

sh_transform-Copy1

January 30, 2022

0.0.1 Solving Poisson Equation on S^2 : $\Delta u = f$

```
[1]: import my_backends
      from my_backends.ducc0_wrapper import *
      import my_shcoeffs
      from my_shcoeffs import SHCoeffs
      ## my_shcoeffs contains expand_adjoint_analysis
```

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```

[2]: import torch

N = 20
from Basis import SPHBasis
from Problem import Problem_Sphere_Poisson
from Model import Model
from Net import Sphere_Net, SPH_Sphere_Net
import seaborn as sns
import matplotlib.pyplot as plt
import math
from math import pi
from spherical_harmonics import *
from torch.distributions.uniform import Uniform
from numpy import arccos
import numpy as np

import pyshtools
from pyshtools.shio import shread
from pyshtools.expand import MakeGridDH
from pyshtools.expand import SHEexpandDH
from pyshtools.spectralanalysis import spectrum

torch.manual_seed(0)
maxiter = 20000
problem = Problem_Sphere_Poisson()

```

```
[3]: N = 20
azimuth = torch.linspace(0.0001, pi, N, requires_grad=False)
polar = torch.linspace(0, 2*pi, 2*N, requires_grad=False)

azimuth, polar = torch.meshgrid(azimuth, polar)
location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1, 1)], dim=1)
location.shape
```

```
[3]: torch.Size([800, 2])
```

```
[4]: def construct_model(net):
    class Poisson1dModel(Model):
        def add_loss_history(self):
#             self.loss_history.append([self.bc_loss, self.pde_loss, self.
↪ predict_error_value])
            self.loss_history.append([self.pde_loss, self.predict_error_value])

        def __init__(self):
            super().__init__(problem=problem, net=net, maxiter=maxiter)

        def inner_sample(self, N=N):
            azimuth = torch.linspace(0.0001, pi, N, requires_grad=False)
            polar = torch.linspace(0, 2*pi, 2*N, requires_grad=False)

            azimuth, polar = torch.meshgrid(azimuth, polar)
            location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,
↪ 1)], dim=1)
            return location # numpy ndarray with shape (N, 2*N)

        def bc_sample(self):
            return torch.tensor([[1., 1.]]) # arbitrary point is okay?

        def init_sample(self):
            pass

        def plot(self, net):
            azimuth = torch.linspace(0, math.pi, N, requires_grad=False)
            polar = torch.linspace(0, 2 * math.pi, 2*N, requires_grad=False)
            azimuth, polar = torch.meshgrid(azimuth, polar)
            location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,
↪ 1)], dim=1)
#             location = self.inner_sample()
            value = net(location) # predicted solution

            value = value.reshape((N, 2*N))

            fig, ax = plt.subplots(1, 1, figsize=(8, 6))
```

```

        sns.set()
        sns.heatmap(value.detach().numpy(), ax=ax, vmin=-0.5, vmax=0.5,
↪cbar=True)
        ax.set_xticks([])
        ax.set_yticks([])
        ax.set_title("Predicted Solution")
        plt.show()

    self.spherical_harmonic_transform(net)

    def spherical_harmonic_transform(self, net):
        azimuth = torch.linspace(0, math.pi, N, requires_grad=False)
        polar = torch.linspace(0, 2 * math.pi, 2*N, requires_grad=False)
        azimuth, polar = torch.meshgrid(azimuth, polar)
        location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,
↪1)], dim=1)
        function_grid = net(location)
        function_grid = function_grid.reshape((N, 2*N))

        coefficients = SHEExpandDH(function_grid.detach().numpy(),
↪sampling=2)
        nl = coefficients.shape[1]
        ls = np.arange(nl)[:10]

        power_per_l = spectrum(coefficients)[:10]
        fig, ax = plt.subplots(1, 1, figsize=(len(ls), 5))
        ax.plot(ls, power_per_l, 'bo')
        ax.plot(ls, power_per_l, 'k-')
        plt.xticks(range(len(ls)))
        ax.grid()
        plt.show()

    def plot_true(self):
        azimuth = torch.linspace(0, math.pi, N, requires_grad=False)
        polar = torch.linspace(0, 2 * math.pi, 2*N, requires_grad=False)
        azimuth, polar = torch.meshgrid(azimuth, polar)
        location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,
↪1)], dim=1)
        value = true_solution(location[:, 0:1], location[:, 1:])

        value = value.reshape((N, 2*N))

        fig, ax = plt.subplots(1, 1, figsize=(8, 6))
        sns.set()
        sns.heatmap(value.detach().numpy(), ax=ax, vmin=-0.5, vmax=0.5,
↪cbar=True)

```

