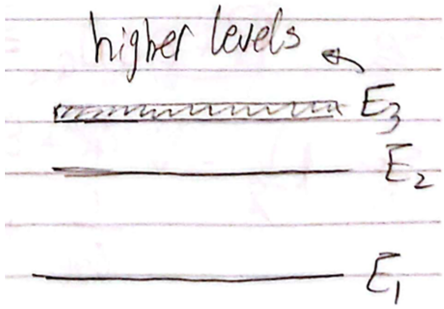
Lecture 5

1 3-Level Laser

- E_1 is the ground state, and E_3 represents all upper states.
- Atoms are pumped from E_1 to E_3, but τ_3 is very short so the atoms quickly decay to E_2, so we can consider that we pump from E_1 to E_2.



1.1 No Input Signal or Small Signal

$$\frac{dN_2}{dt} = R - \frac{N_{21}}{\tau_2}$$

At steady state, $\frac{dN_2}{dt} = 0$, so

$$N_2 = R\tau_{21}$$

If we use $\frac{dN_1}{dt}$, we will get the same result, so we use $N_a = N_1 + N_2$. To get population inversion:

$$\begin{aligned} N_2 - N_1 &= N_2 - (N_a - N_2) = 2N_2 - N_a \\ &= 2R\tau_{21} - N_a \end{aligned}$$

For population inversion, $N_2 > N_1$, so

$$R > \frac{N_a}{2\tau_{21}}$$

1.2 Large Input Signal

Assume steady state, so:

$$\begin{aligned} \frac{dN_2}{dt} &= R - \frac{N_2}{\tau_{21}} - \sigma\phi_\nu N_2 + \sigma\phi_\nu N_1 = 0 \\ \frac{dN_1}{dt} &= -R + \frac{N_2}{\tau_{21}} + \sigma\phi_\nu N_2 - \sigma\phi_\nu N_1 = 0 \end{aligned}$$

Using $N_1 = N_a - N_2$:

$$\begin{aligned} R + \sigma\phi_\nu N_a &= \frac{N_2}{\tau_{21}} + 2\sigma\phi_\nu N_2 \\ \Rightarrow N_2 &= \frac{R + \sigma\phi_\nu N_a}{\frac{1}{\tau_{21}} + 2\sigma\phi_\nu} \end{aligned}$$

population inversion ($N = N_2 - N_1 = 2N_2 - N_a$):

$$\begin{aligned} N &= \frac{2R + 2\sigma\phi_\nu N_a}{\frac{1}{\tau_{21}} + 2\sigma\phi_\nu} - N_a \\ &= \frac{2R - \frac{N_a}{\tau_{21}}}{\frac{1}{\tau_2} + 2\sigma\phi_\nu} = \frac{2R\tau_{21} - N_a}{1 + 2\sigma\phi_\nu\tau_{21}} \\ &= \frac{N_o}{1 + \frac{\phi}{\phi_s}} \end{aligned}$$

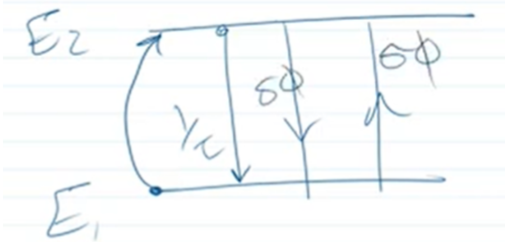
where $N_o = 2R\tau_{21} - N_a$ and $\phi_s = \frac{1}{2\sigma\tau_{21}}$.

Since E_1 is the ground level, it has a large population, so to achieve population inversion, we use very large power which is inefficient. However, in 4-level lasers, both E_1 and E_2 have small number of atoms, so pumping is more efficient.

2 2-Level Laser

- Note that in 3-level & 4-level lasers, we pump from ground level to upper level (E_3), so we only consider spontaneous emission during pumping. While the input signal has energy $\Delta E = E_2 - E_1$ so we consider all light matter interaction.

- In 2-level lasers, we only have 2 levels, so during pumping, we have to consider all light matter interaction (spontaneous emission + stimulated emission + absorption).



