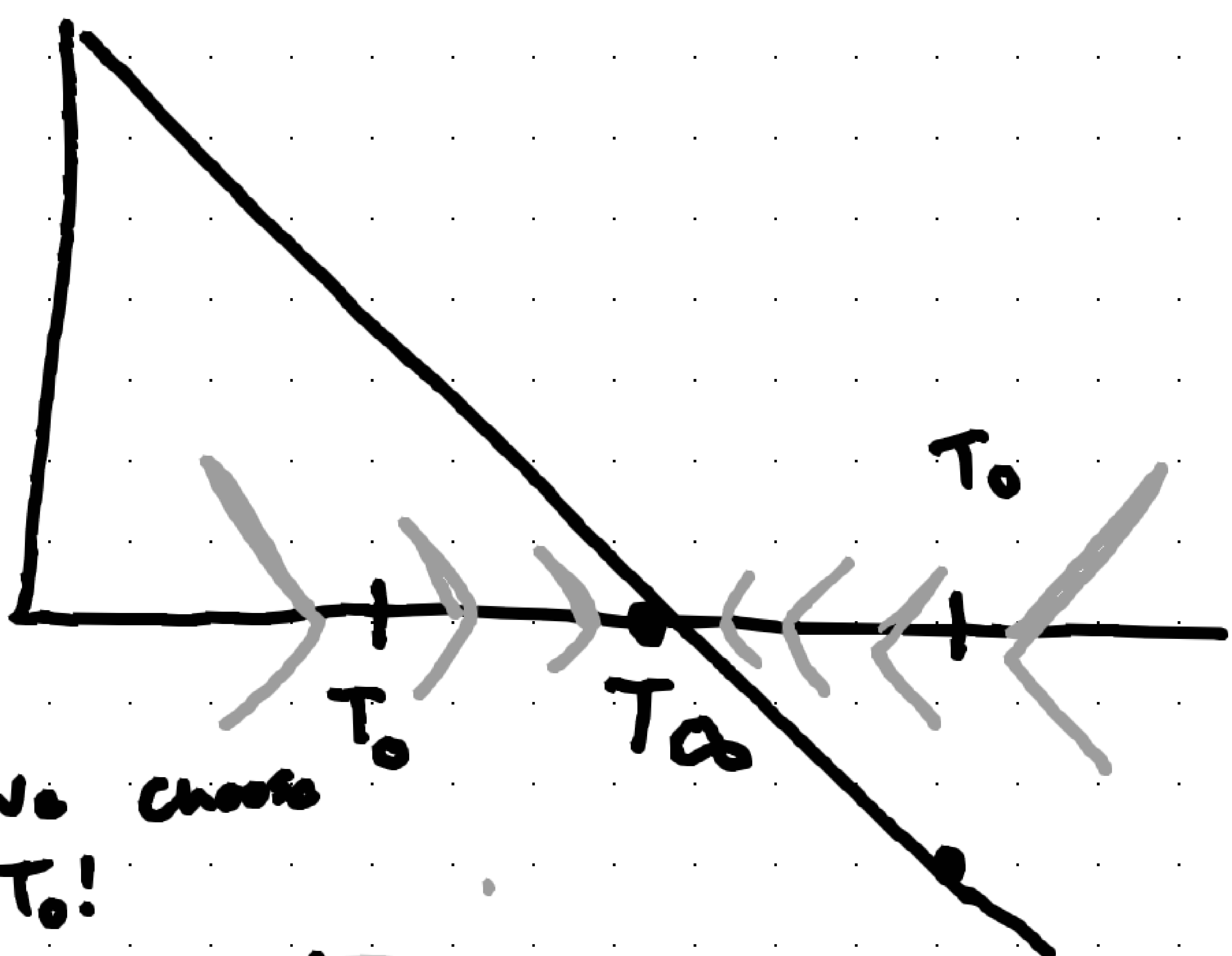
"The coffee will always return to the ambient temperature!)

If a system is unstable, it will always remember

For a system to be truly smart, it must be able to forget. Stable systems can do this.

