

Peirce's Reduction Thesis

Peirce, I argued in chapter 1, endorses the Kantian Insight—that the categories somehow correspond to the logical forms of propositions—until 1902, and he continues to endorse a modified version of the view after 1902, a version modified to accommodate his mature classification of the sciences. In the previous chapter, I indicated that beginning in the mid-1880s Peirce recognizes that the logical forms of propositions correspond to the numbers one, two, three. Although he had an intimation of this idea as early as his 1867 essay “On a New List of Categories,” he could only consistently hold this view by abandoning the Aristotelian analysis of propositional forms and replacing it with an analysis consistent with first-order predicate logic. On Peirce’s mature view, every proposition minimally consists of an index—for example, the “there is something that is,” the \exists_x , or, in Peirce’s Existential Graphs, the line of identity—and a relative term. Peirce calls the index the subject and the relative term itself the predicate. He maintains (1) that the most basic propositional forms are either monadic (a relative term taking one subject, one), dyadic (a relative term taking two subjects, two), or triadic (a relative term taking three subjects, three), and (2) that there is no need to posit any other basic propositional forms.¹ The task of this chapter is to explain why Peirce holds this view.

The two claims (1) and (2) just noted are commonly called Peirce’s reduction thesis, yet there is significant scholarly disagreement about what is meant by “reducibility,” especially as it pertains to mathematical reduction. In addition, the truth of Peirce’s reduction thesis remains contested.² In my judgment, though, we can understand how Peirce came to his view on the basic propositional forms without settling questions about what Peirce means by reduction and without treading into the jungles of minute mathematical analysis. This is in no way to impugn or denigrate the important mathematical work that has been done on Peirce’s reduction thesis. It is only to claim that given Peirce’s commitment to the modified Kantian Insight and our specific interest in Peirce’s phenomenology, we can get to the heart of the matter by reflecting on Peirce’s theory of the proposition and his graphical logic without treading

too deeply into the mathematical underpinnings of his thesis. Accordingly, my aim here is not to prove Peirce's reduction thesis. Rather, my aim is to reconstruct, from Peirce's writings, a line of thought that leads him to embrace the reduction thesis. I shall do so by extrapolating a line of thought supporting four other theses intimately related to his reduction thesis—four theses that in fact entail the reduction thesis. These theses lie at the heart of Peirce's analysis of propositional forms, on which he bases his formal, logical categories of firstness, secondness, and thirdness. Those four other theses are the following, where n is a natural number:

Constructability Thesis: All n -adic propositional forms where $n > 3$ are constructable from triadic propositional forms.

Decomposability Thesis: All n -adic propositional forms where $n > 3$ can be decomposed into triadic propositional forms.

Inconstructability Thesis: (1) Triadic propositional forms cannot be constructed from dyadic propositional forms, and (2) dyadic propositional forms cannot be constructed from monadic propositional forms.

Ingredient Thesis: All triadic propositional forms contain as abstractable logical ingredients both dyadic and monadic propositional forms.

If all of these theses are true, it follows that there are three and only three basic propositional forms: the monadic form (firstness); the dyadic form (secondness); and the triadic form (thirdness). We need not posit any more than these three, and we cannot posit any fewer.

Peirce claims he has an argument for his reduction thesis, though Peirce scholars have struggled to find one. If the present reconstruction of Peirce's line of thinking is accurate, then Peirce may have justly claimed to have a proof for his reduction thesis. His "proof," however, is not the typical sort of proof one would expect to find in a treatise on mathematics. Rather, to use an example from Peirce, it is a "proof" much in the same way one can show that the sum of the angles formed by one line abutting another line is always 180 degrees by drawing a dotted line perpendicular to the point of abutment (see EP 2:207). In other words, it is a diagrammatic proof of the very sort we would expect from Peirce. While there may be grounds to doubt that such diagrammatic proofs are proofs at all, let alone proofs that withstand the rigorous demands of mathematical inquiry, there can be little doubt that Peirce himself thought that such proofs were satisfactory.³ What I have to present here, I should note, is especially indebted to Kenneth Laine Ketner's work (1986 and 1987) on manuscript R 482. In addition, a word of warning is in order: What follows here is both briefly stated and highly abstract. Any reader who is satisfied to simply accept that Peirce's logical graphs entail that we need posit only monadic, dyadic, and triadic propositional forms as basic might prefer to simply move ahead to the next chapter.