2.1 No Input Signal or Small Signal

We can replace the pumping rate R by the absorption rate. We also have stimulated emission resulted from the light matter interaction:

$$\frac{dN_2}{dt} = -\frac{N_{21}}{\tau_2} - \sigma\phi_\nu N_2 + \sigma\phi_\nu(N_a - N_2) = 0$$
$$\Rightarrow N_2 = \frac{\sigma\phi_\nu N_a}{\frac{1}{\tau_2} + 2\sigma\phi_\nu}$$

Population inversion:

$$N = \frac{2\sigma\phi_\nu N_a}{\frac{1}{\tau_{21}} + 2\sigma\phi_\nu} - N_a$$
$$= \frac{-\frac{N_a}{\tau_{21}}}{\frac{1}{\tau_2} + 2\sigma\phi_\nu} < 0$$

We can never make a laser system with 2 levels when using optical pumping.

3 Introduction to Semiconductor Lasers

3.1 Travelling Wave Amplifier (TWA)

- We pump the semiconductor with current till we reach transparency current I_{tr}.
- Trancparency current is the current at which the input power is equal to the output power.
- We put anti-reflection coating on the front and back of the semiconductor to prevent reflection which prevents oscillation.
- We can consider it as an open-loop amplifier.
- Small gain but high BW

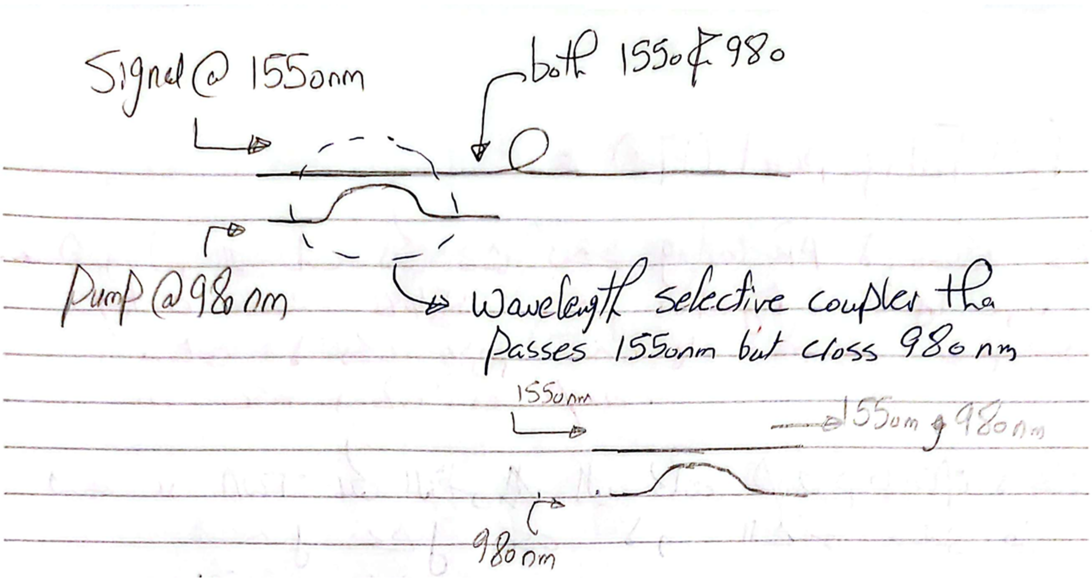
3.2 Fabry-Perot Laser (FP)

- We put a mirror on the front and back of the semiconductor.
- We can consider it as a positive feedback amplifier.

- Adjusting the mirrors (reflections) will determine wether the laser will oscillate or not (amplifier or LASER).
- High gain but low BW

3.3 Erbium Doped Fiber Amplifier (EDFA)

- Regular fiber doped by Erbium, so it is a 3-level laser.
- The input signal is at 1550 nm, and the pump is at 980 nm.
- We use a directional coupler to combine the input signal and the pump.



4 Amplifier Noise

- Spontaneous emission is the main source of noise in amplifiers.
- Note that 1 atom emits 1 photon

Rate per atom:

$$\frac{1}{\tau_{sp}} g_{\nu_o}(\nu)\Delta\nu$$

Number of atoms per second per unit volume:

$$\frac{1}{\tau_{sp}} g_{\nu_o}(\nu)\Delta\nu N_2$$

Energy of spontaneous emission per unit volume per unit time:

$$\frac{1}{\tau_{sp}} g_{\nu_o}(\nu)\Delta\nu N_2 h\nu$$

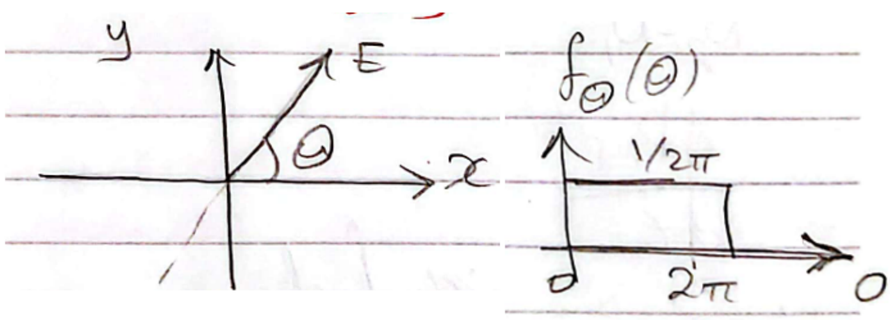
Spontaneous emission is in all directions in a Gaussian shape with a solid angle of $\frac{\lambda^2}{\text{area}}$. However, we are interested in the direction of the input signal, so:

$$\text{volume} * \frac{\text{LASER solid angle}}{4\pi} = \cancel{\text{area}}\Delta z \left(\frac{\lambda^2}{4\pi\cancel{\text{area}}}\right)$$

Spontaneous power in the beam direction:

$$dP_{ASE} = \frac{1}{\tau_{sp}} g_{\nu_o}(\nu)\Delta\nu N_2 h\nu\Delta z \left(\frac{\lambda^2}{4\pi}\right)$$

$E_x = Ecos(\theta)$ and $I_x \propto E^2 cos(\theta)^2$. For unpolarized beam, the polarization can be in any direction, so we need to take the average:



$$< I_x >= I_o \int_0^{2\pi} \frac{1}{2\pi} \cos^2 \theta d\theta = \frac{I_o}{2}$$

Only half the power couples the x-component, so we need to multiply by half:

$$dP_{ASE} = \frac{1}{\tau_{sp}} g_{\nu_o}(\nu) \Delta\nu N_2 h\nu \Delta z \left(\frac{\lambda^2}{4\pi} \right) \frac{1}{2}$$

$\sigma = \frac{\lambda^2}{8\pi\tau_{sp}} g_{\nu_o}(\nu)$ and $N_2 = N_2 \frac{N_2 - N_1}{N_2 - N_1}$, so:

$$\frac{dP_{ASE}}{dz} = \sigma N_2 \frac{N_2 - N_1}{N_2 - N_1} h\nu \Delta\nu$$

$\gamma = \sigma(N_2 - N_1)$:

$$\frac{dP_{ASE}}{dz} = \gamma \frac{N_2}{N_2 - N_1} h\nu \Delta\nu$$

Using spontaneous emission factor $n_{sp} = \frac{N_2}{N_2 - N_1}$:

$$\frac{dP_{ASE}}{dz} = \gamma n_{sp} h\nu \Delta\nu$$

The noise spectrum is very wide, so we pass it through an optical filter of bandwidth B_o so only inband noise is passed.

$$\frac{dP_{ASE}}{dz} = \gamma n_{sp} h\nu B_o$$

Notes:

- Net stimulated emission is proportional with $N_2 - N_1$, but net spontaneous emission is proportional with N_2. Thus $n_{sp} = \frac{N_2}{N_2 - N_1}$ is named the spontaneous emission factor.
- Since $N_2 > N_2 - N_1$, spontaneous emission is dominant, but since it is random, it is called noise.
- Spontaneous emission can happen between any 2 levels, unlike stimulated emission which only happens between E_2 and E_1 from the input signal. Therefore, spontaneous emission has a much larger bandwidth than stimulated emission.

$$\left. \frac{dP}{dz} \right|_{\text{total}} = \gamma P + \gamma n_{sp} h\nu B_o$$

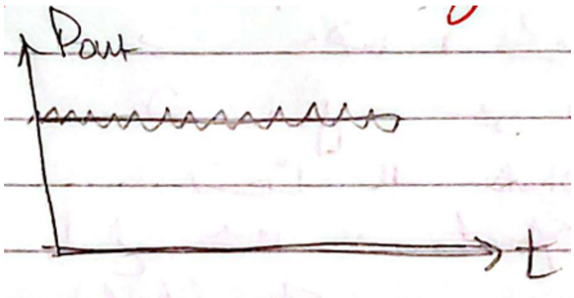
Solving the differential equation assuming $\gamma = \gamma_o$ (s.s gain) and neglecting the gain saturation:

$$P_{out} = G_o P_{in} + n_{sp} h\nu B_o (G_o - 1)$$

Notes:

