

sh_transform-Copy1

January 30, 2022

0.0.1 Solving Poisson Equation on S^2 : $\Delta u = f$

```
[1]: import my_backends
      from my_backends.ducc0_wrapper import *
      import my_shcoeffs
      from my_shcoeffs import SHCoeffs
      ## my_shcoeffs contains expand_adjoint_analysis
```

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```

[2]: import torch

N = 20
from Basis import SPHBasis
from Problem import Problem_Sphere_Poisson
from Model import Model
from Net import Sphere_Net, SPH_Sphere_Net
import seaborn as sns
import matplotlib.pyplot as plt
import math
from math import pi
from spherical_harmonics import *
from torch.distributions.uniform import Uniform
from numpy import arccos
import numpy as np

import pyshtools
from pyshtools.shio import shread
from pyshtools.expand import MakeGridDH
from pyshtools.expand import SHEexpandDH
from pyshtools.spectralanalysis import spectrum

torch.manual_seed(0)
maxiter = 1000
problem = Problem_Sphere_Poisson()

```

```
[3]: N = 20
azimuth = torch.linspace(0.0001, pi, N, requires_grad=False)
polar = torch.linspace(0, 2*pi, 2*N, requires_grad=False)

azimuth, polar = torch.meshgrid(azimuth, polar)
location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1, 1)], dim=1)
location.shape
```

```
[3]: torch.Size([800, 2])
```

```
[4]: def construct_model(net):
    class Poisson1dModel(Model):
        def add_loss_history(self):
#             self.loss_history.append([self.bc_loss, self.pde_loss, self.
→ predict_error_value])
            self.loss_history.append([self.pde_loss, self.predict_error_value])

        def __init__(self):
            super().__init__(problem=problem, net=net, maxiter=maxiter)

        def inner_sample(self, N=N):
            azimuth = torch.linspace(0.0001, pi, N, requires_grad=False)
            polar = torch.linspace(0, 2*pi, 2*N, requires_grad=False)

            azimuth, polar = torch.meshgrid(azimuth, polar)
            location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,
→ 1)], dim=1)
            return location # numpy ndarray with shape (N, 2*N)

        def bc_sample(self):
            return torch.tensor([[1., 1.]]) # arbitrary point is okay?

        def init_sample(self):
            pass

        def plot(self, net):
            azimuth = torch.linspace(0, math.pi, N, requires_grad=False)
            polar = torch.linspace(0, 2 * math.pi, 2*N, requires_grad=False)
            azimuth, polar = torch.meshgrid(azimuth, polar)
            location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,
→ 1)], dim=1)
#             location = self.inner_sample()
            value = net(location) # predicted solution

            value = value.reshape((N, 2*N))

            fig, ax = plt.subplots(1, 1, figsize=(8, 6))
```

```

        sns.set()
        sns.heatmap(value.detach().numpy(), ax=ax, vmin=-0.5, vmax=0.5,
↪cbar=True)
        ax.set_xticks([])
        ax.set_yticks([])
        ax.set_title("Predicted Solution")
        plt.show()

    self.spherical_harmonic_transform(net)

    def spherical_harmonic_transform(self, net):
        azimuth = torch.linspace(0, math.pi, N, requires_grad=False)
        polar = torch.linspace(0, 2 * math.pi, 2*N, requires_grad=False)
        azimuth, polar = torch.meshgrid(azimuth, polar)
        location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,
↪1)], dim=1)
        function_grid = net(location)
        function_grid = function_grid.reshape((N, 2*N))

        coefficients = SHEExpandDH(function_grid.detach().numpy(),
↪sampling=2)
        nl = coefficients.shape[1]
        ls = np.arange(nl)[:10]

        power_per_l = spectrum(coefficients)[:10]
        fig, ax = plt.subplots(1, 1, figsize=(len(ls), 5))
        ax.plot(ls, power_per_l, 'bo')
        ax.plot(ls, power_per_l, 'k-')
        plt.xticks(range(len(ls)))
        ax.grid()
        plt.show()

    def plot_true(self):
        azimuth = torch.linspace(0, math.pi, N, requires_grad=False)
        polar = torch.linspace(0, 2 * math.pi, 2*N, requires_grad=False)
        azimuth, polar = torch.meshgrid(azimuth, polar)
        location = torch.cat([azimuth.reshape(-1, 1), polar.reshape(-1,
↪1)], dim=1)
        value = true_solution(location[:, 0:1], location[:, 1:])

        value = value.reshape((N, 2*N))

        fig, ax = plt.subplots(1, 1, figsize=(8, 6))
        sns.set()
        sns.heatmap(value.detach().numpy(), ax=ax, vmin=-0.5, vmax=0.5,
↪cbar=True)

```

