Experimentation with Varying Innovation Protection and Market Power*

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Abstract

I describe equilibrium firm experimentation behavior for any combination of innovation protection and market power. When experimenting is expensive, firms only do so when they expect to act as a monopolist if successful. In this case, welfare is maximized by increasing innovation protection and decreasing expected market power in order to preclude entry after the initial innovation. If experimenting is cheap, firms may experiment even when they expect additional firm entry. In this case, the welfare-maximizing levels of innovation protection and market power are ambiguous: encouraging additional firm entry after the initial success decreases deadweight loss but causes a duplication of experimentation effort. If consumers share the same value for the new good, a restricted social planner minimizes the duplication of experimentation effort by selecting levels of innovation protection and market power in order to block firm entry after the initial innovation. If consumers have heterogeneous values for the good and the social planner discounts at a low enough rate, they select levels of innovation protection and market power in order to minimize deadweight loss.

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1 Introduction

Firms must invest in costly experimentation in order to discover new innovations. When making experimentation decisions, firms must consider both the level of market power and innovation protection they expect to receive upon successfully innovating. We know that the level of innovation protection and market power vary widely across industries (Levin et al., 1987; De Loecker et al., 2020). Furthermore, we have the ability to affect expected innovation protection and market power through anti-trust and patent protection policy. However, existing models of firm experimentation, such as those by Keller et al. (2005) and Acemoglu et al. (2011), do not include independent measures of both parameters. Therefore, in this paper, I present a simple model of duopoly firm experimentation with independent measures for both innovation protection and marker power. I first solve for equilibrium firm behavior across the plane of all potential combinations of market power and innovation protection. I then solve a constrained social planner's problem in order to derive optimal levels of innovation protection and market power. Welfare trade-offs exist along both axis: high market power induces high deadweight loss; however, low market power may disincentivize experimentation after the initial innovation, leading to monopoly markups. Likewise, high innovation protection causes a duplication of effort; however, low innovation protection may induce free-riding, slowing the process of innovation.

My measure of innovation protection takes the form of a continuous parameter which modifies the difficulty of subsequent innovations rather than a perfect patent which blocks further entry. This decision was motivated by the empirical research conducted by Levin et al. (1987). In their paper, 650 R&D managers from 130 lines of business were asked to score different means of appropriation for their innovations. Patents were deemed less effective than other means of appropriation in the majority of industries. Instead, lead time and secrecy were given the highest scores. Levin and his coauthors also asked the R&D managers how long and how costly duplicating a competitor's innovation was as a portion of the time and cost needed to create the initial innovation. The managers reported a wide range of percentages, suggesting that some innovations can be copied very quickly while others cannot. Patented innovations were also reported as being more difficult to copy. Thus, while patents may not be the main source of appropriation for innovations, they do appear to increase innovation protection. Subsequent studies have reported similar findings (Harabi, 1995; Cohen et al., 2000). Thus, my measure of innovation protection captures the lead time that the initial innovator can expect before future entry, rather than a complete block on future innovations. Increasing innovation protection increases this expected lead time.

I borrow from the price dispersion literature for my measure of market power (Varian, 1980; Burdett and Judd, 1983; Stahl, 1989). When a firm is the first to innovate, they act as a monopolist until further firm entry. However if both firms have innovated and entered the market, their profit flows are a function of the number of "captives" who they can sell to. Captives only see the price posted by one firm or the other, but not both. The rest of the consumers see both posted prices. As in the price dispersion literature, this induces firms to randomize over the prices they post. If there are more captives, they post prices closer to the monopoly price. If there are less captives, they post prices closer to marginal cost. I use the share of captives in the market as my measure of market power. As the share of captives increases from 0 to 1, the firms' profit flows increase from 0 to an even split of the monopoly profits.

In my analysis, I describe all pure and mixed-strategy equilibria. The game essentially plays out in three stages and thus I proceed via backwards induction. Firms know the level of market power in the industry and therefore are aware of their expected profit flows if both firms enter the market. Thus, after the initial innovation, the remaining firm immediately decides to either continue experimenting or to drop out. Firms are more likely to continue experimenting if there is low innovation protection as well as if they expect high levels of market power. This informs each firm's decision to either experiment or wait before either have innovated: if they expect to drop out of the race if they don't innovate first, there is no value in waiting initially. However, if they expect to continue experimenting after the initial innovation, it may make sense to wait in order to benefit from their competitor's innovation.

The game presents four potential strategy profiles: in the *Monopoly* strategy profile, both firms experiment initially, and the remaining firm drops out after the first innovation. In the *Free-Rider* strategy profile, one firm experiments initially while the other waits, only experimenting after the initial innovation is discovered. In the *Full-Experimentation* strategy profile, both firms continuously experiment until they innovate, regardless of whether or not they innovate first. And finally, in the *No-Experimentation* strategy profile, neither firm experiment initially and thus no innovation is ever discovered.

I begin by describing when each strategy profile is an equilibrium for all potential values of innovation protection and market power. If experimenting is expensive, only the Monopoly and No-Experimentation strategy profiles may be equilibria—firms only experiment if they expect to act as a perpetual monopolist if they innovate first. If experimenting is cheap, the Free-Rider and Full-Experimentation strategy profiles also obtain as equilibria for portions of the parameter space—firms may experiment initially knowing that their competitor will eventually also enter the market. Furthermore, the relative positions of the equilibria on the plane are robust. This allows me to make statements about the kinds of firm dynamics we should expect for different levels of innovation protection and market power.

I then analyze the problem of a constrained planner who can only control the level of innovation protection and market power in the market. The optimal levels of both depend on whether or not experimenting is expensive. When experimenting is expensive, it is optimal to increase innovation protection and decrease market power in order to dissuade experimentation after the initial innovation, since firms will only experiment initially if they expect to act as a perpetual monopolist once they innovate. If experimenting is cheap, the optimal levels of innovation protection and market power are ambiguous: experimentation beyond the initial innovation is costly; however, additional firm entry decreases the deadweight-loss in the market. I explore two limiting cases: one in which all consumers share the same value for the good, and another in which the social planner has a low discount rate. In the first case, the market exhibits full coverage, and so market power does not create deadweight loss. Thus, the social planner will select a level of innovation protection and market power in order to minimize the duplication of innovation effort. In the second case, the benefit of decreased deadweight loss overpowers the initial cost of effort duplication. Thus, planner will select a level of innovation protection and market power in order to minimize deadweight loss.

1.1 Related Literature

My project adds to the literature on firm experimentation. As in the seminal paper by Keller et al. (2005), firms decide whether or not to experiment at every point in time. However, in their paper, the rate of discovery is unknown, and thus firms experiment in order to discover whether or not innovation is possible. A successful innovation does not change the rate of discovery for subsequent firms, nor do the firms compete in a market once they innovate. In contrast, I assume that the rate of discovery is known to the firms and that the initial innovation changes this rate. The empirical work by Levin et al. (1987) appears to support such an assumption: the initial innovation may make subsequent ones easier due to spillovers or harder due to innovation protection. Firms also compete for consumers once they enter the market in my model, allowing me to discuss the effect of market power on experimentation.

My paper is more closely related to Wong (2018). The environment in his paper is similar to Keller et al. (2005); however, the share of profits conferred to the initial innovator changes based on the level of patent protection. If patent protection is strong, the initial innovator earns all of the profits; if patent protection is weak, they split the profits with the competing firm. This induces two different inefficiencies: high patent protection causes a harmful duplication of effort, whereas low patent protection causes free riding. My paper captures a similar trade-off along the innovation protection axis, while also capturing another trade-off between effort duplication and deadweight loss along the market power axis. Both are important in conjunction when determining the optimal level of innovation protection and market power.

Acemoglu et al. (2011) analyze a model of experimentation in which the rate of success is known to the firms as is the case in my setting. They assume that the success or failure of experimentation is instantly realized, and that firms would always rather wait to copy an innovation than experiment themselves. In contrast, innovations arrive stochastically when firms experiment in my setting. Additionally, firms only prefer to wait and copy a successful innovation for certain levels of innovation protection and market power. They also implement a patent scheme which perfectly protects innovation, whereas my measure of innovation protection makes innovating more difficult without blocking future innovation entirely.

My project also relates to the patent race literature. Loury (1979) and Lee and Wilde (1980) explore how increasing firm quantity effects experimentation effort in a setting in which the initial innovator can perfectly protect their innovation. In these models, markets are competitive and innovations can be copied instantly, necessitating patents to induce innovation. In my setting, market power and innovation protection vary, allowing for a wider range of potential firm behavior. If innovation protection is high enough, the initial innovator will act as a monopolist and the remaining firm will drop out as in the classic patent race. However, the remaining firm may also remain in the race and enter themselves, or there may be free riding in the initial state if innovation protection is low enough. Fershtman and Markovich (2010) explore firm behavior for a multistage race with and without patenting, but they don't include the possibility of imperfect innovation protection. Choi (1991) and Awaya and Krishna (2021) create patent races with uncertain rates of innovation akin to the setup in models of experimentation; however, again, patents perfectly protect successful innovations.