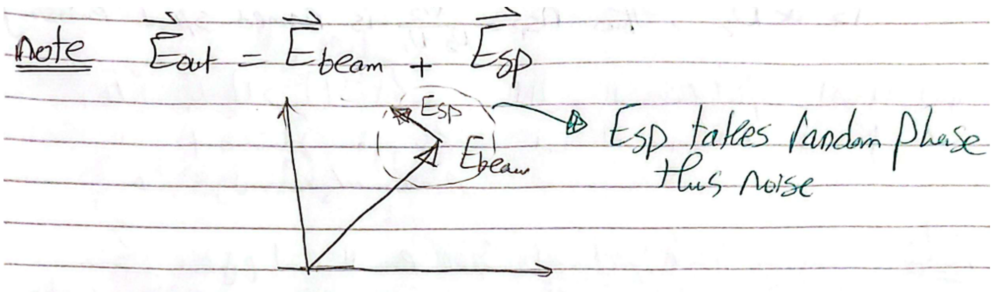
- Sources of noise:

  - **Relative Intensity Noise:** Fluctuations in the LASER output from the spontaneous emission.



  - **Stimulated Emission Noise:** Different material interactions with the same input signal.

- Optical SNR $= \frac{P_{signal}}{P_{noise}} = \frac{G_o P_{in}}{n_{sp} h\nu B_o (G_o - 1)}$, if $G_o >> 1$, then $SNR = \frac{P_{in}}{n_{sp} h\nu B_o}$
- The ν used in the equations is the center of the LASER linewidth.

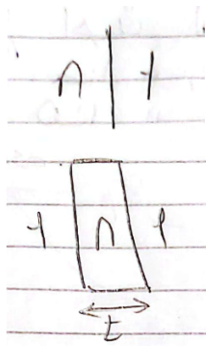


5 LASER

A LASER is an oscillator, so we need to achieve the positive feedback conditions:

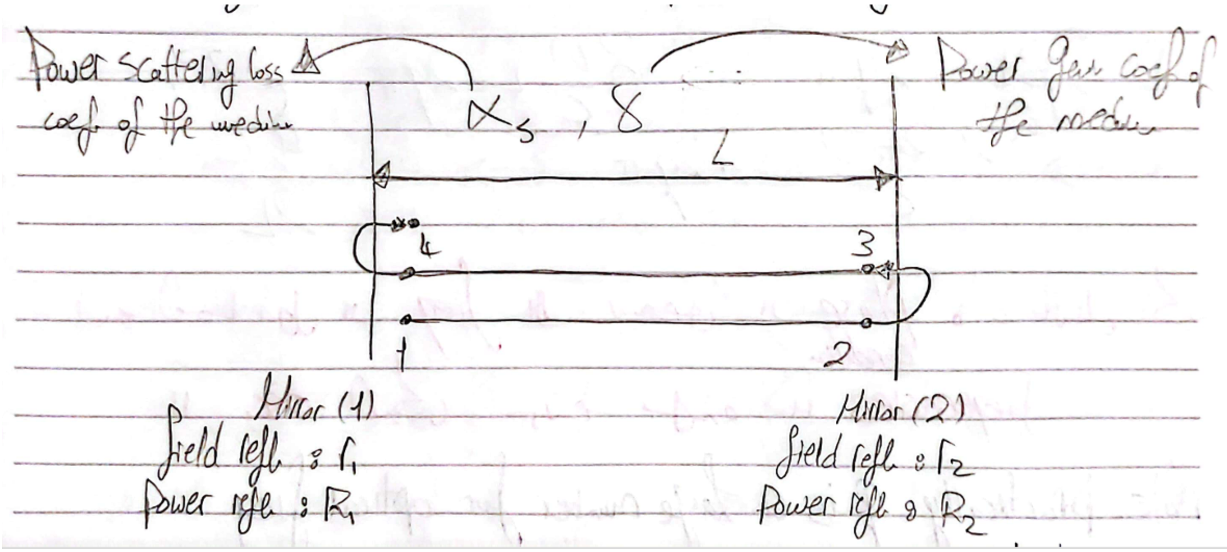
- Starting Noise (spontaneous emission noise).
- Feedback (mirrors).
- $A\beta > 1$ which reaches $A\beta = 1$ due to non-linearities (A is from material amplification, and β is from the refelectance of the mirrors).