```

        ax.set_xticks([])
        ax.set_yticks([])
        ax.set_title("True Solution")
        plt.show()

        self.sht_value(value)

def sht_value(self, function_grid):
    function_grid = function_grid.reshape((N, 2*N))

    coefficients = SHExpandDH(function_grid.detach().numpy(),
↪sampling=2)
    nl = coefficients.shape[1]
    ls = np.arange(nl)[:10]

    power_per_l = spectrum(coefficients)[:10]
    fig, ax = plt.subplots(1, 1, figsize=(len(ls), 5))
    ax.plot(ls, power_per_l, 'bo')
    ax.plot(ls, power_per_l, 'k-')
    plt.xticks(range(len(ls)))
    ax.grid()
    plt.show()

def post_process(self, ax=None):
    if ax is None:
        for losses in self.loss_history:
            for i in range(3):
                losses[i].detach().numpy()
        plt.plot(self.loss_history)
        plt.yscale('log')
        plt.legend(('BC loss', 'pde loss', 'predict error'))
        plt.show()
    else:
        for losses in self.loss_history:
            for i in range(3):
                losses[i].detach().numpy()
        ax.plot(self.loss_history)
        ax.set_yscale('log')
        ax.set_ylim(1e-4, 100)
        ax.legend(('BC loss', 'pde loss', 'predict error'))

def predict_error(self):
    coor = self.inner_sample()
    true = self.problem.ground_truth(coor)
    predict = self.net(coor)
    predict_error = self.pde_loss_f(true, predict)
    return predict_error

```



```

def train(self, ax100=None):
    self.plot_true()
    problem = self.problem
    net = self.net
    opt = self.opt
    maxiter = self.maxiter

    for iter in range(maxiter):
        net.zero_grad()

        coor_inner = self.inner_sample().detach().requires_grad_(True)
        # coor_inner = self.inner_sample().requires_grad_(True)
        infer_value_inner = net(coor_inner)
        # truth_inner, predict_inner = problem.pde(coor_inner,
        # infer_value_inner)
        predict_inner, truth_inner = problem.pde(coor_inner,
        # infer_value_inner)
        ## order, is the lhs even the predicted value???
        ## reason the loss function is going up could be because we are
        # computing the negative gradient, which could happen by switching the order
        # of y, y_pred in orward and backward functions of the loss.

        self.pde_loss = self.pde_loss_f(predict_inner, truth_inner)

        bc_samples = self.bc_sample()
        if bc_samples is None:
            self.bc_loss = torch.tensor(0.)
        else:
            coor_bc = bc_samples.detach().requires_grad_(True)
            infer_value_bc = net(coor_bc)
            truth_bc, predict_bc = problem.bound_condition(coor_bc,
            # infer_value_bc)
            self.bc_loss = self.bc_loss_f(predict_bc, truth_bc)

            init_samples = self.init_sample()
            if init_samples is None:
                self.init_loss = torch.tensor(0.)
            else:
                coor_init = init_samples.detach().requires_grad_(True)
                infer_value_init = net(coor_init)
                truth_init, predict_init = problem.
            # bound_condition(coor_init, infer_value_init)
                self.init_loss = self.bc_loss_f(predict_init, truth_init)

        self.predict_error_value = self.predict_error()
        self.total_loss = self.pde_loss + self.bc_loss + self.init_loss

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        self.add_loss_history()
#         self.total_loss.register_hook(lambda grad: print('gradient',
→grad))

        self.total_loss.backward()

        opt.step()
        opt.zero_grad()
#         scheduler.step()

        if iter % (maxiter // 20) == 0:
            print("iteration {}: loss = {}, pde loss = {}".format(iter,
→self.total_loss, self.pde_loss))

