RLC Resonance

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

At the Resonance

$$\boxed{Z = R}$$

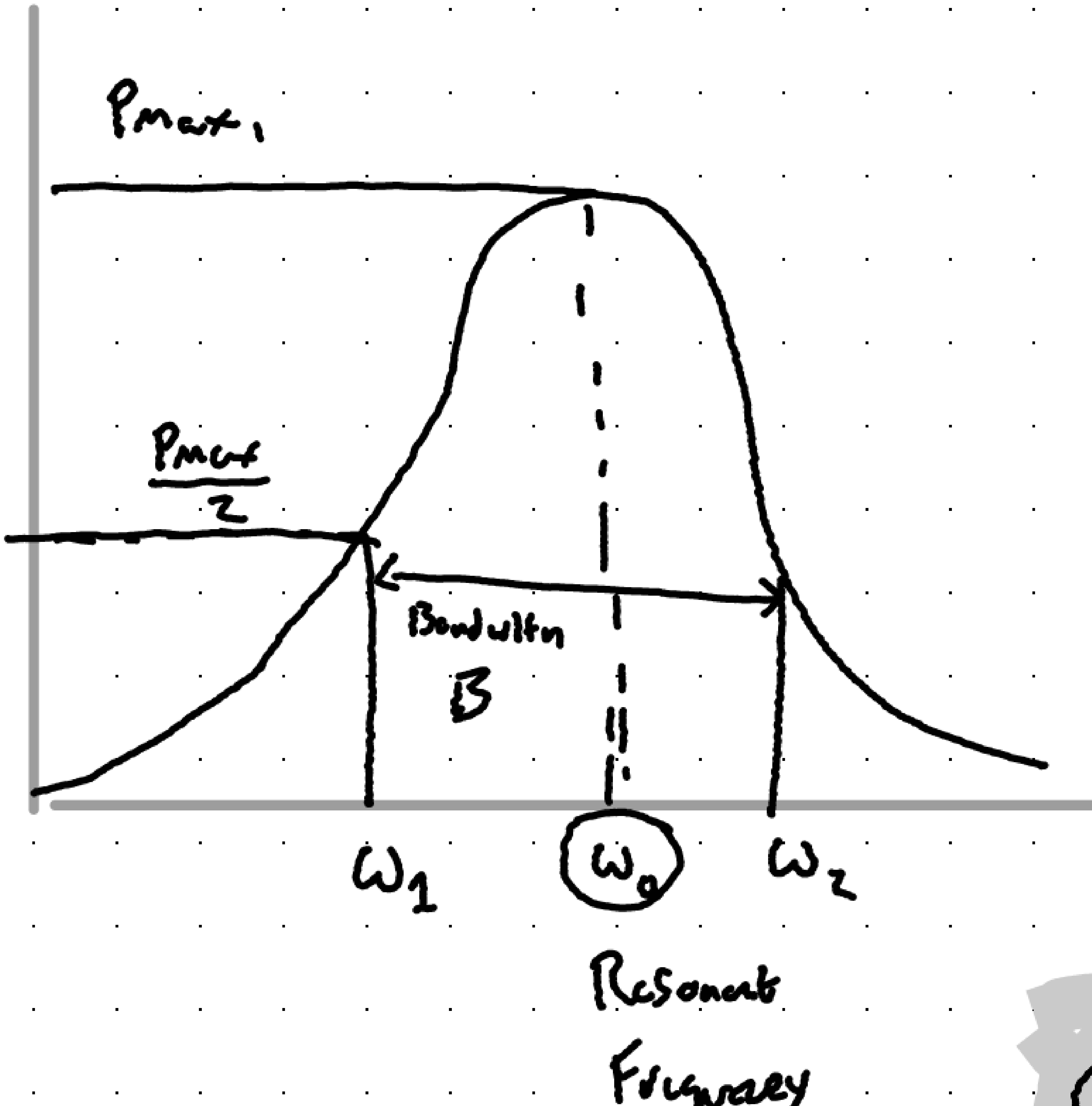
∴ At resonance, $Im(Z) = 0$

$$\omega_0^2 = \frac{1}{LC} \qquad \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

rads/sec

$$2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}}$$



$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

Quality Factor (Q)