5.1 Types of Mirrors



- **Simple Discontinuity:**
  - $\Gamma = \frac{n-1}{n+1}$ (real)
  - The normal case in semiconductor lasers.
- **Dielectric Slab:**
  - Γ can be complex dependeing on time.
  - The normal case in gas.

5.2 Phase Condition



Assuming constant gain (γ):

$$\begin{aligned}
&E_1\\
&E_2 = E_1 e^{-j\beta L} e^{-\frac{\alpha_s}{2}L} e^{\frac{\gamma}{2}L}\\
&E_3 = r_2 E_1 e^{-j\beta L} e^{-\frac{\alpha_s}{2}L} e^{\frac{\gamma}{2}L}\\
&E_4 = r_2 E_1 e^{-2j\beta L} e^{-\alpha_s L} e^{\gamma L}\\
&E_5 = r_1 r_2 E_1 e^{-2j\beta L} e^{-\alpha_s L} e^{\gamma L}
\end{aligned}$$

For oscillation, E_1 should add with E_5:

$$2\beta L = 2q\pi \Rightarrow 2\frac{\omega}{c}L = 2q\pi$$
$$2\frac{2\pi\nu}{c}L = 2q\pi \Rightarrow \nu = \frac{qc}{2L}$$

where q is an integer. Practically q is a large number $\approx 10^2$.

5.3 Gain Condition

$$r_1 r_2 e^{-\alpha_s L} e^{\gamma L} > 1$$

So:

$$e^{\gamma L} > \frac{e^{\alpha_s L}}{r_1 r_2} \Rightarrow \gamma L > \alpha_s L + \ln(\frac{1}{r_1 r_2})$$
$$\gamma > \alpha_s + \frac{1}{L}\ln\left(\frac{1}{r_1 r_2}\right)$$

$r_1 = \sqrt{R_1}$ and $r_2 = \sqrt{R_2}$, so:

$$\gamma > \alpha_s + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

where:

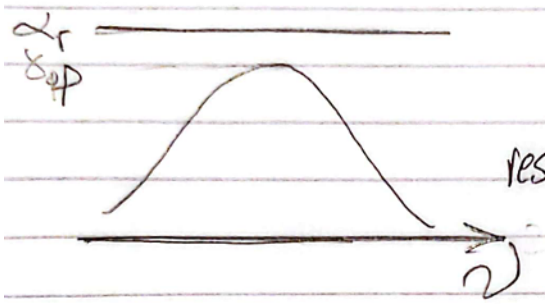
- α_s is the scattering loss and other losses (apart from absorption as it is included in the gain γ).
- $\frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$ is the mirror loss (if $R_1 = R_2 = 1$ this term is 0).
- $\alpha_s + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$ is called the resonator loss (α_r). For amplification, $\gamma > \alpha_r$.
- We assumed that both r_1 and r_2 are real so they only affect the gain condition, but the general case is that they are complex, so they affect the phase condition as well.

6 Conclusion

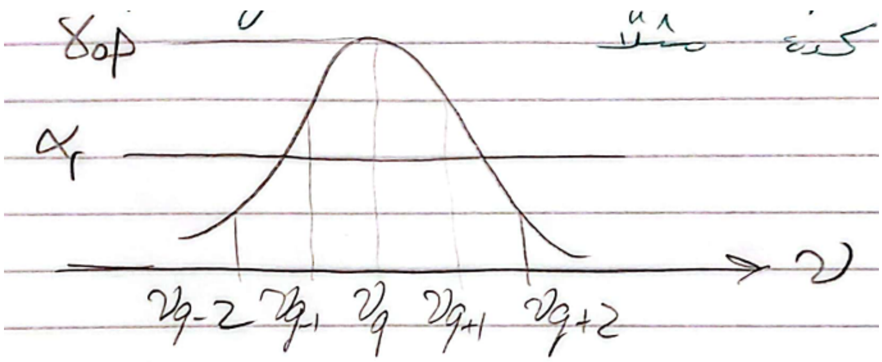
For oscillation, we must satisfy both conditions:

- Phase condition: $\nu = \frac{qc}{2L}$
- Gain condition: $\gamma > \alpha_r$

Gain peak is lower than $\alpha_r \Rightarrow$ No oscillation.



Oscillation only happens at ν_q, ν_{q+1}, ν_{q-1}:



As light intensity increases, the gain decreases, so the gain curve moves down, so some modes will stop oscillating ($\gamma < \alpha_r$) even if they satisfy the phase condition.