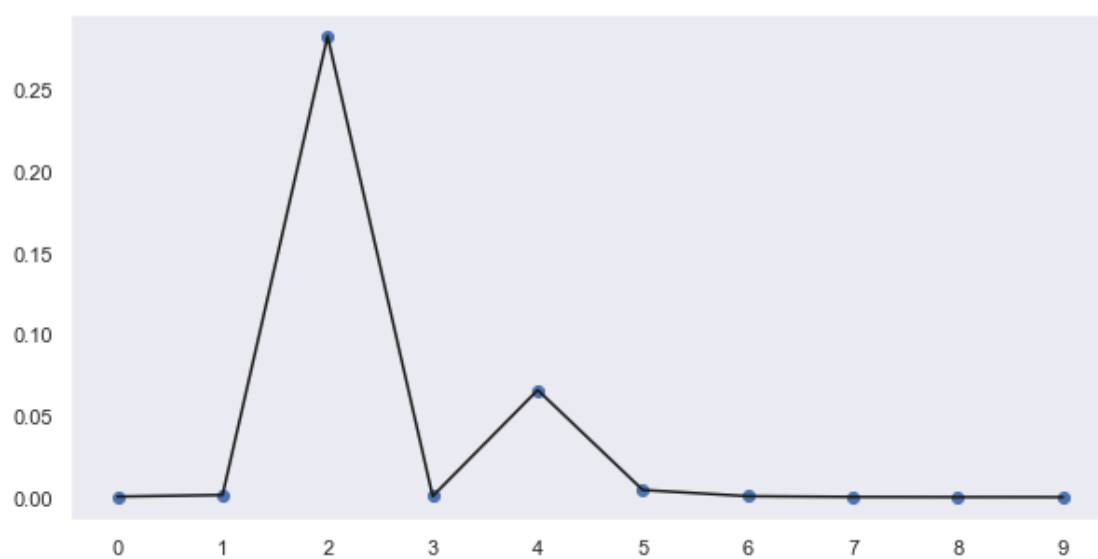
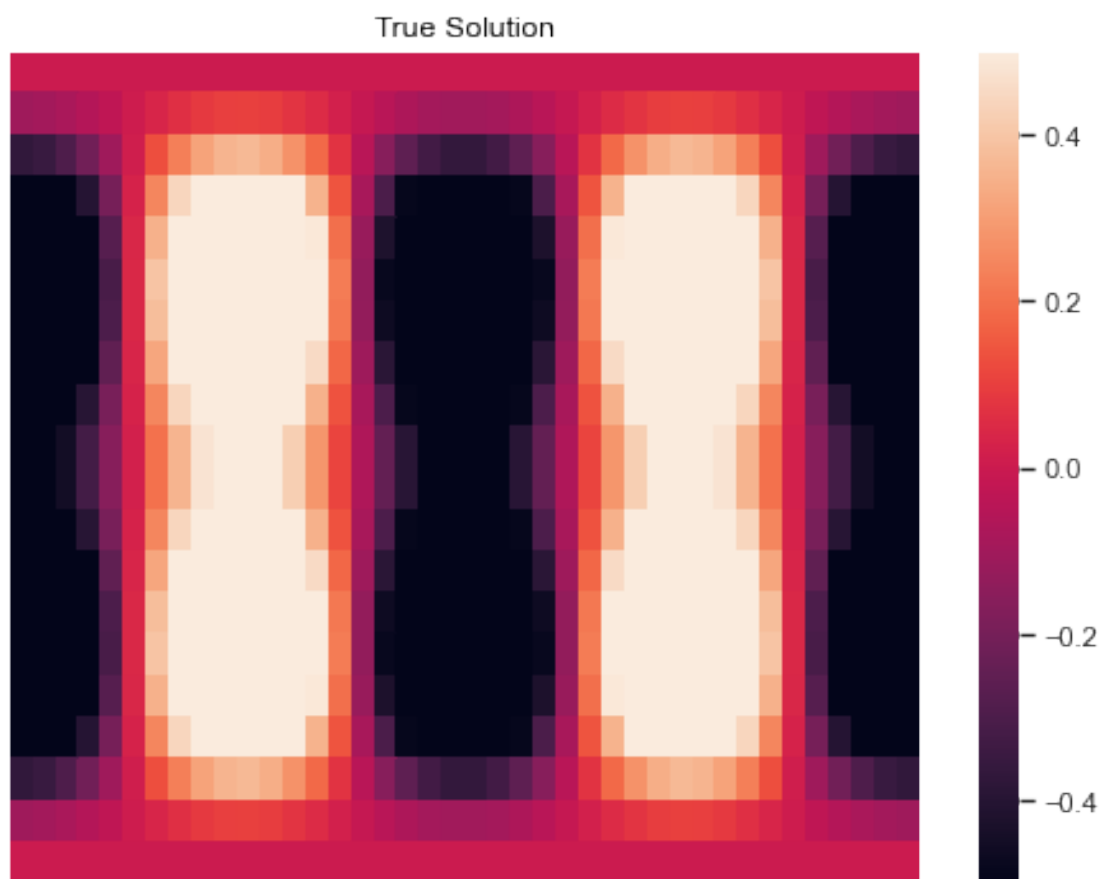
            if iter % int(maxiter / 5) == 0:
                self.plot(net)
            print("iteration {}: loss = {}, pde loss = {}".format(iter, self.
→total_loss, self.pde_loss))
            self.plot(net)

