sh_transform-Copy1

January 30, 2022

0.0.1 Solving Poisson Equation on S^2: $\Delta u = f$

[1]:
```
import my_backends
from my_backends.ducc0_wrapper import *
import my_shcoeffs
from my_shcoeffs import SHCoeffs
## my_shcoeffs contains expand_adjoint_analysis
```

```
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plmbar.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plmbar_d1.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plbar.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plbar_d1.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plmon.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plmon_d1.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plon.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plon_d1.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plmschmidt.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plmschmidt_d1.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plschmidt.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plschmidt_d1.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plegendrea.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plegendrea_d1.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/plegendre.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
```

reu-2021/PDPINN/doc/plegendre_d1.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shexpanddh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/makegriddh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shexpanddhc.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/makegriddhc.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shglq.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shexpandglq.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/makegridglq.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shexpandglqc.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/makegridglqc.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/glqgridcoord.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shexpandlsq.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shexpandwlsq.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/makegrid2d.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/makegridpoint.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/makegridpointc.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shmultiply.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shread2.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shread2error.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shreadjpl.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shreadjplerror.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shcilmtovector.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shvectortocilm.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shcilmtocindex.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-

reu-2021/PDPINN/doc/shcindextocilm.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shrtoc.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shctor.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shadmitcorr.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shconfidence.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shmultitaperse.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shmultitapercse.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shlocalizedadmitcorr.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shreturntapers.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shreturntapersm.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/computedm.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/computedg82.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shfindlwin.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shbiask.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shmtcouplingmatrix.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shbiasadmitcorr.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shmtdebias.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shmtvaropt.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shsjkpg.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shmultitapermaskse.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shmultitapermaskcse.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shreturntapersmap.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/shbiaskmask.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-reu-2021/PDPINN/doc/computedmap.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-

```
reu-2021/PDPINN/doc/curve2mask.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/shbias.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/sphericalcapcoef.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/djpi2.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/shrotatecoef.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/shrotaterealcoef.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/makegravgriddh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/makegravgradgriddh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/makegeoidgriddh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/cilmplusdh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/cilmminusdh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/cilmplusrhohdh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/cilmminusrhohdh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/batohilmdh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/batohilmrhohdh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/downcontfilterma.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/downcontfiltermc.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/normalgravity.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/makemaggriddh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/makecirclecoord.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/makeellipsecoord.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/wigner3j.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/dhaj.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/makemaggradgriddh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
```

```
reu-2021/PDPINN/doc/shrotatetapers.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/slepiancoeffs.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/slepiancoeffstosh.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/shscouplingmatrix.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/shmtvar.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/shslepianvar.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/shscouplingmatrixcap.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/makegravgridpoint.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/makemaggridpoint.doc'
[Errno 2] No such file or directory: '/Users/aparnagupte/Documents/cornell-
reu-2021/PDPINN/doc/makegradientdh.doc'
```

[2]:
```python
import torch

N = 20
from Basis import SPHBasis
from Problem import Problem_Sphere_Poisson
from Model import Model
from Net import Sphere_Net, SPH_Sphere_Net
import seaborn as sns
import matplotlib.pyplot as plt
import math
from math import pi
from spherical_harmonics import *
from torch.distributions.uniform import Uniform
from numpy import arccos
import numpy as np

import pyshtools
from pyshtools.shio import shread
from pyshtools.expand import MakeGridDH
from pyshtools.expand import SHExpandDH
from pyshtools.spectralanalysis import spectrum

torch.manual_seed(0)
maxiter = 1000
problem = Problem_Sphere_Poisson()
```

[3]:
```
N = 20
azimuth = torch.linspace(0.0001, pi, N, requires_grad=False)
polar = torch.linspace(0, 2*pi, 2*N, requires_grad=False)

azimuth, polar = torch.meshgrid(azimuth, polar)
location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1, 1)], dim=1)
location.shape
```

[3]:
```
torch.Size([800, 2])
```

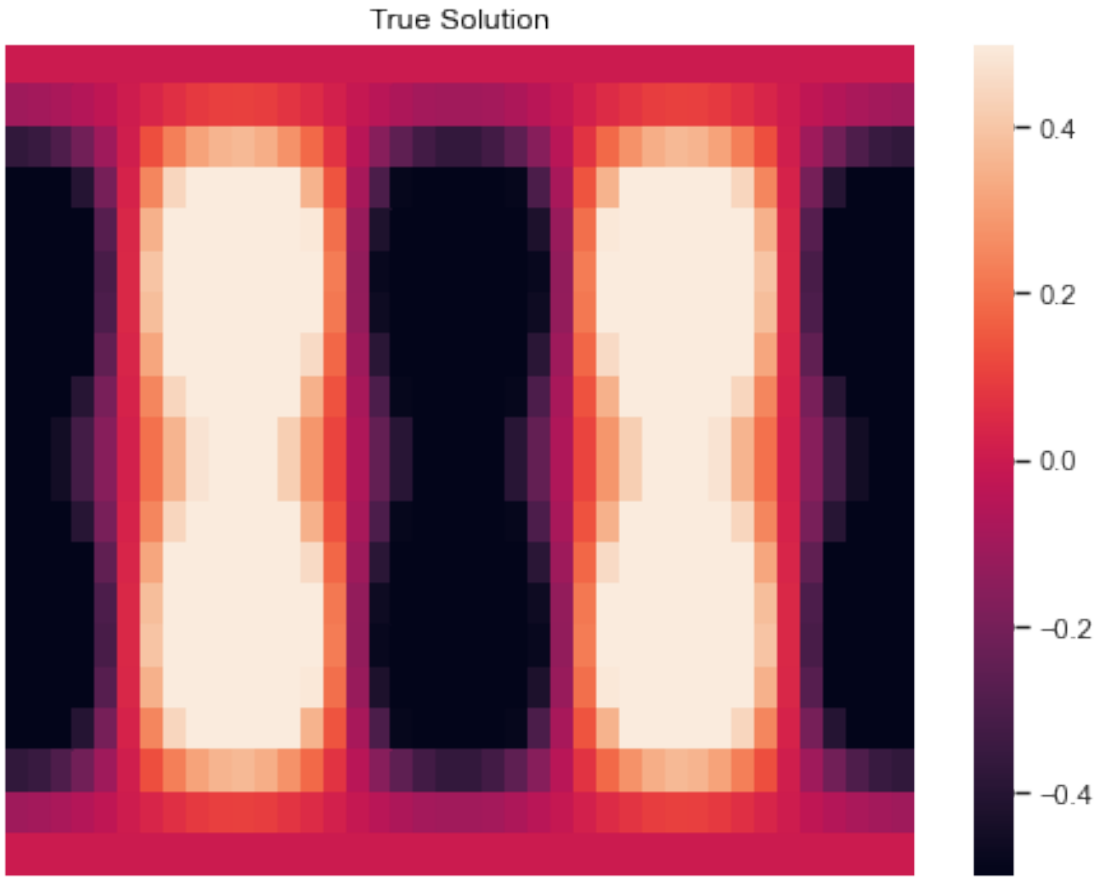
[4]:
```
def construct_model(net):
    class Poisson1dModel(Model):
        def add_loss_history(self):
#             self.loss_history.append([self.bc_loss, self.pde_loss, self.
 ↪predict_error_value])
            self.loss_history.append([self.pde_loss, self.predict_error_value])

        def __init__(self):
            super().__init__(problem=problem, net=net, maxiter=maxiter)

        def inner_sample(self, N=N):
            azimuth = torch.linspace(0.0001, pi, N, requires_grad=False)
            polar = torch.linspace(0, 2*pi, 2*N, requires_grad=False)

            azimuth, polar = torch.meshgrid(azimuth, polar)
            location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,␣
 ↪1)], dim=1)
            return location # numpy ndarray with shape (N, 2*N)

        def bc_sample(self):
            return torch.tensor([[1., 1.]]) # arbitrary point is okay?

        def init_sample(self):
            pass

        def plot(self, net):
            azimuth = torch.linspace(0, math.pi, N, requires_grad=False)
            polar = torch.linspace(0, 2 * math.pi, 2*N, requires_grad=False)
            azimuth, polar = torch.meshgrid(azimuth, polar)
            location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,␣
 ↪1)], dim=1)
#             location = self.inner_sample()
            value = net(location) # predicted solution

            value = value.reshape((N, 2*N))

            fig, ax = plt.subplots(1, 1, figsize=(8, 6))
```