This project relates to the literature on firm entry games as well (Fudenberg et al., 1983; Hoppe and Lehmann-Grube, 2005; Chen et al., 2023). In these models, the cost of technological adoption becomes cheaper over time, and thus a trade-off exists between entering first and acting as a monopolist for a time vs. entering later and adopting the technology at a lower price. The same trade-off exists within my setting; however, the cause is less mechanical. Experimenting after the initial innovation may be less expensive due to spillovers, inducing one firm to wait while the other innovates initially. However, my setting also describes scenarios in which experimenting becomes more difficult after the initial innovation, in which case there is no benefit to free-riding.

2 Environment

The game consists of two identical firms $i \in \{A, B\}$ and a continuous flow of consumers with unit mass. Time is continuous in the sense of discreet time intervals whose length is shrunk infinitesimally as in Hoppe and Lehmann-Grube (2005). Both firms discount with rate ρ. Firms must experiment in order to discover a new good which they can sell to the flow of consumers. Thus, the state of the game at time t, ω_t, is the set of firms that have successfully innovated and entered the market by time t: if $\omega_t = \varnothing$, neither firm has discovered their innovation by t. If $\omega_t = \{A, B\}$, both have. Assume that $\omega_0 = \varnothing$: neither firm has discovered their innovation at the start of the game.

If $i \notin \omega_t$, firm i selects strategy $x_t^i \in \{0, 1\}$, where $x_t^i = 1$ implies that firm i experiments at time t, and $x_t^i = 0$ implies that they wait. Firm i's cost flow at time t is $-cx_t^i$. Thus, while experimenting, firms pay a flow cost c. When $\omega_t = \varnothing$, the instantaneous probability of i discovering their innovation is λx_t^i, $i \in \{A, B\}$. Thus, when neither firm has innovated, the rate of discovery for each is λ when experimenting and 0 otherwise. Let $-i$ be not firm i. When $\omega_t = \{-i\}$, the rate of discovery for firm i at time t is $(\lambda/\sigma)x_t^i$, where $\sigma \in (0, \infty)$ represents the level of innovation protection. If $\sigma \to 0$, $\lambda/\sigma \to \infty$, and thus firm i can discover their innovation instantly. If $\sigma \to \infty$, $\lambda/\sigma \to 0$, and thus i will never innovate no matter how much they experiment.

Once a firm enters the market, it produces a good at marginal cost normalized to 0 and can sell it to the flow of consumers whose value for the good follows a probability distribution $G(p)$. Assume that $G(p)$ presents a unique monopoly price p^* and that $G'(p)$ exists for all p. If $i \in \omega_t$, firm i selects a distribution over potential prices $F_t^i(p)$ at each moment in time. If $\omega_t = \{i\}$, only firm i has entered the market by time t, and thus the entire flow of consumers see the price posted by i. If $\omega_t = \{A, B\}$, both firms have entered the market and will post prices. A portion μ of the consumers will see the price posted by one firm or the other, whereas the remaining $1 - \mu$ proportion will see the price posted by both. I use μ as my measure of market power: as μ increases, more consumers only see one price, allowing firms to impose greater makeups. As μ decreases, more consumers see both prices, causing the firms to compete more intensely.

3 Equilibrium

I begin by solving for the monopoly and duopoly profit flows that firms expect to earn upon successfully innovating. I then use these values to solve for pure and mixed equilibrium experimentation decisions for varying levels of innovation protection and market power.

3.1 Profit Flows

First, define $\pi = (1 - G(p^*))p^*$. Remember that p^* is the unique monopoly price. Thus, If only one firm has innovated and entered the market, they post the price p^* and earn monopoly profits flows π. Now assume that $\omega = \{A, B\}$; both firms have entered the market. We will now nail down the symmetric equilibrium price distribution $F(p)$ and duopoly profit flow. The process is derived from the price dispersion literature, and thus the proofs presented here are similar to those in Varian (1980) and Burdett and Judd (1983). First, we demonstrate that if some consumers see both prices while others only see one, firms will mix over prices in equilibrium.

Lemma 1. *If $\omega = \{A, B\}$ and $\mu \in (0, 1)$, F(p) has no mass points.*

Proof. A.1 $\square$

If one firm plays a price with positive mass, the other can increase their expected payoff by playing a price just below their competitor's mass point, allowing them to capture the mass of non-captives whenever both play their mass point prices. Thus, no price is paid with positive mass.

Now define $\underline{p}$ as the value satisfying

$$\frac{\mu}{2}\pi = \left(1 - \frac{\mu}{2}\right)(1 - G(\underline{p}))\underline{p}. \tag{1}$$

Notice that $\underline{p}$ is the price at which the profit flow for selling to the non-captives and captives is the same as the profit flow when selling to only the captives at the monopoly price. We can now define the support of $F(p)$.

Lemma 2. *If $\omega = \{A, B\}$ and $\mu \in (0, 1)$, the support of $F(p)$ is $[\underline{p}, p^*]$*

Proof. A.2 $\square$

Therefore, the equilibrium price distribution has continuous support over the interval $[\underline{p}, p^*]$. Now we can define the monopoly and duopoly equilibrium price distributions as well as the resulting profit flows.

Proposition 1. *If $\omega = \{i\}$, i charges p^* and the profit flow is π. If $\omega = \{A, B\}$, there are three potential cases:*

- *If $\mu = 1$,*

$$F(p) = \begin{cases} 0 & \text{if} \quad p < p^* \\ 1 & \text{if} \quad p \geq p^* \end{cases}$$

  and profit flows are $\dfrac{\pi}{2}$.

- *If $\mu = 0$,*

$$F(p) = \begin{cases} 0 & \text{if} \quad p < 0 \\ 1 & \text{if} \quad p \geq 0 \end{cases}$$

  and profit flows are 0.

- *If* $\mu \in (0,1)$,

$$F(p) = \begin{cases} 0 & \textit{if} \quad p < 0 \\ 1 - \dfrac{\mu[\pi - (1-G(p))p]}{2(1-\mu)(1-G(p))p} & \textit{if} \quad 0 \leq p \leq p^* \\ 1 & \textit{if} \quad p > p^* \end{cases}$$

and profit flows are $\dfrac{\mu\pi}{2}$.

Proof. A.3 □

If only one firm has entered, they simply charge the monopoly price since the face no competition. Once another firm enters the market, the distribution of prices charged depends on the share of captives. If all consumers are captives, each only sees one firm's price, and so both firms charge the monopoly price. Thus, the firms split the monopoly profits. If none of the consumers are captives, all see both prices, and thus competition reduces to a Bertrand pricing game. Firms post a price of 0, their marginal cost, and subsequently earn no profits. If some consumers are captives while others are not, the firms will charge a distribution of prices. Since firms earn $(\mu/2)\pi$ profit flows when charging the monopoly price p^*, this must be the the expected profit flow for any price in the support of the equilibrium distribution. This fact reveals equilibrium duopoly profit flows and allows us to derive the equilibrium price distribution.

3.2 Pure-Strategy Equilibria

In this section, I define the potential equilibrium strategy profiles and derive inequalities that describe when each strategy profile is an equilibrium for a given combination of market power and innovation protection. Let $V_i(x_i, x_{-i}, \omega)$ be firm i's value function given their own strategy, the strategy of their competitor, and the state of the game. We begin by defining our equilibrium concept.

Definition 1. *A pure-strategy equilibrium is a sequence of actions* x_A^*, x_B^* *such that*

$$V_i(x_i^*, x_{-i}^*, \omega) \geq V_i(x_i, x_{-i}, \omega) \quad \textit{for} \quad i \in \{A, B\}$$

for all potential strategies x_i, x_{-i} *for each state* ω *at every point in time.*

Thus, we restrict our analysis to Markov-perfect equilibria. Throughout the remainder of the paper, I shorten $V_i(x_i^*, x_{-i}^*, \omega)$ to V_ω. The state-transition diagram in figure 1 describes the progression between states of the game. The game begins in state $\varnothing$, as neither firm has yet discovered their innovation. At every moment in time, the game may move to state $\{A\}$, $\{B\}$, or remain in state $\varnothing$, depending on whether or not the firms experiment. Once the game is in state $\{i\}$, firm i sells to the flow of consumers at the monopoly price while

