# Lecture 6

1 Gain Saturation

- For oscillation, we have to satisfy the phase condition ($\nu = \frac{qc}{2L}$), so only longitudinal modes are allowed. Also, we have to satisfy the gain condition ($\gamma > \alpha_s + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$).

- During operation, we lose some of the population inversion and the gain curve moves down until it reaches steady state. At steady state, only the modes that satisfy the gain condition will oscillate.

- Settling means that the gain is equal to α_r at the oscillating frequency.
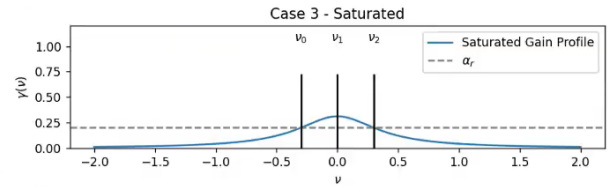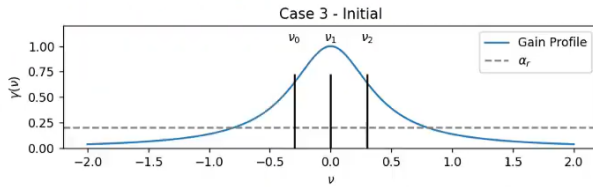
1.1 Homogeneous Broadening

If the center of the Lorentzian (ν_o) coincides with one of the longitudinal modes, only ν_o will oscillate.
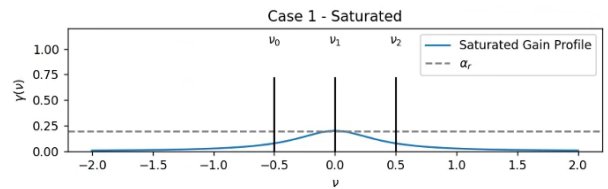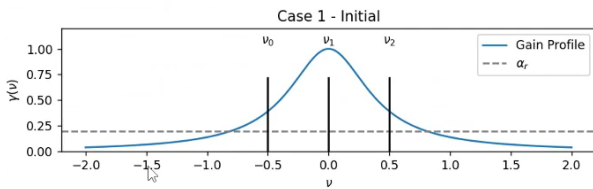
If the center of the Lorentzian (ν_o) does not coincide with one of the longitudinal modes AND the longitudinal modes are not symmetrically distributed around ν_o, the closest mode to ν_o will oscillate.
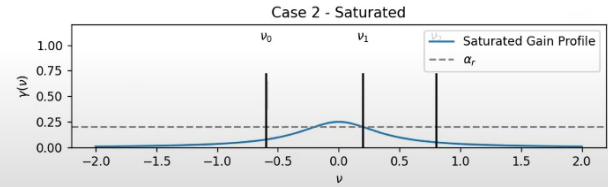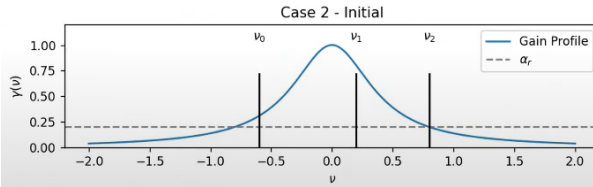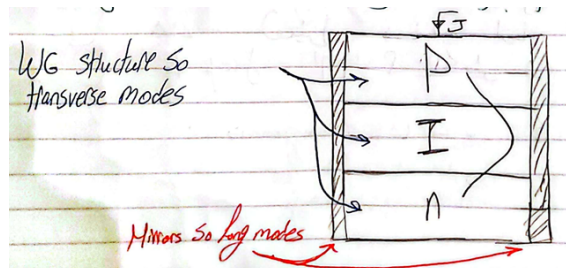

If the center of the Lorentzian (ν_o) does not coincide with one of the longitudinal modes AND the longitudinal modes are symmetrically distributed around ν_o, both modes around ν_o will oscillate.


Note: In semiconductor lasers, PIN has a waveguide structure, so there are transverse modes as well.