```

        ax.set_xticks([])
        ax.set_yticks([])
        ax.set_title("True Solution")
        plt.show()

        self.sht_value(value)

    def sht_value(self, function_grid):
        function_grid = function_grid.reshape((N, 2*N))

        coefficients = SHExpandDH(function_grid.detach().numpy(),
→sampling=2)
        nl = coefficients.shape[1]
        ls = np.arange(nl)[:10]

        power_per_l = spectrum(coefficients)[:10]
        fig, ax = plt.subplots(1, 1, figsize=(len(ls), 5))
        ax.plot(ls, power_per_l, 'bo')
        ax.plot(ls, power_per_l, 'k-')
        plt.xticks(range(len(ls)))
        ax.grid()
        plt.show()

    def post_process(self, ax=None):
        if ax is None:
            for losses in self.loss_history:
                for i in range(3):
                    losses[i].detach().numpy()
            plt.plot(self.loss_history)
            plt.yscale('log')
            plt.legend(('BC loss', 'pde loss', 'predict error'))
            plt.show()
        else:
            for losses in self.loss_history:
                for i in range(3):
                    losses[i].detach().numpy()
            ax.plot(self.loss_history)
            ax.set_yscale('log')
            ax.set_ylim(1e-4, 100)
            ax.legend(('BC loss', 'pde loss', 'predict error'))

    def predict_error(self):
        coor = self.inner_sample()
        true = self.problem.ground_truth(coor)
        predict = self.net(coor)
        predict_error = self.pde_loss_f(true, predict)
        return predict_error

```



```

def train(self, ax100=None):
    self.plot_true()
    problem = self.problem
    net = self.net
    opt = self.opt
    maxiter = self.maxiter

    for iter in range(maxiter):
        net.zero_grad()

        coor_inner = self.inner_sample().detach().requires_grad_(True)
        # coor_inner = self.inner_sample().requires_grad_(True)
        infer_value_inner = net(coor_inner)
        # truth_inner, predict_inner = problem.pde(coor_inner,
        # infer_value_inner)
        predict_inner, truth_inner = problem.pde(coor_inner,
        # infer_value_inner)
        ## order, is the lhs even the predicted value???
        ## reason the loss function is going up could be because we are
        # computing the negative gradient, which could happen by switching the order
        # of y, y_pred in orward and backward functions of the loss.

        self.pde_loss = self.pde_loss_f(predict_inner, truth_inner)

        bc_samples = self.bc_sample()
        if bc_samples is None:
            self.bc_loss = torch.tensor(0.)
        else:
            coor_bc = bc_samples.detach().requires_grad_(True)
            infer_value_bc = net(coor_bc)
            truth_bc, predict_bc = problem.bound_condition(coor_bc,
            # infer_value_bc)
            self.bc_loss = self.bc_loss_f(predict_bc, truth_bc)

            init_samples = self.init_sample()
            if init_samples is None:
                self.init_loss = torch.tensor(0.)
            else:
                coor_init = init_samples.detach().requires_grad_(True)
                infer_value_init = net(coor_init)
                truth_init, predict_init = problem.
                # bound_condition(coor_init, infer_value_init)
                self.init_loss = self.bc_loss_f(predict_init, truth_init)

        self.predict_error_value = self.predict_error()
        self.total_loss = self.pde_loss + self.bc_loss + self.init_loss

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```

        self.add_loss_history()
        self.total_loss.register_hook(lambda grad: print('gradient', ↵
↵grad))

        self.total_loss.backward()

        opt.step()
        opt.zero_grad()
        # scheduler.step()

        if iter % (maxiter // 20) == 0:
            print("iteration {}: loss = {}, pde loss = {}".format(iter, ↵
↵self.total_loss, self.pde_loss))

            if iter % int(maxiter / 5) == 0:
                self.plot(net)

