Physical Origins of Gate Errors in Transmon Qubits

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We derive a physical model for amplitude, angle and off-resonance errors in 1-qubit gates on superconducting transmon qubits of the type found in IBM Quantum Experience devices. We show how to leverage the model with experiments to determine these errors using OpenQASM and OpenPulse instructions. This allows us to calculate more precisely the major source of error in specific circuits, rather than using averaged approximations to gate fidelity from Randomized Benchmarking experiments.

I. INTRODUCTION

Quantum Computing has recently emerged from Quantum Information Theory as a way to provide the next generation of computers, allowing the development of algorithms that can suppress time complexity exponentially for certain problems [1], and provide smaller speedups for wider classes of problems.
Superconducting Quantum Computing (SQC) is one of the more promising efforts to implement a physical Quantum Computer, a so-called Noisy Intermediate-Scale Quantum Computer (NISQ). In this paper, we examine the Hamiltonian of widely-studied implementation of SQC, using transmon qubits driven by AWG microwaves pulses acting as the gates.

A. Mathematics

¡¡ Could have more derivation/ explanation here¿¿ Single qubit gates on transmon qubits are implemented with a monochromatic microwave drive pulse with angular frequency ω_d and phase offset γ, ie with an electric field component

$$\mathbf{E(t)} = \mathbf{E_0(t)} \cos(\omega_\mathbf{d} \mathbf{t} + \gamma), \tag{1}$$

where $\mathbf{E_0(t)} \in \mathbb{R}^\mathbf{3}$.
The theoretical ideal drive pulse Hamiltonian in the Rotating Wave Approximation (RWA) is then (from open-source Qiskit textbook/ many sources)

$$\hat{H} = \frac{\hbar\Delta}{2}\hat{\sigma}_z + \frac{\hbar\Omega_R(t)}{2}(\cos(\gamma)\hat{\sigma}_x + \sin(\gamma)\hat{\sigma}_y), \tag{2}$$

where:

- $\Omega_R(t) \propto \|\mathbf{E_0(t)}\|_\mathbf{2}$ is known as the *Rabi Frequency.*
- $\Delta := \omega_{01} - \omega_d$, where $\omega 01 = E(|0\rangle \to |1\rangle)/\hbar$.
- $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$ are the Pauli operators.

By varying γ, we can implement an arbitrary . This leaves us to choose ω_d and $\Omega_R(t)(via\mathbf{E_0(t)})$, as well as the total duration of the pulse, to maximise gate fidelity. From recent theory, we should choose $\omega_d = \omega_{01}$, a short pulse duration (to minimise circuit duration, and hence time-dependent (non-Markovian) errors) and $\Omega_R(t)$ as a DRAG pulse. This is for purposes of minimising 'leakage' etc. For ease of calculation, let's assume $\Omega R(t)$ is constant, so that

$$\begin{aligned} \theta &= 2E_C \left(\tfrac{E_J}{8E_C}\right)^{1/4}_0 {}^T \Omega_R(t) \mathrm{d}t & (3) \\ &\propto \Omega T & (4) \end{aligned}$$

where E_C and E_J are the Cooper Box and Josephson Junction energies respectively. Then an ideal single resonant pulse with no error ($\Delta = 0$) gives the gate

$$\begin{aligned} U &= e^{-i\theta(\cos(\gamma)\hat{\sigma}_x + \sin(\gamma)\hat{\sigma}_y)} & (5) \\ &= Z(-2\gamma) \cdot X(2\theta) \cdot Z(2\gamma), & (6) \end{aligned}$$

where $X(\cdot), Z(\cdot)$ are Pauli rotations, and taking $\hbar = 2$ w.l.o.g, demonstrating single-qubit gate universality even for a fixed θ by this pulse. Then the noisy version of this gate is

$$\begin{aligned} \tilde{U} &= e^{-i\hat{H}}. \text{ Adding in assumptions,} & (7) \\ &= e^{-i[\Delta\hat{\sigma}_z + \theta(\cos(\gamma)\hat{\sigma}_x + \sin(\gamma)\hat{\sigma}_y)]}. & (8) \end{aligned}$$

We examine the matrix exponential to obtain a Taylor Series:

$$\tilde{U} = U + \Delta i \textstyle\sum_{k=0}^{\infty} \left[\frac{(-1)^{k+1}\theta^{2k}}{(2k+1)!}\right] + 2\delta\theta \sum_{k=0}^{\infty} \left[\frac{(-1)^{k}\theta^{2k}}{(2k)!}\right] + ...$$

(9)

[1] R. Cleve, A. Ekert, C. Macchiavello, and M. Mosca, Quantum algorithms revisited 10.1098/rspa.1998.0164 (1997), arXiv:quant-ph/9708016.

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