#         self.post_process()

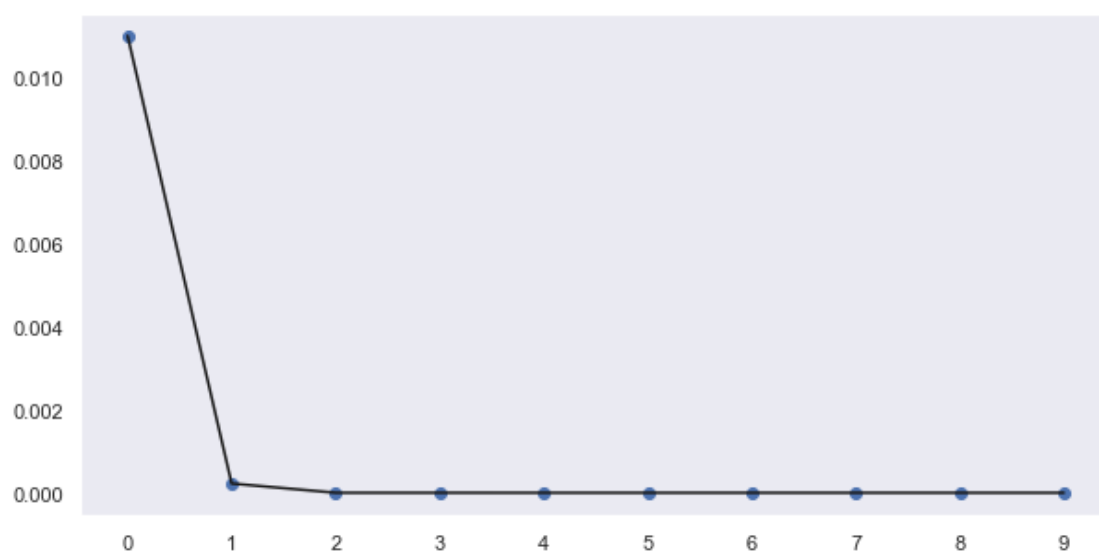
    return Poisson1dModel()