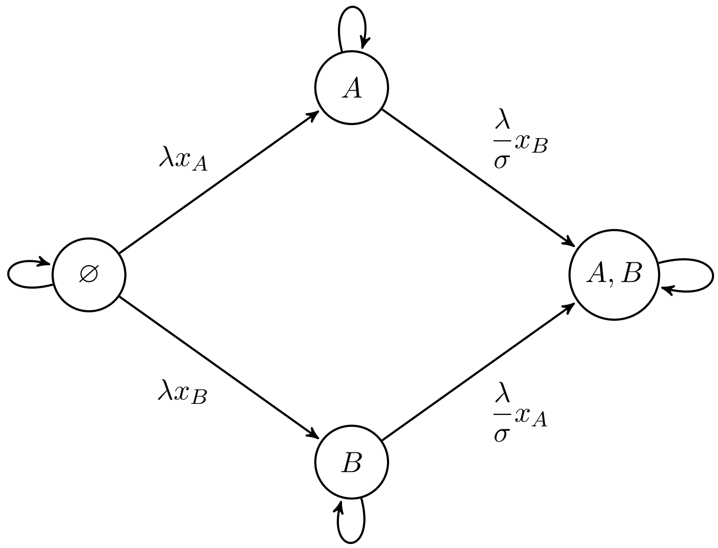


Figure 1: State-transitions of the game

firm $-i$ decides whether or not to experiment. If $-i$ does experiment, the game moves to state $\{A, B\}$ at rate λ/σ. If both firms enter the market, the game reaches state $\{A, B\}$ and remains there for the rest of time. Both firms continuously play equilibrium distributions of prices based on μ, the proportion of captive consumers. The following set of value functions capture this progression.

$$\rho V_{\varnothing} = \max_{x_i} -cx_i + \lambda x_i(V_{\{i\}} - V_{\{\varnothing\}}) + \lambda x_{-i}(V_{\{-i\}} - V_{\{\varnothing\}}) \tag{2}$$

$$\rho V_{\{-i\}} = \max_{x_i} -cx_i + \frac{\lambda}{\sigma}x_i(V_{\{A,B\}} - V_{\{-i\}}) \tag{3}$$

$$\rho V_{\{i\}} = \pi + \frac{\lambda}{\sigma}x_{\{-i\}}(V_{\{A,B\}} - V_{\{i\}}) \tag{4}$$

$$\rho V_{\{A,B\}} = \frac{\mu}{2}\pi \tag{5}$$

Equation 2 describes the asset value of being in the $\varnothing$ state for firm i. The firm will pay a flow when they experiment and move to the state in which they alone have innovated and entered the market at a rate λ. Likewise, they move into the state in which only their competitor has innovated with rate λ when their competitor experiments. Equation 3 describes the asset value for i in the state which only their competitor has innovated. Firm i will decide whether or not to experiment based on the level of innovation protection enjoyed by their competitor as well as the level of market power they expect to face if they innovate. When experimenting, they move into the state in which both firms have innovated with rate λ/σ. Equation 4 describes the asset value for i of being in the state in which they alone have innovated. They earn monopoly profits, which we derived in proposition 1, until their competitor enters, at which point their profit flows will depend on the level of market power. Equation 5 describes the asset value of being in the state in which both firms have innovated, which is just the duopoly profit flow we derived in proposition 1.

There are four potential pure strategy profiles that may be equilibria of the game. In the No-Experimentation strategy profile, neither firm experiments in the initial state and thus neither firm ever innovates. In the Monopoly strategy profile, both firms experiment initially until one succeeds. At this point, the remaining firm ceases to experiment and thus the firm who succeeded acts as a perpetual monopolist. In the Free-Rider strategy profile, one firm experiments initially while the other waits. After the experimenting firm innovates, the remaining firm begins experimenting until they also innovate. In the Full-Experimentation strategy profile, both firms begin experimenting immediately and continue to do so until they innovate regardless of the other firm's behavior. I formally define each of these equilibrium below.

Definition 2. *The potential equilibrium pure strategy profiles of the game are as follows:*

- ***No Experimentation****:* $x_A = x_B = 0$ *in state* $\varnothing$.
- ***Monopoly****:* $x_A = x_B = 1$ *in state* $\varnothing$ *and* $x_{-i} = 0$ *in state* $\{i\}$.
- ***Free-Rider****:* $x_i = 1$ *while* $x_{-i} = 0$ *in state* $\varnothing$*, and* $x_{-i} = 1$ *in state* $\{i\}$.
- ***Full Experimentation****:* $x_A = x_B = 1$ *in state* $\varnothing$*, and* $x_{-i} = 1$ *in state* $\{i\}$.

In what follows, I define curves which partition the parameter space into sets which obtain these different strategy profiles in equilibrium. I proceed via backwards induction: I solved for the equilibrium profit flows in proposition 1, and thus I will now describe the portion of the parameter space in which the remaining firm ceases to experiment after the initial innovation is discovered. We can rewrite equation 5 as

$$V_{\{A,B\}} = \frac{\mu\pi}{2\rho}.$$

The value of entering the state in which both firms have innovated is just the discounted duopoly profit flows. Plugging this value into equation 3 and solving for $V_{\{-i\}}$ gives us

$$V_{\{-i\}} = \max_{x_i} \frac{(\frac{\lambda}{\sigma} \cdot \frac{\mu\pi}{2\rho} - c)x_i}{\rho + \lambda/\sigma}.$$

The value of being in the state in which only your competitor has innovated depends on whether or not you decide to experiment. If you don't experiment, the state has no value. If you do experiment, the value is the difference between the discounted profit flows earned from entering the market vs. the current cost of experimenting. Thus, firm i only experiments if this difference is positive. Therefore, i's best response in state $\{-i\}$ is

$$x_i = \begin{cases} 1 & \text{if} \quad \mu \geq \dfrac{2c\rho}{\lambda\pi}\sigma, \\ 0 & \text{otherwise.} \end{cases}$$

Now, define

$$\mu_M(\sigma) = \frac{2c\rho}{\lambda\pi}\sigma.$$

The curve is labeled μ_M since it describes the minimum level of market power necessary to avoid the Monopoly strategy profile in equilibrium and encourage experimentation after the initial innovation. We will now reparameterize the model in terms of the ratios c/π and ρ/λ as they determine equilibrium behavior rather than c, π, ρ, and λ independently. Thus, define

$$\tilde{c} = \frac{c}{\pi} \quad \text{and} \quad \tilde{\rho} = \frac{\rho}{\lambda}.$$

I will refer to $\tilde{c}$ as the relative cost of experimentation and $\tilde{\rho}$ as the relative discount rate. Thus, $\mu_M(\sigma)$ can be rewritten as

$$\mu_M(\sigma) = 2\tilde{\rho}\tilde{c}\sigma. \tag{6}$$

I comment on μ_M's behavior along the $\sigma - \mu$ plane in the following remark.

Remark 1. $\lim_{\sigma \to 0} \mu_M = 0$, $\dfrac{\partial \mu_M}{\partial \sigma} > 0$, *and the remaining firms drops out for any* $\mu \in [0, 1]$ *if* $\sigma > \dfrac{1}{2\tilde{c}\tilde{\rho}}$.

Figure 2 captures these characteristics. Each are intuitive: as $\sigma \to 0$, the firm can copy the initial innovation almost instantly. Thus, they require infinitesimal market power to induce them to experiment, since they know they won't have to experiment for long. This connects to the general relationship between μ and σ: as innovation protection increases, firms expect to experiment longer before they innovate. Thus, they require a higher level of market power once they enter in order to recoup their experimentation costs. This relates to Schumpeter's critique of perfect competition as it relates to innovation (1942): firms require some level of market power in order to encourage costly experimentation. In this model, if market power is too low, the remaining firm drops out after the initial innovation, leading to a monopoly. Therefore, paradoxically, low market power may induce higher markups, since the initial innovator is more likely to act as a perpetual monopolist. The final point in remark 1 highlights the fact that if innovation protection is strong enough, the remaining firm drops out no matter how much market power they expect upon entry. In this case, the game reduces to a simple patent race where the initial innovator is perfectly protected from future entry.

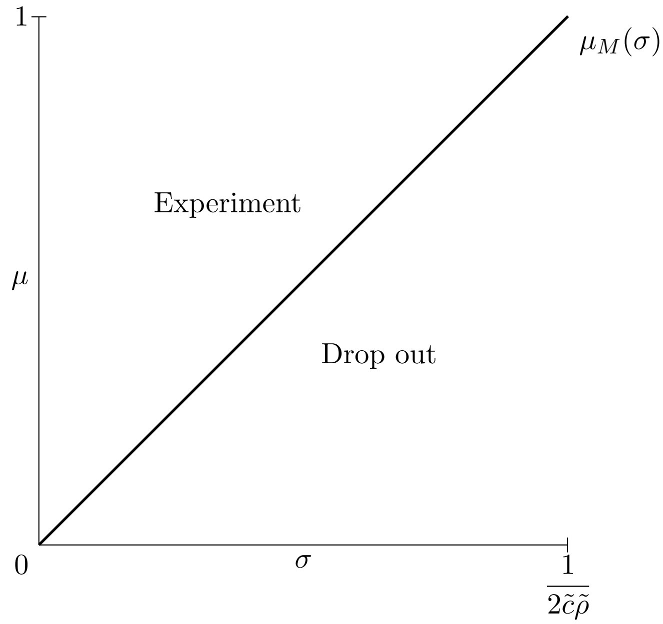


Figure 2: Best-response for the remaining firm after the initial innovation.

