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Tighter Security for Group Key Agreement in the Random Oracle Model

Bachelor Thesis

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Abstract

TODO: How to adapt abstract? What should it contain?

The Messaging Layer Security (MLS) protocol, recently standardized in RFC 9420 [2], aims to provide efficient asynchronous group key establishment with strong security guarantees. TreeKEM is the construction underlying MLS and a variant of it was proven adaptively secure in the Ran- dom Oracle Model (ROM) with a polynomial loss in security in [1]. The proof makes use of the Generalized Selective Decryption (GSD) security game introduced in [6], adapted to the public-key setting. GSD security is closely related to the security of TreeKEM and the encryption scheme used in TreeKEM was proven to be GSD secure in the ROM under the standard assumption of IND-CPA security, implying a proof of security for TreeKEM (a sketch of this proof was provided in [1] for the TreeKEM variant).

TODO: describe results

Contents

Chapter 1

Introduction

TODO: more accessible introduction on why this is important We all rely on messaging applications like WhatsApp, Signal, etc. in our daily lives and take it for granted that our messages will be transmitted securely (**TODO:** "see it as a prerequisite" maybe better?). **TODO:** smoother transition to talking about protocols? For two parties, the Double Ratchet protocol is a common solution (**TODO:** true?) to transmit messages securely and efficiently. For more than two parties this problem was only solved recently with the MLS protocol.

The Messaging Layer Security (MLS) protocol, recently standardized in RFC 9420 [2], aims to provide efficient asynchronous group key establishment with strong security guarantees. The main component of MLS, which is the source of its important efficiency and security properties, is a protocol called TreeKEM (initially proposed in [3]). In essence, TreeKEM, as adopted from its predecessors, structures a group of users as a binary tree with the group key at the root and all group members as leaves. Group members may then compute the group key, update it or add/remove other members with a number of operations logarithmic in the group size.

As for any scheme, it is important to have formal security guarantees for TreeKEM based on precise hardness assumptions. Providing security definitions for the scheme already helps to describe exactly what assumptions are made on the capabilities of an adversary and what kind of security one should expect when using the scheme in practice. Moreover, proofs of (reasonably tight) security under these definitions serve as a guide to implementors on what values to choose for the security parameters of the scheme and provide strong justification that there are no flaws in its design. Given that a major vision for the MLS protocol is for it to be used by messaging applications and that it has support from several large companies ([4], [5]), it has the potential to be used by a huge number of users. Thus, it is important to better understand the security of MLS and hence also of TreeKEM.

One choice that can be made when defining the security of TreeKEM is whether the adversary is modeled as *selective* or *adaptive*. In the former case, the adversary must provide all the interactions it will have with the protocol and when it will attempt to break the scheme at the beginning of the security game, while in the latter case the adversary can make its decisions based on responses from previous interactions. Clearly, the adaptive setting is much closer to how an attack would unfold in practice, so it is desirable to prove security against adaptive adversaries. However, achieving this without too much of a blow-up in the security loss is a challenge since one often resorts to guessing actions performed by the adversary.

The Generalized Selective Decryption (GSD) security game ([6]) was introduced precisely to analyze adaptive security for protocols based on a graph-like structure (as is the case with TreeKEM). In [1], a variant of TreeKEM was proven adaptively secure in the Random Oracle Model (ROM) with a security loss in $\mathcal{O}((n \cdot Q)^2)$, where n is the number of users and Q the number of protocol operations performed by these users. The proof mainly relies on showing that the encryption scheme employed in TreeKEM, a slight modification of an arbitrary IND-CPA secure encryption scheme, is GSD secure in the ROM.

TODO: describe results and contribution in detail

Q: Should all necessary concepts be explained in detail in "Preliminaries" and new contributions later?

Chapter 2

Preliminaries

TODO: add security definitions and explanations

TODO: define GSD game

TODO: define IND-CPA security

TODO: explain the ROM

Chapter 3

Example Chapter

Dummy text.

3.1 Example Section

Dummy text.

3.1.1 Example Subsection

Dummy text.

Example Subsubsection

Dummy text.