model = construct_model(Sphere_Net([3, 150, 50, 50, 50, 16, 1]))
model.train()

```

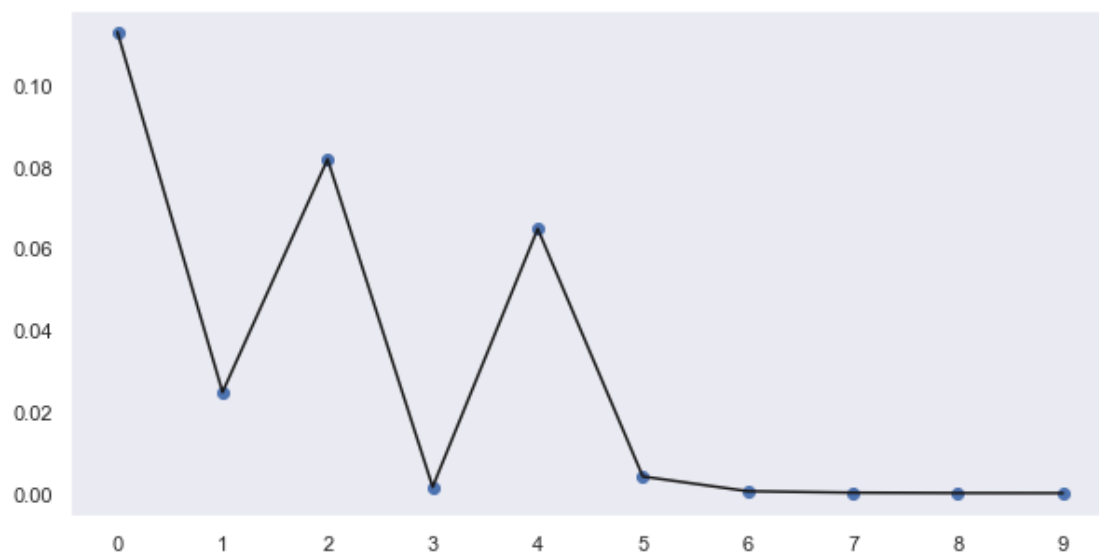
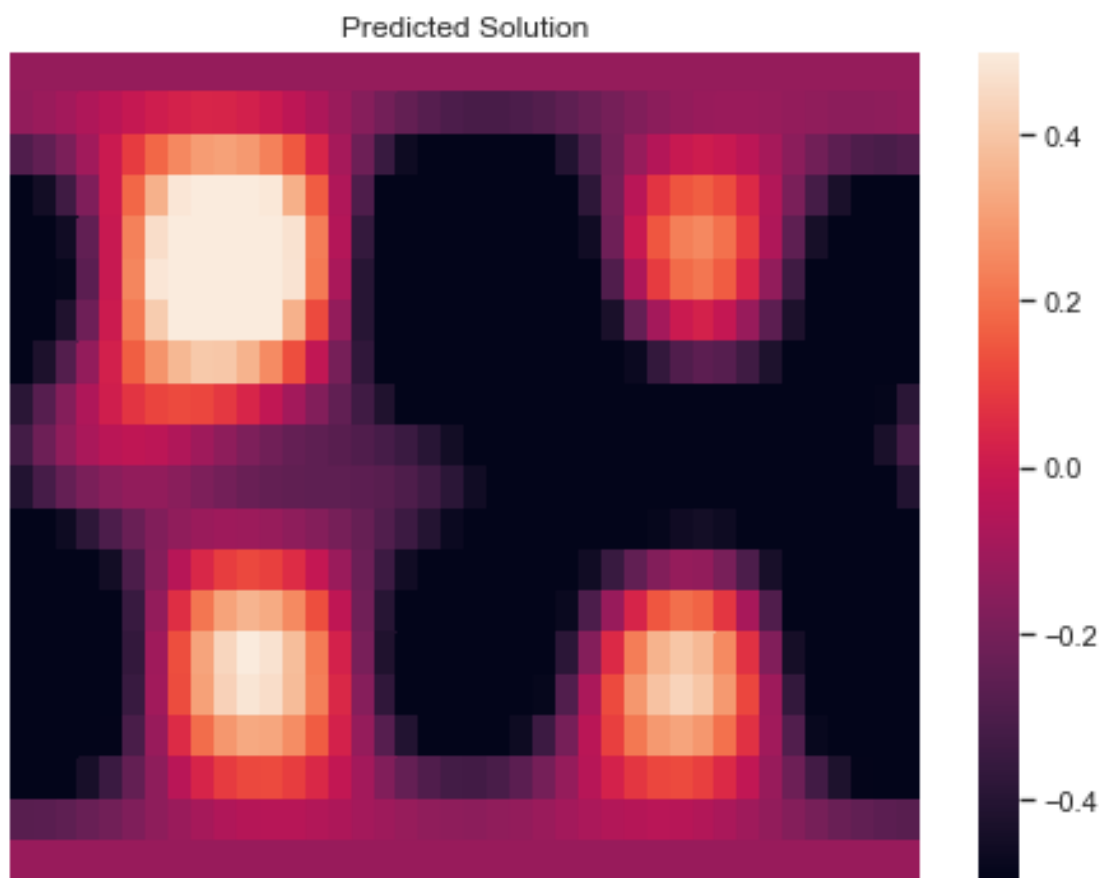


iteration 0: loss = 9527871.188298473, pde loss = 9527870.878506223