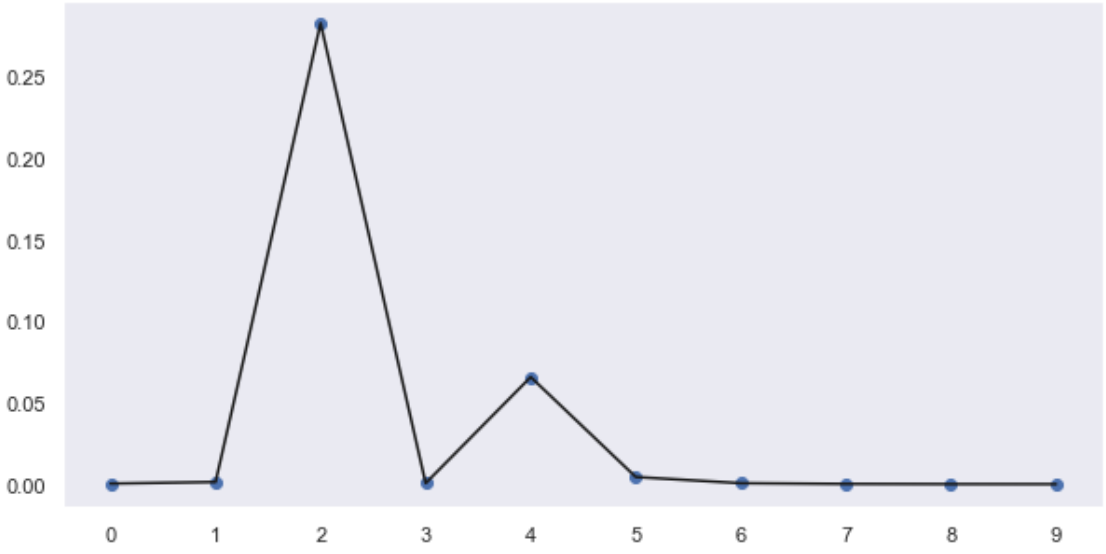
```
            sns.set()
            sns.heatmap(value.detach().numpy(), ax=ax, vmin=-0.5, vmax=0.5,␣
↪cbar=True)
            ax.set_xticks([])
            ax.set_yticks([])
            ax.set_title("Predicted Solution")
            plt.show()

            self.spherical_harmonic_transform(net)

        def spherical_harmonic_transform(self, net):
            azimuth = torch.linspace(0, math.pi, N, requires_grad=False)
            polar = torch.linspace(0, 2 * math.pi, 2*N, requires_grad=False)
            azimuth, polar = torch.meshgrid(azimuth, polar)
            location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,␣
↪1)], dim=1)
            function_grid = net(location)
            function_grid = function_grid.reshape((N, 2*N))

            coefficients = SHExpandDH(function_grid.detach().numpy(),␣
↪sampling=2)
            nl = coefficients.shape[1]
            ls = np.arange(nl)[:10]

            power_per_l = spectrum(coefficients)[:10]
            fig, ax = plt.subplots(1, 1, figsize=(len(ls), 5))
            ax.plot(ls, power_per_l, 'bo')
            ax.plot(ls, power_per_l, 'k-')
            plt.xticks(range(len(ls)))
            ax.grid()
            plt.show()


        def plot_true(self):
            azimuth = torch.linspace(0, math.pi, N, requires_grad=False)
            polar = torch.linspace(0, 2 * math.pi, 2*N, requires_grad=False)
            azimuth, polar = torch.meshgrid(azimuth, polar)
            location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,␣
↪1)], dim=1)
            value = true_solution(location[:, 0:1], location[:, 1:])

            value = value.reshape((N, 2*N))

            fig, ax = plt.subplots(1, 1, figsize=(8, 6))
            sns.set()
            sns.heatmap(value.detach().numpy(), ax=ax, vmin=-0.5, vmax=0.5,␣
↪cbar=True)
```

```
            ax.set_xticks([])
            ax.set_yticks([])
            ax.set_title("True Solution")
            plt.show()

            self.sht_value(value)

        def sht_value(self, function_grid):
            function_grid = function_grid.reshape((N, 2*N))

            coefficients = SHExpandDH(function_grid.detach().numpy(),␣
↪sampling=2)
            nl = coefficients.shape[1]
            ls = np.arange(nl)[:10]

            power_per_l = spectrum(coefficients)[:10]
            fig, ax = plt.subplots(1, 1, figsize=(len(ls), 5))
            ax.plot(ls, power_per_l, 'bo')
            ax.plot(ls, power_per_l, 'k-')
            plt.xticks(range(len(ls)))
            ax.grid()
            plt.show()

        def post_process(self, ax=None):
            if ax is None:
                for losses in self.loss_history:
                    for i in range(3):
                        losses[i].detach().numpy()
                plt.plot(self.loss_history)
                plt.yscale('log')
                plt.legend(('BC loss', 'pde loss', 'predict error'))
                plt.show()
            else:
                for losses in self.loss_history:
                    for i in range(3):
                        losses[i].detach().numpy()
                ax.plot(self.loss_history)
                ax.set_yscale('log')
                ax.set_ylim(1e-4, 100)
                ax.legend(('BC loss', 'pde loss', 'predict error'))

        def predict_error(self):
            coor = self.inner_sample()
            true = self.problem.ground_truth(coor)
            predict = self.net(coor)
            predict_error = self.pde_loss_f(true, predict)
            return predict_error
```