Example Paragraph Dummy text.

Example Subparagraph Dummy text.

Chapter 4

Tighter GSD security

TODO: Motivate GSD

Following the general approach used in [1] to prove the security of (a variant of) TreeKEM in the ROM, we first prove a result on the GSD security of an IND-CPA secure encryption scheme. We do this specifically for the DHIES scheme. Moreover, we will make some notable modifications to the publik key GSD game defined in [1], to allow for results to be applied to TreeKEM more directly. We motivate the modifications made later in Section 5 on page 11.

4.1 Seeded GSD with Dependencies

We call our adaptation of GSD security *Seeded GSD with Dependencies* (SD-GSD).

TODO: Explain definition in words. **TODO:** Motivate restrictions.

We will refer to the following as the *SD-GSD experiment* or the *SD-GSD game* (the two are interchangeable).

Q: Where best to define things like *hidden* seeds or *corrupted* nodes?

Definition 4.1 (The SD-GSD Game/Experiment) *Let $\lambda \in \mathbb{N}$ a security parameter. **Q:** Where to define λ? Let $\Pi = (\mathrm{Gen}, \mathrm{Enc}, \mathrm{Dec})$ a public key encryption scheme such that the key space is a subset of the message space. Let $H_{\mathrm{gen}} \colon \{0,1\}^\lambda \to ?, H_{\mathrm{dep}} \colon \{0,1\}^\lambda \to \{0,1\}^\lambda$ two KDFs. Define the experiment $\mathrm{Exp}^{\mathrm{SD-GSD}}_{\mathcal{A},\Pi}$ for an adversary $\mathcal{A}$:*

1. *The adversary $\mathcal{A}$ outputs $n \in \mathbb{N}$ and a list of dependencies $D = \{(a_i, b_i)\}_{i=1}^m \in [n]^2$. For each $v \in [n]$:*

   (i) • ***Case $v = b_i$ for some i (v is the target of some dependency):*** *set $s_v = H_{\mathrm{dep}}(s_{a_i})$.*

- ***Otherwise:*** *sample* $s_v \leftarrow \{0,1\}^\lambda$.

We call s_v *the* seed *of the node* v.

(ii) Compute $(sk_v, pk_v) = \mathrm{Gen}(H_{\mathrm{gen}}(s_v))$.

Set $\mathcal{C} = E = \varnothing$. *We call the graph* $([n], E)$ *a* GSD graph *of* size n.

2. $\mathcal{A}$ *may adaptively do the following queries:*
   - reveal*: Given* $v \in [n]$, $\mathcal{A}$ *is given* pk_v.
   - encrypt*: Given two distinct nodes* $u, v \in [n]$, (u, v) *is added to* E *and* $\mathcal{A}$ *is given* $c \leftarrow \mathrm{Enc}_{pk_u}(s_v)$.
   - corrupt*: Given* $v \in [n]$, $\mathcal{A}$ *is given* s_v *and* v *is added to* $\mathcal{C}$. *We call such a node* $v \in \mathcal{C}$ corrupted. *The remaining nodes are* safe *and we say their seeds are* hidden *(even if a corrupted node happens to have the same seed).*
3. $\mathcal{A}$ *outputs a node* $v \in [n]$. *We call* v *the* challenge node. *A bit* $b \leftarrow \{0,1\}$ *is sampled and* $\mathcal{A}$ *is given*

$$r = \begin{cases} H_{\mathrm{dep}}(s_v) & b = 0 \\ s & b = 1 \end{cases},$$

   where $s \leftarrow \{0,1\}^\lambda$. $\mathcal{A}$ *may continue to do queries as before.*
4. $\mathcal{A}$ *outputs a bit* b'. *The output of the experiment is defined to be* 1 *if* $b' = b$, *and* 0 *otherwise.*

During execution of the above experiment, we require that the adversary adhere to the following:

- *The challenge node always remains sink.*
- *The challenge node is never reachable from any corrupted node in* $(V, E \cup D)$.
- reveal *is never queried on the challenge node.*
- *The graphs* (V, E) *and* (V, D) *always remain acyclic and without self-loops.*
- *All paths in the graph* (V, D) *are vertex disjoint.*

We call an adversary for the above experiment that adheres to the above rules a SD-GSD adversary.