Now we can express the best responses for firms in the initial state of the game. First, assume that $\mu < \mu_M(\sigma)$: the remaining firm drops out. In this case, equations 3 and 4 simplify to

$$V_{\{-i\}} = 0 \quad \text{and} \quad V_{\{i\}} = \frac{\pi}{\rho}.$$

The value for firm i in the state in which their competitor has innovated is 0, since i's best response is to not experiment after the initial innovation. However, if i innovates first, they earn monopoly profits for the rest of time since their competitor will cease to experiment. Thus, equation 2 simplifies to

$$V_\varnothing = \max_{x_i} \frac{(\lambda \cdot \frac{\pi}{\rho} - c)x_i}{\rho + \lambda(x_i + x_{-i})}.$$

If neither firm has innovated, both decide whether or not to experiment based on the value of acting as a perpetual monopolist and the rate at which they expect to innovate. Notice that this problem is not strategic: since the remaining firm will drop out after the initial innovation, firms cannot learn from each other. Therefore, the best response for firm i in state $\varnothing$ when $\mu < \mu_M(\sigma)$ is

$$x_i = \begin{cases} 1 & \text{if} \quad c \leq \dfrac{\lambda\pi}{\rho}, \\ 0 & \text{otherwise.} \end{cases}$$

Let $\bar{c} = \dfrac{\pi}{\tilde{\rho}}$. We can now describe the unique equilibrium behavior whenever $\mu < \mu_M$.

Proposition 2. *If $\mu < \mu_M(\sigma)$ and $c > \bar{c}$, a point (σ, μ) exhibits the No-Experimentation* *strategy profile in equilibrium. If $\mu < \mu_M(\sigma)$ and $c \leq \bar{c}$, a point (σ, μ) exhibits the Monopoly strategy profile in equilibrium.*

The proposition is an immediate result of the firm's best response function and thus proof is omitted. If $\mu < \mu_M$, neither firm conducts R&D if costs are too high, while both conduct R&D in state $\varnothing$ otherwise. Since $\mu < \mu_M$, the remaining firm drops out after the initial success, leaving the initial innovator to act as a monopolist.

The following corollary notes that costs above $\bar{c}$ prevent experimentation for any combination of μ and σ.

Corollary 1. *No experimentation occurs if $c > \bar{c}$.*

Since costs above $\bar{c}$ prevent experimentation even when firms expect to act as a monopolist for the rest of time if successful, it also prevents initial experimentation when firms expect future entry from their competitor. Thus, for the rest of the paper, I assume that $c < \bar{c}$.

Now assume that $\mu > \mu_M(\sigma)$ for a point (σ, μ): the remaining firm experiments after the initial innovation is discovered. In this case, equations 2 can be written as

$$V_\varnothing = \max_{x_i} \frac{\lambda x_i V_{\{i\}} + \lambda x_{-i} V_{\{-i\}} - c x_i}{\rho + \lambda(x_i + x_{-i})}$$

with

$$V_{\{-i\}} = \frac{\frac{\lambda\mu\pi}{2\rho\sigma} - c}{\rho + \lambda/\sigma} \quad \text{and} \quad V_{\{i\}} = \frac{\frac{\lambda\mu\pi}{2\rho\sigma} + \pi}{\rho + \lambda/\sigma}.$$

Note that the value for i in the state in which $-i$ has innovated is now positive: firm i will experiment until they innovate and enter, as the present value of their expected profit flow is greater than the cost of experimenting. The value of the state in which i innovates first is now lower than in the monopoly equilibrium, since they expect $-i$ to eventually innovate and partially erode their profit flow.

Also note that this problem is now strategic: the firm earns a positive payoff whether or not they are the leader, and thus firm i's experimentation decision may depend on the decision of their competitor. If their competitor experiments initially, they may prefer to wait and learn from the leader's success rather than pay now for the chance to be a monopolist for a time. Therefore, a firm's best-response will depend on the strategy played by it's competitor.

First, I solve for firm i's best response in state $\varnothing$ given that $x_{-i} = 0$. In this case, firm i's best response will be

$$x_i = \begin{cases} 1 & \text{if} \quad \dfrac{\frac{\pi + \frac{\lambda\mu\pi m}{\sigma 2\rho}}{\rho + \lambda/\sigma} - c}{\rho + \lambda} \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, firm i should experiment when $-i$ is waiting as long as the value of experimenting is greater than 0. Rewriting the above inequality in terms of μ and σ, we define

$$\mu_N(\sigma) = 2\tilde{c}\tilde{\rho} - 2\tilde{\rho}(1 - \tilde{c}\tilde{\rho})\sigma. \tag{7}$$

The curve is labeled μ_N since it describes the minimum level of market power necessary to avoid the No-Experimentation strategy profile in equilibrium. The following remark pins down the behavior of equation 7 in the $\sigma - \mu$ plane.

Remark 2. $\lim_{\sigma \to 0} \mu_N > 0$ *and* $\frac{\partial \mu_N}{\partial \sigma} < 0$.

The first part of the remark is obvious: if firms cannot protect their innovation at all, they must expect some level of market power in order to invest in initial experimentation. The second part of the remark can be proven by taking the derivative and using the fact that $c \leq \bar{c}$; it is intuitive as well: as the level of innovation protection increases, the minimum level of market power necessary to induce initial experimentation falls. This is because firms expect to act as a monopolist for a longer period of time as innovation protection increases, and so they are willing to earn lower profits after their competitor eventually enters.

Next, I solve for firm i's best response given that $x_{-i} = 1$. In this case, firm i's best response will be

$$x_i = \begin{cases} 1 & \text{if} \quad \dfrac{\frac{\pi + \frac{\lambda}{\sigma} \cdot \frac{\mu\pi}{2\rho}}{\rho + \lambda/\sigma} + \frac{-c + \frac{\lambda}{\sigma} \cdot \frac{\mu\pi}{2\rho}}{\rho + \lambda/\sigma} - c}{\rho + 2\lambda} > \dfrac{\frac{-c + \frac{\lambda}{\sigma} \cdot \frac{\mu\pi m}{2\rho}}{\rho + \lambda/\sigma}}{\rho + \lambda} \\ 0 & \text{otherwise} \end{cases}.$$

If $-i$ is experimenting, i should experiment only if the benefit of potentially acting as a monopolist for a time and increasing the rate of innovation are greater than the benefits of waiting to copy your competitor's innovation. The next expression defines the minimum level of market power necessary in order to satisfy the inequality.

$$\mu_F(\sigma) = 2\tilde{c}(\tilde{\rho} + 1) - 2[(1 + \tilde{\rho})(1 - \tilde{c}\tilde{\rho}) + \tilde{c}]\sigma. \tag{8}$$

The curve is labeled μ_F since it describes the minimum level of market power necessary to avoid the Free-Rider strategy profile in equilibrium. The following remark describes the behavior of equation 8 in the $\sigma - \mu$ plane.

Remark 3. $\lim_{\sigma \to 0} \mu_F > \lim_{\sigma \to 0} \mu_N$ *and* $\frac{\partial \mu_F}{\partial \sigma} < \frac{\partial \mu_N}{\partial \sigma} < 0$.

Proof is again omitted due to simplicity. The first part of the remark states that if there is no innovation protection, a firm requires more market power to experiment when their competitor is already experimenting vs. when they are not. This is because if there is no innovation protection and your competitor is experimenting, you will be happy to wait and copy their innovation rather than experiment yourself. The second part states that innovation protection exerts a greater marginal effect on the minimum level of market power firms require to experiment when their competitor is already experimenting vs. when they are not.

The following lemma demonstrates that μ_M, μ_N, and μ_F curves always meet at one point. I refer to this combination of market power and innovation protection as the indifference point, since firms expect no profits at the point regardless of the strategy profile.

Lemma 3. *The curves μ_M, μ_N, and μ_F meet at a unique point $(\hat{\sigma}, \hat{\mu})$ on the $\sigma - \mu$ plane, where*

$$\hat{\sigma} = \frac{\tilde{c}}{1 + \tilde{c}(1 - \tilde{\rho})} \quad \text{and} \quad \hat{\mu} = 2\tilde{c}\tilde{\rho}\hat{\sigma}.$$

Proof. A.4 □

To understand this result intuitively, remember that μ_M describes the points in which the firm who innovates second expects no profits. Similarly, the μ_N function describes the points at which the firm who innovates first expects no profits. The μ_F function describes points where a firm is indifferent between innovating first or second. Thus, if μ_M and μ_N intersect, both the first and second innovator expect no profits, and so a firm will be indifferent between innovating first or second. Therefore, μ_F must intersect μ_M and μ_N at that point as well. More generally, any two of these functions intersecting implies the intersection of the third. This is important, as it gives structure to the relative positions of equilibrium strategy profiles.

In the next lemma, I argue that if the cost of experimenting is high enough, $\hat{\mu} > 1$, which is not within the $\sigma - \mu$ plane sine $\mu \leq 1$: the share of captives cannot exceed 100%. This is important, as the Full Experimentation and Free-Rider strategy profiles are only potential equilibria for points above the indifference point. Define $\hat{c}$ as cost beyond which $\hat{\mu} > 1$. The following lemma demonstrates that this level of cost is below $\bar{c}$, the level of cost beyond which no innovation is possible anywhere on the $\sigma - \mu$ plane.