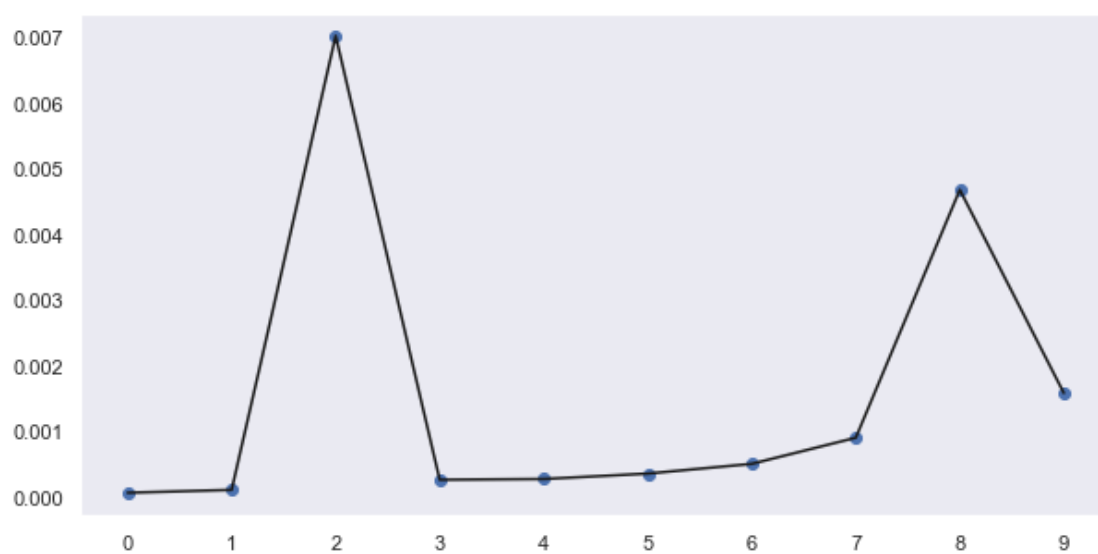
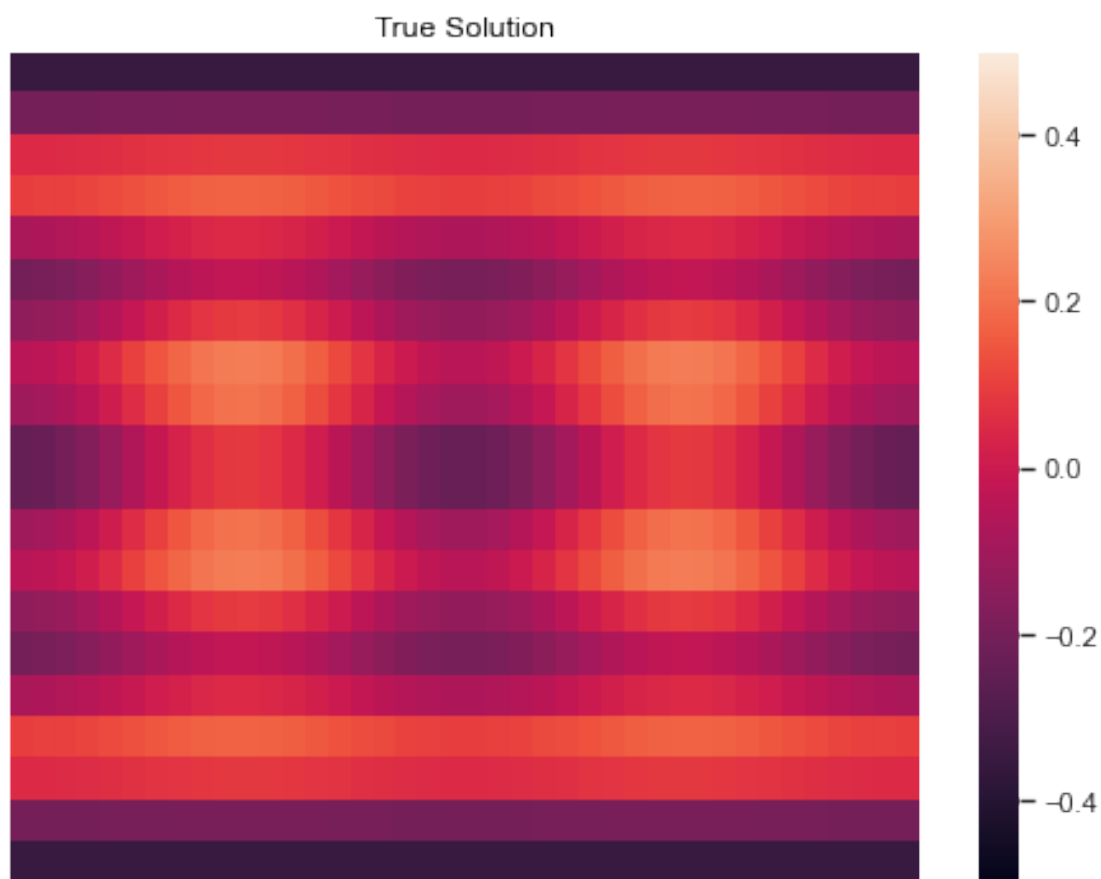
        self.plot(net)

