IBMQ GATE ERROR CALCULATIONS FOR CALIBRATION EXPERIMENTS

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ABSTRACT

Derivation of gate angle and amplitude error functions for superconducting transmon qubits of the type found in IBM Quantum Experience devices.

Keywords Quantum Computing · Quantum Information Theory · Gate Errors · IBM Quantum Experience

1 Theory

Using the Hamiltonian for a transmon qubit**??**, we can derive that an AWG pulse of amplitude Ω, period T and phase γ, at the drive frequency ω_D, gives rise to the unitary

$$U = e^{-\frac{i}{2}\Omega T(\cos(\gamma)\sigma_x + \sin(\gamma)\sigma_y)} \tag{1}$$

Letting $\theta = \Omega T$ and adding the corresponding errors, we have the noisy unitary $\tilde{U}$ is

$$e^{-\frac{i}{2}(\theta+\delta\theta)(\cos(\gamma+\delta\gamma)\sigma_x + \sin(\gamma+\delta\gamma)\sigma_y)} \tag{2}$$

where $\delta\theta, \delta\gamma$ are commonly referred to as (multiples of) amplitude and angle error, respectively. Now letting $\theta = \pi/2$ as in IBM's machines and applying the Baker-Campbell-Hausdorff formula, we get

Lemma 1.1

$$\begin{aligned} \tilde{U}(\gamma) &= e^{\frac{i}{2}(\gamma+\delta\gamma)\sigma_z} e^{\frac{i}{2}(\frac{\pi}{2}+\delta\theta)\sigma_x} e^{-\frac{i}{2}(\gamma+\delta\gamma)\sigma_z} && (3) \\ &= Z_{-(\gamma+\delta\gamma)} \cdot X_{\pi/2+\delta\theta} \cdot Z_{\gamma+\delta\gamma} && (4) \end{aligned}$$

after identifying Pauli rotations, and noting carefully that

Corollary 1.1.1 *The last line in particular allows us to enact a Z gate* ***virtually*** *by choosing γ.*

Corollary 1.1.2 *The final (leftmost) Z gate does not affect the statistics of a Pauli-Z measurement.*

Let us now assume the error $\delta\theta$ depends only on θ, and is hence constant for our purposes. This is a fair assumption since $\theta = \frac{\pi}{2}$ is calibrated by inferring a suitable Ω for fixed T on IBMQ backends, and hence any error should be independent of γ. Let us further assume for now that $\delta\gamma$ is *homoscedastic*, that is, independent of γ and θ .

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1.1 U_2

For an ideal U_2 gate,we have

$$U_2(\phi,\lambda) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -e^{i\lambda} \\ e^{i\phi} & e^{i(\phi+\lambda)} \end{pmatrix} \tag{5}$$

$$= Z_{\phi+\pi/2}\cdot X_{\pi/2}\cdot Z_{\lambda-\pi/2} \tag{6}$$

So the empirical (noisy) version is implemented by

$$\tilde{U_2}(\phi,\lambda) = Z_{\lambda+\phi}\cdot\tilde{U}(\lambda-\frac{\pi}{2}) \tag{7}$$

$$= Z_{\lambda+\phi}\cdot\left[Z_{-(\lambda-\pi/2+\delta\gamma)}\cdot X_{\pi/2+\delta\theta}\cdot Z_{\lambda-\pi/2+\delta\gamma}\right] \tag{8}$$

$$\implies \tilde{U_2}^n(\phi,\lambda) = Z_{n(\lambda+\phi)}\cdot\left[\tilde{U}(n\lambda+(n-1)\phi-\frac{\pi}{2})\cdot\ldots\cdot\tilde{U}(2\lambda+\phi-\frac{\pi}{2})\cdot\tilde{U}(\lambda-\frac{\pi}{2})\right] \tag{9}$$

Assuming multiple $\tilde{U}_2$ gates in 9 are implemented independently, and recalling the leftmost Z gates are not implemented due to 1.1.2 throughout.

1.2 U_3

For an ideal U_3 gate,we have

$$\tag{10}$$

$$U_3(\theta,\phi,\lambda) = \begin{pmatrix} \cos() & -e^{i\lambda}\sin() \\ e^{i\phi}\sin() & e^{i(\phi+\lambda)}\cos() \end{pmatrix} \tag{11}$$

$$= Z_{\phi-\pi/2}\cdot X_{\pi/2}\cdot Z_{\pi-\theta}\cdot X_{\pi/2}\cdot Z_{\lambda-\pi/2} \tag{12}$$

$$\implies \tilde{U}_3(\theta,\phi,\lambda) = Z_{\lambda+\phi-\theta}\cdot\left[\tilde{U}(\lambda+\pi/2-\theta)\cdot\tilde{U}(\lambda-\pi/2)\right] \tag{13}$$

$$= Z_{\phi-\pi/2-\delta\gamma}\cdot X_{\pi/2}\cdot Z_{\pi-\theta}\cdot X_{\pi/2}\cdot Z_{\lambda-\pi/2+\delta\gamma} \tag{14}$$

Note carefully how there is no $\delta\gamma$ error in the middle $Z_{\pi-\theta}$ due to cancellation. This means angle error is self-correcting for 'sandwiched' Z gates under our assumptions.

1.3 X_θ

$$\begin{aligned} X_\theta = exp(-i\sigma_x) &= \begin{pmatrix} \cos & -i\sin \\ -i\sin & \cos \end{pmatrix} \text{ by Euler's Formula.} \\ &= U_3(\theta,-\pi/2,\pi/2) \\ \implies \tilde{X}_\theta &= Z_\theta\cdot\left[\tilde{U}(\pi-\theta)\cdot\tilde{U}(0)\right] \end{aligned} \tag{15}$$

1.3.1 Experiment Probabilities

Assuming always that we start with our system initialised in the $|0\rangle$ state, we calculate the probability of measuring $|0\rangle$ after $\tilde{X}_\theta$:

$$2\tilde{\mathcal{P}}(0)-1 = \sin^2(\delta\theta)+\cos^2(\delta\theta)\cos(\theta) \tag{16}$$

Whereas the theoretical quantity is

$$2\mathcal{P}(0)-1 = \cos(\theta) \tag{17}$$

$$\implies \frac{2\tilde{\mathcal{P}}(0)-1}{2\mathcal{P}(0)-1} = \sin^2(\delta\theta)\sec(\theta)+\cos^2(\delta\theta) \tag{18}$$

No we compute the probability $\mathcal{P}_{|1\rangle}\left(\tilde{X}^n_{\pi/2}\right)$ of measuring $|1\rangle$ after n applications of $\tilde{X}_{\pi/2}$

$$\mathcal{P}_{|1\rangle}\left(\tilde{X}_{\pi/2}\right) = \frac{1}{2}-\frac{1}{2}\cos\left[\left(\delta\theta+\frac{\pi}{2}\right)n\right] \tag{19}$$

$$\implies \text{F.T.}\left(\mathcal{P}\right)(\omega) = \frac{1}{2}\delta(\omega)-\frac{\pi}{2}\left[\delta(\omega-(\pi/2+\delta\theta))+\delta(\omega+\pi/2+\delta\theta)\right] \tag{20}$$

So $\delta\theta$ can empirically be determined via FFT.