1.2 Inhomogeneous Broadening

Recall that each group of atoms has a Lorentzian gain curve, but overall the gain curve is Gaussian. As population inversion decreases, the gain curve moves down for each group of atoms at the longitudinal modes, so we notice some dips in the overall Gaussian gain curve.
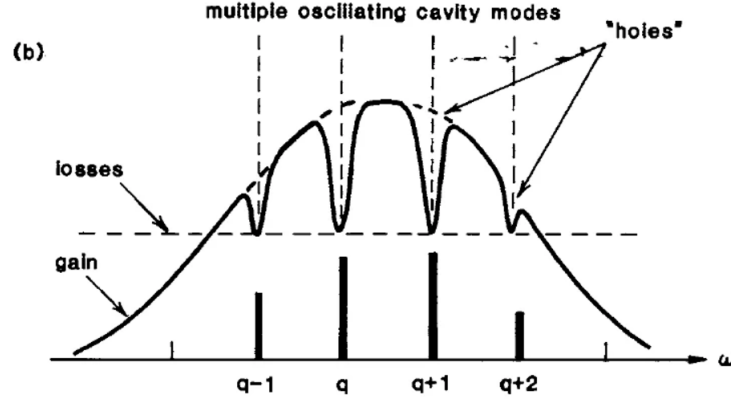
- For inhomogenous broadening, all longitudinal modes that initially satisfy the gain condition will oscillate.

- For inhomogenous broadening, the number of oscillating modes is $\frac{B}{\nu_F}$, where B is the gain bandwidth (where $\gamma > \alpha_r$) and ν_F is the free spectral range.

- The output power for each resonnaing mode is different which leads to mode competition.

- Inhomogeneous broadening oscillate at multiple frequencies even if ν_o coincides with a longitudinal mode.

- For ideal LASER, the output frequency, amplitude, and phase are constant.

1.3 Space Dependance

- Inside the LASER cavity, we have a standing wave, so at certain points we have ϕ_{max}, so gain saturation occurs, while at other points we have ϕ_{min} and gain saturation does not occur. This is called spatial hole burning.

- $\gamma = \frac{\sigma N_o}{1 + \frac{\phi}{\phi_s}}$, where ϕ is a function in space.

- If $R_1 = R_2 = 1$, the standing wave is from 0 to ϕ_{max}.

- If $R < 1$, the standing wave is between ϕ_{min} to ϕ_{max}.

- The problem with spatial hole burning is that the points with ϕ_{min} will never change when changing ϕ_{in}, and no gain saturation will occur at these points, so multiple modes will oscillate (even in ideal homogeneous broadening).

- Note that the longitudinal modes are not a perfect impulse and have bandwidth $(\nu_q + \Delta\nu)$ due to cavity losses.

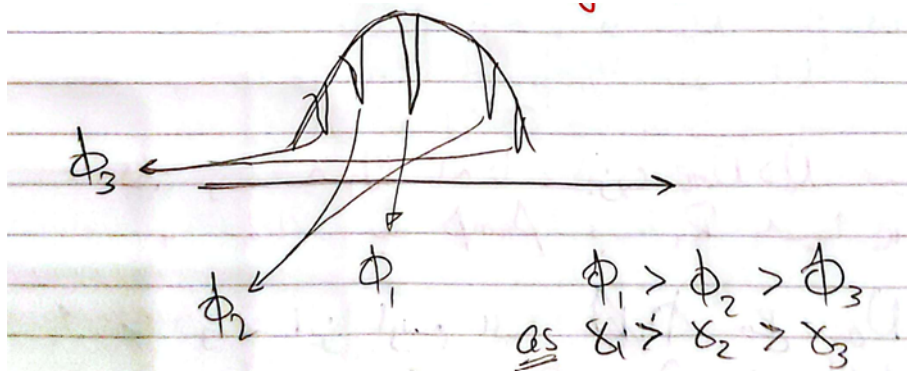- Therefore, the output is not monochromatic even if we do not have spatial hole burning.

2 LASER Oscillator

To get oscillation $\rightarrow \gamma = \alpha_r$:

$$\frac{\sigma N_o}{1 + \frac{\phi}{\phi_s}} = \alpha_r \rightarrow \frac{\phi}{\phi_s} = \frac{\sigma N_o}{\alpha_r} - 1$$

$$\phi = \phi_s \left(\frac{\sigma N_o}{\alpha_r} - 1 \right) = \phi_s \left(\frac{\gamma_o}{\alpha_r} - 1 \right)$$

As gain decreases, ϕ decreases, which explains why in inhomogenous broadening, the frequencies with small $g(\nu)$ will have small ϕ.

Notes:

- If $\gamma_o \gg \alpha_r$, the line has large power, so output power depends on the difference between γ_o and α_r.

