sh_transform-Copy1

January 30, 2022

0.0.1 Solving Poisson Equation on S^2: $\Delta u = f$

[1]:
```
import my_backends
from my_backends.ducc0_wrapper import *
import my_shcoeffs
from my_shcoeffs import SHCoeffs
## my_shcoeffs contains expand_adjoint_analysis
```

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```

[2]:
```python
import torch

N = 20
from Basis import SPHBasis
from Problem import Problem_Sphere_Poisson
from Model import Model
from Net import Sphere_Net, SPH_Sphere_Net
import seaborn as sns
import matplotlib.pyplot as plt
import math
from math import pi
from spherical_harmonics import *
from torch.distributions.uniform import Uniform
from numpy import arccos
import numpy as np

import pyshtools
from pyshtools.shio import shread
from pyshtools.expand import MakeGridDH
from pyshtools.expand import SHExpandDH
from pyshtools.spectralanalysis import spectrum

torch.manual_seed(0)
maxiter = 20000
problem = Problem_Sphere_Poisson()
```

[3]:
```
N = 20
azimuth = torch.linspace(0.0001, pi, N, requires_grad=False)
polar = torch.linspace(0, 2*pi, 2*N, requires_grad=False)

azimuth, polar = torch.meshgrid(azimuth, polar)
location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1, 1)], dim=1)
location.shape
```

[3]:
```
torch.Size([800, 2])
```

[4]:
```
def construct_model(net):
    class Poisson1dModel(Model):
        def add_loss_history(self):
#             self.loss_history.append([self.bc_loss, self.pde_loss, self.
 ↪predict_error_value])
            self.loss_history.append([self.pde_loss, self.predict_error_value])

        def __init__(self):
            super().__init__(problem=problem, net=net, maxiter=maxiter)

        def inner_sample(self, N=N):
            azimuth = torch.linspace(0.0001, pi, N, requires_grad=False)
            polar = torch.linspace(0, 2*pi, 2*N, requires_grad=False)

            azimuth, polar = torch.meshgrid(azimuth, polar)
            location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,␣
 ↪1)], dim=1)
            return location # numpy ndarray with shape (N, 2*N)

        def bc_sample(self):
            return torch.tensor([[1., 1.]]) # arbitrary point is okay?

        def init_sample(self):
            pass

        def plot(self, net):
            azimuth = torch.linspace(0, math.pi, N, requires_grad=False)
            polar = torch.linspace(0, 2 * math.pi, 2*N, requires_grad=False)
            azimuth, polar = torch.meshgrid(azimuth, polar)
            location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,␣
 ↪1)], dim=1)
#             location = self.inner_sample()
            value = net(location) # predicted solution

            value = value.reshape((N, 2*N))

            fig, ax = plt.subplots(1, 1, figsize=(8, 6))
```

```
            sns.set()
            sns.heatmap(value.detach().numpy(), ax=ax, vmin=-0.5, vmax=0.5,␣
↪cbar=True)
            ax.set_xticks([])
            ax.set_yticks([])
            ax.set_title("Predicted Solution")
            plt.show()

            self.spherical_harmonic_transform(net)

        def spherical_harmonic_transform(self, net):
            azimuth = torch.linspace(0, math.pi, N, requires_grad=False)
            polar = torch.linspace(0, 2 * math.pi, 2*N, requires_grad=False)
            azimuth, polar = torch.meshgrid(azimuth, polar)
            location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,␣
↪1)], dim=1)
            function_grid = net(location)
            function_grid = function_grid.reshape((N, 2*N))

            coefficients = SHExpandDH(function_grid.detach().numpy(),␣
↪sampling=2)
            nl = coefficients.shape[1]
            ls = np.arange(nl)[:10]

            power_per_l = spectrum(coefficients)[:10]
            fig, ax = plt.subplots(1, 1, figsize=(len(ls), 5))
            ax.plot(ls, power_per_l, 'bo')
            ax.plot(ls, power_per_l, 'k-')
            plt.xticks(range(len(ls)))
            ax.grid()
            plt.show()


        def plot_true(self):
            azimuth = torch.linspace(0, math.pi, N, requires_grad=False)
            polar = torch.linspace(0, 2 * math.pi, 2*N, requires_grad=False)
            azimuth, polar = torch.meshgrid(azimuth, polar)
            location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,␣
↪1)], dim=1)
            value = true_solution(location[:, 0:1], location[:, 1:])

            value = value.reshape((N, 2*N))

            fig, ax = plt.subplots(1, 1, figsize=(8, 6))
            sns.set()
            sns.heatmap(value.detach().numpy(), ax=ax, vmin=-0.5, vmax=0.5,␣
↪cbar=True)
```

```
            ax.set_xticks([])
            ax.set_yticks([])
            ax.set_title("True Solution")
            plt.show()

            self.sht_value(value)

        def sht_value(self, function_grid):
            function_grid = function_grid.reshape((N, 2*N))

            coefficients = SHExpandDH(function_grid.detach().numpy(),␣
↪sampling=2)
            nl = coefficients.shape[1]
            ls = np.arange(nl)[:10]

            power_per_l = spectrum(coefficients)[:10]
            fig, ax = plt.subplots(1, 1, figsize=(len(ls), 5))
            ax.plot(ls, power_per_l, 'bo')
            ax.plot(ls, power_per_l, 'k-')
            plt.xticks(range(len(ls)))
            ax.grid()
            plt.show()

        def post_process(self, ax=None):
            if ax is None:
                for losses in self.loss_history:
                    for i in range(3):
                        losses[i].detach().numpy()
                plt.plot(self.loss_history)
                plt.yscale('log')
                plt.legend(('BC loss', 'pde loss', 'predict error'))
                plt.show()
            else:
                for losses in self.loss_history:
                    for i in range(3):
                        losses[i].detach().numpy()
                ax.plot(self.loss_history)
                ax.set_yscale('log')
                ax.set_ylim(1e-4, 100)
                ax.legend(('BC loss', 'pde loss', 'predict error'))

        def predict_error(self):
            coor = self.inner_sample()
            true = self.problem.ground_truth(coor)
            predict = self.net(coor)
            predict_error = self.pde_loss_f(true, predict)
            return predict_error
```

```
        def train(self, ax100=None):
            self.plot_true()
            problem = self.problem
            net = self.net
            opt = self.opt
            maxiter = self.maxiter

            for iter in range(maxiter):
                net.zero_grad()

                coor_inner = self.inner_sample().detach().requires_grad_(True)
#                 coor_inner = self.inner_sample().requires_grad_(True)
                infer_value_inner = net(coor_inner)
#                 truth_inner, predict_inner = problem.pde(coor_inner,␣
↪infer_value_inner)
                predict_inner, truth_inner = problem.pde(coor_inner,␣
↪infer_value_inner)