        # self.post_process()

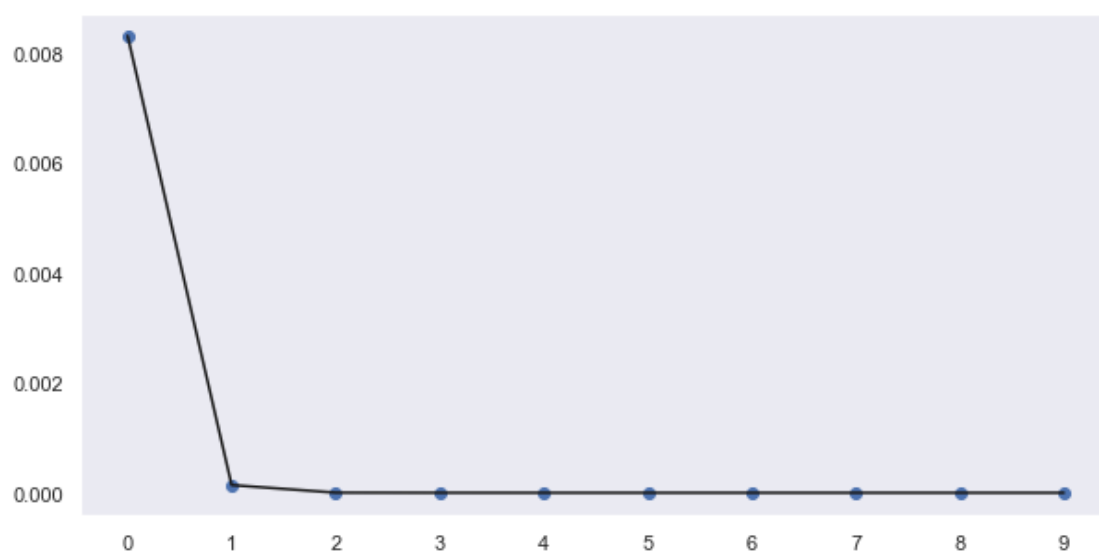
    return Poisson1dModel()