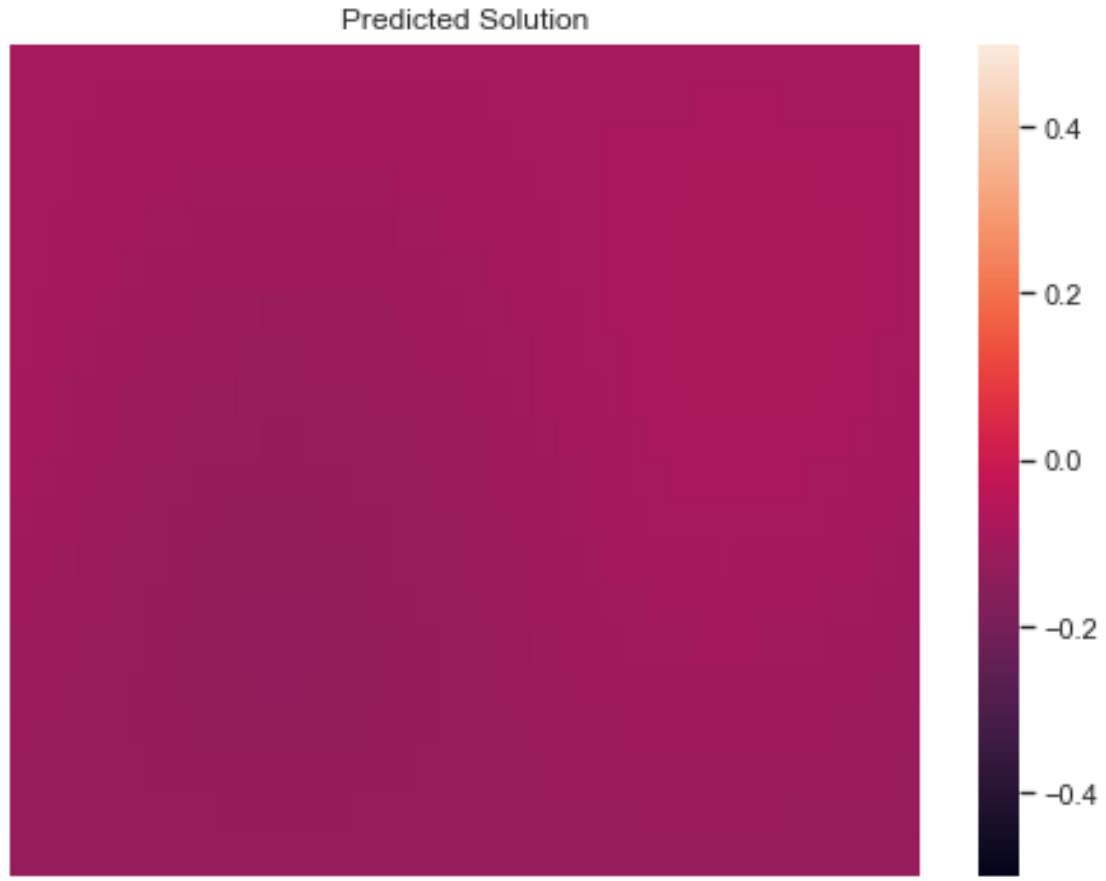
```
        def train(self, ax100=None):
            self.plot_true()
            problem = self.problem
            net = self.net
            opt = self.opt
            maxiter = self.maxiter

            for iter in range(maxiter):
                net.zero_grad()

                coor_inner = self.inner_sample().detach().requires_grad_(True)
#                 coor_inner = self.inner_sample().requires_grad_(True)
                infer_value_inner = net(coor_inner)
#                 truth_inner, predict_inner = problem.pde(coor_inner,␣
↪infer_value_inner)
                predict_inner, truth_inner = problem.pde(coor_inner,␣
↪infer_value_inner)
                ## order, is the lhs even the predicted value???
                ## reason the loss function is going up could be because we are␣
↪computing the negative gradient, which could happen by switching the order␣
↪of y, y_pred in orward and backward functions of the loss.

                self.pde_loss = self.pde_loss_f(predict_inner, truth_inner)

                bc_samples = self.bc_sample()
                if bc_samples is None:
                    self.bc_loss = torch.tensor(0.)
                else:
                    coor_bc = bc_samples.detach().requires_grad_(True)
                    infer_value_bc = net(coor_bc)
                    truth_bc, predict_bc = problem.bound_condition(coor_bc,␣
↪infer_value_bc)
                    self.bc_loss = self.bc_loss_f(predict_bc, truth_bc)

                init_samples = self.init_sample()
                if init_samples is None:
                    self.init_loss = torch.tensor(0.)
                else:
                    coor_init = init_samples.detach().requires_grad_(True)
                    infer_value_init = net(coor_init)
                    truth_init, predict_init = problem.
↪bound_condition(coor_init, infer_value_init)
                    self.init_loss = self.bc_loss_f(predict_init, truth_init)

                self.predict_error_value = self.predict_error()
                self.total_loss = self.pde_loss + self.bc_loss + self.init_loss
```