                ## order, is the lhs even the predicted value???
                ## reason the loss function is going up could be because we are␣
↪computing the negative gradient, which could happen by switching the order␣
↪of y, y_pred in orward and backward functions of the loss.

                self.pde_loss = self.pde_loss_f(predict_inner, truth_inner)

                bc_samples = self.bc_sample()
                if bc_samples is None:
                    self.bc_loss = torch.tensor(0.)
                else:
                    coor_bc = bc_samples.detach().requires_grad_(True)
                    infer_value_bc = net(coor_bc)
                    truth_bc, predict_bc = problem.bound_condition(coor_bc,␣
↪infer_value_bc)
                    self.bc_loss = self.bc_loss_f(predict_bc, truth_bc)

                init_samples = self.init_sample()
                if init_samples is None:
                    self.init_loss = torch.tensor(0.)
                else:
                    coor_init = init_samples.detach().requires_grad_(True)
                    infer_value_init = net(coor_init)
                    truth_init, predict_init = problem.
↪bound_condition(coor_init, infer_value_init)
                    self.init_loss = self.bc_loss_f(predict_init, truth_init)

                self.predict_error_value = self.predict_error()
                self.total_loss = self.pde_loss + self.bc_loss + self.init_loss
```

```
                self.add_loss_history()
#                 self.total_loss.register_hook(lambda grad: print('gradient',␣
 ↪grad))
                self.total_loss.backward()

                opt.step()
                opt.zero_grad()
#                 scheduler.step()

                if iter % (maxiter // 20) == 0:
                    print("iteration {}: loss = {}, pde loss = {}".format(iter,␣
 ↪self.total_loss, self.pde_loss))

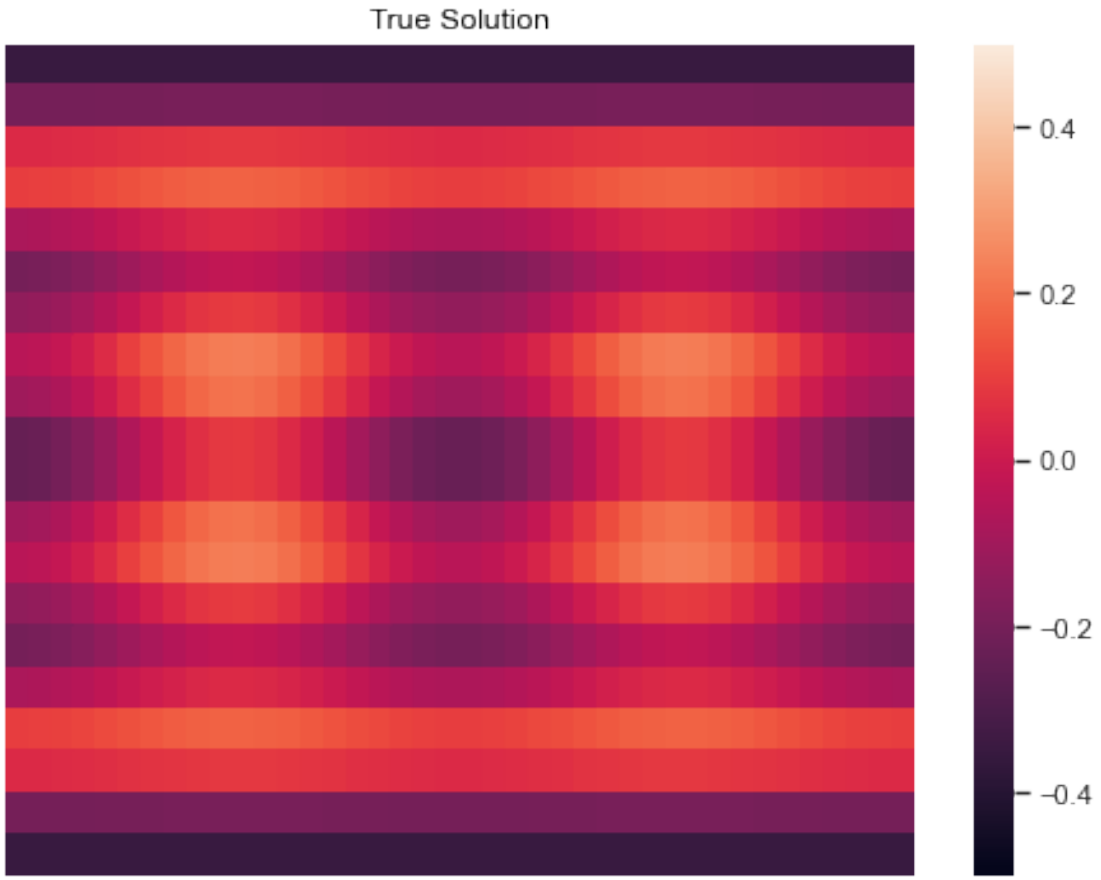
                if iter % int(maxiter / 5) == 0:
                    self.plot(net)

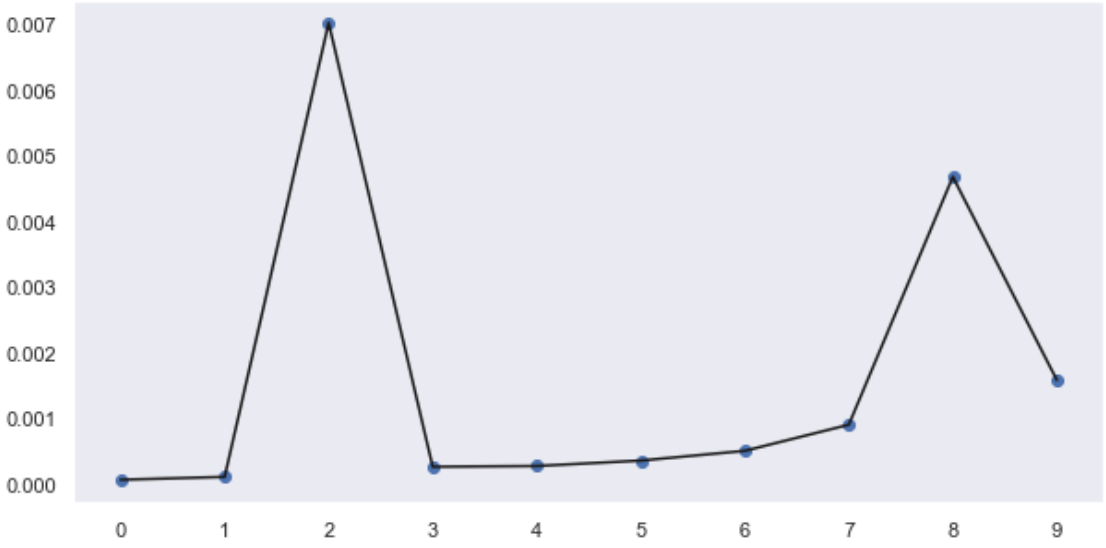
            self.plot(net)

#             self.post_process()

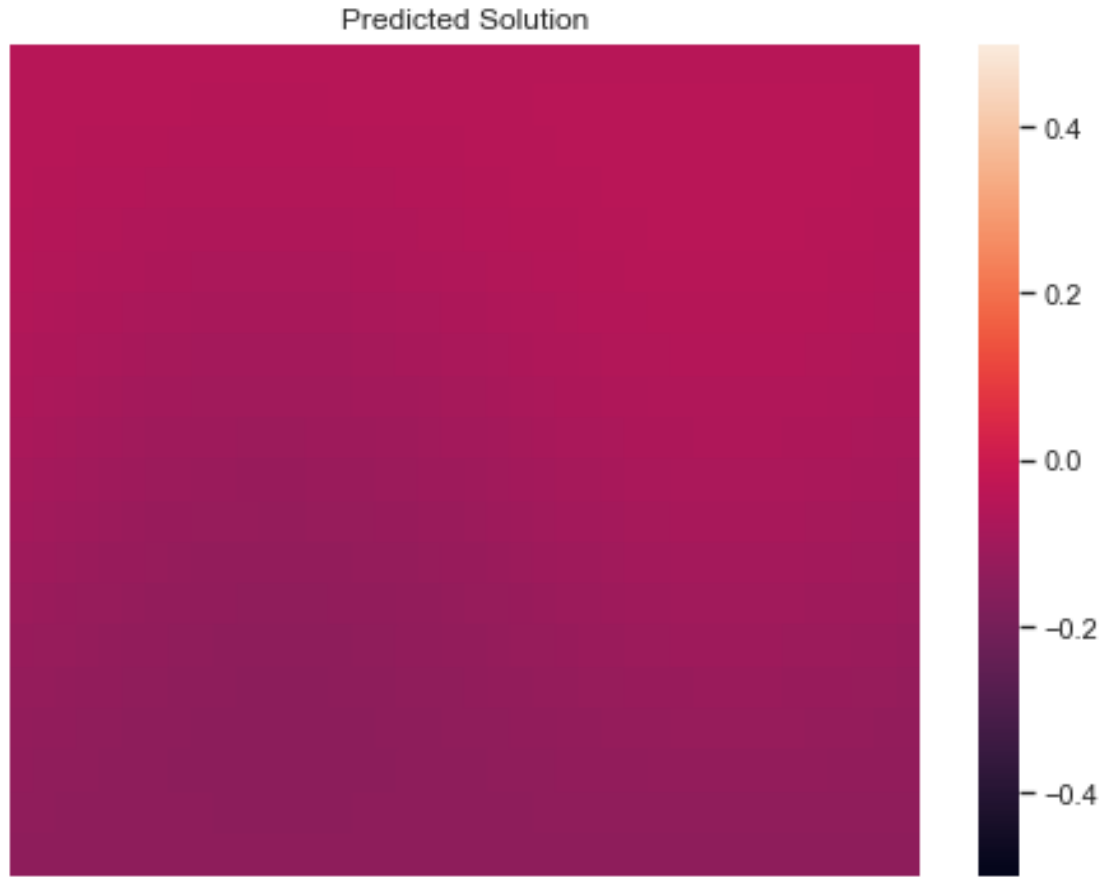
    return Poisson1dModel()

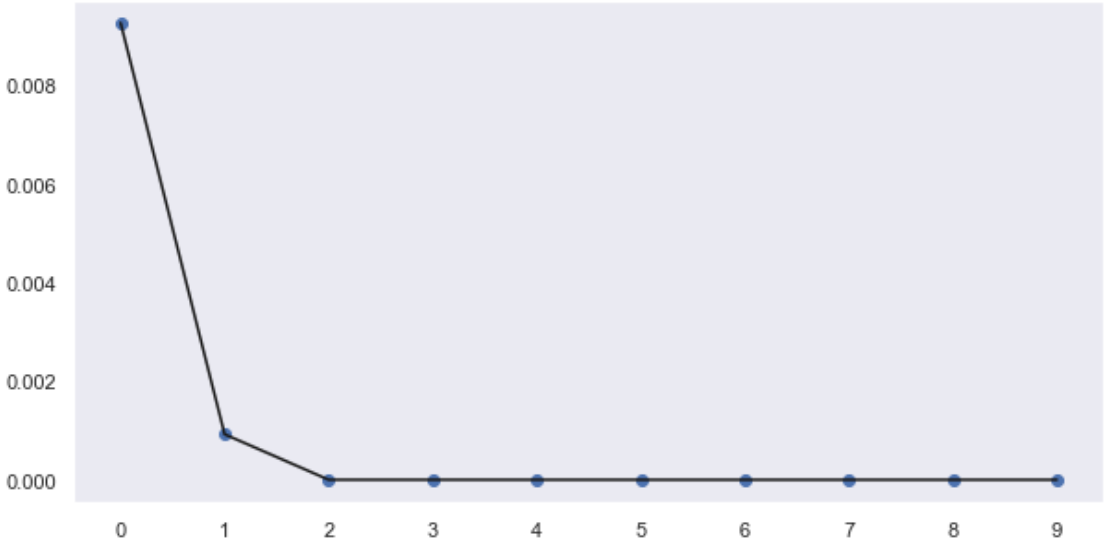
model = construct_model(Sphere_Net([3, 150, 50, 50, 50, 16, 1]))
model.train()
```






```
iteration 0: loss = 1.203897369349317, pde loss = 1.1832642231825155
```






```
iteration 1000: loss = 1.859887968009802, pde loss = 1.8531682972856782
```

```
iteration 2000: loss = 2.3484622926895553, pde loss = 2.3479531620374927
iteration 3000: loss = 1.522098811876158, pde loss = 1.5138988416382473