So, let's get it "right"

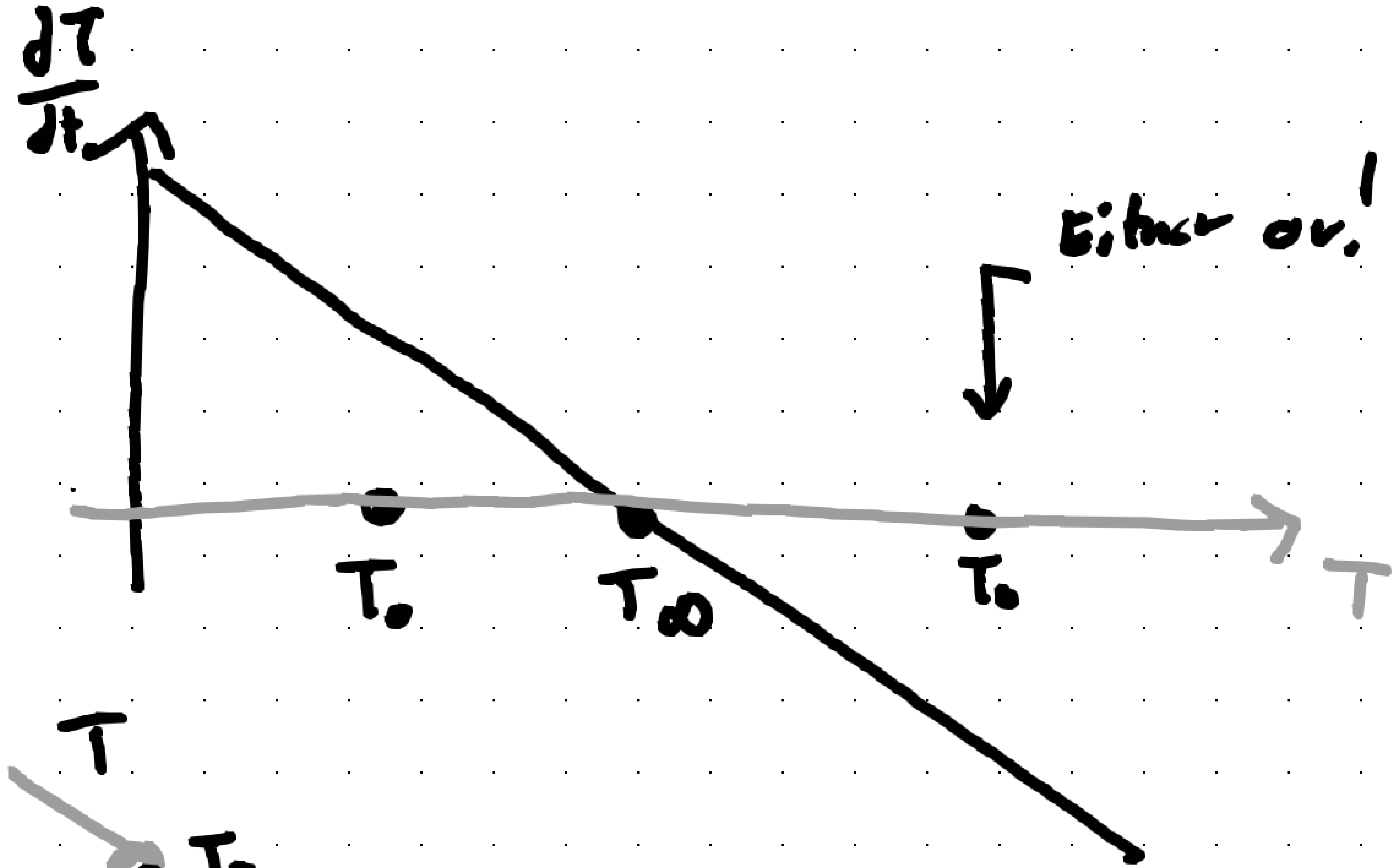
$$\frac{dT}{dt} = -K(T - T_\infty)$$



We choose T_0!
(we can bring whatever coffee we want into the room)

$T_0 \rightarrow T_\infty$... Yaay!

T_0 returns to ambient, with no memory of I.C. Stable system!