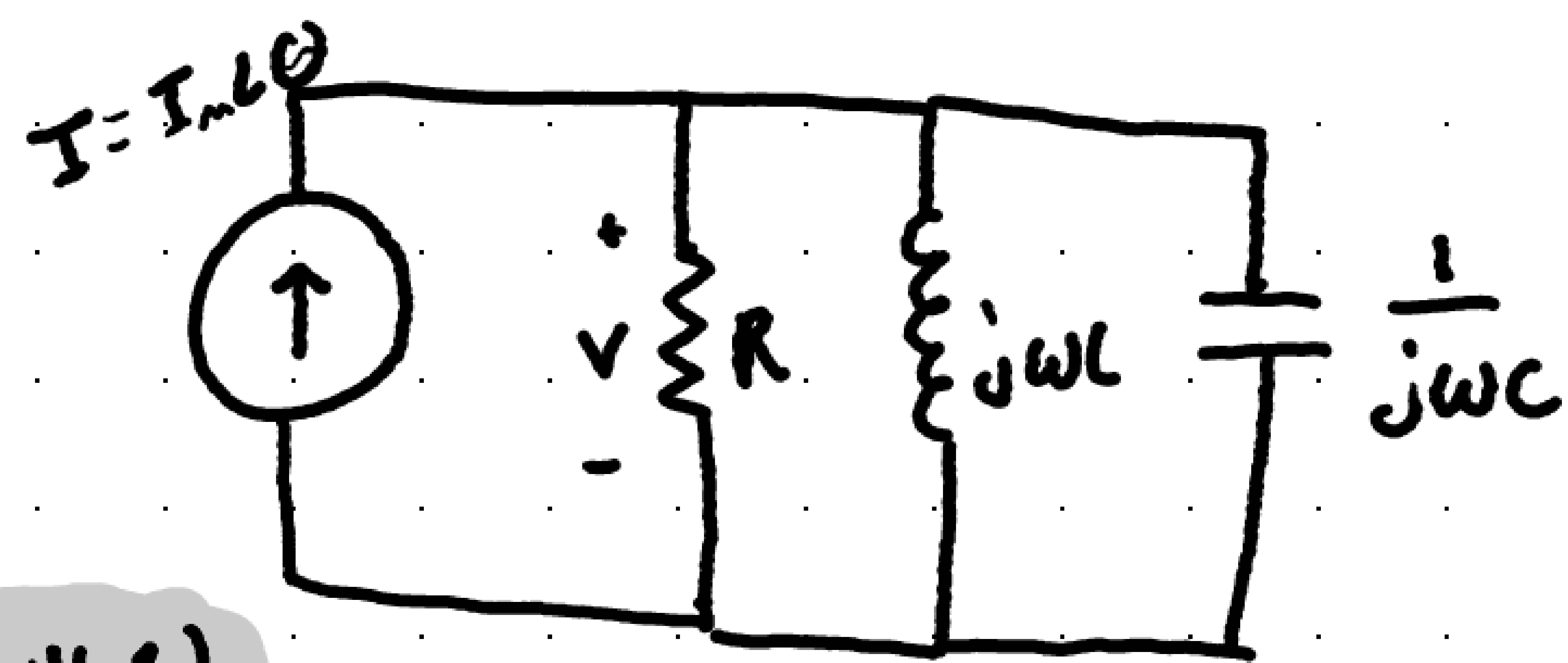
$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}}$$

$$\boxed{Q = \frac{\omega_0}{B}}$$

B = Bandwith

$$\frac{P_{max}}{2} = \frac{1}{2}\frac{V_m^2}{R}$$

Parallel Resonance



(Admittance)

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

Resonance occurs when the imaginary part of Y is **zero**.