Lemma 4. $\hat{c} \in (0, \bar{c})$.

Proof. A.5 □

We can now describe the relative positions of equilibrium strategy profiles across the plane of potential innovation protection and market power. Using lemma 3, we can also show that the relative positions are robust and that equilibrium strategy profiles are unique across the plane. I begin with the case in which $c > \hat{c}$.

Proposition 3. *If $c \in (\hat{c}, \bar{c}]$, a point (σ, μ) presents the following strategy profile in equilibrium:*

- *Monopoly if $\mu < \mu_M(\sigma)$*

- *No Experimentation if* $\mu \geq \mu_M(\sigma)$.

Proof. A.6 □

If experimentation costs are high, there are only two potential equilibrium strategy profiles. If firms expect to act as a monopolist if they innovate first, both initially experiment. However, if they expect the remaining firm to experiment after the initial innovation, neither wish to experiment initially. This is because when costs are high, the only way for firms to recoup their experimentation costs is to act as a perpetual monopolist. The next proposition describes the potential equilibria of the game given that the cost of experimentation isn't too high. In this case, each strategy profile obtains in equilibrium for a unique subset of the parameter space.

Proposition 4. *If* $c \leq \hat{c}$, *a point* (σ, μ) *presents the following strategy profile in equilibrium:*

- *Monopoly if* $\mu < \mu_M(\sigma)$.
- *No Experimentation if* $\mu \geq \mu_M(\sigma)$ *and* $\mu < \mu_N(\sigma)$.
- *Full Experimentation if* $\mu \geq \mu_M(\sigma)$ *and* $\mu > \mu_F(\sigma)$.
- *Free Rider if* $\mu_N(\sigma) < \mu \leq \mu_F(\sigma)$.

Proof. A.7 □

Figure 3 presents the potential equilibria of the game on the innovation protection—market power plane given $c < \hat{c}$. Generally speaking, low innovation protection and low market power prevent either firm from experimenting: firms expect their competitor to easily copy their innovation and erode profits in the market. Thus, neither wish to innovate first. Low innovation protection and high market power induce free-riding: there is enough market power to encourage the initial innovation, but since copying the innovation is easy, a firm would rather wait to copy their competitor instead of experimenting initially. Thus, one firm experiments initially while the other waits. If innovation protection is high while market power is low, the game simplifies to a patent race: both firms experiment initially since they know that the firm who does not innovate first will drop out. If both innovation protection and market power are high, both firms experiment until they innovate, regardless of the actions of their competitor: high innovation protection discourages free-riding, while high market power encourages experimentation after the initial innovation.

The following corollary states that the relative location of equilibrium strategy profiles is consistent for varying levels of relative costs and discount rates.

Corollary 2. *The relative position of equilibrium strategy profiles is robust around the indifference point.*

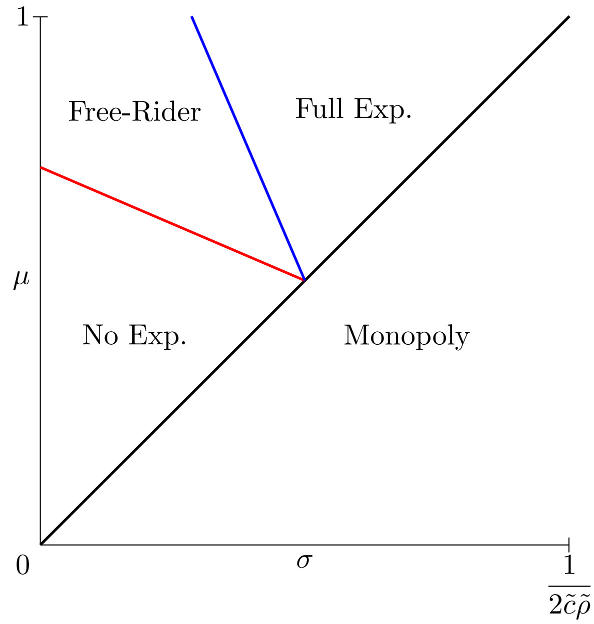


Figure 3: Equilibrium strategy profiles across the innovation protection—market power plane given $c < \hat{c}$.

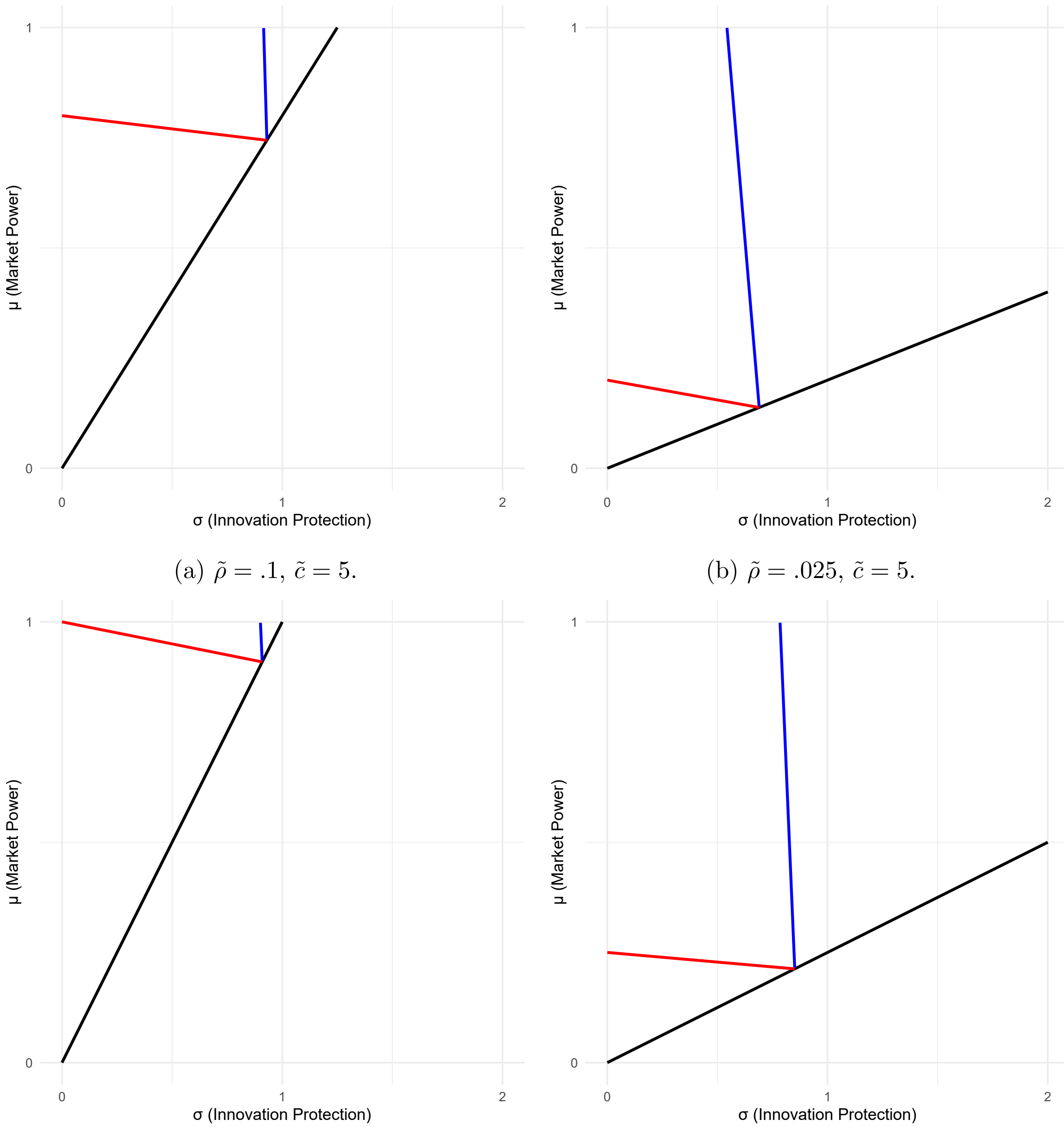
Figure 4 demonstrate the robustness of relative equilibrium locations. The equilibrium strategy profile areas change with changes in $\tilde{c}$ and $\tilde{\rho}$. However, their relative positions remain consistent.

3.3 Mixed-Strategy Equilibria

In this section I relax the restriction to pure strategies. As we will see, this only affects potential equilibrium strategy profiles for points where the Free-Rider strategy profile is an equilibrium. This is because the Free-Rider strategy profile is asymmetric: one firm experiments while the other does not. Asymmetric pure-strategy equilibria such at these may be deemed unrealistic, as they imply some level of coordination between the agents who play different strategies. Thus, I solve for the symmetric mixed-strategy equilibrium to complement the existing asymmetric pure-strategy equilibrium. To begin, redefine $x_i \in [0, 1]$ as the probability that i experiments at a point in time. I once again restrict my attention to Markov-perfect equilibria.