iteration 4000: loss = 1.3806398997265152, pde loss = 1.3805483812910044
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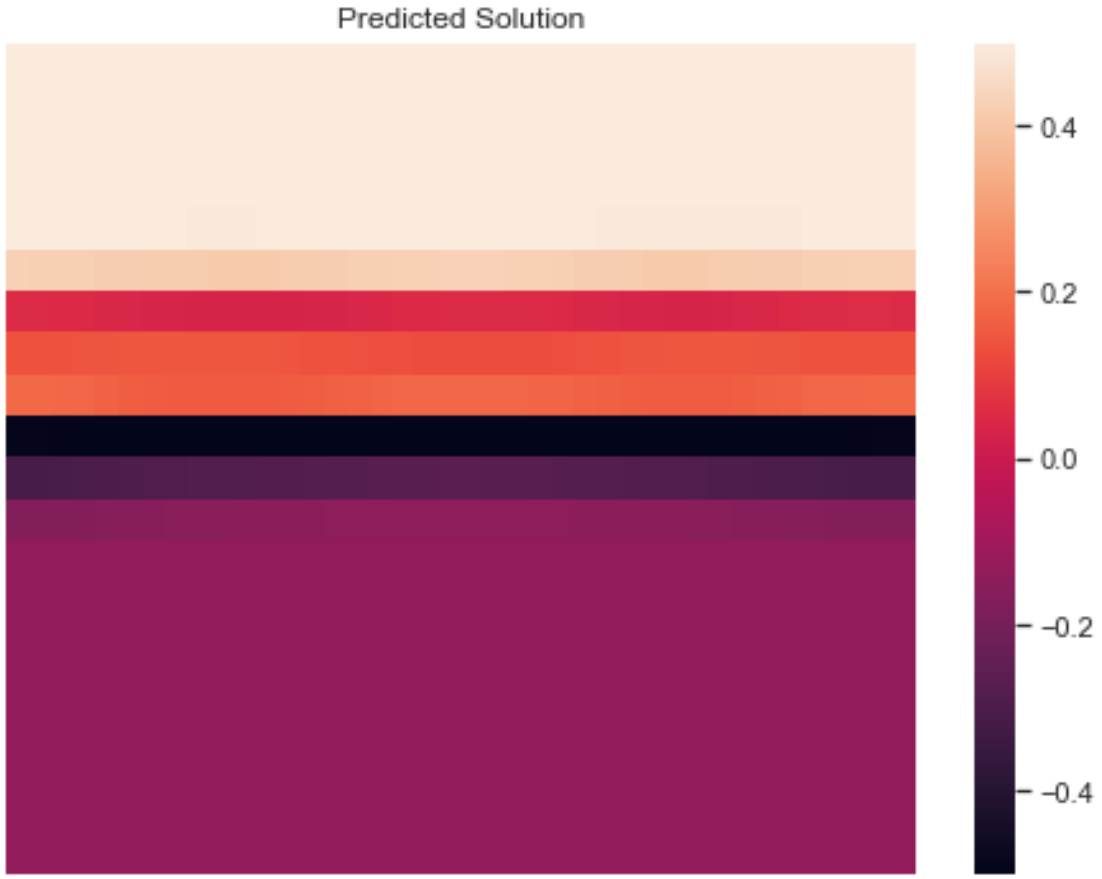

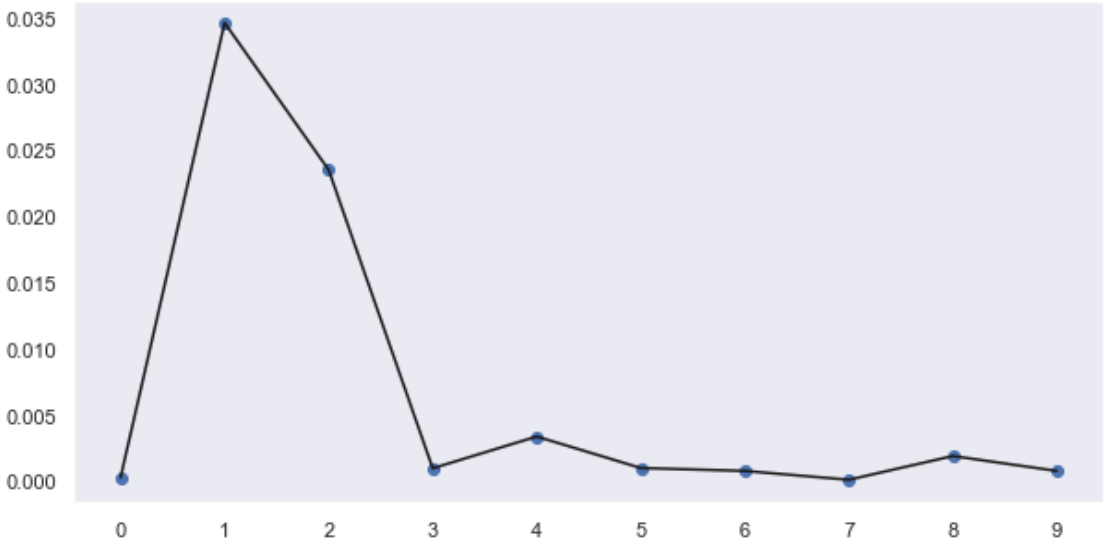

```
iteration 5000: loss = 0.6851448967416255, pde loss = 0.6580203614492385
iteration 6000: loss = 0.0033858933616783206, pde loss = 0.003354537847894683
iteration 7000: loss = 0.0009703454277984804, pde loss = 0.0009599239682063001
iteration 8000: loss = 0.000596357040467299, pde loss = 0.0005962713454961798
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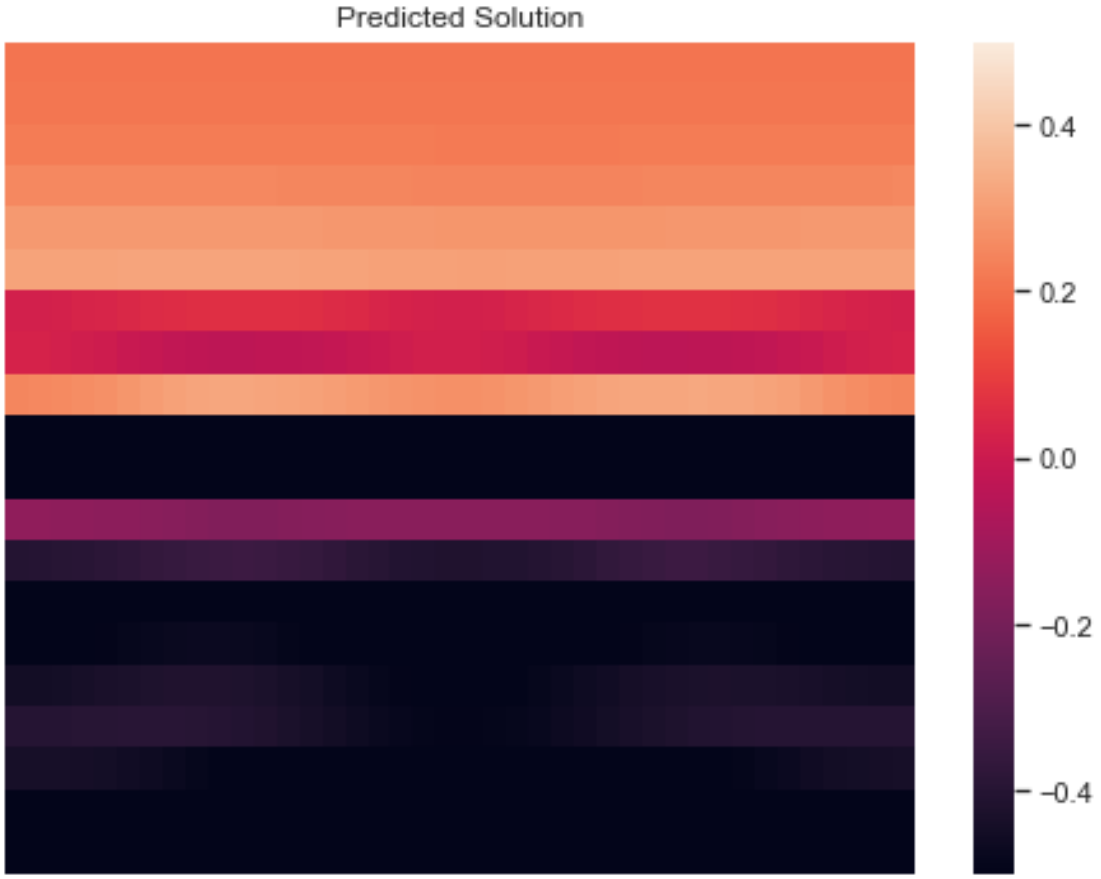

