Algorithm 1 Compute an invariant subspace or ensure that no such subspace is

INPUT: a list $\mathcal{M} = \{M_1, \ldots, M_r\}$ of square matrices of order n

OUTPUT: a basis of a non trivial $\mathcal{M}$-invariant subspace or `None`

function INVARIANT_SUBSPACE($\mathcal{M}$)
  compute $E_1, \ldots, E_k$ the generalized eigenspaces of a random linear combination in the M_i's.
  for $1 \leq j \leq k$ **do**
    if $\dim(E_j) = 1$ **and** $\mathrm{Inv}_{\mathcal{M}}(E_j) \neq \mathbb{C}^n$ **then** return $\mathrm{Inv}_{\mathcal{M}}(E_j)$
  if $k = n$ **then** return `None`
  $j :=$ index for which $\dim(E_j) > 1$
  compute $M_i^{[E_j]}$ for each $1 \leq i \leq r$ where $M^{[E_j]}$ is defined by:

$$T^{-1}MT =: \left(\begin{array}{c|c|c} * & * & * \\ \hline * & M^{[E_j]} & * \\ \hline * & * & * \end{array}\right)$$

with T a transition matrix from the canonical basis to a basis adapted to $\mathbb{C}^n = E_1 \oplus \cdots \oplus E_k$

  compute $K := \bigcap_{i=1}^{r} \mathrm{Ker}\left(M_i^{[E_j]} - \lambda_{M_i,j}\right)$
  while $K \neq \{0\}$ **do**
    take non zero $v \in K$
    if $\mathrm{Inv}_{\mathcal{M}}(v) \neq \mathbb{C}^n$ **then** return $\mathrm{Inv}_{\mathcal{M}}(v)$
    compute $M \in \mathrm{Alg}(\mathcal{M})$ such that $M^{[E_j]}v \in E_j \backslash \mathbb{C}v$
    replace K by $K \cap \mathrm{Ker}\left(M^{[E_j]} - \lambda_{M,j}\right)$
  compute $\mathcal{B}$ a basis of the algebra generated by $\mathcal{M}$
  return INVARIANT_SUBPACE($\mathcal{B}$)