We now need to know the process time constant (PTC). This will tell us how long we have to wait.

$$\frac{dT}{dt} = -K(T - T_\infty)$$

°C °C

s

So this must be

$$K = \frac{1}{t}$$

Let's re-arrange this into system form.

$$\frac{dT}{dt} + KT = KT_\infty, \quad t = 0, \quad T = T_0$$

Ambient system

Forced Response

Money:

$\$ = \text{debt}$

$ is unstable

↳ Always Remembers Everything

↳ Including Initial conditions

$$\frac{d\$}{dt} \propto \$$$

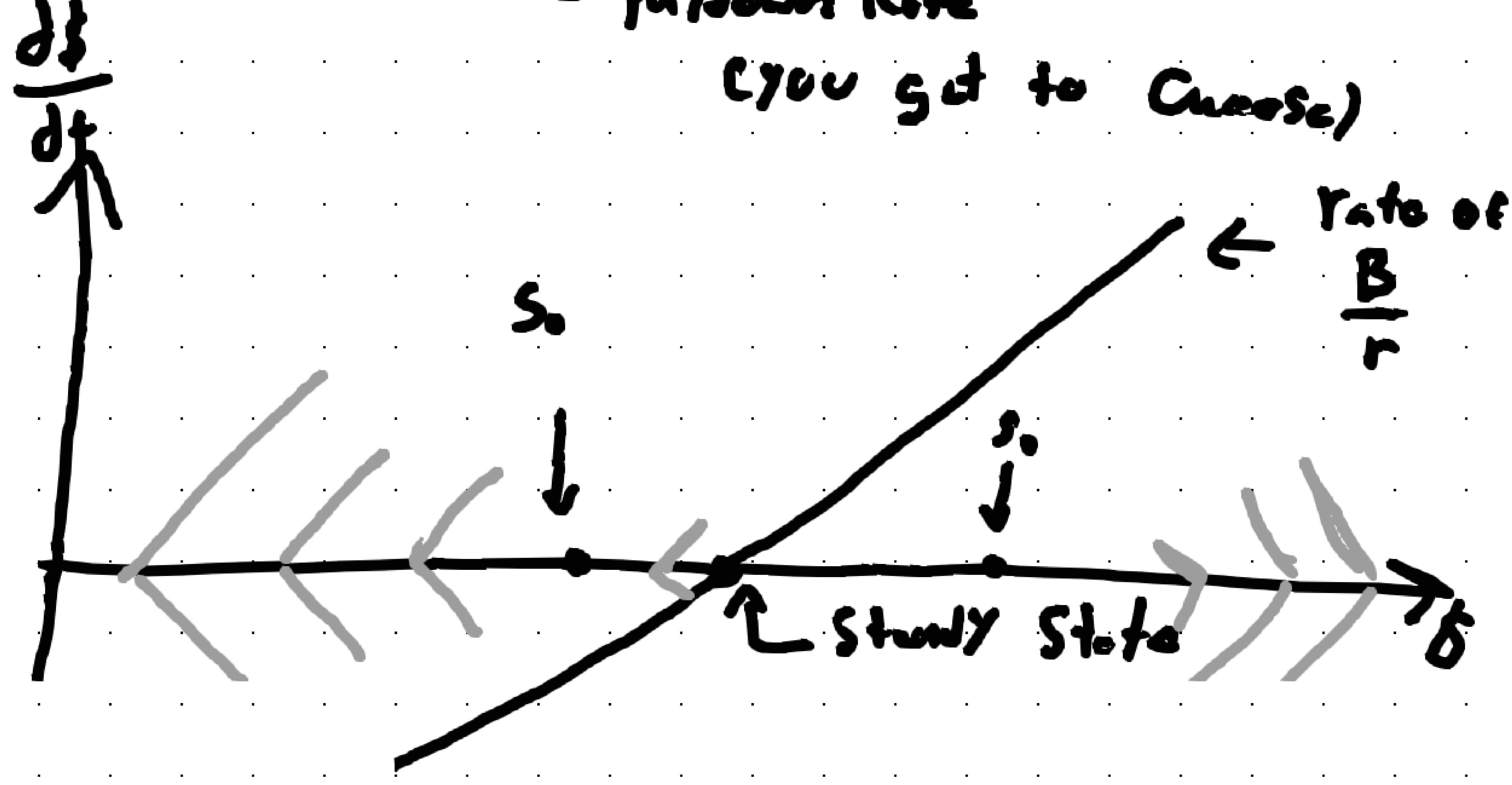
Rate of change of debt

Amount of money you owe.

Interest Rate (Controlled by the bank)

$$\frac{d\$}{dt} = r\$ - B, \qquad r=0, \ \$=\$_0$$

Pay Down Rate (you get to choose)



Say you don't want to pay down, or fall behind on your debt...

$S_0 = \frac{B}{r}$, If r is 10%

$B = r S_0$

owe 10$

Rate of 10%

$B = 0.1 \times 10 = $ 1$ per year

↳ $S_0 > \frac{B}{r}$ and so on...

↳ $S_0 < \frac{B}{r}$