Propositional Forms

As I showed in the previous chapter, on Peirce's mature theory of propositions, every proposition minimally consists of two parts. The first is an index or logical subject; the second is a relative term, logical predicate, or rheme. In his graphical logic, the index is represented by a line of identity, and the relative term is represented by a letter. Accordingly and as I showed in chapter 1, a simple proposition such as "there is something that is black" can be graphed by drawing a line of identity and hooking it onto the letter B, which stands for the predicate is black. When we have such a simple graph, the line of identity shows that we have an unsaturated predicate term—*x* is black—since one end of the line of identity is a loose end (see figure 3.1). This unsaturated (or incomplete) propositional form is a monad since there is only one loose end.

We can saturate (or complete) the predicate by adding a grammatical (not logical) subject to the proposition, such as stove. If we do so, we get the graph for "the stove is black" (see figure 3.2). As there are no loose ends, we then have a saturated (or complete) proposition formed by compounding two monadic propositional forms. Grammatically, the graph states that the stove is black. Logically, though, on Peirce's view it should be interpreted as "something is a black stove" (see EP 2:173, 1903) or, as we might put it, there is something that is both black and a stove. Peirce calls a saturated or completed proposition such as this a medad because no line of identity has any loose ends: "In a complete proposition there are no blanks. It may be called a medad, or medadic relative" (LI 190, 1897).

Of course, there are also predicate terms that can take two logical subjects, such as the term resembles. Adding subjects to the relative term resembles so as to form a proposition, we would have the letter R with a line of identity extending from the front, the grammatical subject, and another line of identity extending from the back, the grammatical direct object (see figure 3.3).



FIGURE 3.1 *x* is black (i.e., something is black)



FIGURE 3.2 *x* is black and a stove (i.e., something is a black stove)



FIGURE 3.3 *x* resembles *y* (i.e., something resembles something)

The graph states that something resembles something, or *x* resembles *y*. Here we have two loose ends and so a dyadic propositional form. If we were to add a noun term such as “is Felix the Cat,” we could saturate one of the loose ends much as we combined or compounded “something is black” (i.e., *x* is black) and “something is a stove” (i.e., *y* is a stove) by asserting the identity of *x* and *y* so as to get “the stove is black.” That is, we could combine “*x* resembles *y*” and “*z* is Felix the Cat” (figure 3.4) by asserting the identity of *y* and *z* so as to get “*x* resembles Felix the Cat,” as in figure 3.5.

This graph is now monadic, as it has one loose end, and so it is still not fully saturated. We could complete it by adding a grammatical subject such as “*w* is Junius the Cat” and then asserting the identity of *w* and *x*, so as to get “Junius the Cat resembles Felix the Cat” (see figure 3.6). This is a medad, a complete or fully saturated proposition.

In addition to monadic and dyadic propositional forms, there are also triadic and tetradic propositional forms. In the previous chapter, I showed that the graph for “*x* resembles *y* by way of *z*.” figure 3.7 is the graph for “*x* gives *y* to *z*.”



FIGURE 3.4 *x* resembles *y*, and *z* is Felix the Cat (i.e., something resembles something, and something is Felix the Cat)

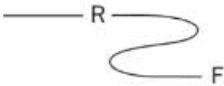


FIGURE 3.5 *x* resembles *y*, *z* is Felix the Cat, and *y* is identical to *z* (i.e., something resembles Felix the Cat)

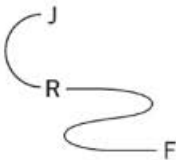


FIGURE 3.6 *x* resembles *y*, *z* is Felix the Cat, *w* is Junius the Cat, *w* is identical to *x*, and *y* is identical to *z* (i.e., Junius the Cat resembles Felix the Cat)

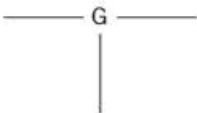


FIGURE 3.7 *x* gives *y* to *z*

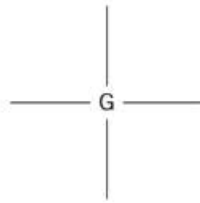


FIGURE 3.8 x gives y to z at time t

As this graph has three loose ends, it is a triad. As an example of a tetrad, we might let G stand for ——— gives ——— to ——— at time ———. If we graph it, we have a tetradic propositional form (see figure 3.8).

