Lecture 1

1 Introduction

- **Light Scattering:** The reflection and transmission of light by a medium.
- **Light Interactions:** This only happens when photon energy is higher than the energy gap of the material. This leads to the excitation of the electrons in the material.
- Laser light is coherent meaning there is no phase change with time.

2 Classical Electron Oscillator Model (CEO)

We can model the forces between nucleus and electron as a mass spring system. There is an attraction force between the electron and the nucleus. The repulsion force is the centrifugal force, so:

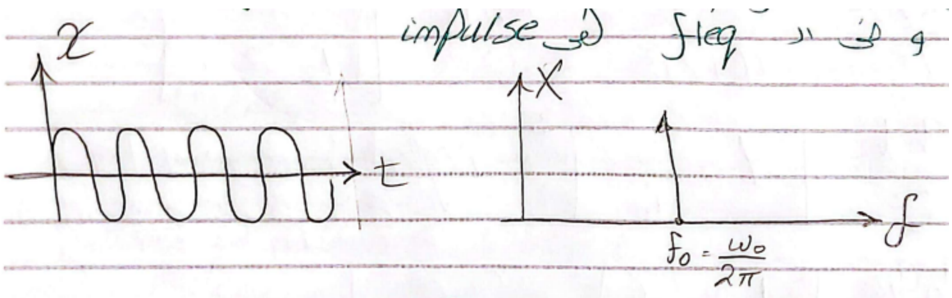
$$\frac{kq_1q_2}{r^2} = \frac{mv^2}{r}$$

2.1 Natural Response (No External Force)

2.1.1 Undamped Oscillation

$$m\frac{d^2x}{dt^2} = -kx$$
$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0$$
$$x = A\cos(\omega_0 t + \phi)$$

where $\omega_0 = \sqrt{\frac{k}{m}}$ is the natural frequency.



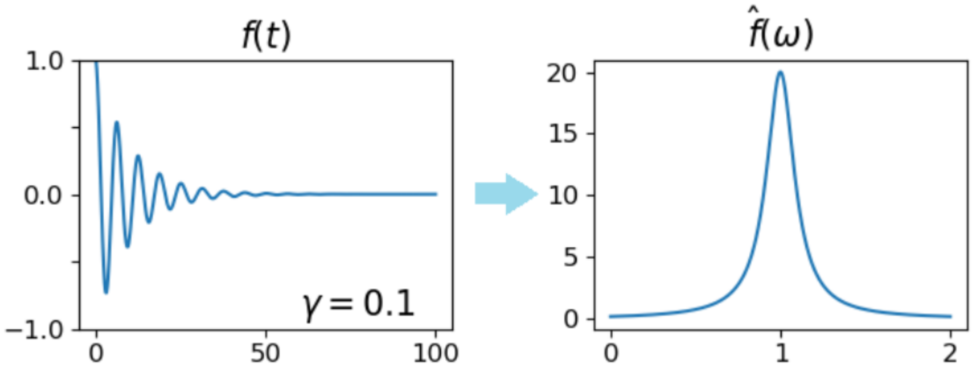
- Since there are no losses, we get a pure sinusoidal wave in time domain or an impulse in frequency domain.
- Light can only interact at the resonance frequency of the material.
- Similar to a pendulum, energy can be calculated at the point of maximum displacement where KE = 0, given by: $E = \frac{1}{2}k_s x_{max}^2$.

2.2 Damped Oscillation

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$
$$\frac{d^2x}{dt^2} + \eta\frac{dx}{dt} + \omega_0^2 x = 0$$
$$x = Ae^{-\frac{\eta}{2}t}\cos(\omega_d t + \phi)$$

where:

- $\omega_d = \sqrt{\omega_0^2 - \frac{\eta^2}{2}}$ is the damped frequency.
- $\eta = \frac{b}{m}$ is the damping factor.