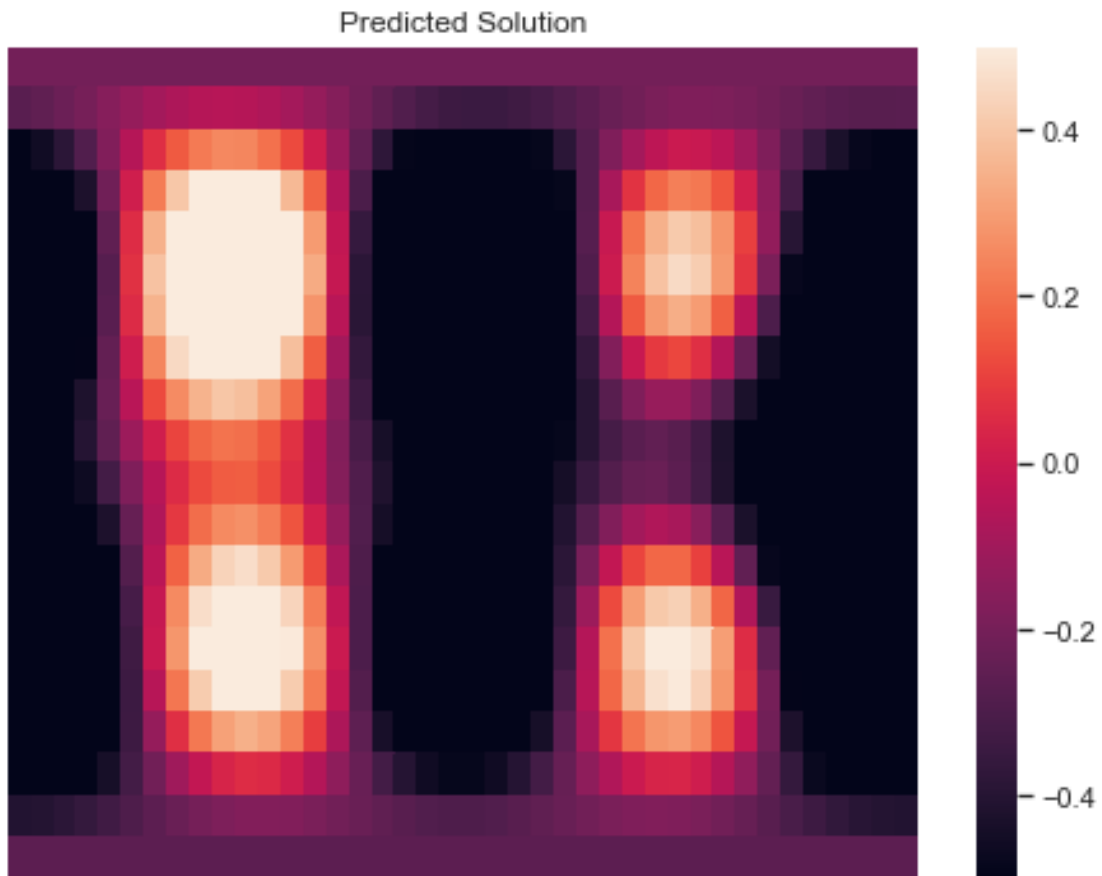


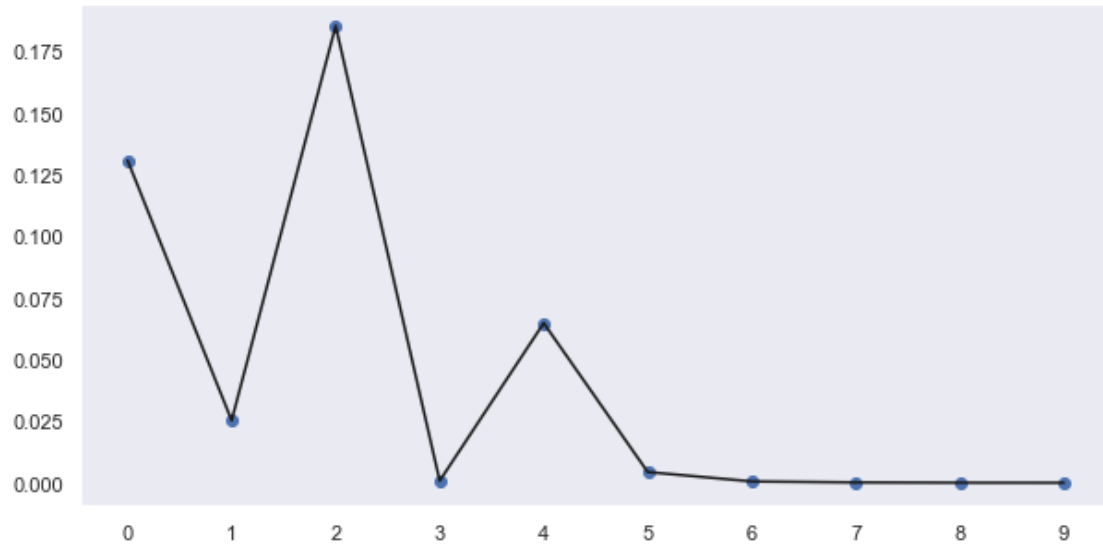
iteration 50: loss = 2822221.705288708, pde loss = 2822221.6271554604

iteration 100: loss = 233848.7516770983, pde loss = 233848.70896156