First, I argue that firms do not mix over strategies for points (σ, μ) exhibiting the Monopoly, Full-Experimentation, or No-Experimentation strategy profiles in equilibrium. This is because either experimenting or waiting is dominant in each state for such points. Dominated strategies are never part of an equilibrium, and thus firms will not mix over strategies in these areas.

Proposition 5. *If (σ, μ) exhibit the Monopoly, Full-Experimentation, or No-Experimentation*



(a) $\tilde{\rho} = .1$, $\tilde{c} = 5$.

(b) $\tilde{\rho} = .025$, $\tilde{c} = 5$.

(c) $\tilde{\rho} = .05$, $\tilde{c} = 8$.

(d) $\tilde{\rho} = .05$, $\tilde{c} = 2$.

Figure 4: Equilibrium strategy profiles with changes in relative experimentation cost $\tilde{c}$ and relative discount rate $\tilde{\rho}$.

strategy profile in equilibrium, firms will not mix over experimentation decisions.

Thus, firm behavior for such points mirrors the behavior when restricted to pure-strategies. I now show that this is not the case for points exhibiting the Free-Rider strategy profile in equilibrium. Since there is no dominant strategy, players may mix over experimenting and waiting.

Proposition 6. *If the Free-Rider strategy profile is an equilibrium, there exists a symmetric*

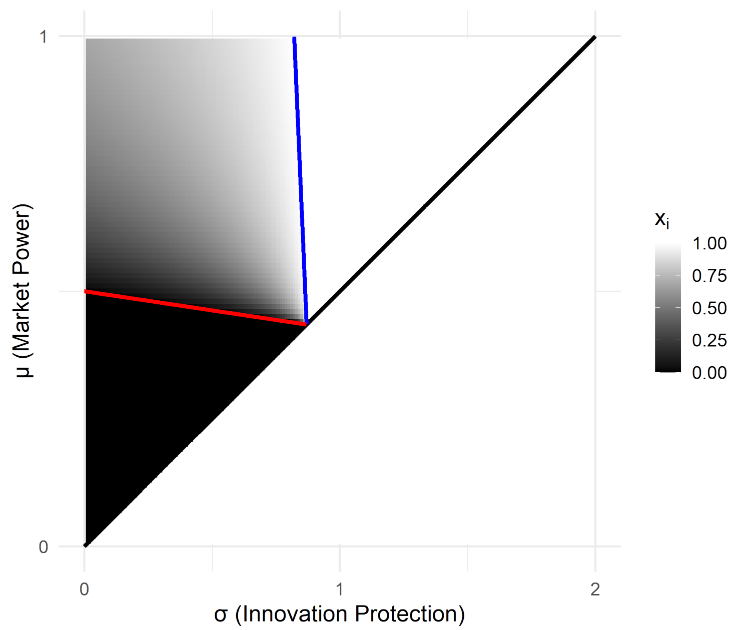


Figure 5: Probability of experimenting in $\varnothing$ state.

mixed strategy equilibrium in which both firms play

$$x_i = \frac{(-c + \lambda V_{\{i\}})(\rho + 2\lambda)}{\lambda(-c + \lambda V_{\{i\}} + \lambda V_{\{-i\}})}$$

in the $\varnothing$ state and $x_i = 1$ in state $-i$.

Proof. A.8 □

In order for a firm to mix over experimenting and waiting, their competitor must play a mixed strategy which makes the firm indifferent between experimenting and waiting. Since firms are symmetric, this mixed strategy is identical for both firms. Figure 5 presents x_i across the $\sigma - \mu$ plane in the state before either firm has innovated. If the No-Experimentation strategy profile is an equilibrium, the probability of experimenting is 0, as waiting is the dominant strategy. If the Full-Experimentation strategy profile is an equilibrium, the probability of experimenting is 1, as experimenting is the dominant strategy. However if the Free-Rider strategy profile is an equilibrium, $x_i \in (0, 1)$ in the symmetric equilibrium, as firms mix over experimenting and waiting. Points closer to the Full-Experimentation equilibria present experimentation probabilities closer to 1, whereas points closer to the No Experimentation equilibria present probabilities closer to 0.

The following corollary describes the comparative statistics for x_i in mixed Free-Rider strategy profile. The probability of experimenting is increasing in both the level of market power and innovation protection.

Corollary 3. *x_i in the mixed Free-Rider strategy profile is increasing in μ and σ.*

As innovation protection increases, firms expect to save less by waiting to copy their competitor. Thus, in equilibrium, they experiment more. The comparative statics concerning market power are less intuitive. Remember that if a firm waits to innovate, they slow down the rate at which the initial innovation is discovered. This in turn delays the duopoly profit flow that they eventually receive. This flow is scaled by μ, and so this delay hurts more as μ, market power, increases. This is why firms are more likely to experiment as μ increases.

4 Constrained Social Planner

I now define the problem of a constrained social planner with the ability to select σ and μ, the level of innovation protection and market power. I begin by solving for the level of welfare across the $\sigma - \mu$ plane. My welfare measure is the sum of discounted firm profits and consumer value. I start by solving for the level of welfare for each strategy profile. Let w_1 and w_2 be the product market welfare flows given that one or two firms have innovated respectively. Let W_n be the planner's value for being in the state in which n firms have innovated. Thus, the planner's value functions will be

$$\rho W_0 = -c(x_A + x_B) + \lambda(x_A + x_B)(W_1 - W_0) \tag{9}$$

$$\rho W_1 = w_1 - cx_i + \frac{\lambda}{\sigma} x_i (W_2 - W_1) \tag{10}$$

$$\rho W_2 = w_2 \tag{11}$$

Equation 9 describes the asset value of being in the state in which neither firm has innovated. There is a welfare loss from firms spending money on experimenting, and the game moves out of the state and into the state in which one firm has innovated with a rate based on the experimentation decisions of the firms. Equation 10 describes the asset value of being in the state in which one firm has innovated. There is a welfare gain of w_1 and a welfare loss if the remaining firm decides to continue experimenting. The game eventually moves to the state in which both have innovated with a rate based on the level of innovation protection. Equation 11 describes the asset value of being in the state in which both firms have innovated, which is just the product-market welfare flow w_2.

We can now solve for the value of each strategy profile at the beginning of time. I restrict my analysis to pure strategy equilibria; however, the results we derive would still hold if we also included analysis of the mixed Free-Rider strategy profile. The following set of equations capture the present value for each strategy profile:

$$W_0|NoExp. = 0 \tag{12}$$

$$W_0|Monopoly = \frac{-2c}{\rho + 2\lambda} + \frac{2\lambda w_1}{(\rho + 2\lambda)\rho} \tag{13}$$

$$W_0|FreeRider = \frac{-c}{\rho + \lambda} + \frac{\lambda(w_1 - c)}{(\rho + \lambda)(\rho + \lambda/\sigma)} + \frac{\lambda \cdot \frac{\lambda}{\sigma} w_2}{(\rho + \lambda)(\rho + \lambda/\sigma)\rho} \tag{14}$$

$$W_0|FullExp. = \frac{-2c}{\rho + 2\lambda} + \frac{2\lambda(w_1 - c)}{(\rho + 2\lambda)(\rho + \lambda/\sigma)} + \frac{2\lambda \cdot \frac{\lambda}{\sigma} w_2}{(\rho + 2\lambda)(\rho + \lambda/\sigma)\rho} \tag{15}$$

Each equation is organized as follows: terms represent the present value of being in each future state of the game. The numerator of each term is the rate at which the game reaches the state multiplied by the welfare value of reaching the state. The denominator represents the rate at which that state is discounted.

I now solve for the product-market welfare flows w_1 and w_2 to complete the analytical representation of the value of each strategy profile. The derivation is similar to that found in Stahl (1989).

Lemma 5. *Welfare profit flows are*

$$w_1 = \pi + \int_{p^*}^{\infty} 1 - G(p)\, dp$$

and

$$w_2 = w_1 + \int_{\underline{p}}^{p^*} pG'(p)\overline{F}(p)\, dp$$

with

$$\overline{F}(p) = \mu F(p) + (1 - \mu)(1 - (1 - F(p))^2).$$

Proof. A.9 □

The value w_1 is simply the sum of monopoly profits and consumer values above the monopoly profit price. The value w_2 is the sum of w_1 plus the additional sales made to consumers with values below the monopoly price. The following corollary to lemma 5 confirms that the game captures our intuition concerning the effects of market power on welfare: $w_2 \geq w_1$ since there is less deadweight loss once both firms have innovated and entered the market. The effect is scaled by μ, our market power parameter. The μ parameter impacts welfare through two channels: first, as μ decreases, a greater share of consumers see the price posted by both firms, and thus there is a greater chance that they see a price below their value for the good holding $F(p)$ constant. Additionally, decreasing μ shifts the mass of $F(p)$ towards lower prices, increasing the chances that both captives and non-captives see a price below their value for the good.

Corollary 4. $w_2 \geq w_1$ *and* $\dfrac{\partial w_2}{\partial \mu} \leq 0$.