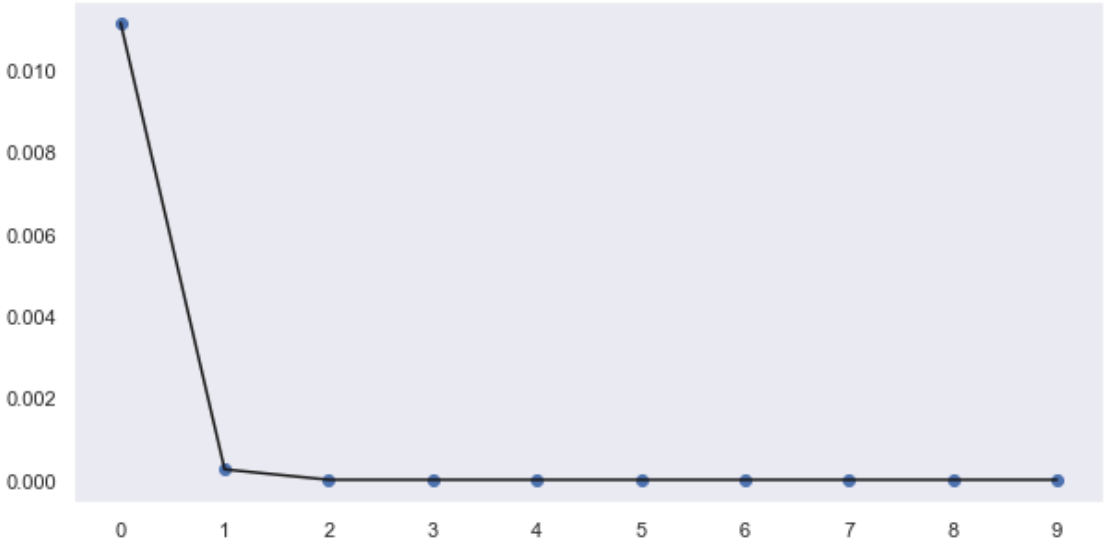
```
                self.add_loss_history()
#                 self.total_loss.register_hook(lambda grad: print('gradient',␣
 ↪grad))
                self.total_loss.backward()

                opt.step()
                opt.zero_grad()
#                 scheduler.step()

                if iter % (maxiter // 20) == 0:
                    print("iteration {}: loss = {}, pde loss = {}".format(iter,␣
 ↪self.total_loss, self.pde_loss))

                if iter % int(maxiter / 5) == 0:
                    self.plot(net)
            print("iteration {}: loss = {}, pde loss = {}".format(iter, self.
 ↪total_loss, self.pde_loss))
            self.plot(net)

#             self.post_process()

    return Poisson1dModel()

model = construct_model(Sphere_Net([3, 150, 50, 50, 50, 16, 1]))
model.train()
```





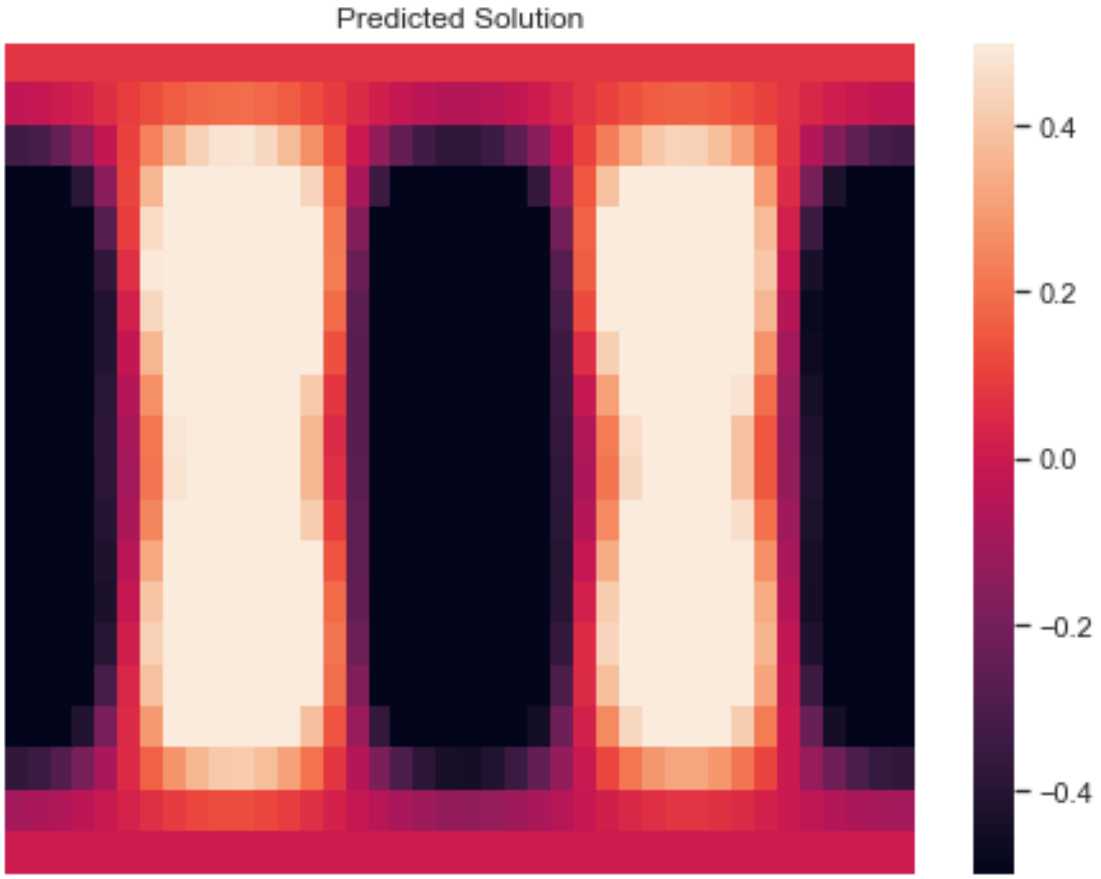

```
iteration 0: loss = 361522.2924683019, pde loss = 361521.9826760515
```

