

MCMC2: Lab Session 2

Coding and output (Supplementary Material)

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In question 3 we are interested in fitting the non-Markovian model to some observed data.

When writing an MCMC algorithm the first thing to do is to write what is the target distribution's density, ie the density of the distribution that we want to sample from. In our case it will be the posterior distribution of the infection rate (β), the scale parameter of the Gamma distribution (γ) and the set of the infection times \mathbf{I} .

$$\begin{aligned}\pi(\beta, \gamma, \mathbf{I} | \mathbf{R}) &\propto \prod_{j \neq a}^n (\beta/N) I_{i_j} \exp \left\{ - \int_{i_a}^{r_b} (\beta/N) S_t I_t \, dt \right\} \\ &\times \prod_{j=1}^n f(R_j - I_j | \gamma) \\ &\times \pi(\beta) \pi(\gamma)\end{aligned}$$

where $f(\cdot)$ denotes the pdf of a Gamma distribution. That means

$$\begin{aligned}\pi(\beta, \gamma, \mathbf{I} | \mathbf{R}) &\propto \prod_{j \neq a}^n (\beta/N) I_{i_j} \exp \left\{ - \int_{i_a}^{r_b} (\beta/N) S_t I_t \, dt \right\} \\ &\times \prod_{j=1}^n \left(\frac{1}{\Gamma(\alpha)} \gamma^\alpha (R_j - I_j)^{\alpha-1} \exp\{-(R_j - I_j)\gamma\} \right) \\ &\times \frac{1}{\Gamma(\lambda_\beta)} \nu_\beta^{\lambda_\beta} \beta^{\lambda_\beta-1} \exp\{-\beta \nu_\beta\} \frac{1}{\Gamma(\lambda_\gamma)} \nu_\gamma^{\lambda_\gamma} \gamma^{\lambda_\gamma-1} \exp\{-\gamma \nu_\gamma\}\end{aligned}$$

Full conditional distributions

1.

$$\pi(\beta|\gamma, \mathbf{I}, \mathbf{R}) \propto \beta^{n-1} \exp \left\{ -\beta \left(\int_{i_a}^{r_b} \frac{1}{N} S_t I_t + \nu_\beta \right) \right\} \beta^{\lambda_\beta - 1} \exp \{ -\beta \nu_\beta \}$$

2.

$$\pi(\gamma|\beta, \mathbf{I}, \mathbf{R}) \propto \gamma^{\alpha n} \exp \left\{ -\gamma \left(\sum_{j=1}^n (R_j - I_j) + \nu_\gamma \right) \right\} \gamma^{\lambda_\gamma - 1} \exp \{ -\gamma \nu_\gamma \}$$

3.

$$\pi(\mathbf{I}|\mathbf{R}, \beta, \gamma) \propto \prod_{j \neq a}^n I_{i_j} \prod_{j=1}^n (R_j - I_j)^{\alpha - 1} \exp \left\{ - \int_{i_a}^{r_b} \frac{\beta}{N} S_t I_t \, dt \right\} \exp \left\{ -\gamma \left(\sum_{j=1}^n (R_j - I_j) + \nu_\gamma \right) \right\}$$

It is then easy to see that

$$\beta|\gamma, \mathbf{I}, \mathbf{R} \sim \text{Ga} \left(n - 1 + \lambda_\beta, \nu_\beta + \int_{i_a}^{r_b} \frac{1}{N} S_t I_t \right)$$

and therefore can do a Gibbs step for β .

Likewise for the scale parameter of the Gamma infectious period distribution

$$\gamma|\beta, \mathbf{I}, \mathbf{R} \sim \text{Ga} \left(\alpha n + \lambda_\gamma, \nu_\gamma + \sum_{j=1}^n (R_j - I_j) \right)$$

and hence, can do a Gibbs step.

Unfortunately, the full conditional distribution of the infection times given the removal times and β and γ does not look like that it comes from a well known distribution and therefore we can do a Metropolis-Hastings step as we did for the Markovian epidemic model.