From a grammatical point of view, however, there seems to be a problem with these graphs. Unlike the relationship of resembling, which is symmetrical,⁴ in that x resembles y iff y resembles x (or, as Peirce had put it in “On a New List,” where x and y are correlates), we distinguish among the giver, the gift, and the recipient. There is, however, nothing in the graphs as written above that makes it clear which line of identity corresponds to the person giving, which to the gift, and which to the recipient. Yet from a formal and logical point of view Peirce regards the grammatical accidents of language to be logically dispensable. He writes,

in a triadic fact, say, for example

A gives B to C

we make no distinction in the ordinary logic of relations between the *subject nominative*, the *direct object*, and the *indirect object*. We say that the proposition has three *logical subjects*. We regard it as a mere affair of English grammar that there are six ways of expressing this:

A gives B to C

B enriches C at expense of A

C thanks A for B

A benefits C with B

C receives B from A

B leaves A for C

(EP 2:170–171, 1903)

In point of fact, from a purely formal and logical point of view, the letters themselves are entirely insignificant. We can simply replace them with dots, which we will suppose in what follows (for reasons that will become evident as we proceed) to simply stand for the relative term “is identical to” and in the case of monadic propositional forms to stand for any common noun. Doing so gives us monadic, dyadic, triadic, and tetradic propositional forms (see figure 3.9).

With these observations in place, we are in a position to turn to Peirce’s theses.

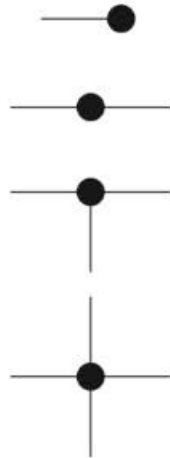


FIGURE 3.9 Monadic, dyadic, triadic, and tetradic propositional forms

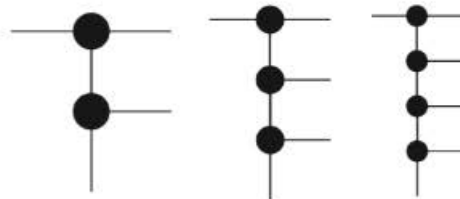


FIGURE 3.10 Construction of n -adic predicates (where n is greater than three) from triads

The Constructability Thesis

Thus far I have restricted my considerations to monadic, dyadic, triadic, and tetradic propositional forms. I have only been presenting ways of representing these forms from a purely formal point of view. I have not argued that we need any or all of these in order to have a complete set of the basic forms of propositions. Clearly, though, a tetrad—that is, a propositional form with four loose ends—can be constructed from triads by compounding (or connecting the loose ends of) triads. Moreover, we can construct pentads, hexads, heptads, etc. simply by adding triads (see figure 3.10).

And this is all there is to Peirce's constructability thesis.

The Decomposability Thesis

Obviously, there are tetradic propositional forms as well, such as “ x gives y to z at time t ,” as I showed earlier. This is important to stress: Peirce does think that there are relative terms that take four or more logical subjects. All he denies is that the tetrad is a basic propositional form. (See Schneider 1952 and Vaught 1986, and see Hausman 1988.) Recall that what counts for the adicity of a graph is how many loose ends it has. Now consider the graph for a tetrad

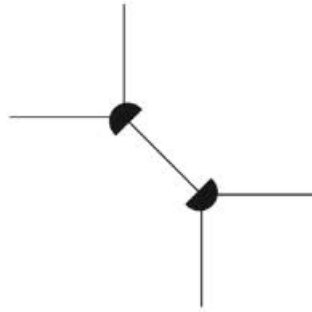


FIGURE 3.11 Decomposition of tetrad into two triads

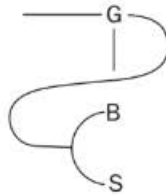


FIGURE 3.12 Someone gives a black stove to someone

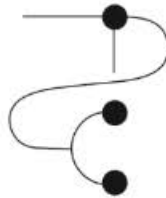


FIGURE 3.13 Purely formal abstraction of figure 3.12

provided earlier, a dot with four lines of identity extending from it. And now suppose that we were to take that dot in the center, cut it in half except for a single fiber, and then stretch that fiber out as in figure 3.11 (see SS 199, 1906).