Resonant frequency is still $\omega_0 = \frac{1}{\sqrt{LC}}$

For Parallel Resonance

Half Power (cutoff) Frequencies

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Bandwidth

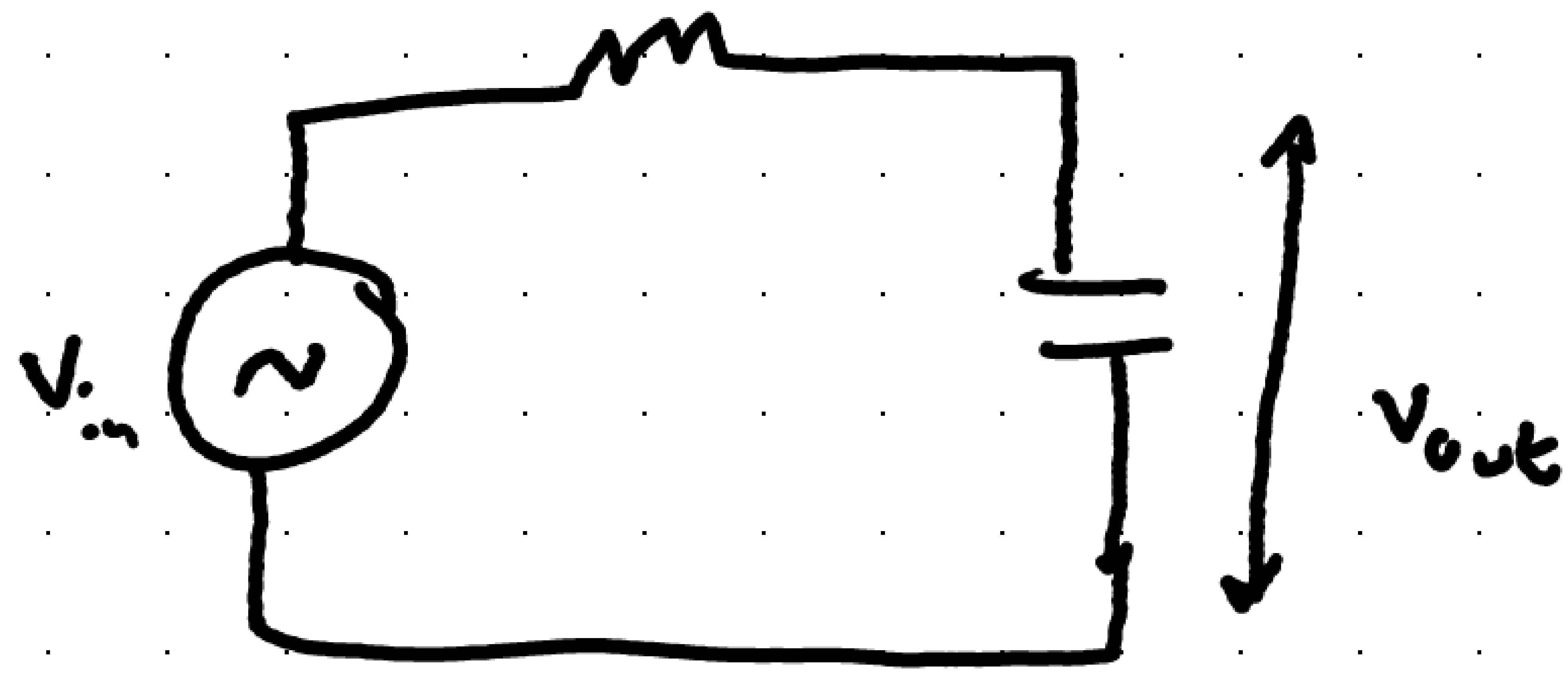
$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