- The presence of losses creates an underdamped time response and a broadened impulse in the frequency domain.
- **Why underdamped?** Since $\eta = \frac{1}{\tau}$ where τ is the electron lifetime (in nano range), while ω_0 is in the Giga range, so $\omega_0 >> \frac{\eta}{2}$.
- Due to the broadening caused by the losses, there is a bandwidth where light can interact with the material.
- Assume that maximum displacement is the envelope of the damped oscillation, so:

$$x_{max} = Ae^{-\frac{\eta}{2}t}$$
$$U = \frac{1}{2}k_s A^2 e^{-\eta t}$$

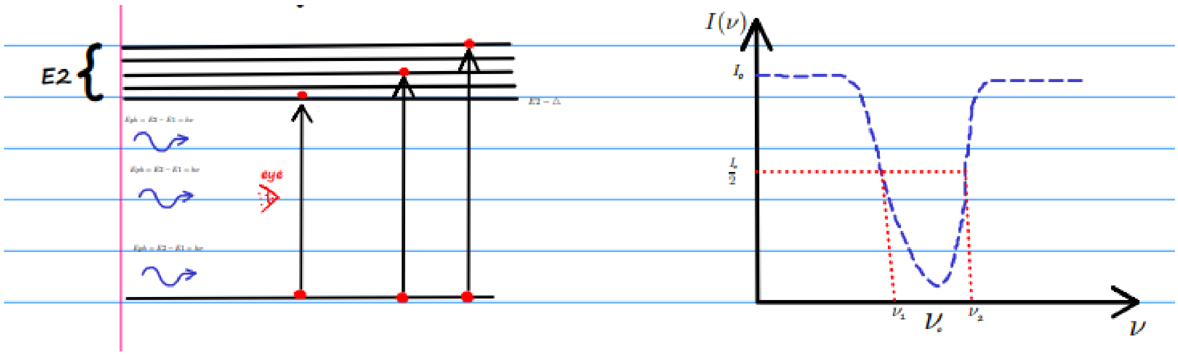
2.2.1 Losses

- **Radiation:** The accelaeration of the electron causes an electromagnetic wave to be emitted.
- **Collision:** The electron collides with the lattice and loses energy.
- The electron lifetime is the average time an electron remains in a specific energy state before transitioning to another state.
- Similar to time constant in an RC circuit, $e^{-\eta t} = e^{-\frac{t}{\tau}}$
- $\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}$, where τ_r is the radiative lifetime and τ_{nr} is the non-radiative lifetime.

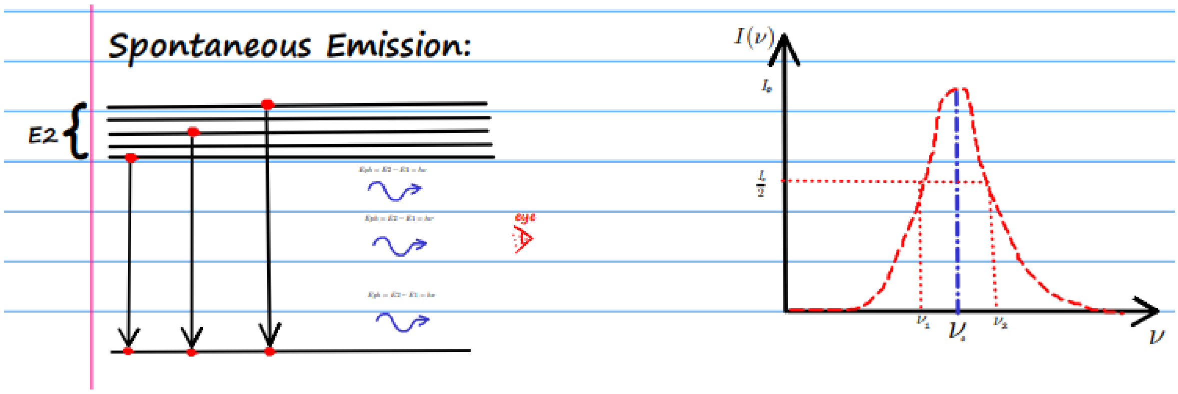
2.3 Quantum-Mechanical Approach of Broadening

Recall that Heisenberg's Uncertainty Principle states that we cannot know the exact energy at a specific time ($\Delta E \Delta t \geq \frac{\hbar}{2}$). This means that there is no exact energy level ($E_2 + \Delta E$) for the electron to transition to, so each electron can gain different energy and still transition. Recall that the energy of a photon is frequency dependent ($E = h\nu$), so light-matter interaction can happen at a range of frequencies (broadening).