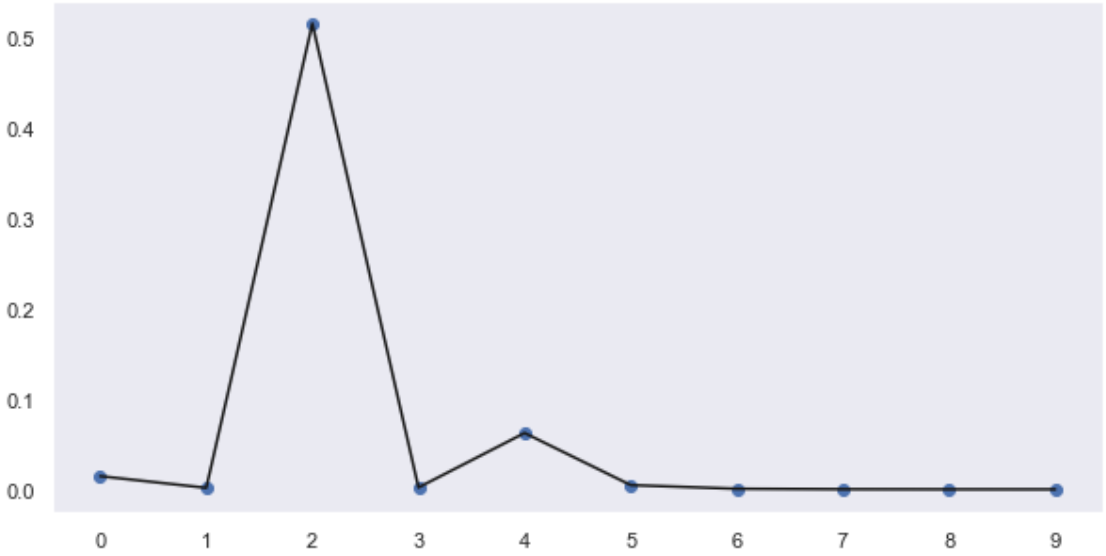




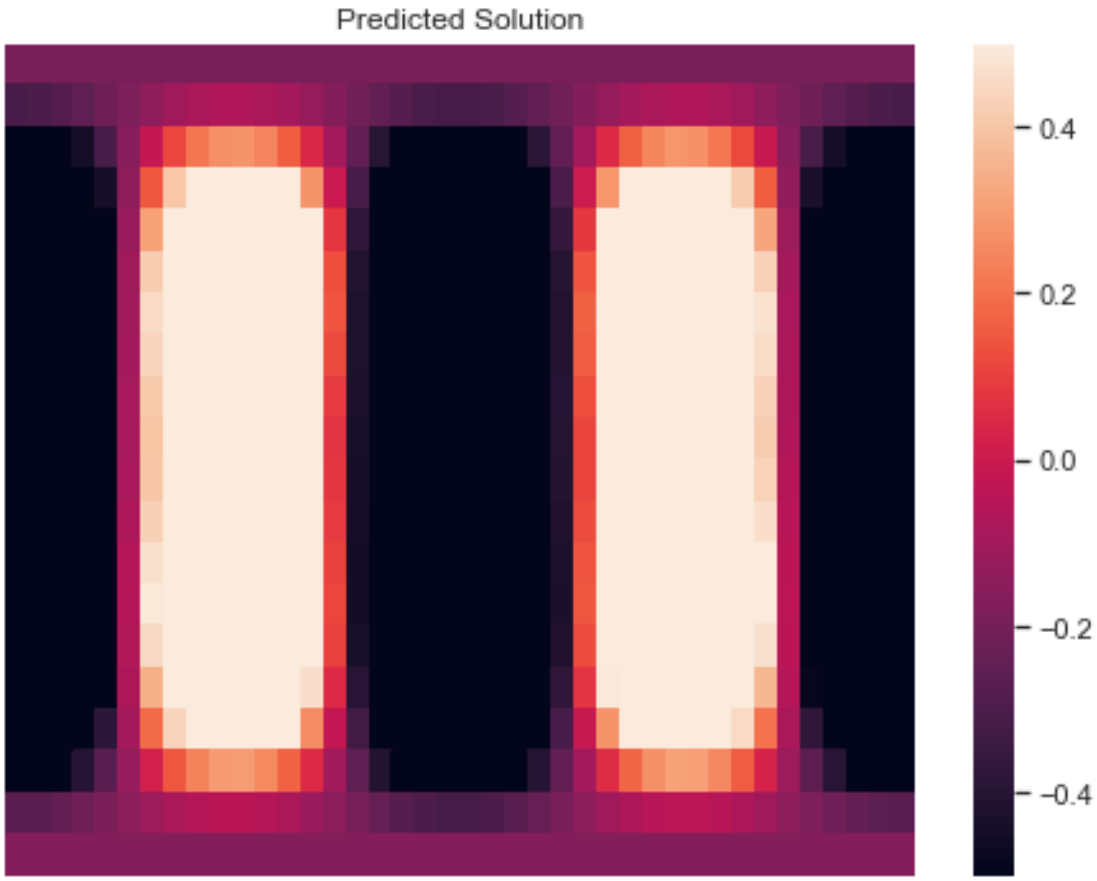

```
iteration 50: loss = 98457.04244040311, pde loss = 98456.67537255943
```