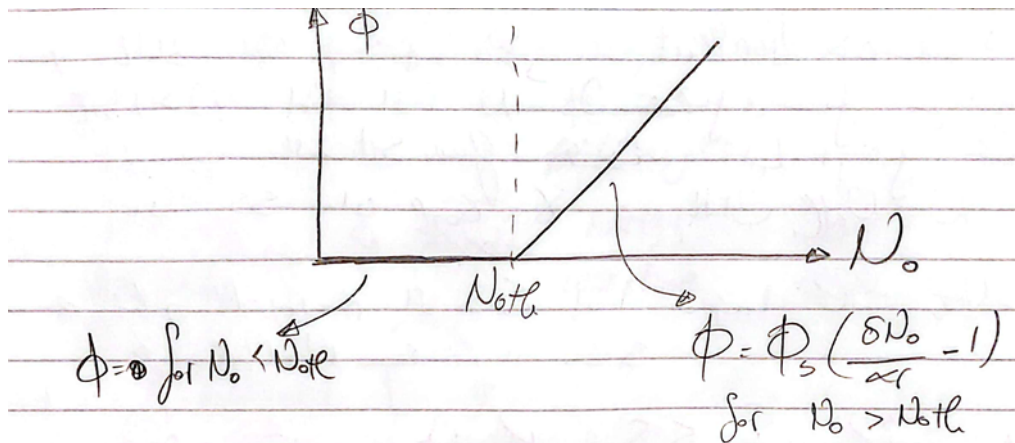- We have to pump till $\gamma > \alpha_r$, as nonlinearity will decrease γ until it reaches α_r.

Threshold condition: $\gamma_{o_{th}} = \alpha_r$:

$$\sigma N_o = \alpha_r \rightarrow N_{o_{th}} = \frac{\alpha_r}{\sigma}$$

$N_{o_{th}}$ indicates R_{th} as there is a relation between them obtained from the rate equations.
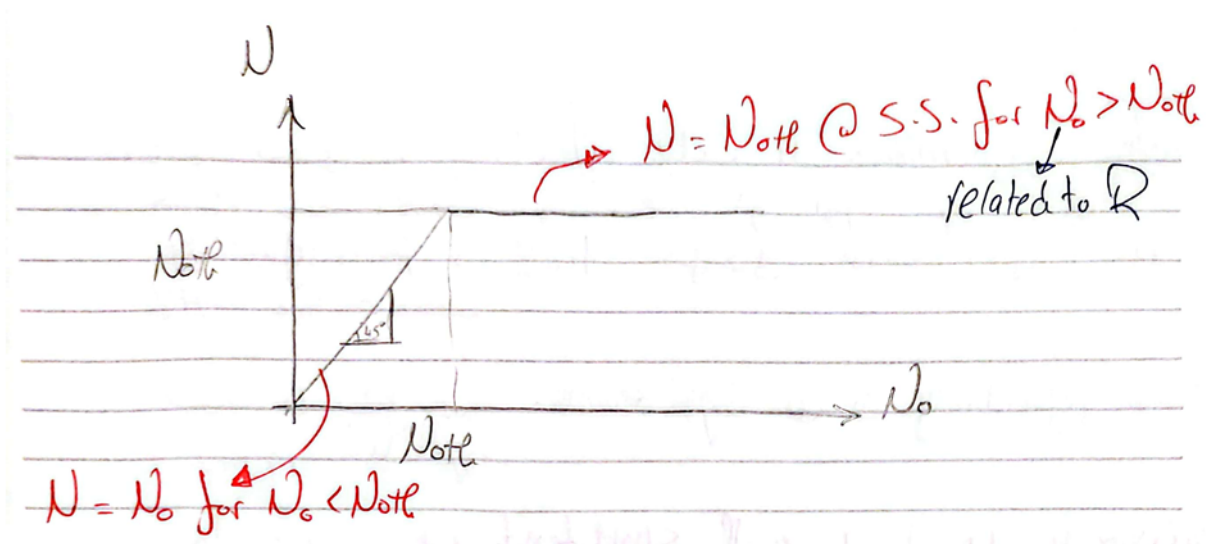
To draw ϕ vs N_o:

- If $N_o < N_{o_{th}} \rightarrow \gamma_o < \alpha_r$, no oscillation and LASER output is zero (ignoring spontaneous emission).