model = construct_model(Sphere_Net([3, 150, 50, 50, 50, 16, 1]))
model.train()

```

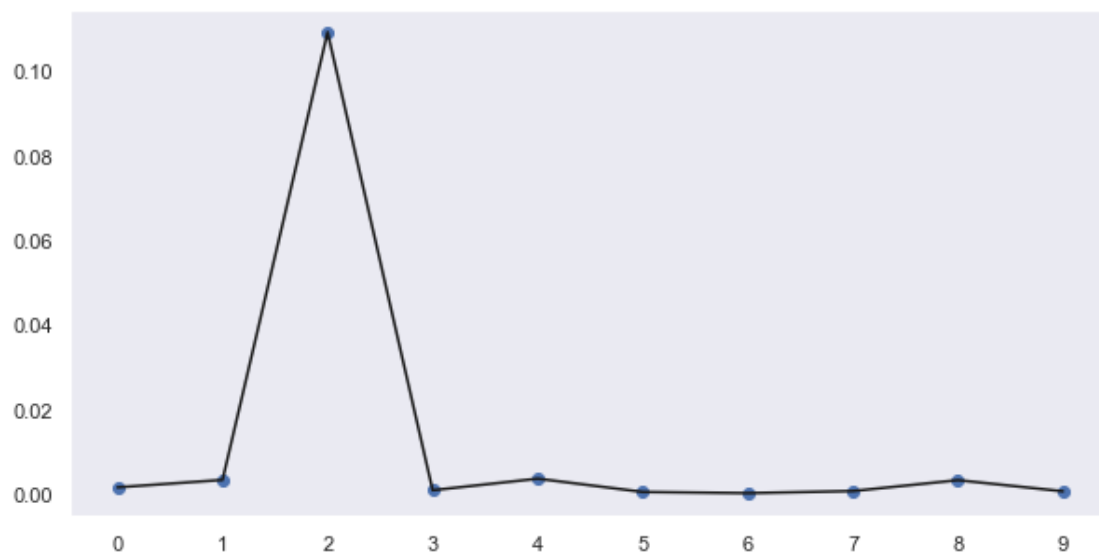
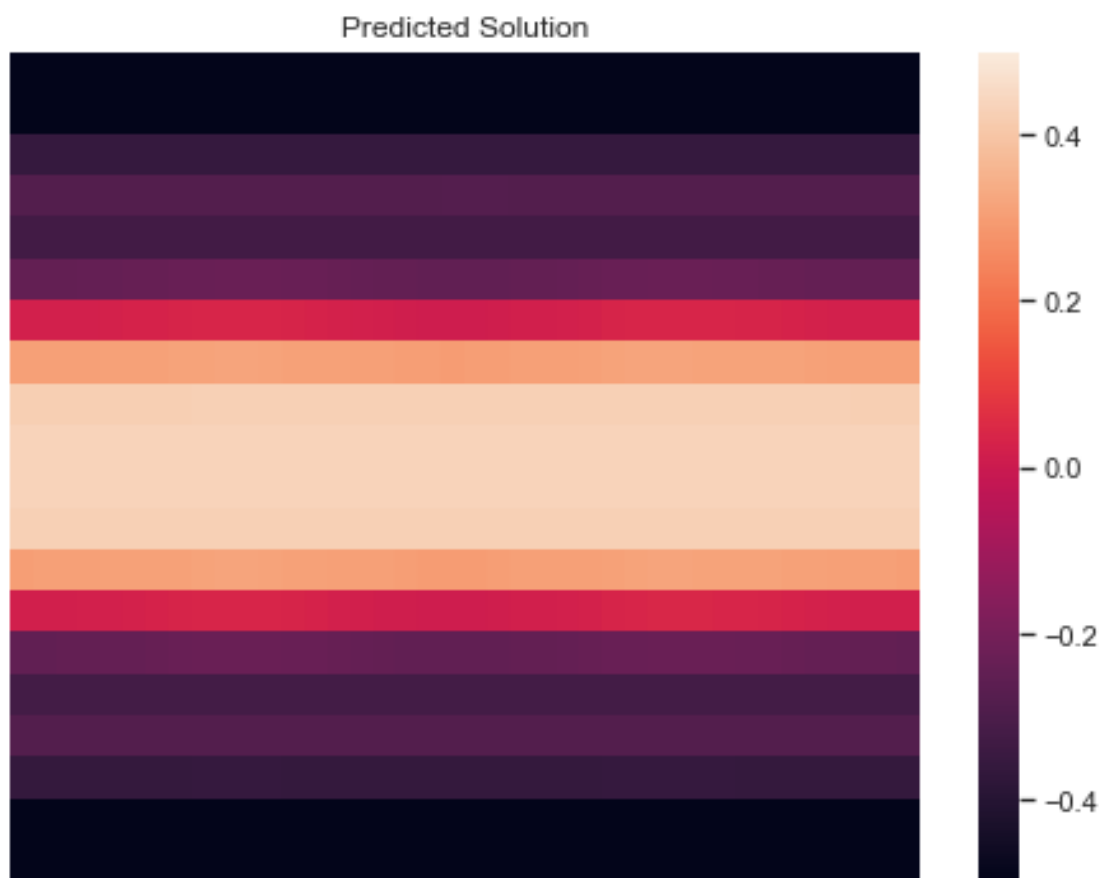


iteration 0: loss = 110.12125396728516, pde loss = 110.10062408447266