Quality factor

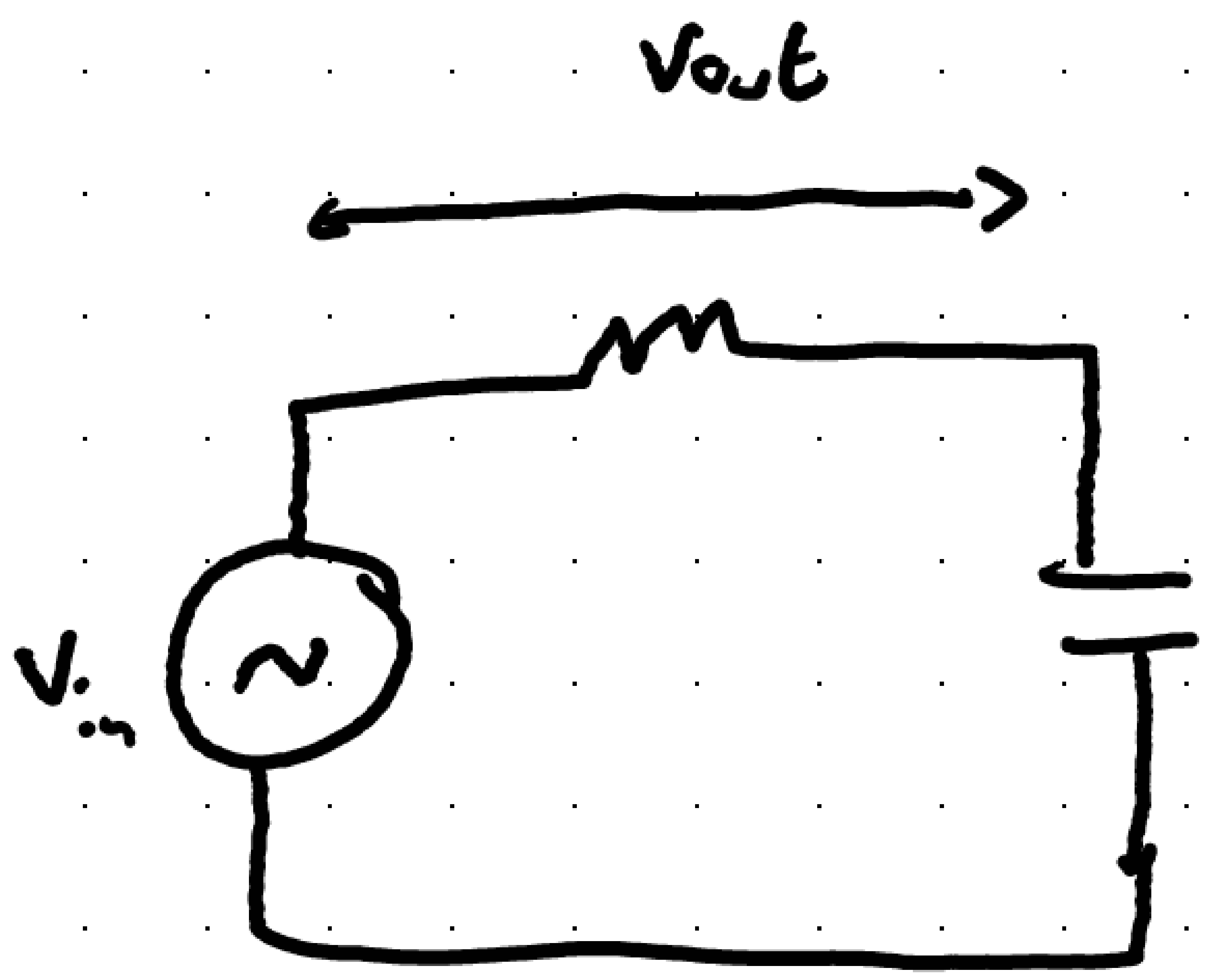
$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