If we observe the light intensity of the source, we find a decrease in intensity due to the absorption of the material in the frequency range around resonance frequency (ν_0).



When an electron goes to a higher energy level, it will eventually come back to the ground state. This causes the emission of a photon with the same frequency as the absorbed photon.



Note that if there was no losses, the drop and rise in intensity would be an impulse.

2.4 Forced Response

Note: We can neglect the magnetic force:

$$\frac{|F_{mag}|}{|F_{elec}|} = \frac{qvB}{qE} = \frac{vB}{E} = \frac{v}{c} << 1$$

Applied Sinusoidal Electric Field:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + k_s x = qE$$
$$\frac{d^2x}{dt^2} + \eta\frac{dx}{dt} + \omega_0^2 x = \frac{qE}{m}$$

Assume steady state (replace $\frac{d}{dt}$ with $j\omega$):

$$-\omega^2 x + j\eta\omega x + \omega_0^2 x = \frac{qE}{m}$$
$$x(\omega_0^2 - \omega^2 + j\eta\omega) = \frac{qE}{m}$$
$$x((\omega_0 - \omega)(\omega_0 + \omega) + j\eta\omega) = \frac{qE}{m}$$

Assume that $\omega \approx \omega_0$:

$$\begin{aligned}
x\left(2\omega(\omega_0-\omega)+j\eta\omega\right) &= \frac{qE}{m}\\
\frac{\omega\eta}{j}x\left(\frac{2j(\omega_0-\omega)}{\eta}-1\right) &= \frac{qE}{m}\\
-\frac{\omega\eta}{j}x\left(1+\frac{2j(\omega-\omega_0)}{\eta}\right) &= \frac{qE}{m}\\
x = \frac{-jqE}{m\omega\eta}\frac{1}{1+\frac{2j(\omega-\omega_0)}{\eta}}
\end{aligned}$$

Notes:

- We want to obtain material parameters, such as relative permitivity (ϵ) or susceptibility (χ) to study the light-matter interaction in the material.
- Note that the x in the dipole moment ($p = qx$) is the Δx from equilibrium position (electron shell).
- Polarization is the dipole moment per unit volume ($P = N_a qx$), where N_a is the number of atoms per unit volume.
- For linear polarization, $P = \epsilon_0 \chi E$.

From polarization equations:

$$\begin{aligned}
P = \epsilon_0\chi E &= \frac{-jN_a q^2 E}{m\omega\eta}\frac{1}{1+\frac{2j(\omega-\omega_0)}{\eta}}\\
\chi &= \frac{-jN_a q^2}{m\omega\eta\epsilon_0}\frac{1}{1+\frac{2j(\omega-\omega_0)}{\eta}}
\end{aligned}$$

Let $\delta = \frac{2(\omega-\omega_0)}{\eta}$:

$$\chi = \frac{-jN_a q^2}{m\omega\eta\epsilon_0}\frac{1}{1+j\delta}$$

Multiply by the conjugate:

$$\begin{aligned}
\chi &= \frac{-jN_a q^2}{m\omega\eta\epsilon_0}\frac{1-j\delta}{1+\delta^2}\\
\chi' &= \frac{N_a q^2}{m\omega\eta\epsilon_0}\frac{-\delta}{1+\delta^2}\\
\chi'' &= j\frac{N_a q^2}{m\omega\eta\epsilon_0}\frac{-1}{1+\delta^2}
\end{aligned}$$

Drawing the susceptibility:

- Let $\chi_0 = \frac{N_a q^2}{m\omega\eta\epsilon_0}$.
- $\delta = \frac{2(\omega-\omega_0)}{\eta}$