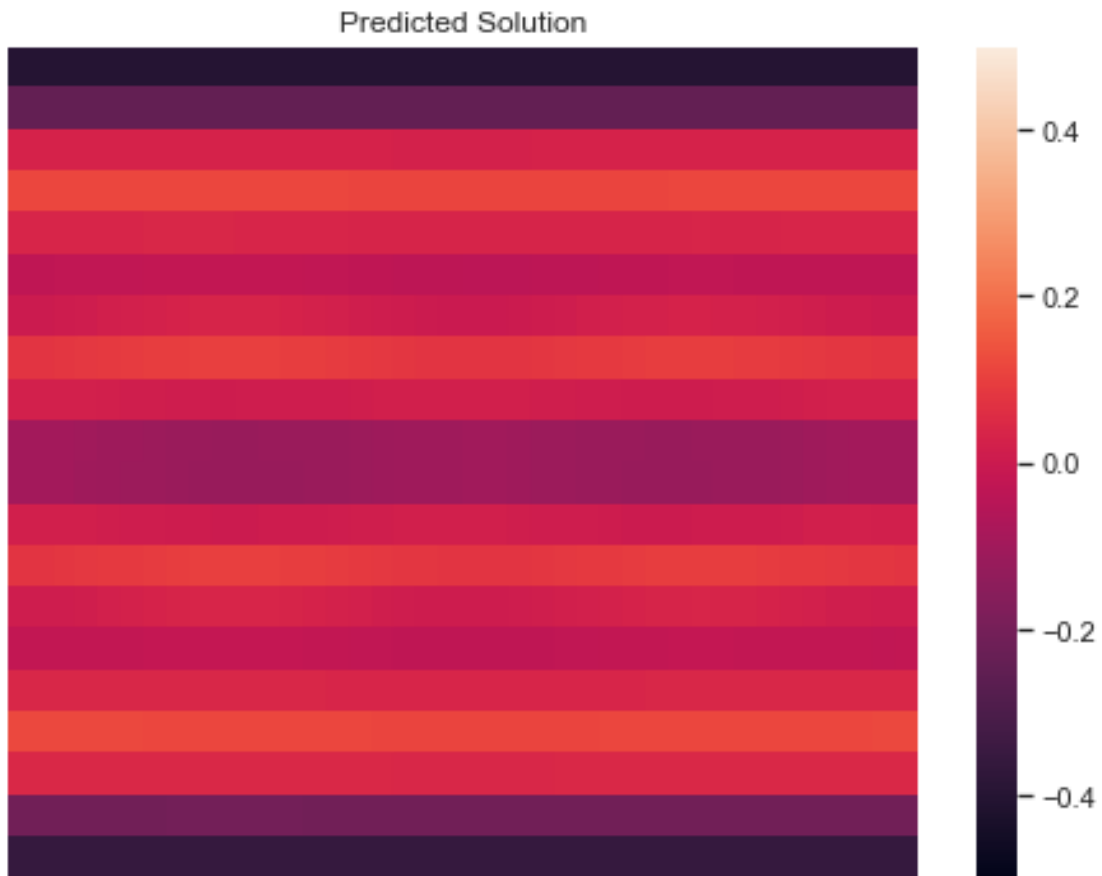


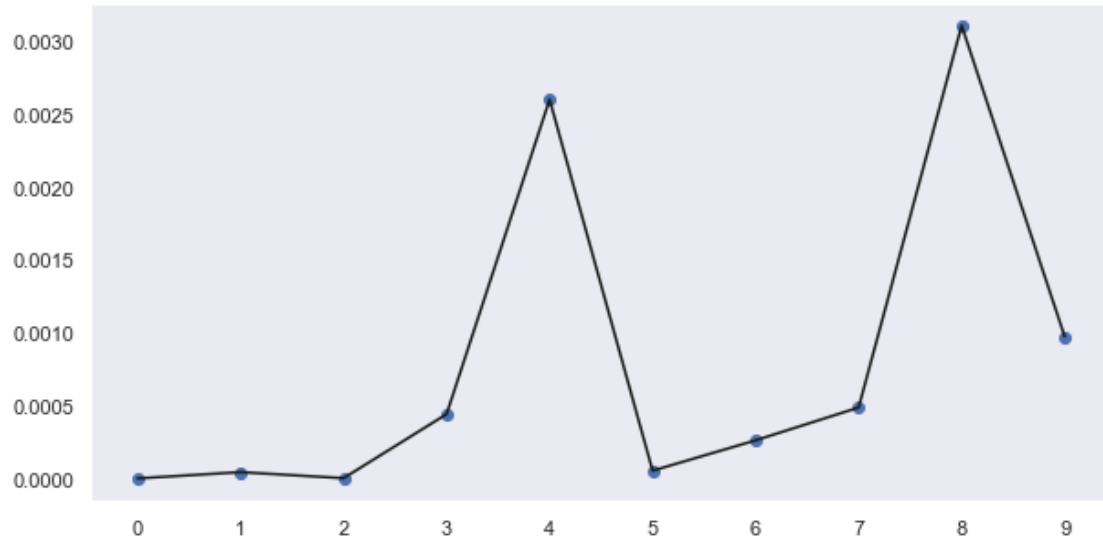
iteration 1000: loss = 8.848636627197266, pde loss = 8.848491668701172

iteration 2000: loss = 8.795053482055664, pde loss = 8.795042991638184