Since we are only interested in the security of the SD-GSD game for the case where H_{gen} and H_{dep} are random oracles, we directly assume in our definition that the KDFs are modelled as such.

Definition 4.2 (SD-GSD security in the ROM) *A public key encryption scheme* Π *is* $(t, \varepsilon, N, \delta)$-SD-GSD secure *if for any SD-GSD adversary* $\mathcal{A}$ *constructing a*

GSD graph of size at most N and indegree at most δ and running in t time we have

$$\mathrm{Adv}^{\mathrm{SD-GSD}}_{\Pi}(\mathcal{A}) \coloneqq 2 \cdot \left| \Pr\left[\mathrm{Exp}^{\mathrm{SD-GSD}}_{\mathcal{A},\Pi} = 1\right] - \frac{1}{2} \right| \leq \varepsilon$$

when H_{gen} and H_{dep} are random oracles.

4.2 Proving SD-GSD security for DHIES

Theorem 4.3 *Let $N, \delta \in \mathbb{N}$ arbitrary. Let Π_{DH} the DHIES scheme instantiated with a symmetric key encryption scheme Π_s. Let H_{DH} the KDF and $\mathbb{G}$ the group used in Π_{DH}. If Π_s is (t, ε)-EAV secure, the DDH problem is (t, ε)-hard in $\mathbb{G}$ and H_{DH} is modelled as a random oracle, then Π_{DH} is $(\tilde{t}, \tilde{\varepsilon}, N, \delta)$-SD-GSD secure.*

Proof Let $\mathcal{A}$ an arbitrary SD-GSD adversary running in time $\tilde{t}$. For an execution of $\mathrm{Exp}^{\mathrm{SD-GSD}}_{\mathcal{A},\Pi_{\mathrm{DH}}}$, we call the seed s_v of a node v in the GSD graph *hidden*, if v is not reachable by any node in $\mathcal{C}$. Additionally, we say "$\mathcal{A}$ *wins*" to denote the event $\mathrm{Exp}^{\mathrm{SD-GSD}}_{\mathcal{A},\Pi_{\mathrm{DH}}} = 1$.

As usual with random oracles we proceed by a case distinction on whether they were queried on some interesting value. Accordingly, let Q_x denote the event that $\mathcal{A}$ queries H_x on a hidden seed for $x \in \{\mathrm{gen}, \mathrm{dep}\}$. (**Q:** What if corrupted seed is queried and it happens to coincide with a hidden seed?) Then we can write

$$\begin{aligned}
\Pr[\mathcal{A} \text{ wins}] &= \Pr\left[\mathcal{A} \text{ wins} \wedge Q_{\mathrm{dep}}\right] + \Pr\left[\mathcal{A} \text{ wins} \wedge \overline{Q_{\mathrm{dep}}}\right] \\
&\overset{(*)}{=} \Pr\left[\mathcal{A} \text{ wins} \wedge Q_{\mathrm{dep}}\right] + \frac{1}{2} \\
&\leq \Pr[Q_s] + \frac{1}{2},
\end{aligned}$$

where $Q_s \coloneqq Q_{\mathrm{gen}} \cup Q_{\mathrm{dep}}$ (s for *seed*).

TODO: Justify (*). (And perhaps name it better?)

The heart of the proof is to bound $\Pr[Q_s]$. When the adversary first triggers Q_s by querying the seed of some node v, it can only have learned the seed through encryptions $c_1 \leftarrow \Pi_{\mathrm{DH}}.\mathrm{Enc}_{pk_{u_1}}(s_v), \ldots, c_d \leftarrow \Pi_{\mathrm{DH}}.\mathrm{Enc}_{pk_{u_d}}(s_v)$ where $(u_1, v), \ldots, (u_d, v)$ are edges in the GSD graph. The proof in [1] simply argued that this is not too likely if these encryptions were made with an IND-CPA secure scheme. In the context of the DHIES scheme we can say more about these encryptions and achieve a better reduction loss. Let $x_i = \log_g(pk_{u_i})$. Each encryption c_i is a tuple of the form $\langle g^{y_i}, \Pi_s.\mathrm{Enc}_{k_i}(s_v)\rangle$ where $k_i = H_{\mathrm{DH}}(g^{x_i \cdot y_i})$. Now we can again do a case distinction on whether H_{DH} was queried for some group element $g^{x_i \cdot y_i}$ or not.