Proof. A.10 □

Now that we can express welfare within a given strategy profile, we can express the problem facing the constrained social planner. First, we express the problem for the case in which experimentation costs are high: $c > \hat{c}$. The planner faces the following problem:

$$\max_{\sigma,\mu} W_0 = \begin{cases} W_0|Monopoly & \text{if} \quad \mu < \mu_M(\sigma) \\ 0 \quad \text{if} \quad \mu \geq \mu_M(\sigma). \end{cases} \tag{16}$$

If experimenting is expensive, the optimal levels of market power and innovation protection are simple to describe: a constrained social planner will set innovation protection high and market power low so that the remaining firms drops out of the race after the initial innovation is discovered. Since firms only experiment if they expect to act as monopolists, the only way for the social planner to avoid the No-Experimentation strategy profile is to pick a point in which the Monopoly strategy profile is the equilibrium. The following proposition formalizes this result.

Proposition 7. *If $c \in (\hat{c}, \bar{c}]$, the social planner selects (σ, μ) such that $\mu < \mu_M(\sigma)$.*

This result confirms our intuition concerning innovation protection: if experimenting is very expensive, we should increase innovation protection in order to block subsequent innovations so that firms are incentivized to innovate initially. It also highlights the role that competition policy can play: firms are more likely to experiment initially in markets which they perceive as competitive as well, since additional firms will not enter if they are the first to innovate.

Now I address the case in which experimentation is cheap. In this case, the optimal level of innovation protection and market power is less obvious. There are welfare trade-offs between the Monopoly, Free-Rider, and Full Experimentation strategy profiles. For example, there is a duplication of experimentation effort within the Free-Rider and Full Experimentation strategy profiles that does not occur within the monopoly profile. However, there is greater deadweight loss within the monopoly strategy profile than their is in either the Free-Rider or Full Experimentation profile. I now state the planner's problem in the case in which $c \leq \hat{c}$:

$$\max_{\sigma,\mu} W_0 = \begin{cases} W_0|Monopoly \quad \text{if} \quad \mu < \mu_M(\sigma) \\ W_0|FullExp \quad \text{if} \quad \mu \geq \mu_M(\sigma) \text{ and } \mu \geq \mu_F(\sigma) \\ W_0|FreeRider \quad \text{if} \quad \mu_N(\sigma) < \mu \leq \mu_F(\sigma) \\ 0 \quad \text{if} \quad \mu \geq \mu_M(\sigma) \text{ and } \mu < \mu_N(\sigma) \end{cases} . \tag{17}$$

We cannot immediately eliminate any strategy profiles other than the No-Experimentation profile as the potential optimal. As mentioned before, each pose different trade-offs with one another. However, we can narrow down optimal levels of innovation protection and market power *within* strategy profiles. The next lemma states that within the Free-Rider and Full-Experimentation strategy profiles, welfare is decreasing in market power and innovation protection. The intuition is simple enough: market power increases deadweight loss, while innovation protection increases the duplication of experimentation effort.

Lemma 6. $\frac{\partial W_0|FreeRider}{\partial \sigma} < 0$, $\frac{\partial W_0|FreeRider}{\partial \mu} \leq 0$, $\frac{\partial W_0|FullExp}{\partial \sigma} < 0$, *and* $\frac{\partial W_0|FullExp}{\partial \sigma} < 0$

The proof is trivial and thus is omitted. Increasing innovation protection increases duplication costs since subsequent firm entry is more difficult. Similarly, increasing market power within equilibria decreases welfare since it increases deadweight loss as proven in corollary 4. Note that these comparative statics only hold within strategy profiles: decreasing innovation protection and market power may also change the equilibrium strategy profile. For example, If innovation protection and market power are low enough, the equilibrium strategy profile will either be No Experimentation or Monopoly.

We will now use the previous lemmas to describe the optimal policy for the constrained planner in a few limiting cases. In the first case, I assume that $G(p)$ is degenerate—that is, that all consumers value the good equally. This is the setup in many of the original papers exploring price dispersion (Varian, 1980; Burdett and Judd, 1983). In this case, there is no dead-weight loss, since demand doesn't slope downwards—firms never charge a price above the consumer's homogeneous value for the good since firms know that they won't sell any goods at such a price. Thus, there is not an increase in welfare once the second firm enters the market. In other words, the duplication of experimentation has no social benefit, and thus the planner will choose a level of innovation protection and market power so that the market presents the Monopoly strategy profile in equilibrium.

Proposition 8. *If $c < \hat{c}$ and $G(p)$ is degenerate, the social planner selects (σ, μ) such that $\mu < \mu_M(\sigma)$.*

Proof. A.11 □

Proposition 8 demonstrates a case in which the costs of experimentation duplication outweigh the benefits of additional firm entry. The following proposition demonstrates that the opposite is also possible. We now assume that consumers value the good differently along the support of the firm's price distribution. Thus, adding firms will present welfare gains. Since the welfare gains of the increased competition are felt for the rest of time, if the social planner discounts at a low enough rate, they will select the indifference point for the level of innovation protection and market power. This is the point at which market power is minimized while still allowing for additional firm entry.

Proposition 9. *If $c < \hat{c}$, $G(p)$ is not degenerate over $(\underline{p}, p^*)$, and ρ is sufficiently low, a constrained social planner will select $(\sigma, \mu) = (\hat{\sigma}, \hat{\mu})$, the indifference point.*

Proof. A.12 □

As the discount rate falls, the benefits from the decrease in deadweight loss surmount the costs of duplication. The social planner selects their level of innovation protection and market power with the sole purpose of minimizing deadweight loss in the long-run, the indifference point.

5 Conclusion

I have presented a model of experimentation with independent measures of both innovation protection and market power. Both parameters vary widely across industries, and both are important policy tools. However, existing models of experimentation do not analyze the effect of both independently. I described pure and mixed-strategy equilibrium firm behavior for every combination of innovation protection and market power. I then presented a constrained social planner's problem. When experimenting is expensive, the planner selects low levels of innovation protection and high levels of innovation protection in order to preclude firm entry after the initial innovation. In contrast, the optimal level of innovation protection and market power is ambiguous whenever experimentation is cheap. The planner must balance the effort duplication inherent in some equilibria with the higher deadweight loss present in others.

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A Proofs

A.1 Lemma 1

Proof. Assume that there is a price $\hat{p}$ played with positive mass. I will show that a firm can deviate from this strategy and earn strictly greater profits. Suppose that firm i deviates by playing $\hat{p}-\epsilon$ with the same positive probability instead. Thus, the expected change in firm i's profits would be

$$\begin{aligned}
&Pr(p_{-i} > \hat{p}-\epsilon, p_{-i} \neq \hat{p})(1-\mu/2)((1-G(\hat{p}-\epsilon))(\hat{p}-\epsilon)) \\
&- Pr(p_{-i} > \hat{p})(1-\mu/2)(1-G(\hat{p}))\hat{p} \\
&+ Pr(p_{-i} < \hat{p}-\epsilon)\mu/2(1-G(\hat{p}-\epsilon))(\hat{p}-\epsilon) \\
&- Pr(p_{-i} < \hat{p})\mu/2(1-G(\hat{p}))\hat{p} \\
&+ Pr(p_{-i} = \hat{p})(1-\mu/2)((1-G(\hat{p}-\epsilon))(\hat{p}-\epsilon)) \\
&- Pr(p_{-i} = \hat{p})1/2(1-G(\hat{p}))\hat{p}.
\end{aligned}$$

As $\epsilon \to 0$, the sum of the first four terms goes to 0. However the sum of the last two terms is strictly positive. When the $-i$ charges $\hat{p}$ and i charges $\hat{p}-\epsilon$, i increases their payoff since they capture all of the non-captives rather than splitting them. $\square$

A.2 Lemma 2

Proof. Charging $p > p^*$ is always dominated by charging p^*: you are more likely to sell to the non-captives and you also earn a higher profit when selling to only your captives since p^* is the unique monopoly price. Likewise, charging $p < \underline{p}$ is always dominated by charging p^*: by definition, the profit from selling to just your captives at the monopoly price is greater than selling to both your captives and the non-captives at a price below $\underline{p}$. Now assume that $[p_1, p_2] \subset [\underline{p}, p^*]$, and that $\hat{p} \in (p_1, p_2)$ is not in the support of $F(p)$. Notice that a firm could improve their expected profit by charging $\hat{p} \in (p_1, p_2)$ instead of p_2: If there competitor charges a price greater than p_1, the firm gets to sell to the non captives at a higher price $\hat{p} > p_2$. Similarly, if their competitor charges $p < p_2$, they still only sell to their captives but again at a higher price. $\square$

A.3 Proposition 1

Proof. In the $\{i\}$ state, in which only i has innovated, i maximizes profits by charging the unique monopoly price p^*.

Now assume we are in the $\{A, B\}$ state and suppose that $\mu = 1$. Thus, all consumers are captives. In this case, both firms act as a local monopoly and should thus charge the monopoly price p^*. They subsequently split the market, earning a profit flow of $\pi/2$. When $\mu = 0$, all consumers see both prices. Thus, competition simplifies to a Bertrand game in which both firms charge a price equal to marginal cost, which is 0 in this case. The resulting profit flows are thus 0.