If we do so, then we can see that the tetrad can be decomposed into two triads. But why should we be allowed to do this? To answer that question, it is important to recall that we are abstracting away from the relative terms themselves and what they mean. Once we do so, we will observe that when a dot conjoins lines of identity the location of the dot is utterly arbitrary. To see why, consider the graph for “someone gives a black stove to someone” (figure 3.12). If, from a purely formal and logical point of view, we abstract away from the relative terms themselves and replace them with dots, we are left with three dots standing for the terms, and at one of the dots (the dot replacing the *G*) we will notice that three different lines intersect there (see figure 3.13).

Yet a more careful inspection of the graph will show that there are actually two dots where three lines intersect. The first is the dot replacing the letter for the relative term *G*. But there is also an intersection of lines of identity at the point where the three lines of identity for what is given, for what is black, and

for what is a stove all connect together. Although in the graph this is rather more like a point than a dot, we can nevertheless regard that point as a dot for the relative term “is identical to both and.” In fact, we can regard any point on any intersection of lines of identity as a dot standing for the relation of “being identical to” or “being identical to both and.” (Note that the very last point on an unsaturated line of identity cannot be treated as a point of intersection between lines but any other point may be.) Accordingly, by cutting the dot in half and stretching out a single fiber, we are simply positing an arbitrary *A*, defining it as identical to *w* and *x* and to *y* and *z*, and stating both that *w* is identical to *x* and *A* and that *A* is identical to *y* and *z*. In fact, this is no more than a roundabout way of doing what we did at the start of this chapter when we compounded the graph for “*x* resembles *y*” and “*z* is Felix the Cat” by claiming that *y* and *z* are identical. Here, we are simply positing an additional *A* that is identical to both *y* and *z* and then using *A* to conjoin the lines of identity representing *y* and *z*. Crucially, when we do so we introduce no new elements into the construction because *A* is defined as identical to what is already part of the construction and, as I will show, triads are logical ingredients of every tetrad. Analogously, we may add whatever we wish to any formula so long as we do not alter its basic formal structure. To $1 + 1 = 2$ we may make an addition like so: $(1 + 1) \times 3 = 3 \times 2$, and to any arbitrary proposition *p* we may make an addition like so: $p \vee q$. As nothing in the formal structure of the diagram is changed by introducing an arbitrary *A*, the relationship of tetridentity (identity among four things) can be decomposed into two teridentities (identity among three). Obviously, we can do the same for pentads, hexads, heptads, etc. Just as these can all be constructed from triads, they can also be decomposed into triads. This is Peirce's decomposability thesis.

The Inconstructability Thesis

The foregoing line of thought may lead us to believe that triadic propositional forms may be reduced to dyadic propositional forms. For we ought to be able to analyze “*x* is identical both to *y* and to *z*” into “*x* is identical to *y* and *y* is identical to *z*.” In other words, it would seem as though our graph for a triad should be identical to a graph compounding two dyads. That is, letting dots stand for the relative terms, it would seem as though from a purely logical and formal view the two graphs in figure 3.14 are identical.



FIGURE 3.14 (Left) *x* is identical to both *y* and *z*; (Right) *x* is identical to *y*, *z* is identical to *w*, and *y* and *z* are identical

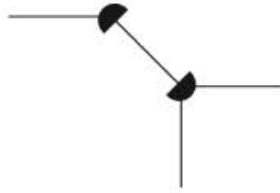


FIGURE 3.15 Attempted decomposition of figure 3.14 (left) into two dyads

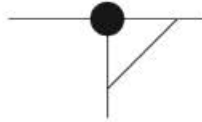


FIGURE 3.16 Insertion of an arbitrary line of identity into figure 3.14 (left), resulting in no new kinds of intersections into the graph

But these are manifestly not identical. In order to see why, simply try to decompose the triad so as to get the graph with two dots. If we cut the dot and stretch the fiber as we did to decompose tetridentity into teridentity, we are still left with a triad (see figure 3.15).