- If such a query was made, then $\mathcal{A}$ solved the DH challenge (g^{x_i}, g^{y_i}). (Remember that we assumed that v is the first node for which Q_s is triggered and if the seed of v is hidden, then so are the seeds of the nodes u_i. Thus the adversary has not yet queried $H_{\text{gen}}(s_{u_i})$ and doesn't know any of the exponents x_i.)
- If no such query was made, then from $\mathcal{A}$'s perspective all the k_i are independent, uniformly random keys and it still was able to learn s_v from the EAV secure encryptions $\Pi_s.\text{Enc}_{k_1}(s_v), \ldots, \Pi_s.\text{Enc}_{k_d}(s_v)$.

We can bound the probability of either of these events occuring using hardness of the DDH problem in $\mathbb{G}$ and EAV security of Π_s, respectively.

To this end, we call a group element $k \in \mathbb{G}$ a *hidden DH key* if $k = pk_u^{y_{u,v}}$, where (u, v) is an edge in the GSD graph, u is not reachable from any node and $y_{u,v}$ is the exponent chosen in the DHIES encryption of s_v (i.e. $\mathcal{A}$ was given a ciphertext of the form $\langle g^{y_{u,v}}, \ldots \rangle$ when it queried encrypt(u, v)). Now analogously to above let Q_{DH} the event that $\mathcal{A}$ queries H_{DH} on a hidden DH key and let F_{DH} the event that $\mathcal{A}$ triggers Q_{DH} before having triggered Q_s. Then we have

$$\Pr[Q_s] = \Pr[Q_s \wedge F_{\text{DH}}] + \Pr\left[Q_s \wedge \overline{F_{\text{DH}}}\right].$$

We show in Lemma 4.4 that

$$\Pr[Q_s \wedge F_{\text{DH}}] \leq \ldots$$

and in Lemma 4.5 that

$$\Pr\left[Q_s \wedge \overline{F_{\text{DH}}}\right] \leq \ldots$$

Then

$$\Pr[\mathcal{A} \text{ wins}] \leq x + \frac{1}{2},$$

so

$$\text{Adv}_{\Pi}^{\text{SD}-\text{GSD}}(\mathcal{A}) \leq 2 \cdot |x| = \tilde{\varepsilon}.$$

TODO: How to argue about $\tilde{t}$? □

Lemma 4.4 *Let $\mathcal{A}$ an SD-GSD adversary. Let $\Pi_{\text{DH}}, H_{\text{DH}}, \mathbb{G}$ and the events $Q_s, Q_{\text{DH}}, F_{\text{DH}}$ as in the statement and proof of Theorem 4.3 on the previous page and assume that the DDH problem is (t, ε)-hard in $\mathbb{G}$. Then*

$$\Pr[Q_s \wedge F_{\text{DH}}] \leq \ldots.$$

Proof

Lemma 4.5 *Let $\mathcal{A}$ an SD-GSD adversary. Let $\Pi_{\text{DH}}, H_{\text{DH}}, \ldots$ and the events $Q_s, Q_{\text{DH}}, F_{\text{DH}}, \ldots$ as in the statement and proof of Theorem 4.3 on the preceding page and assume that Π_s is (t, ε)-EAV secure. Then*

$$\Pr\left[Q_s \wedge \overline{F_{\text{DH}}}\right] \leq \ldots.$$

Proof

Chapter 5

Application to TreeKEM

TODO: Reduce TreeKEM security to GSD security.

Appendix A

Dummy Appendix

You can defer lengthy calculations that would otherwise only interrupt the flow of your thesis to an appendix.

Bibliography

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