Max & Min for χ':

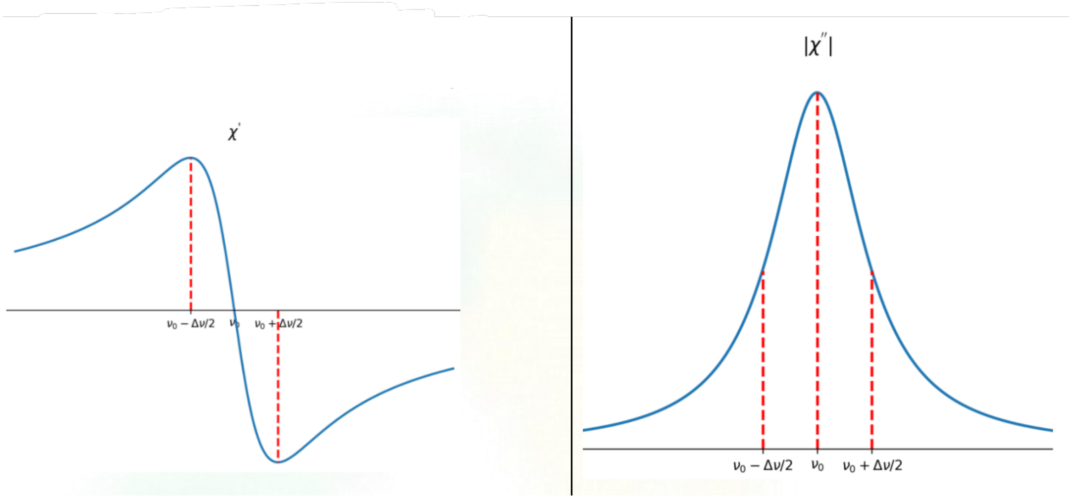
$$\frac{d\chi'}{d\delta} = \chi_0\frac{1-\delta^2}{(1+\delta^2)^2} = 0 \rightarrow \delta = \pm 1$$

$$\omega = \omega_0 \pm \frac{\eta}{2}$$

Max χ':

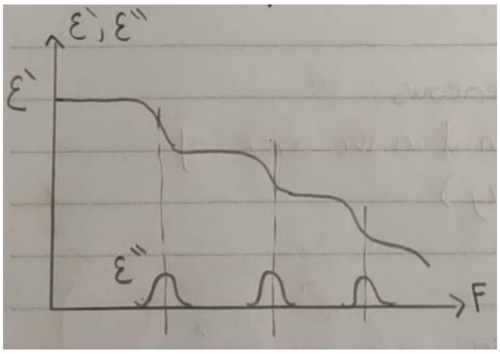
$$\frac{d\chi''}{d\delta} = \chi_0 \frac{-2\delta}{(1+\delta^2)^2} = 0 \rightarrow \delta = 0$$

$$\omega = \omega_0$$



From previous equations, we can draw the relative permitivity using the susceptibility:

$$\epsilon_r = 1 + \chi = 1 + \chi' + j\chi''$$



Proof that χ'' leads to attenuation:

$$k = k_0\sqrt{\epsilon_r} = k_0\sqrt{1+\chi'+\chi''}$$
$$= k_0\sqrt{(1+\chi')\left(1+j\frac{\chi''}{1+\chi'}\right)}$$
$$= k_0\sqrt{1+\chi'}\sqrt{1+j\frac{\chi''}{1+\chi'}}$$

Note that $\sqrt{1+\chi'} = \sqrt{\epsilon'} = n$, so:

$$k_0 n\sqrt{1+j\frac{\chi''}{n^2}} \approx k_0 n\left(1+j\frac{\chi''}{2n^2}\right)$$

Therefore:

- lossless propagation constant: $k_0 n$
- lossy propagation constant (γ): $\gamma = k_0\frac{\chi''}{2n}$ (Always negative in classical model)

Recall that wave propagation has e^{jkx} term, so the real term in k will be the phase term, while the imaginary term will be the attenuation term.