Лабораторная работа 3 (2 часть)

Решение уравнений параболического типа

(2D уравнение теплопроводности)

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Для решения дана следующая задача:

$$\frac{\partial U}{\partial t} = \alpha(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2})$$
$$0 \leqslant x \leqslant L_1,\ 0 \leqslant y \leqslant L_2,\ t \geqslant 0$$

С условиями:

$$U(x, y, 0) = sin(\pi x) \cdot sin(\pi y)$$
$$U(0, y, t) = 0,\ U(L_1, y, t) = 0$$
$$U(x, 0, t) = 0,\ U(x, L_2, t) = 0$$

Цель:

- явная двухслойная схема (FTCS метод)
- метод продольно-поперечных прогонок

In [146]:

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import cm
from mpl_toolkits.mplot3d import Axes3D
import math
```

In [168]:

```
alph = 1
l_x, l_y = 1, 1
N_x, N_y = 10, 10

time_sum = 1
time_1 = 0.01
time_2 = 0.05
time_3 = 0.1
time_4 = 0.25

d_1 = 0.1
d_2 = 0.5
d_3 = 0.6
d_4 = 2.5
```

In [169]:

```
def border_left_x(y, t):
    return 0
def border_right_x(y, t):
    return 0
def border_left_y(x, t):
    return 0
def border_right_y(x, t):
    return 0
def fun_initial(x, y):
    return math.sin(math.pi * x) * math.sin(math.pi * y)
```

In [206]:

```
def explicit_schem(d):
    step_x = l_x / N_x
    step_y = l_y / N_y
    step_time = step_x**2 * d / alph*2
    N_t = int(time_sum / dt)
    print("Диффузионное число = ", d)
    u = np.zeros((N_t + 1, columns_x + 1, columns_y + 1))

    for i in range(1, N_x):
        for j in range(1, N_y):
            u[0][i][j] = fun_initial(i * step_x, j * step_y)

    for i in range(N_t + 1):
        for j in range(N_y + 1):
            u[i][0][j] = border_left_y(j * step_y, i * step_time)
            u[i][N_x][j] = border_right_y(j * step_y, i * step_time)

    for i in range(N_t + 1):
        for j in range(N_x + 1):
            u[i][j][0] = border_left_x(j * step_x, i * step_time)
            u[i][j][N_y] = border_right_x(j * step_x, i * step_time)

    for i in range(N_t):
        for j in range(1, N_x):
            for k in range(1, N_y):
                u[i+1][j][k]=(d*(u[i][j+1][k]-2*u[i][j][k]+u[i][j-1][k])
                              + d * (u[i][j][k+1]-2*u[i][j][k]+u[i][j][k-1])
                              + u[i][j][k])
    return u, step_time
```

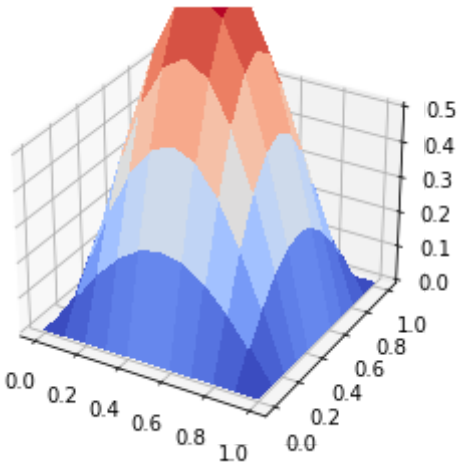
In [207]:

```
def draw(step_t, u: list, time):
    print("Время = ", time)
    a, b, c = np.shape(u)
    x = np.linspace(0, length_x, b)
    y = np.linspace(0, length_y, c)
    X, Y = np.meshgrid(x, y)
    fig= plt.figure()
    ax = fig.gca(projection='3d')
    ax.set_zlim(0, 0.5)
    surf = ax.plot_surface(X, Y, u[int(time / step_t), :], cmap=cm.coolwarm, linewidth=0, a
    plt.show()
```

In [208]:

```
u, dt = explicit_schem(d_1)
draw(dt, u, time_1)
```

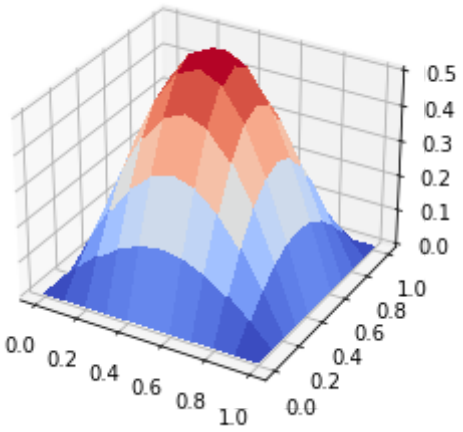
```
Диффузионное число =  0.1
Время =  0.01
```



In [209]:

```
u, dt = explicit_schem(d_1)
draw(dt, u, time_2)
```

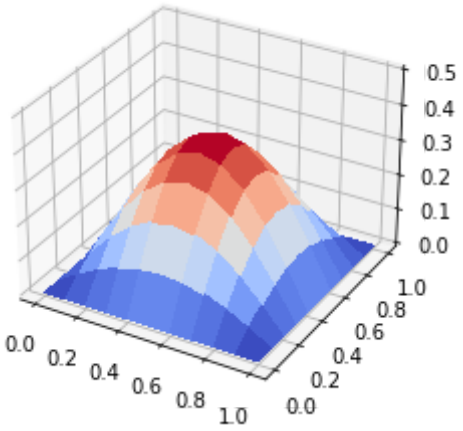
```
Диффузионное число =  0.1
Время =  0.05
```



In [210]:

```
u, dt = explicit_schem(d_1)
draw(dt, u, time_3)
```

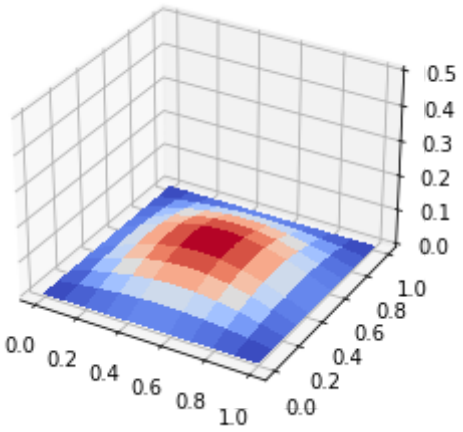
```
Диффузионное число =  0.1
Время =  0.1
```



In [211]:

```
u, dt = explicit_schem(d_1)
draw(dt, u, time_4)
```

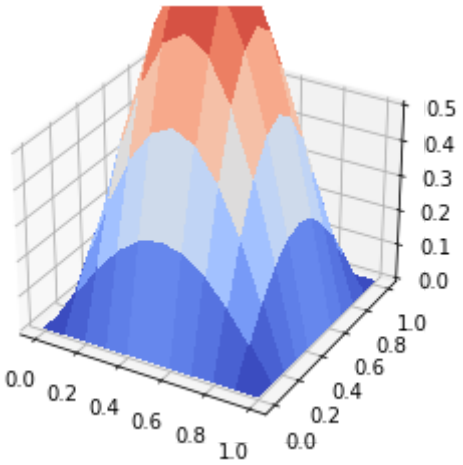
```
Диффузионное число =  0.1
Время =  0.25
```



In [212]:

```
u, dt = explicit_schem(d_2)
draw(dt, u, time_1)
```

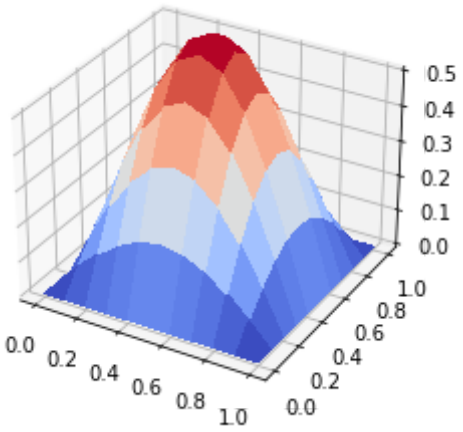
```
Диффузионное число =  0.5
Время =  0.01
```



In [213]:

```
u, dt = explicit_schem(d_2)
draw(dt, u, time_2)
```

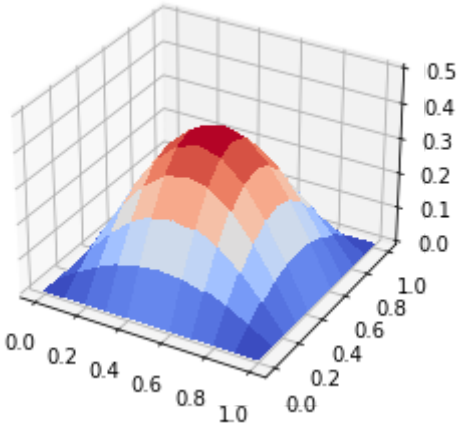
```
Диффузионное число =  0.5
Время =  0.05
```



In [214]:

```
u, dt = explicit_schem(d_2)
draw(dt, u, time_3)
```

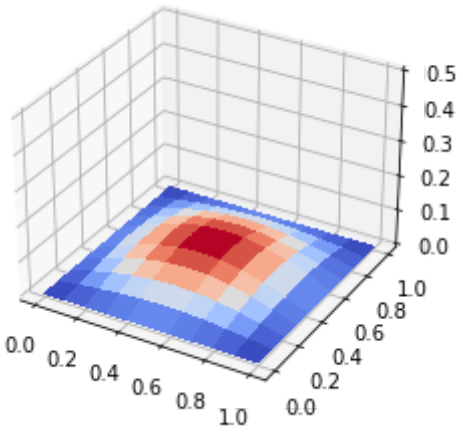
```
Диффузионное число =  0.5
Время =  0.1
```



In [215]:

```
u, dt = explicit_schem(d_2)
draw(dt, u, time_4)
```

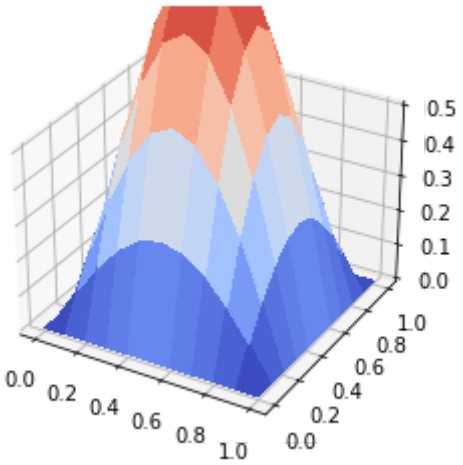
```
Диффузионное число =  0.5
Время =  0.25
```



In [216]:

```
u, dt = explicit_schem(d_3)
draw(dt, u, time_1)
```

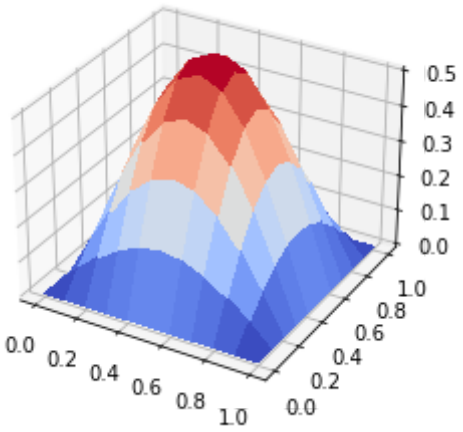
```
Диффузионное число =  0.6
Время =  0.01
```



In [217]:

```
u, dt = explicit_schem(d_3)
draw(dt, u, time_2)
```

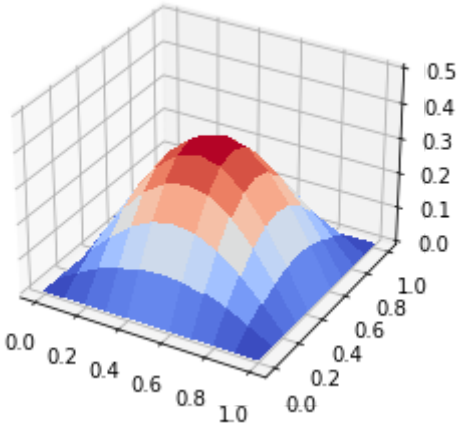
```
Диффузионное число =  0.6
Время =  0.05
```



In [218]:

```
u, dt = explicit_schem(d_3)
draw(dt, u, time_3)
```

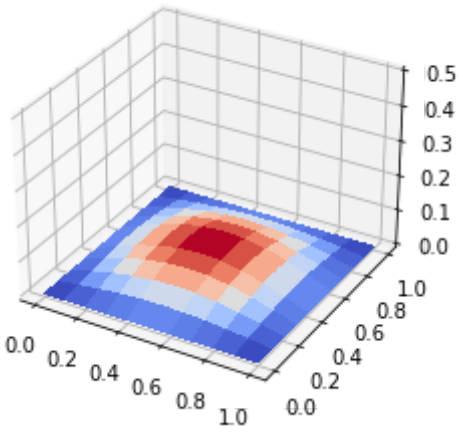
```
Диффузионное число =  0.6
Время =  0.1
```



In [219]:

```
u, dt = explicit_schem(d_3)
draw(dt, u, time_4)
```

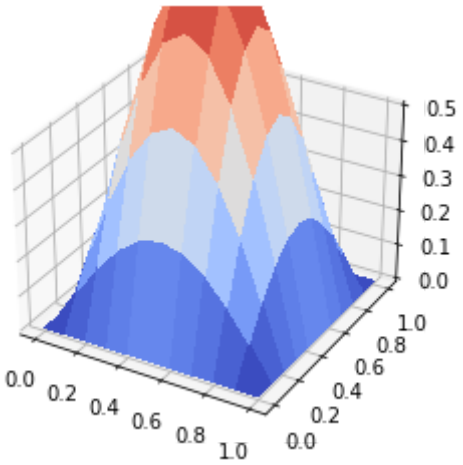
```
Диффузионное число =  0.6
Время =  0.25
```



In [220]:

```
u, dt = explicit_schem(d_4)
draw(dt, u, time_1)
```

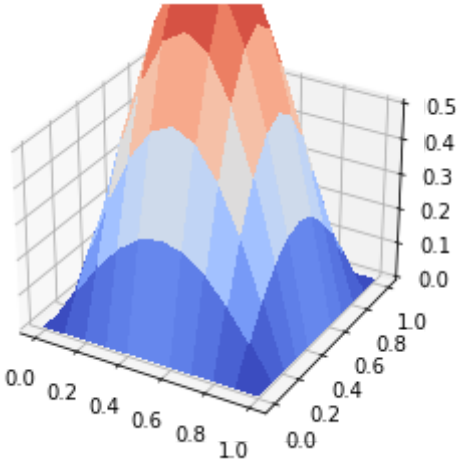
```
Диффузионное число =  2.5
Время =  0.01
```



In [221]:

```
u, dt = explicit_schem(d_4)
draw(dt, u, time_2)
```

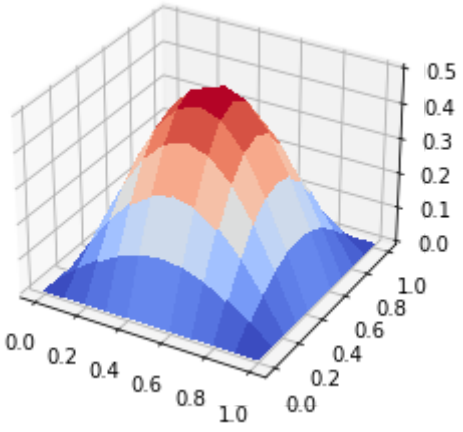
```
Диффузионное число =  2.5
Время =  0.05
```



In [222]:

```
u, dt = explicit_schem(d_4)
draw(dt, u, time_3)
```

```
Диффузионное число =  2.5
Время =  0.1
```



In [223]:

```
u, dt = explicit_schem(d_4)
draw(dt, u, time_4)
```

```
Диффузионное число =  2.5
Время =  0.25
```