Similarly, if we try to construct the triad merely from sets of dyads, we will find it impossible to do so. No matter how many dyads are compounded, we are always left with another dyad (unless we connect the loose ends, in which case we would have a medad, as explained later). Granted as much, it would seem to follow that something went wrong in our previous analysis.

In order to see that nothing has gone wrong, we must keep in mind that any arbitrary point on a branch in the triadic figure is identical to any other arbitrary point on a branch in the same triad (recalling that we are treating all dots and points except those at the end of a line of identity as standing for the relation of identity), and we can see that they are identical by simply positing an arbitrary *A* and defining it as identical to each point. Introducing the arbitrary *A* introduces nothing new into the graph. Peirce writes, “every line of identity ought to be considered as bristling with microscopic points of teridentity” (SS 199, 1906). Notice that he is merely claiming the line of identity ought to be considered as such, not that it in fact has such microscopic points, for if it did have such microscopic points there would be no genuine monads or dyads, a topic I shall take up later. Nonetheless, if we do regard the line of identity as bristling with such points, we can draw a line from that one point to any other point on the line of identity, as in figure 3.16.

Consistent with our foregoing analysis of tetradic relatives, all we must do to warrant drawing this line is postulate some arbitrary *A* defined as identical to both points. Most important, when we draw this new line, we do not introduce any new sorts of intersections: Every point of intersection is triadic. The same is not true, however, of the figure of compounded dyadic propositional



FIGURE 3.17 Insertion of an arbitrary line of identity into figure 3.14 (right), resulting in the introduction of teridentity into the graph



FIGURE 3.18 Connection of two monads

forms. For here, if we wish to represent the identity of any point on the line to any other arbitrary point on the line, we will need to draw a line as in figure 3.17.

In drawing this line, however, we have introduced two intersections of teridentity whereas there were no such intersections in our previous graph of the compounded dyads. A new feature has been introduced into the graphical construction that was not part of the original graph, namely, teridentity. That is, teridentity cannot be constructed from binary identity (or what Peirce at one points calls binidentity) without the introduction of teridentity. This is the first part of Peirce's inconstructability thesis: Triads cannot be constructed from dyads. Any attempt to do so would require introducing a triad of teridentity.

It might be objected at this point that we could also draw two lines from any one arbitrary point on a line of identity in the triad to two different points on a different line of identity. In that case, it would seem we do introduce a new element into the propositional form, namely, a tetrad. In response, however, notice we can decompose the tetrad into two triads. As seen earlier, though, we cannot decompose the triadic points introduced in the dyadic graph into dyads. Introducing the tetrad into the triadic graph introduces no new basic formal elements into the graph, but introducing the triad into the dyadic graph does.

The same point applies to the attempt to construct binary identities out of monads. On this score, it is important to remember that monads only have one loose end. Consequently, although the dots at intersections are arbitrary, in monads they stand for any common noun and mark that no lines of identity can be conjoined to one end of the line of identity. With this in mind, trying to construct a binary identity from monads will readily be seen to be futile. For if we take two monads and connect them, we are left with no loose ends whatsoever (see figure 3.18).

In fact, the resultant graph is equivalent to the diagram for "the stove is black" above, if we let dots take the place of the terms stove and black and bend the line of identity. It is a medad, or a graph with no loose ends whatsoever. It also elucidates the second part of Peirce's inconstructability thesis.

The Ingredient Thesis

Let us recur for a moment to the tetrad we constructed from triads in the discussion of the constructability thesis. Let us suppose for a moment, though, that we joined together two of their loose ends, as in figure 3.19.

In this case, we have constructed a dyad from a set of triads. In a similar way, we can construct a monad from a triad by joining two of the triad's loose ends (see figure 3.20).

Moreover, since we can construct a triad from a pentad in a way similar to the way we just made a monad from a triad, we could create any n -adic propositional form from pentads alone (see figure 3.21).