iteration 150: loss = 162822.02433125436, pde loss = 162822.0229378121
iteration 200: loss = 111429.26090596861, pde loss = 111429.25590294471

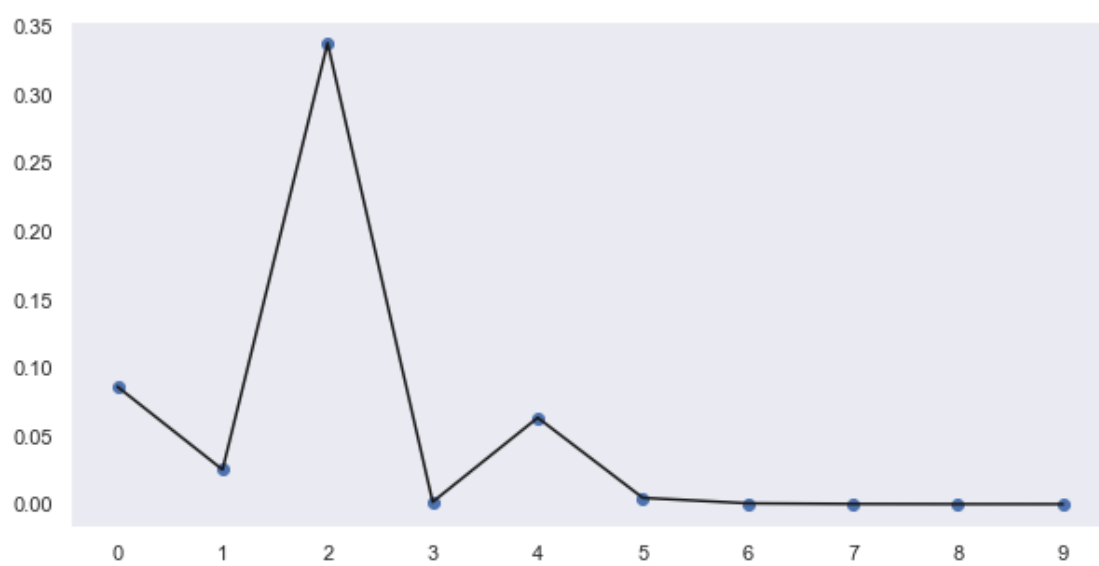
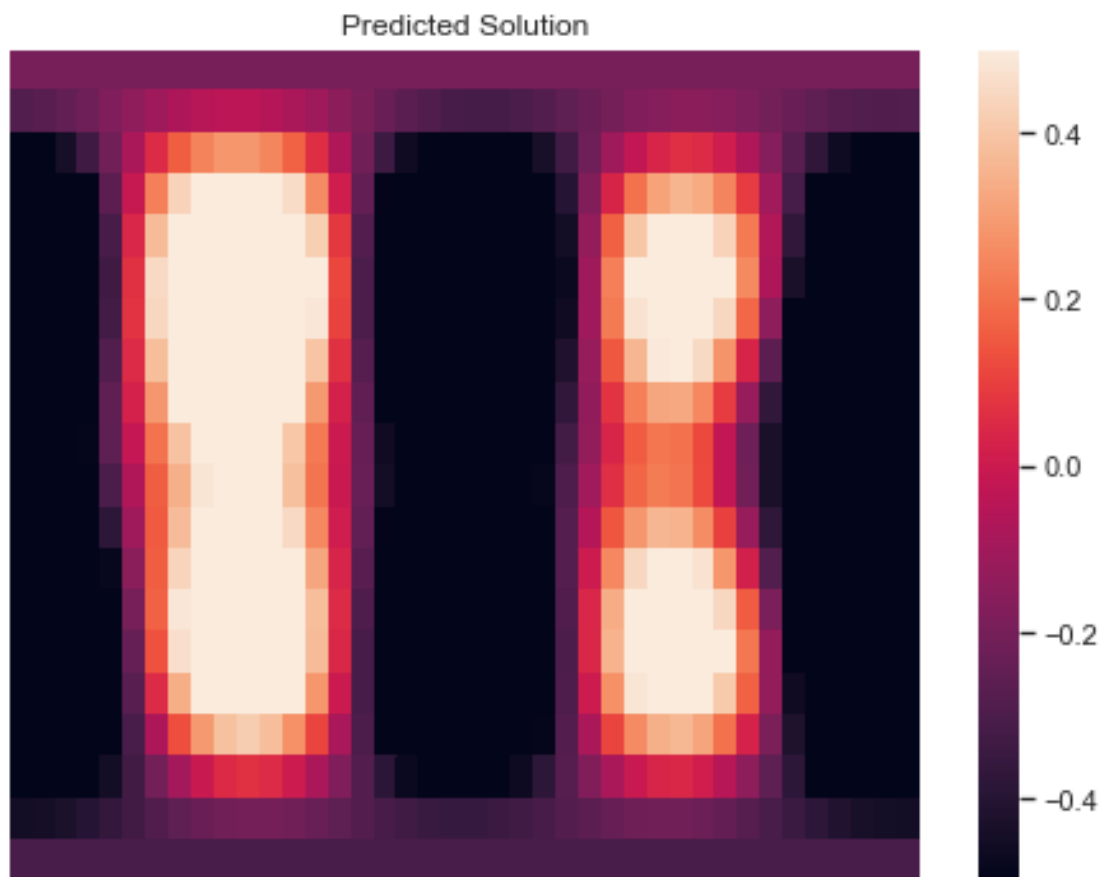