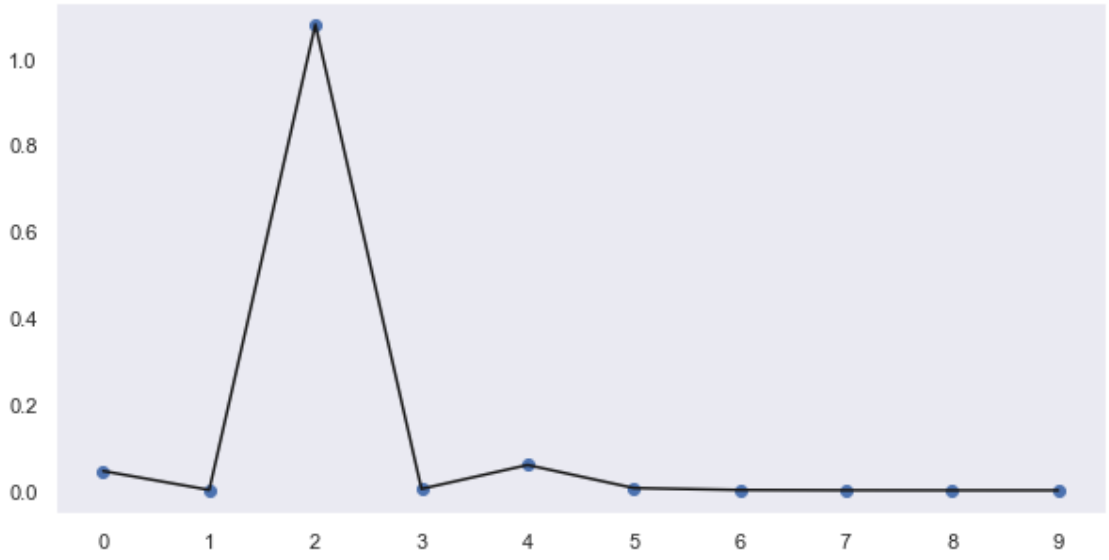
```
iteration 100: loss = 10384.206067373852, pde loss = 10384.180668527137
iteration 150: loss = 4548.434056686272, pde loss = 4548.42947718458
iteration 200: loss = 2160.8990709491754, pde loss = 2160.8986303934926
```



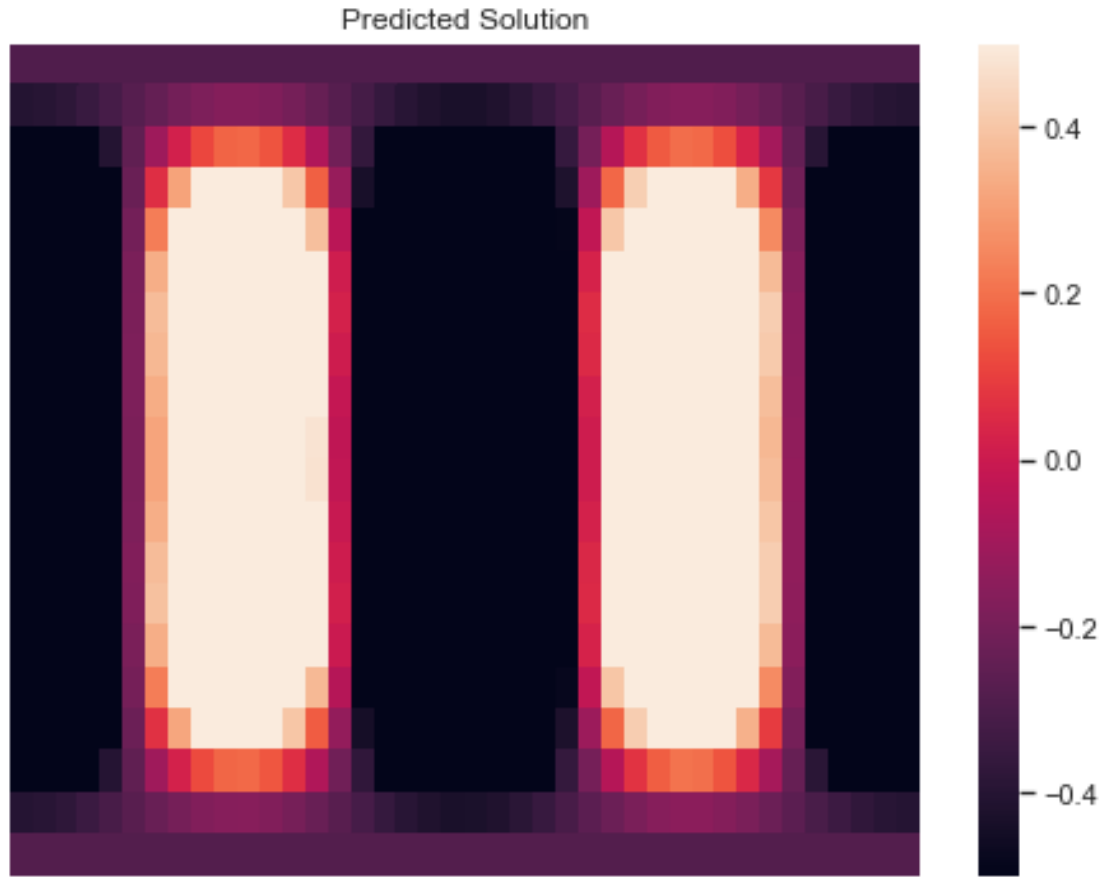


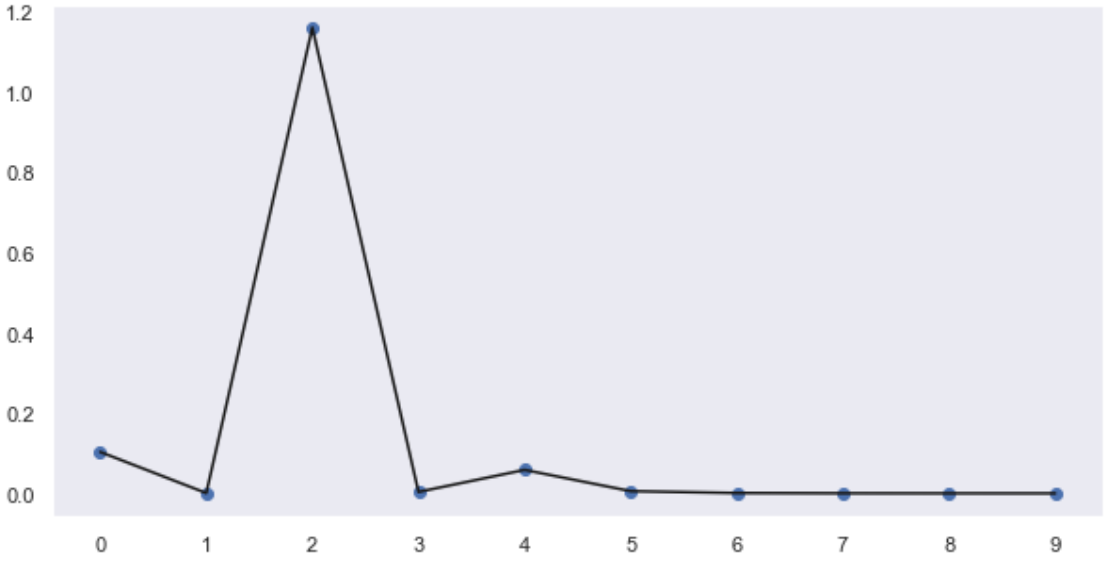
```
iteration 250: loss = 1286.7726562200519, pde loss = 1286.7548185820165
iteration 300: loss = 592.1937526877683, pde loss = 592.1658923499
iteration 350: loss = 262.5455967690713, pde loss = 262.5295502711005
iteration 400: loss = 86.20073048365018, pde loss = 86.18131189574203
```