Now suppose that $\mu \in (0,1)$. By lemma 2, we know that an equilibrium $F(p)$ has continuous support in $[p^*, \underline{p}]$. Thus, $F(p) = 0$ for $p < 0$ and $F(p) = 1$ for $p > p^*$. By lemma 1, we know that $F(p)$ has no mass points, and so ties happen with probability 0. Thus, there is no chance that both players play p^*, and so the payoff for playing p^* must be

$$\frac{\mu}{2}\pi.$$

If a firm plays the monopoly price, they will only sell to their captives. But then this must be the expected profit flow for every price in the support of $F(p)$ in order for $F(p)$ to be an equilibrium. Thus, for $p \in [\underline{p}, p^*]$, we know that

$$\frac{\mu}{2}\pi = \frac{\mu}{2}(1 - G(p))p + (1 - \mu/2)(1 - F(p))(1 - G(p))p.$$

Solving for $F(p)$ gives us

$$F(p) = 1 - \frac{\mu[\pi - (1 - G(p))p]}{2(1 - \mu)(1 - G(p))p}.$$

□

A.4 Lemma 3

Proof. Notice that for

$$\sigma = \frac{\tilde{c}}{1 + \tilde{c}(1 - \tilde{\rho})},$$

we have

$$\mu_M(\sigma) = \mu_N(\sigma) = \mu_F(\sigma) = \frac{2\tilde{c}^2\tilde{\rho}}{1 + \tilde{c}(1 - \tilde{\rho})}.$$

Thus, all three curves meet at this point. Furthermore, since the functions are linear with differing slopes in the $\sigma - \mu$ plane, this is the only point in which they all intersect. □

A.5 Lemma 4

Proof. First, notice that if $c = 0$, $\hat{\mu} = 0$. If $c = \bar{c}$, then $\hat{\mu} = 2$. Also,

$$\frac{\partial \hat{\mu}}{\partial c} > 0.$$

Therefore, by the intermediate value theorem, there exists a $c \in (0, \bar{c})$ such that $\hat{\mu} = 1$. □

A.6 Proposition 3

Proof. If $c \in (\hat{c}, \bar{c}]$, the indifference point falls above $\mu = 1$, and thus is not on the σ–μ plane. By remarks 2 and 3, we know that the Free-Rider and Full-Experimentation strategy profiles are only equilibria above the indifference point. Thus, they are not present at any point on the plane whenever $cin(\hat{c}, \bar{c}]$. This only leaves the No-Experimentation strategy profile as the equilibrium if $\mu \geq \mu_M(\sigma)$. □

A.7 Proposition 4

Proof. Since $c \leq \hat{c}$, the indifference point falls on the plain. By lemma 3, the curves μ_M, μ_N, and μ_F all intersect and thus all strategy profiles obtain as potential equilibria. $\square$

A.8 Proposition 6

Proof. Firms will only mix over experimenting and waiting if the expected payoff for both is equivalent. Thus, x_i must satisfy

$$x_i \cdot \frac{-c + \lambda V_{\{i\}} + \lambda V_{\{-i\}}}{\rho + 2\lambda} + (1 - x_i) \cdot \frac{-c + \lambda V_{\{i\}}}{\rho + \lambda} = x_i \cdot \frac{\lambda V_{\{-i\}}}{\rho + \lambda}.$$

Solving for x_i obtains

$$x_i = \frac{(-c + \lambda V_{\{i\}})(\rho + 2\lambda)}{\lambda(-c + \lambda V_{\{i\}} + \lambda V_{\{-i\}})}.$$

$\square$

A.9 Lemma 5

Proof. Let w^p, w^c be producer and consumer welfare. Then $w_1^p = \pi$ and

$$w_1^c = \int_{p^*}^{\infty} 1 - G(p)\, dp.$$

Thus,

$$w_1 = \pi + \int_{p^*}^{\infty} 1 - G(p)\, dp. \tag{18}$$

To find w_2^c, notice that the probability of the lowest price observed by the consumer who sees both is p is given by $1 - (1 - F(p))^2$. Thus,

$$\begin{aligned} w_2^c = \mu &\int_{\underline{p}}^{p^*} \int_{p}^{\infty} (1 - G(x)))\, dx\, dF(p) \\ &+ (1 - \mu) \int_{\underline{p}}^{p^*} \int_{p}^{\infty} (1 - G(x))\, dx\, d(1 - (1 - F(p))^2). \end{aligned}$$

Define $\overline{F}(p) = \mu F(p) + (1 - \mu)(1 - (1 - F(p))^2)$. Thus, via integration by parts,

$$w_2^c = w_1^c + \int_{\underline{p}}^{p^*} (1 - G(p))\overline{F}(p)\, dp. \tag{19}$$

Solving for w_2^p,

$$\begin{aligned} w_2^p = \mu &\int_{\underline{p}}^{p^*} (1 - G(p))p\, dF(p) \\ &+ (1 - \mu) \int_{\underline{p}}^{p^*} (1 - G(p))p\, d(1 - (1 - F(p))^2) \end{aligned}$$

Via integration by parts,

$$w_2^p = w_1^p - \int_{\underline{p}}^{p^*} [(1 - G(p))p]' \, \overline{F}(p) \, dp. \tag{20}$$

Using equations 2 and 3, we find that

$$w_2 = w_1 + \int_{\underline{p}}^{p^*} pG'(p)\overline{F}(p) \, dp. \tag{21}$$

□

A.10 Corollary 4

Proof. First, notice that $\overline{F}(p) > 0$. Also, since $G(p)$ is a probability distribution, $G'(p) \geq 0$. Therefore,

$$\int_{\underline{p}}^{p^*} pG'(p)\overline{F}(p) > 0$$

and so $w_2 \geq w_1$. For the second part of the corollary, notice that

$$\frac{\partial \overline{F}(p)}{\partial \mu} = [F^2(p) - F(p)] \cdot \frac{\partial F(p)}{\partial \mu}.$$

Notice that $F^2(p) - F(p) < 0$ for $p \in (\underline{p}, p^*)$ and that

$$\frac{\partial F(p)}{\partial \mu} = -\frac{\pi - (1 - G(p))p}{2(1 - \mu)^2(1 - G(p))p)} < 0$$

for $p \in (\underline{p}, p^*)$. Thus, $\frac{\partial \overline{F}(p)}{\partial \mu} < 0$ over the domain of integration. Additionally, from equation 1, we know that $\frac{\partial \underline{p}}{\partial \mu} < 0$. Thus the domain of integration also grows with μ. Thus, $\frac{\partial w_2}{\partial \mu} \leq 0$. □

A.11 Proposition 8

Proof. When $G(p)$ is degenerate, all consumers value the good at the same price p. This must then be our monopoly price p^*. Thus, for $p \in (\underline{p}, p^*)$, $G(p) = 0$ and so $G'(p) = 0$. Thus, $w_1 = w_2$, and so for the rest of the proof I replace w_2 with w_1. I will now show that $V_0|Monopoly > V_0|FullExp$. By lemma 6, we know that

$$V_0|FullExp < \lim_{\sigma \to 0} V_0|FullExp = \frac{-2c}{\rho + 2\lambda} + \frac{2\lambda w_1}{(\rho + 2\lambda)\rho}.$$

However, notice that this is just $V_0|Monopoly$. Therefore

$$V_0|Monopoly - V_0|FullExp > 0,$$

and so $V_0|Monopoly > V_0|FullExp$. Similarly, notice that

$$V_0|FreeRider < \lim_{\sigma \to 0} V_0|FreeRider = \frac{-c}{\rho + \lambda} + \frac{\lambda w_1}{(\rho + \lambda)\rho}.$$

Therefore,

$$\begin{aligned} V_0|Monopoly - V_0|FreeRider &> \frac{-2c}{\rho+2\lambda} + \frac{2\lambda w_1}{(\rho+2\lambda)\rho} - \left[\frac{-c}{\rho+\lambda} + \frac{\lambda w_1}{(\rho+\lambda)\rho}\right] \\ &= \frac{\lambda w_1 - c\rho}{(\rho+2\lambda)(\rho+\lambda)}. \end{aligned}$$

By lemma **??**, $\lambda w_1 - \rho c > \lambda\pi - \rho c$, and since $c < \bar{c} = \frac{\lambda\pi}{\rho}$, $\lambda\pi - \rho c > 0$. Thus, $V_0|Monopoly > V_0|FreeRider$. □

A.12 Proposition 9

Proof. (sketch) As ρ decreases, the final terms in the expressions of $V_0|Monopoly$, $V_0|FreeRider$, and $V_0|FullExp$ tend to infinity. Thus, if ρ is low enough, the behavior in the prior terms becomes inconsequential. The final term is scaled by w_2 in the Free-Rider and Full-Experimentation strategy profiles, and by w_1 in the Monopoly strategy profile. Since $w_2 > w_1$, the social planner will not select (σ, μ) such that the Monopoly profile is an equilibrium. Since w_2 is minimized at $(\hat{\sigma}, \hat{\mu})$, they select this point. □