Here we come to a potentially devastating two-pronged objection to Peirce's isolation of monadic, dyadic, and triadic propositional forms based on the figures above. The first prong is that there is no need to posit monadic and dyadic propositional forms because they can be constructed from triads, as just shown. The second prong is that there is not even a need to posit triadic propositional forms since monads, dyads, and triads could all be derived from pentadic propositional forms, also as just shown. I will leave it as an exercise to the reader to show that we need not even posit pentadic propositional forms, for heptadic ones would do just as well. In fact, Peirce had realized just this in 1897: "[A]ny perissid, or odd-ad (except a monad), can by repetition produce a relative of any *adinity*" (LI 203).

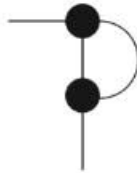


FIGURE 3.19 Construction of a dyad from two triads

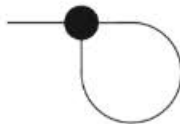


FIGURE 3.20 Construction of a monad from a triad

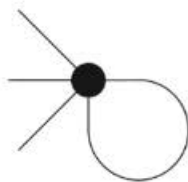


FIGURE 3.21 Construction of a triad from a pentad

Peirce has a reply to these worries. With respect to the first prong, he states that a triad "connects three objects, *A*, *B*, *C*, however indefinite *A*, *B*, and *C* may be. There must, then, be one of the three, at least, say *C*, which establishes a relation between the other two, *A* and *B*. The result is that *A* and *B* are in dyadic relation, and *C* may be ignored, even if it cannot be supposed absent" (EP 2:364–365, 1905). Peirce's reference to what is ignored is important here. His point is that we can attend to the relation of *A* and *B* without considering the relation of *A* and *B* to *C*. When we do this, we engage in a sort of attentive analysis, what Peirce in "On a New List" calls *prescission* and about which I shall have more to say in chapter 5.

When we engage in this sort of attentive analysis of the triadic propositional form, we see that a dyad is nevertheless present in every triad even though it is a part of that triadic propositional form. As Peirce states, the dyad is an ingredient of the triadic relation. We can abstract the dyad from the triad. Peirce's claim is that even though we cannot attend to a dyadic feature of a triad (*A* and *B*) without supposing it to be part of a triad, we can ignore a feature of the triadic construction and so recognize that there is a dyad as a part of the triad. This ought to be fairly obvious from the earlier reflections on *teridentity*. If *x* is identical to both *y* and *z* (*teridentity*), then *x* is identical to *y*. We can consider the identity of *x* and *y* without considering the identity of *x* and *y* to *z*.

This line of reply also applies to the second prong of the objection. Take any pentad. Just as we decomposed tetrads into two triadic propositional forms, we can decompose any pentad into three triads. We can then consider each of the triads on its own, independently of its relation to the pentad. When we do so, we see that a triad is nevertheless a logical ingredient of every pentad. Moreover, we see that we can construct any pentad from triads. Consequently, it is necessary to posit triadic propositional forms as ingredients of every pentad, but it is not necessary to posit pentadic propositional forms since they can be constructed from triads.

Furthermore, every dyad has as a logical ingredient monads, and every triad has as a logical ingredient dyads and monads. Notice that here we find the same *prescindibility* relations among monads, dyads, and triads that Peirce had first described in "On a New List" and that were discussed in the previous chapter. Just as every correlative function of the copula also involves the grounding function, so every dyad involves a monad as a logical ingredient separable by attending to the monad. Moreover, just as every representational function of the copula also involves the grounding and correlative functions, so too every triad also involves monads and dyads. However, once again, whereas that derivation in "On a New List" was based on the functions of the copula, the derivation here is based on Peirce's mature logic of relatives.

The upshot for Peirce is that we cannot suppose there to be triads without also recognizing in triads dyadic and monadic elements by ignoring one or two of the lines of the triad. Moreover, we cannot suppose there to be pentads

without recognizing in them triadic, dyadic, and monadic elements by ignoring one or more of the lines. Cannot this same point be used to show that there are tetradic ingredients in every pentad? The answer is yes. But recall, also, that we can construct tetrads from—and decompose tetrads into—triads. Hence, those tetradic elements can be decomposed into triadic ingredients. In contrast, we cannot construct triads from dyads, nor can we construct dyads from monads.