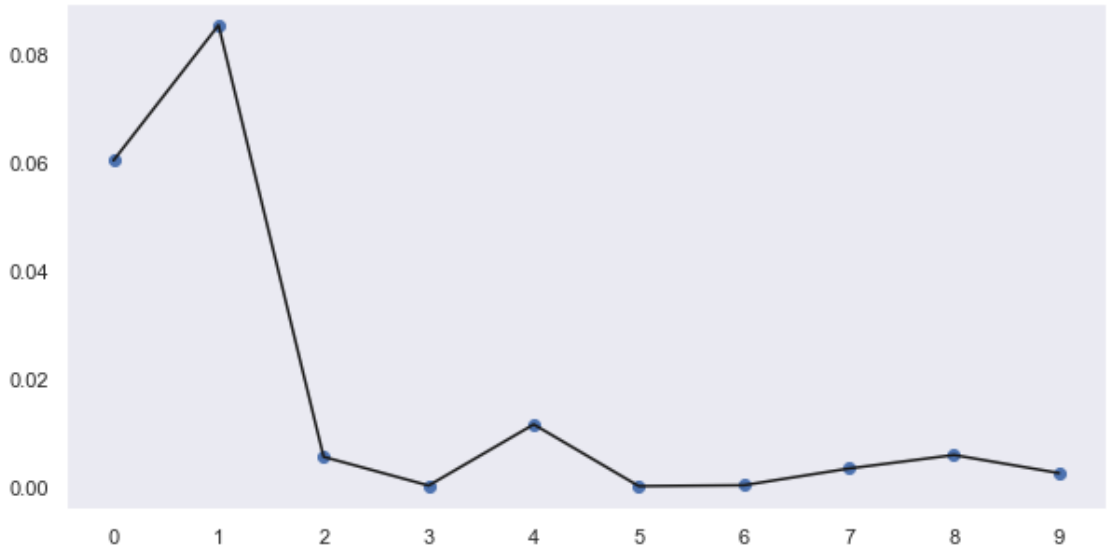


```
iteration 9000: loss = 0.0009333612354088453, pde loss = 0.0008975277735293255
iteration 10000: loss = 0.00011771211218054753, pde loss = 0.0001170551653422853
iteration 11000: loss = 0.08214224210257624, pde loss = 0.051667407958460795
iteration 12000: loss = 0.0034232207189835617, pde loss = 0.003421608595059429
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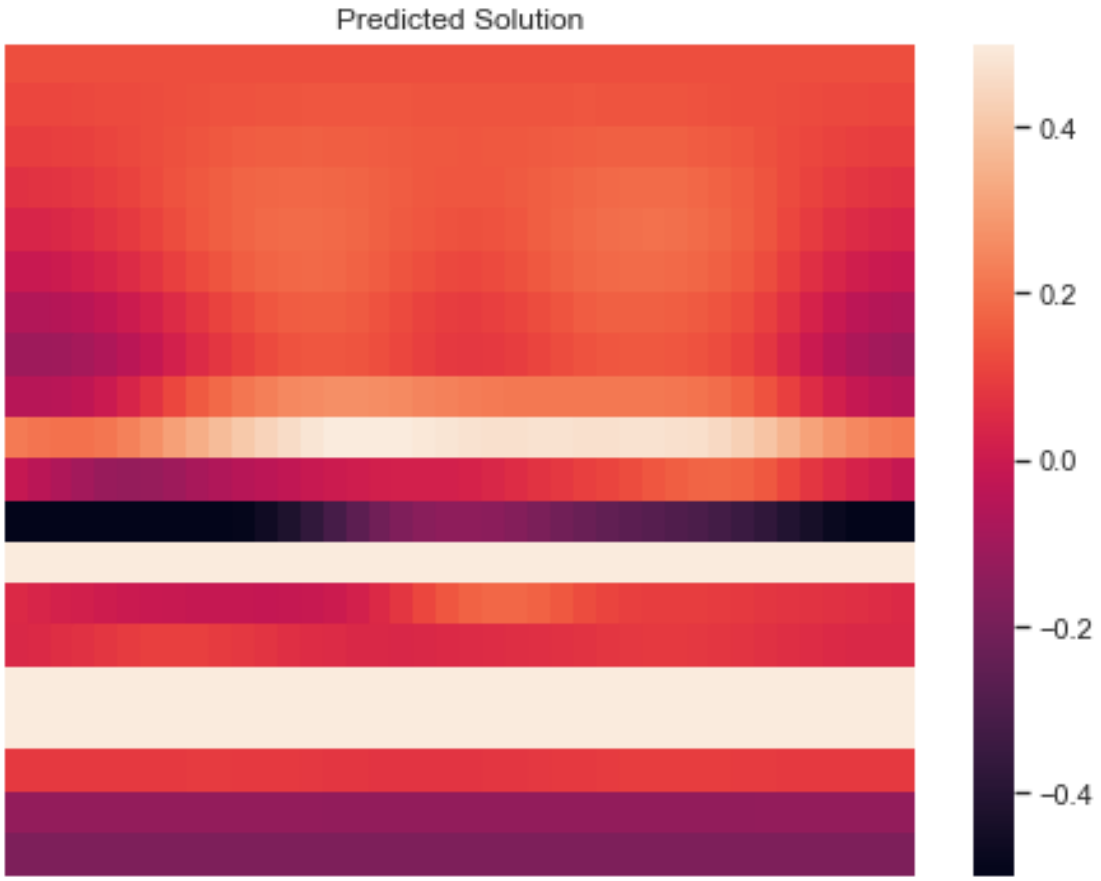
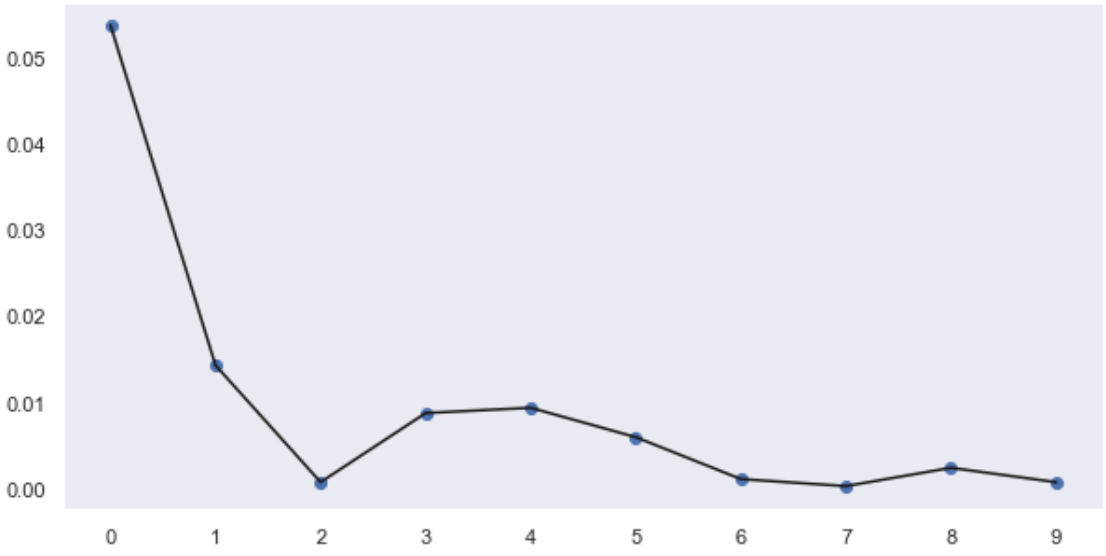
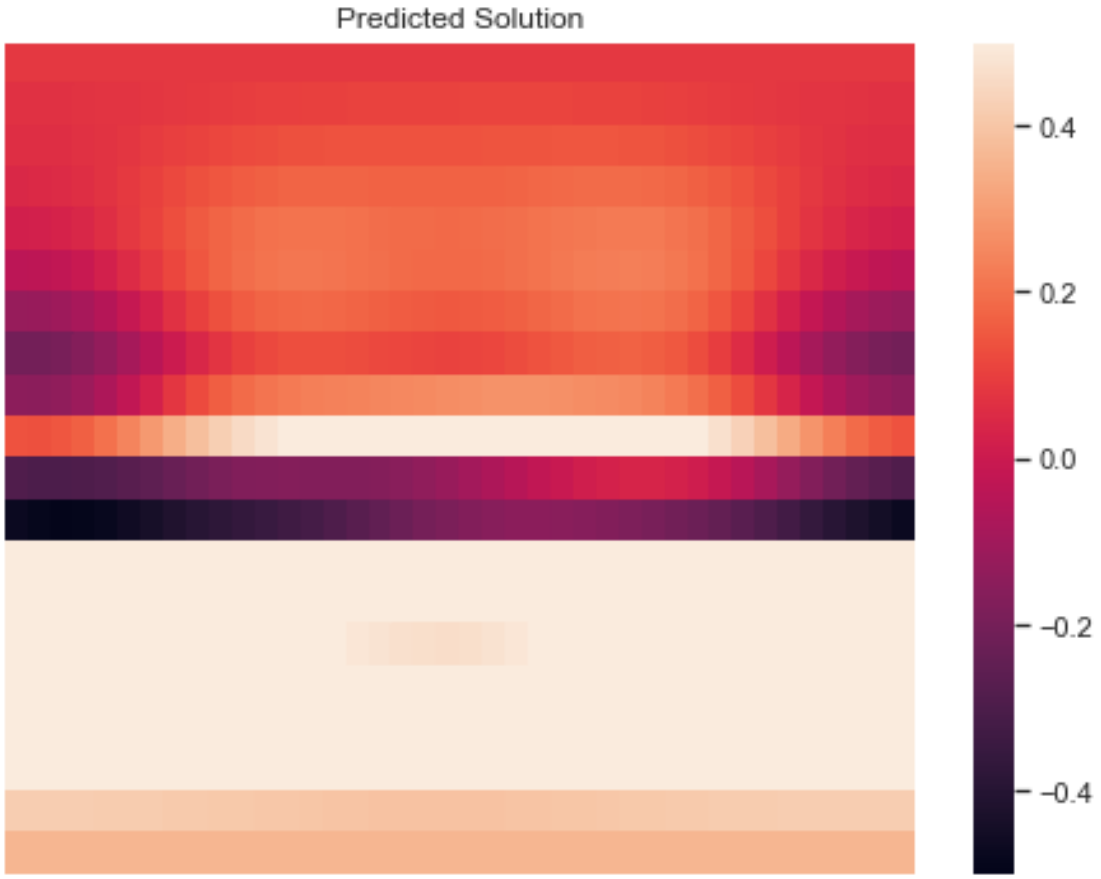
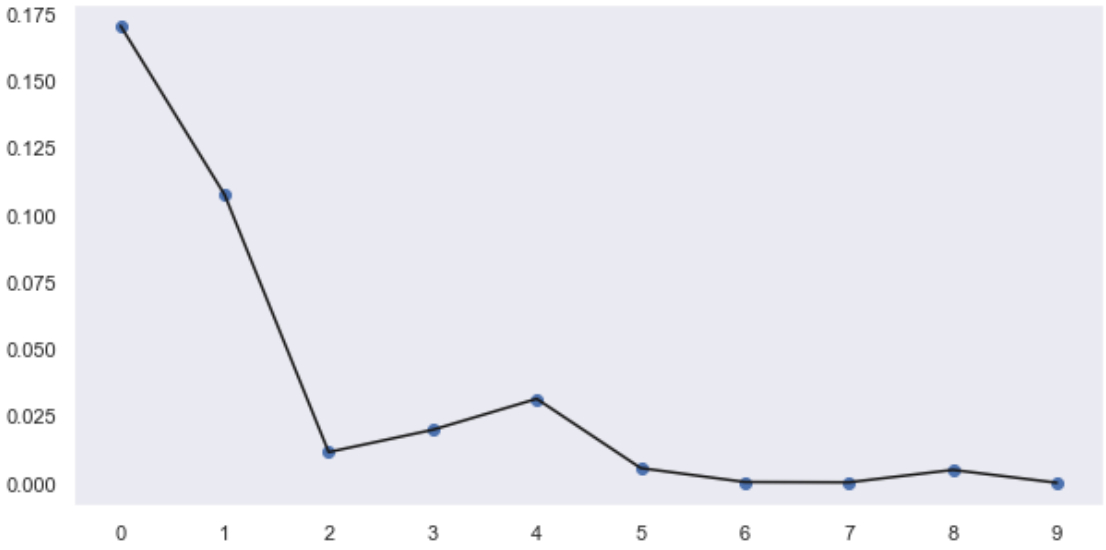




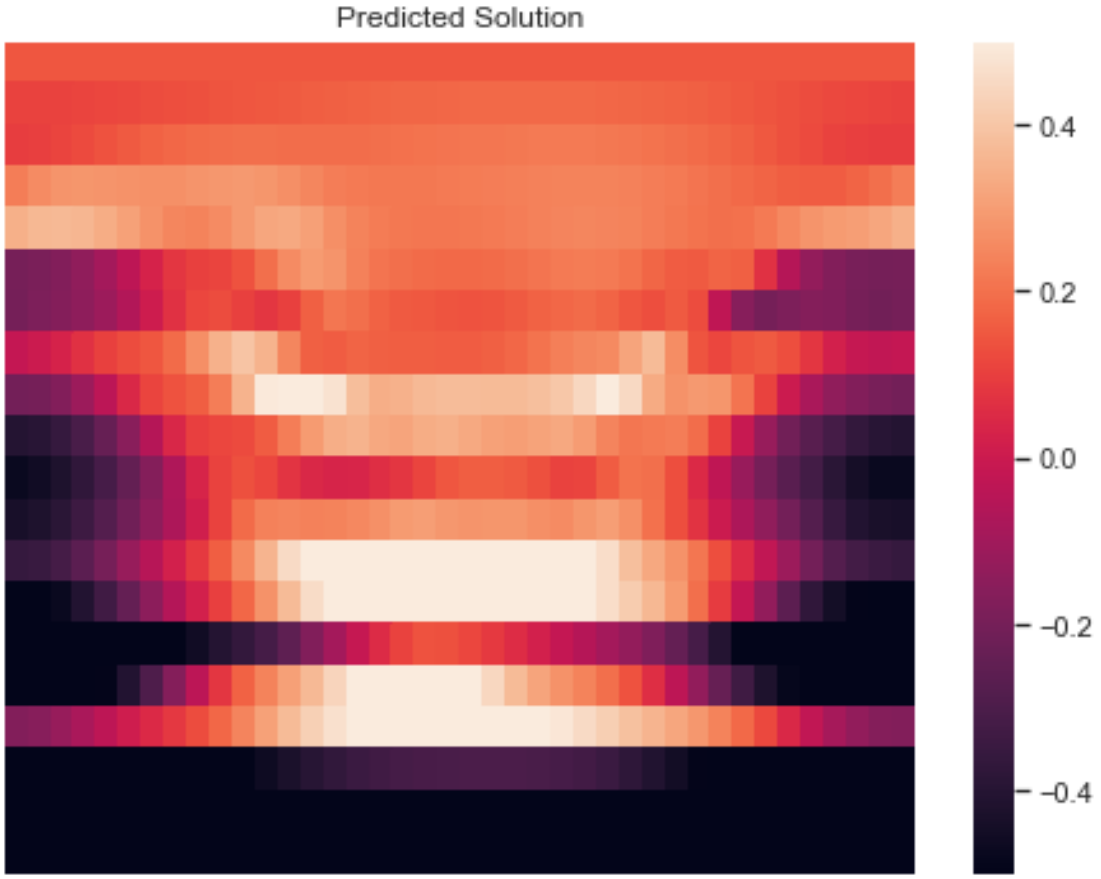

```