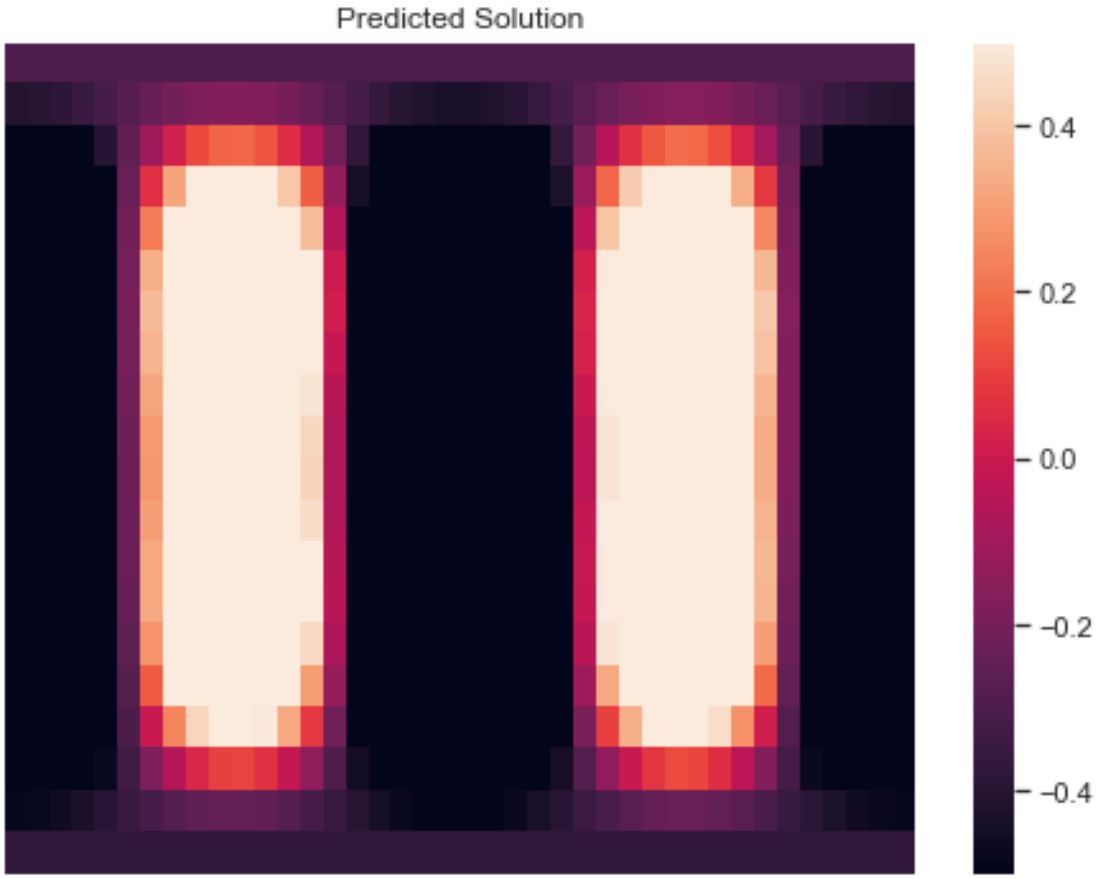


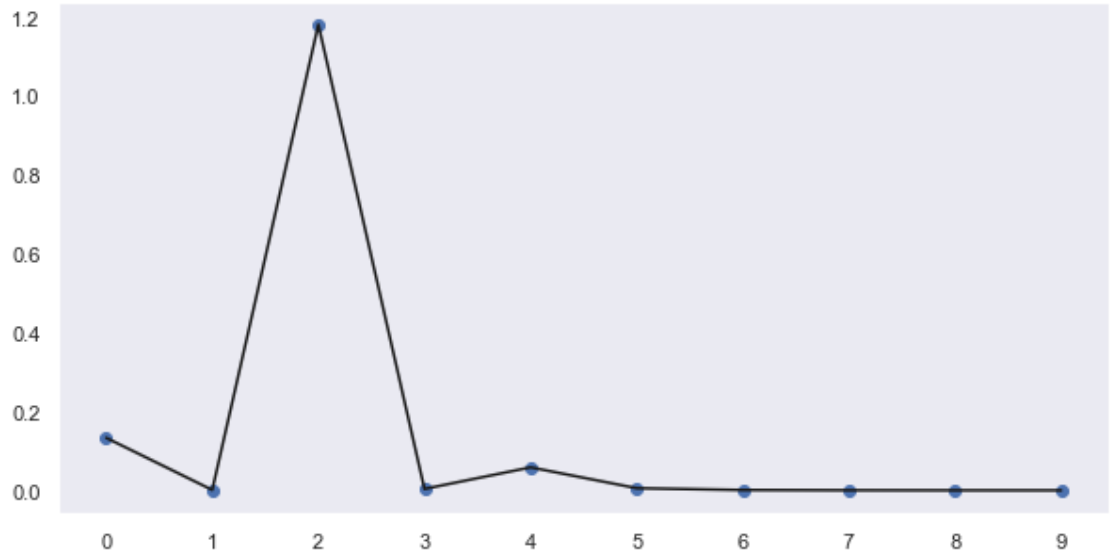

```
iteration 450: loss = 17.18527183475359, pde loss = 17.174316727355258
iteration 500: loss = 10.3922890533281, pde loss = 10.385109188645842
iteration 550: loss = 18.897870045591212, pde loss = 18.89261277715496
iteration 600: loss = 52.351558180265656, pde loss = 52.34711735948973
```