```
iteration 250: loss = 76585.11751519011, pde loss = 76585.11623157906  
iteration 300: loss = 63892.23007525978, pde loss = 63892.23003839584  
iteration 350: loss = 73399.39524248152, pde loss = 73399.39472608498  
iteration 400: loss = 59531.16960418875, pde loss = 59531.1681578626
```



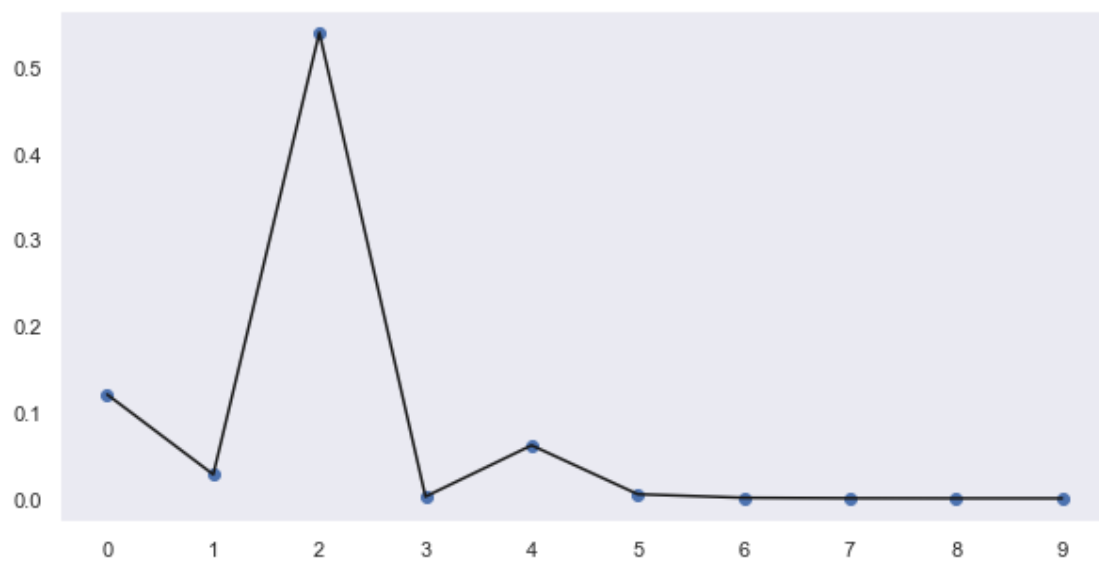
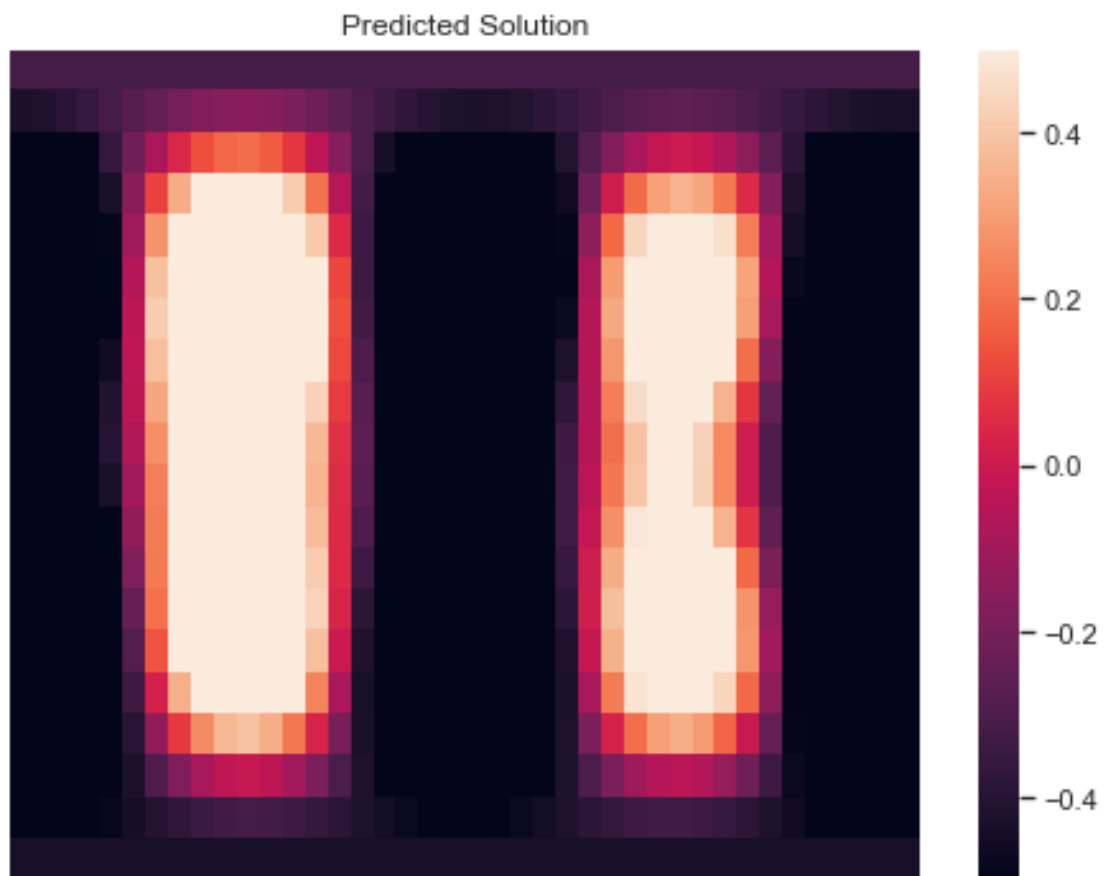


```
iteration 450: loss = 48357.43455277659, pde loss = 48357.43307976619
iteration 500: loss = 38135.38011989802, pde loss = 38135.38011696618
iteration 550: loss = 40753.44494531205, pde loss = 40753.44412608625
iteration 600: loss = 50936.099323112074, pde loss = 50936.09734875711
```



iteration 650: loss = 30267.53424078872, pde loss = 30267.531311510535

iteration 700: loss = 21508.01699910481, pde loss = 21508.0161782524
iteration 750: loss = 15913.811568012472, pde loss = 15913.810325743525
iteration 800: loss = 45766.42071710669, pde loss = 45766.41998679397



```
iteration 850: loss = 11174.952080697916, pde loss = 11174.950831421349  
iteration 900: loss = 12841.100777906946, pde loss = 12841.100268372275  
iteration 950: loss = 10277.753791146459, pde loss = 10277.753174653872  
iteration 999: loss = 16312.765207052189, pde loss = 16312.763524223077
```