- If $N_o = N_{o_{th}}$, oscillation starts and $\phi = \phi_s \left(\frac{\sigma N_o}{\alpha_r} - 1 \right)$.

To draw N vs N_o:

- If $N_o < N_{o_{th}}$, there is no oscillation, so there is no effect on N, so $N = N_o$.

- If $N_o > N_{o_{th}}$, there is oscillation. From transient POV, N should decrease due to gain saturation till it reaches $N_{o_{th}}$, so from steady state POV, $N = N_{o_{th}}$.

The figure shows a plot with N on the vertical axis and N_o on the horizontal axis. Annotations read:

$N = N_{oth} \text{ @ S.S. for } N_o > N_{oth}$ (related to R)

$N = N_o \text{ for } N_o < N_{oth}$

with a $45°$ line and N_{oth} marked on both axes.

2.1 Power Analysis

$$\phi = \frac{\# \text{ atoms}}{\text{area . time}} \rightarrow \phi h\nu = \frac{\text{energy}}{\text{area . time}} = \frac{\text{power}}{\text{area}} = I$$

$$I_{out} = \phi h\nu T$$

where T is the power reflection coefficient of the output mirror.

Notes:

- LASER is a gain medium between 2 mirrors, so we make 1 of the mirrors fully reflective and the other partially reflective to obtain output.

- We usually select mirrors with a very close power reflectivity (100% and 97%). This helps us make this approximation: $\phi^+ \approx \phi^- \approx \frac{\phi}{2}$.
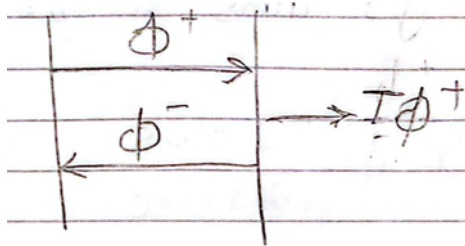
$$I_{out} = \frac{T}{2}\phi h\nu = \frac{T}{2}\phi_s \left(\frac{\sigma N_o}{\alpha_r} - 1 \right) h\nu$$

Notes:

- If $R_1 = R_2 \rightarrow \phi^+ = \phi^-$, so we have a pure standing wave. If $R_1 \neq R_2 \rightarrow \phi^+ \neq \phi^-$, so we have both standing and travelling waves.

- As R increases, α_r decreases, so it is better to use mirrors with high reflectivity to have a smaller threshold.

- However, if reflectivity increases, transmission decreases, so we have a smaller output power.

- There is a trade-off between decreasing cavity losses and increasing output power.

- To get the optimum point, we will differentiate I_{out} with respect to T and set it to zero.

$$\alpha_r = \alpha_s + \frac{1}{2L} \ln \left(\frac{1}{R_1 R_2} \right)$$

Let $R_1 = R_2 = R = 1 - T$:

$$\alpha_r = \alpha_s - \frac{1}{2L} \ln(1 - T)$$

$ln(1 + x) \approx x$ if $x << 1$:

$$\alpha_r = \alpha_s - \frac{1}{2L}(-T) = \frac{2\alpha_s L + T}{2L}$$

So:

$$I_{out} = \frac{h\nu\phi_s}{2} \left(\frac{\gamma_o}{\alpha_r} - 1 \right) T$$
$$= \frac{h\nu\phi_s}{2} \left(\frac{2\gamma_o L}{2\alpha_s L + T} - 1 \right) T$$

Let $g_o = 2\gamma_o L$ and $\mathscr{L} = 2\alpha_s L$:

$$I_{out} = \frac{h\nu\phi_s}{2} \left(\frac{g_o T}{\mathscr{L} + T} - T \right)$$

Differentiate I_{out} with respect to T and set it to zero:

$$\frac{\partial I_{out}}{\partial T} = \frac{h\nu\phi_s}{2} \left[\frac{(\mathscr{L} + T)g_o - g_o T}{(\mathscr{L} + T)^2} - 1 \right]$$
$$= \frac{h\nu\phi_s}{2} \left[\frac{\mathscr{L} g_o}{(\mathscr{L} + T)^2} \right] = 0$$

So:

$$\mathscr{L} g_o = (\mathscr{L} + T)^2 \Rightarrow T = \sqrt{\mathscr{L} g_o} - \mathscr{L}$$