iteration 3000: loss = 3.8146893978118896, pde loss = 3.814676523208618
iteration 4000: loss = 3.1626296043395996, pde loss = 3.162492036819458

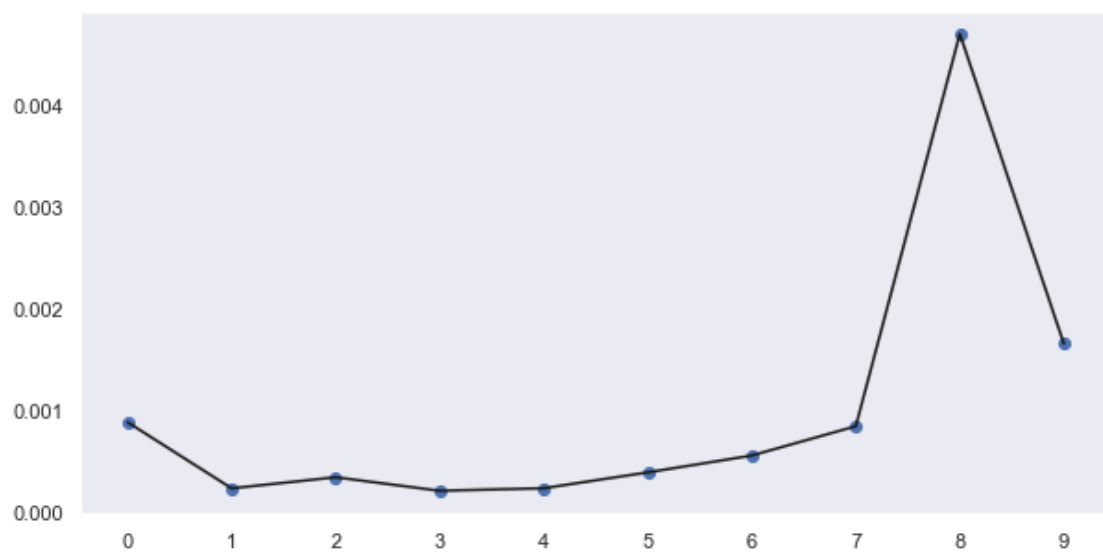
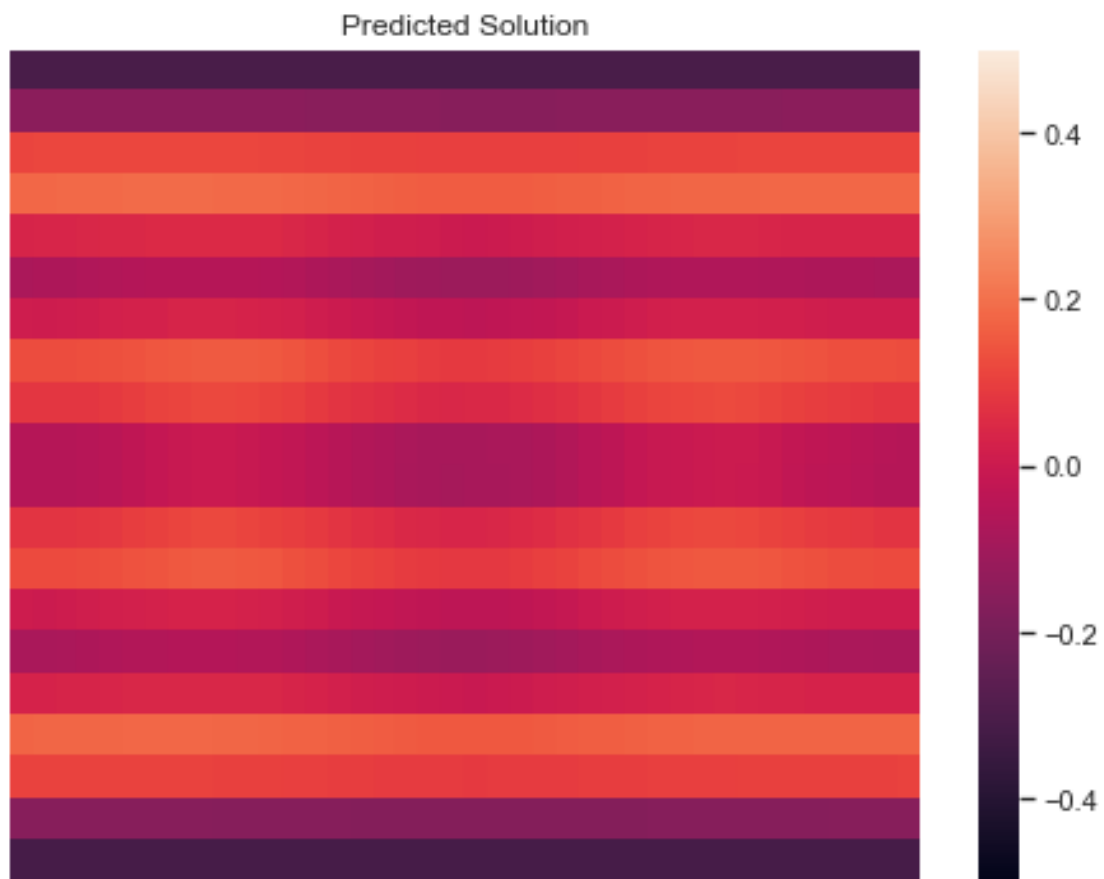