iteration 13000: loss = 0.00018756592270321515, pde loss = 0.0001836039182187228
```

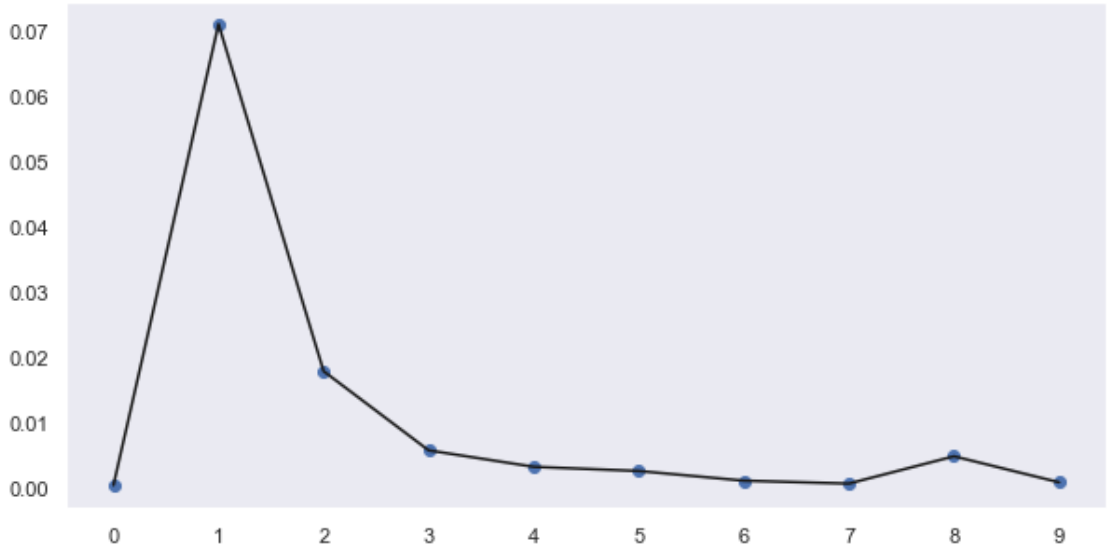
```
iteration 14000: loss = 0.00017947147030774604, pde loss =
0.00015425621900924116
iteration 15000: loss = 0.0012775890824350499, pde loss = 0.0011690195493947685
iteration 16000: loss = 0.00010405729898320957, pde loss =
0.00010359925141711088
```






```
iteration 17000: loss = 0.020741895142036636, pde loss = 0.007764762568312843
iteration 18000: loss = 6.143961779787547e-05, pde loss = 3.82741765632794e-05
iteration 19000: loss = 0.0005996484540962134, pde loss = 0.00019526033416394928
```

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