Resonant Frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



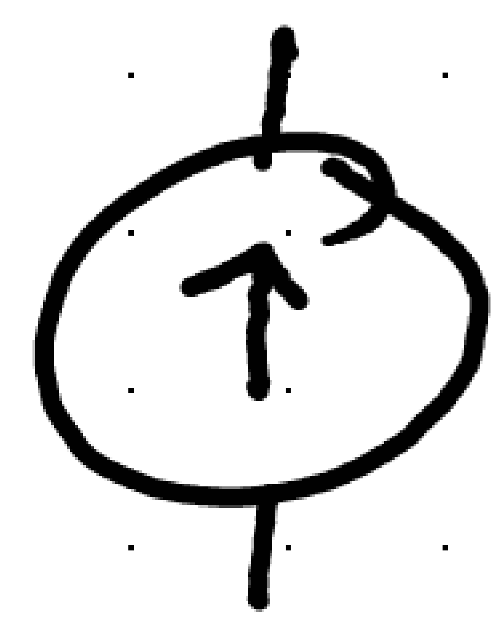
Low Pass Filter

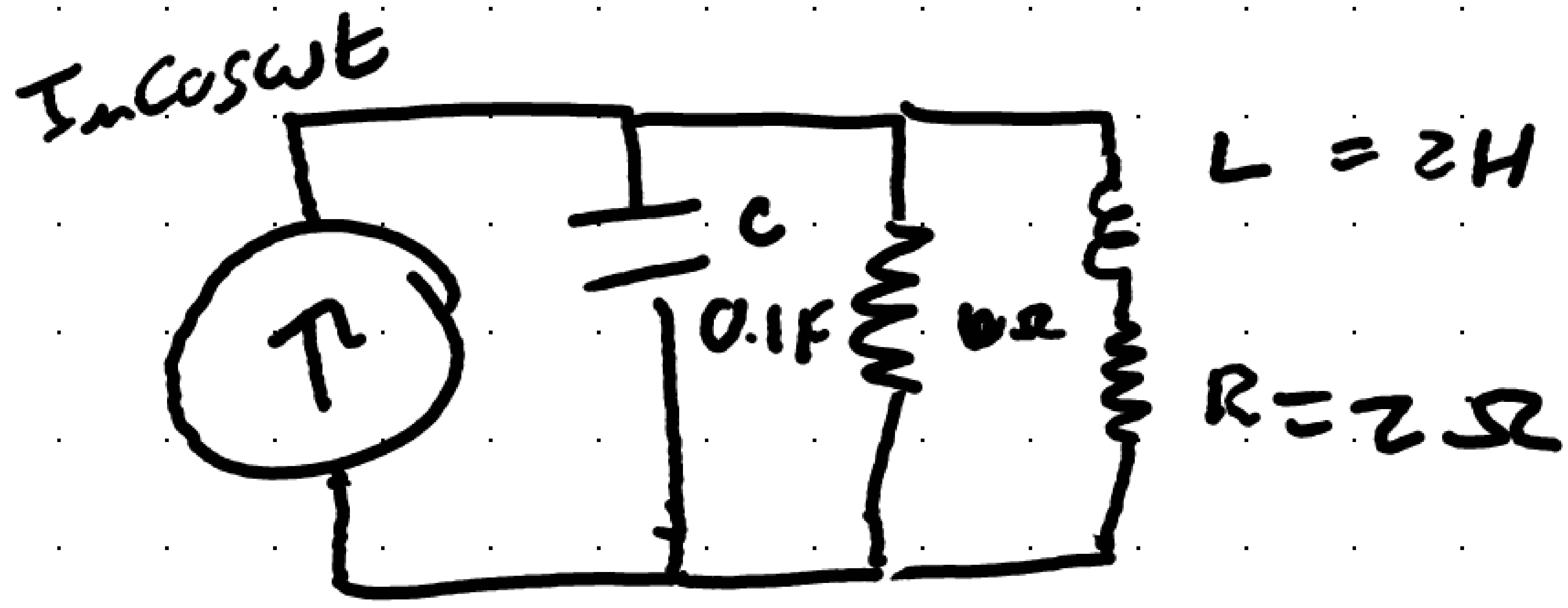


High Pass Filter

Source Transformation Formula

$$I = \frac{V}{R}$$





$$Y = \frac{1}{10} + j\omega(0.1) + \frac{1}{2 + 2j\omega}$$

$$Y = \frac{1}{10} + j\omega(0.1) + \frac{2 + 2j\omega}{4 + 4\omega^2}$$

$$Y = \underbrace{\left(0.1 + \frac{2}{4 + 4\omega^2}\right)}_{RE} + j\underbrace{\left(0.1\omega - \frac{2}{4 + 4\omega^2}\right)}_{Im}$$

The resonance occurs when $Im(Y) = 0$

$$0.1\omega_0 - \frac{2\omega_0}{4 + 4\omega_0^2} = 0$$

$$4 + 4\omega_0^2 = 20$$

$\omega_0 = 2$ rad/s