```
iteration 5000: loss = 0.1482522040605545, pde loss = 0.1482393592596054  
iteration 6000: loss = 0.07854381948709488, pde loss = 0.07854123413562775  
iteration 7000: loss = 0.05587850883603096, pde loss = 0.0558757409453392  
iteration 8000: loss = 0.09840525686740875, pde loss = 0.09840524941682816
```



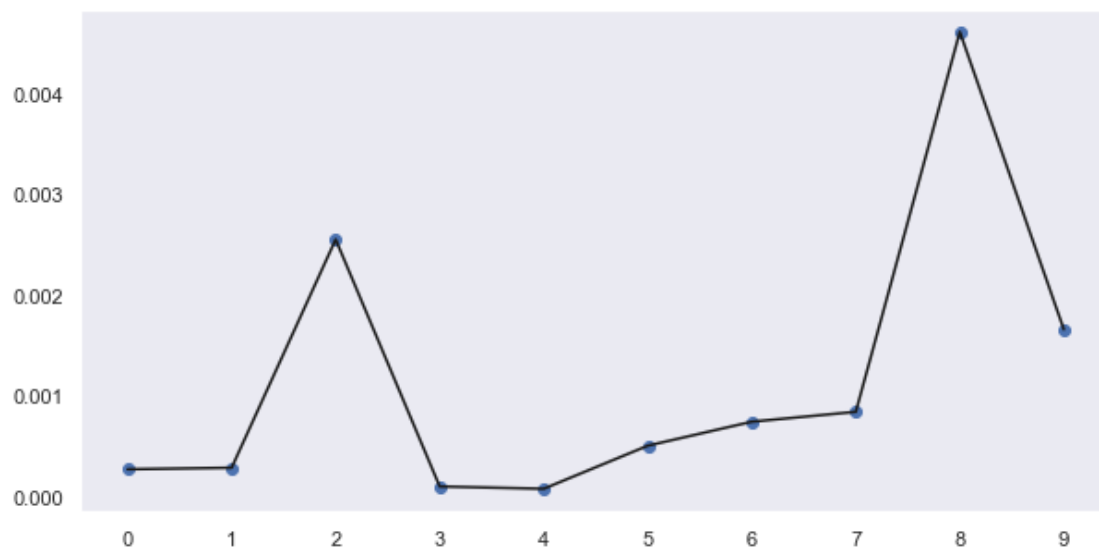
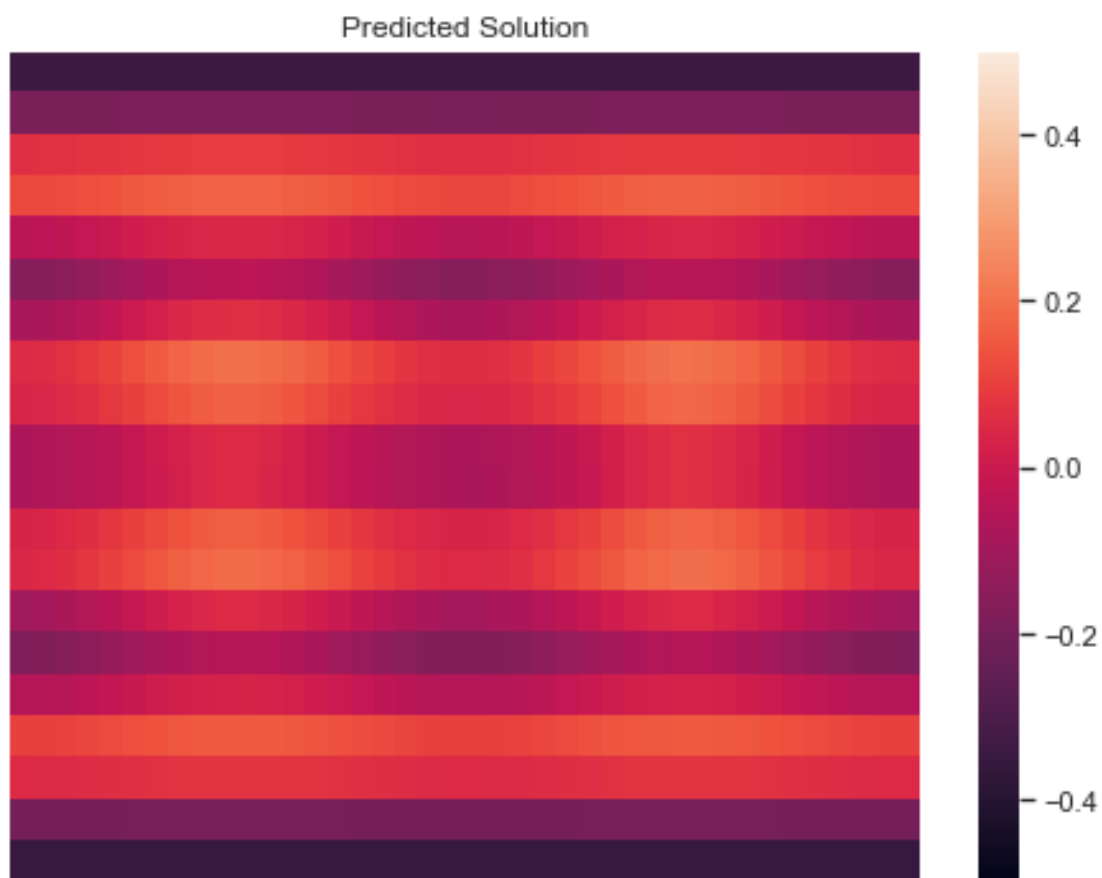


```
iteration 9000: loss = 0.01820532977581024, pde loss = 0.01820450834929943
iteration 10000: loss = 0.0316828191280365, pde loss = 0.03168274834752083
iteration 11000: loss = 0.0024282499216496944, pde loss = 0.0024280990473926067
iteration 12000: loss = 0.11794360727071762, pde loss = 0.11793964356184006
```



iteration 13000: loss = 0.006411933805793524, pde loss = 0.006411828566342592

iteration 14000: loss = 0.0031465827487409115, pde loss = 0.0031465108040720224
iteration 15000: loss = 0.01487185899168253, pde loss = 0.014870703220367432
iteration 16000: loss = 0.0024545667693018913, pde loss = 0.0024544575717300177



iteration 17000: loss = 0.012341991998255253, pde loss = 0.012340063229203224
iteration 18000: loss = 0.002915848046541214, pde loss = 0.002915820572525263
iteration 19000: loss = 0.0032142442651093006, pde loss = 0.003213283373042941