From Monads, Dyads, and Triads to Firstness, Secondness, and Thirdness

There is one more step to take in this discussion. It is to take the basic propositional forms of being monadic, being dyadic, and being triadic and to treat them as substances. When we do so, they become firstness, secondness, and thirdness.⁵ These are Peirce's purely formal and logical categories. To get his categories of firstness, secondness, and thirdness, we treat the basic propositional forms as substances or things by adding the suffix “-ness” to them, much as we transform the predicate term being red to redness. Peirce calls this practice of transforming predicate terms such as being red into secondary substances such as redness hypostatic abstraction, and he regards it as essential to mathematics and the sciences (see chapter 5; CP 5.449, 1905, NEM 3[2]:917, 1904, CP 4.332, 1905, PPM 133, 1903; and see Zeman 1982 and Legg 1999). When we perform hypostatic abstraction on the sorts of basic propositional forms there are, we get firstness, secondness, and thirdness. These are Peirce's mature formal and logical categories. I emphasize that these are formal categories because, as I will show in later chapters, Peirce also concedes that there is a second set of material categories. I also emphasize that they are logical because, consistent with Peirce's commitment to the modified Kantian Insight as discussed in chapter 1, they ought to have phenomenological correspondents. One question I shall take up in the coming chapters is what those correspondents are.

Before continuing, a word about medads is in order. It is, hopefully, clear that medads are not basic propositional forms. They are formed by the compounding of other propositional forms, such as two monads, or from the identity of the logical subjects of higher-adicity propositional forms, such as the graph for “ x is identical to x ” (see figure 3.22).

Peirce does not make use of the notion of a medad in his phenomenology, and I am here suggesting why that is so (though see Hausman 1990 and 1993,



FIGURE 3.22 Construction of a medad from a dyad (i.e., x is identical to itself)

121–124). Since Peirce accepts the modified Kantian Insight and medads are not basic propositional forms, zeroness (so to speak) will not be a purely formal and logical category. Moreover, there can be no phenomenon that is medadic because it would be nothing at all, “*absolutely* instantaneous, thunderless, unremembered, and *altogether* without effect” (CP 1.292, 1906, emphases added). Presumably for this reason, Peirce holds that the only way we can even talk about a medad is by treating it as a monad: “[I]n the case of the Medad . . . the possibility of scribing the Graph upon an Area is the only Valency the Spot has,—the only circumstance that brings it and other thoughts together. For this reason, we can, without other than a Verbal inconsistency, due to the incompleteness of our Terminology, speak of a Medad as a Monad” (NEM 4:322, 1906).

Degeneracy and the Ingredient Relations

Before concluding, a few comments on aspects of Peirce's categoriology that can cause some perplexity are in order. The first issue is that Peirce sometimes writes of degenerate secondness and thirdness (see, for example, CP 1.473, 1896, and CP 1.521–544, 1903). The language of degeneracy is borrowed from mathematics. Consider, for example, a triangle. If we imagine a triangle, we think of each of the angles as measuring some positive, nonzero quantity (e.g., an isosceles right triangle that has angles of ninety, forty-five, and forty-five degrees). Suppose, though, that one of the angles of a triangle were zero; if it were, we would have a line. This is a degenerate case of a triangle. Similarly, we think of circles as having some radius of a positive, nonzero quantity. If a circle had a radius of zero, we would be left with a point, and this is a degenerate case of a circle. Analogously, we can have degenerate secondness if we imagine that the two subjects of a dyadic propositional form are the same, for example, *x* is identical to itself, diagrammed earlier. This is not genuine secondness because the logical subjects are not in any way distinct from each other (see Kruse 1991). Similarly, we can have degenerate thirdness if two of the logical subjects are identical to each other (as in the construction of a monad from a triad earlier) or if three of them are identical to each other (e.g., *x* is identical to itself and itself). I should note that although I have been treating all of the dots for nonmonadic terms as standing for the relation of identity, it does not follow that the forms amount to no more than “*x* is identical to *x*,” for the lines of identity stand for different variables and may attach to different other predicate terms. Accordingly, when a dyad is not a medad formed from one dyad (as with *x* is identical to *x*) it has the form of *x* is identical to *y*. If that is not obvious, consider that “Hesperus is identical to Phosphorus” tells us something that “Hesperus is identical to Hesperus” does not.