Predicted Solution
0.4
0.2
0.0
−0.2
−0.4

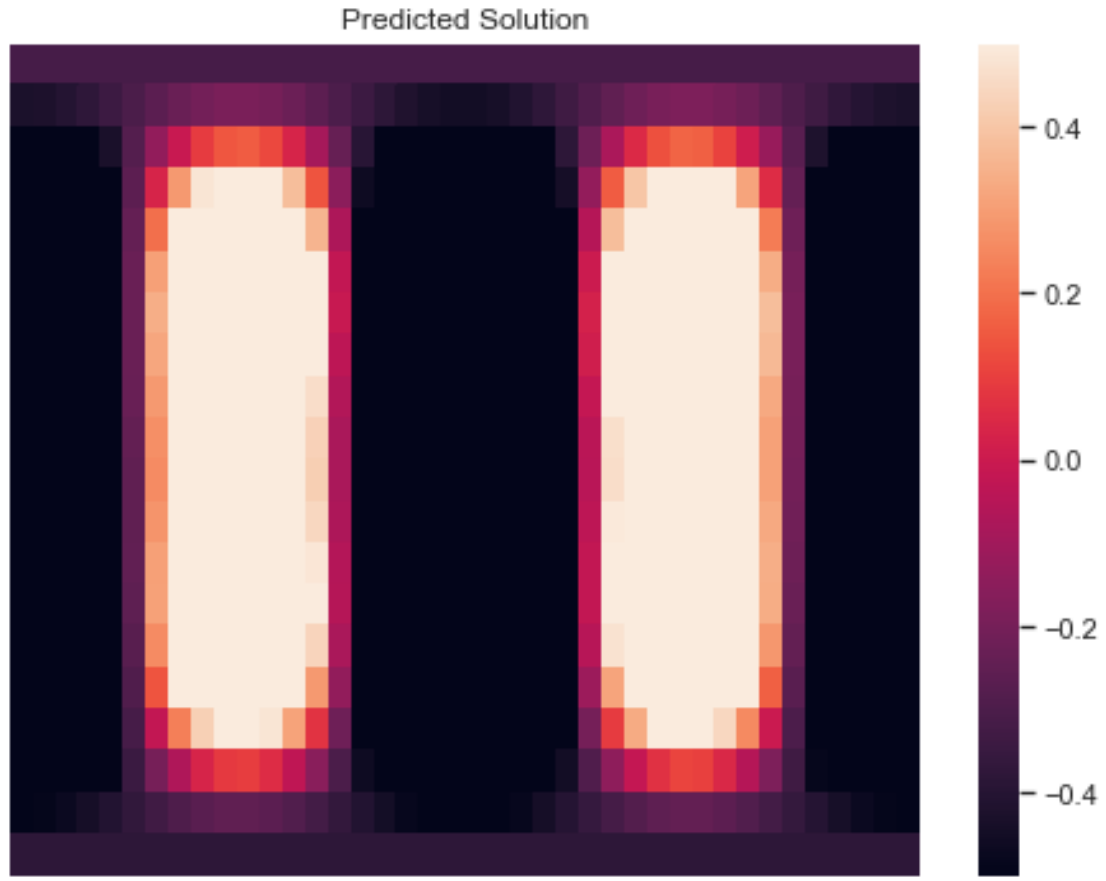

1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
8
9

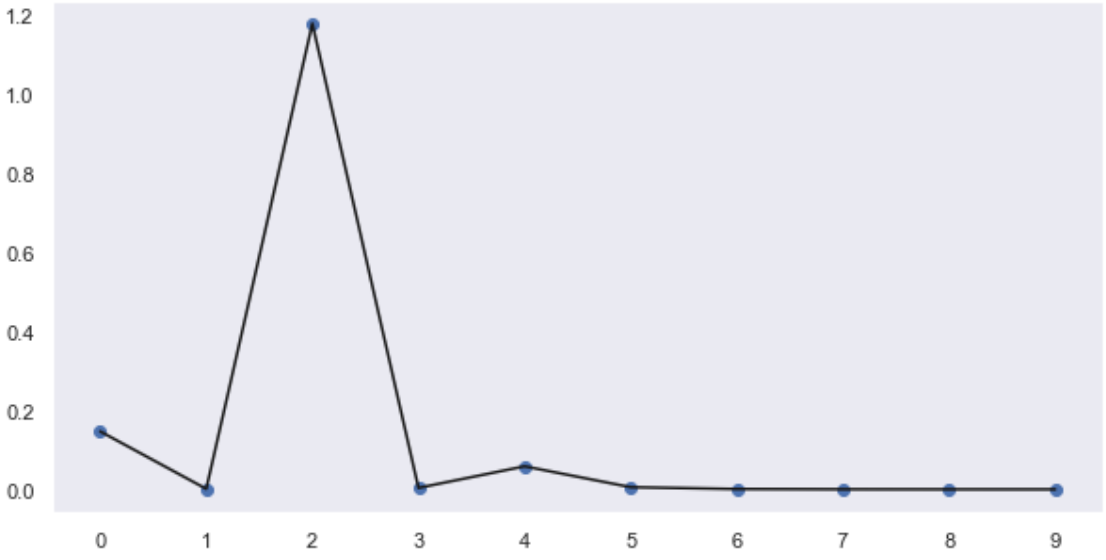
```
iteration 650: loss = 7.715222550287824, pde loss = 7.713912846950795
iteration 700: loss = 123.13300599708396, pde loss = 123.1326942322828
iteration 750: loss = 276.1689212169499, pde loss = 276.168601079326
iteration 800: loss = 109.41288907504591, pde loss = 109.41288683269696
```






```
iteration 850: loss = 62.22911802640285, pde loss = 62.229115153757924
iteration 900: loss = 34.48842689318271, pde loss = 34.488401283980146
iteration 950: loss = 92.7329928478175, pde loss = 92.73299283025305
iteration 999: loss = 10.400129987294267, pde loss = 10.399922246742708
```


Predicted Solution
0.4
0.2
0.0
−0.2
−0.4


1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
8
9