IBMQ GATE ERROR CALCULATIONS FOR CALIBRATION EXPERIMENTS

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ABSTRACT

Derivation of gate angle and amplitude error functions for superconducting transmon qubits of the type found in IBM Quantum Experience devices. All equalities involving a matrix are up to a global phase multiple and where appropriate, conventions mach those in IBMQ documentation.

Keywords Quantum Computing · Quantum Information Theory · Gate Errors · IBM Quantum Experience

1 Theory

Using the Hamiltonian for a transmon qubit**??**, we can derive that an AWG pulse of amplitude Ω, period T and phase γ, at the drive frequency ω_D, gives rise to the unitary

$$U = e^{-\frac{i}{2}\Omega T(\cos(\gamma)\sigma_x + \sin(\gamma)\sigma_y)} \tag{1}$$

Letting $\theta = \Omega T$ and adding the corresponding errors, we have the noisy unitary $\tilde{U}$ is

$$e^{-\frac{i}{2}(\theta+\delta\theta)(\cos(\gamma+\delta\gamma)\sigma_x + \sin(\gamma+\delta\gamma)\sigma_y)} \tag{2}$$

where $\delta\theta, \delta\gamma$ are commonly referred to as (multiples of) amplitude and angle error, respectively. Now letting $\theta = \pi/2$ as in IBM's machines and applying the Baker-Campbell-Hausdorff formula, we get

Lemma 1.1

$$\tilde{U}(\gamma) = e^{\frac{i}{2}(\gamma+\delta\gamma)\sigma_z} e^{\frac{i}{2}(\frac{\pi}{2}+\delta\theta)\sigma_x} e^{-\frac{i}{2}(\gamma+\delta\gamma)\sigma_z} \tag{3}$$
$$= Z_{-(\gamma+\delta\gamma)} \cdot X_{\pi/2+\delta\theta} \cdot Z_{\gamma+\delta\gamma} \tag{4}$$

after identifying Pauli rotations, and noting carefully that

Corollary 1.1.1 *The last line in particular allows us to enact a Z gate* ***virtually*** *by choosing γ.*

Corollary 1.1.2 *The final (leftmost) Z gate does not affect the statistics of a Pauli-Z measurement.*

Let us now assume the error $\delta\theta$ depends only on θ, and is hence constant for our purposes. This is a fair assumption since $\theta = \frac{\pi}{2}$ is calibrated by inferring a suitable Ω for fixed T on IBMQ backends, and hence any error should be independent of γ. Let us further assume for now that $\delta\gamma$ is *homoscedastic*, that is, independent of γ and θ .

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1.1 U_2

For an ideal U_2 gate,we have

$$U_2(\phi,\lambda) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -e^{i\lambda} \\ e^{i\phi} & e^{i(\phi+\lambda)} \end{pmatrix} \tag{5}$$

$$= Z_{\phi+\pi/2} \cdot X_{\pi/2} \cdot Z_\lambda \tag{6}$$

So the empirical (noisy) version is implemented by

$$\tilde{U_2}(\phi,\lambda) = Z_{\lambda+\phi+\pi/2} \cdot \tilde{U}(\lambda) \tag{7}$$

$$= Z_{\lambda+\phi+\pi/2} \cdot \left[Z_{-(\lambda+\delta\gamma)} \cdot X_{\pi/2+\delta\theta} \cdot Z_{\lambda+\delta\gamma}\right] \tag{8}$$

$$\implies \tilde{U_2}^n(\phi,\lambda) = Z_{n(\lambda+\phi+\pi/2)} \cdot \left[\tilde{U}(n\lambda + (n-1)(\phi+\frac{\pi}{2})) \cdot \ldots \cdot \tilde{U}(2\lambda+\phi+\frac{\pi}{2}) \cdot \tilde{U}(\lambda)\right] \tag{9}$$

Assuming multiple $\tilde{U}_2$ gates in 9 are implemented independently, and recalling the leftmost Z gates are not implemented due to 1.1.2 throughout.

1.2 U_3

For an ideal U_3 gate,we have

$$U_3(\theta,\phi,\lambda) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -e^{i\lambda}\sin\left(\frac{\theta}{2}\right) \\ e^{i\phi}\sin\left(\frac{\theta}{2}\right) & e^{i(\phi+\lambda)}\cos\left(\frac{\theta}{2}\right) \end{pmatrix} \tag{10}$$

$$(= Z_\phi \cdot X_{\pi/2} \cdot Z_{\pi-\theta} \cdot X_{\pi/2} \cdot Z_{\lambda+\pi}) \tag{11}$$

$$= Z_{\phi+\pi} \cdot X_{\pi/2} \cdot Z_{\pi+\theta} \cdot X_{\pi/2} \cdot Z_\lambda \tag{12}$$

$$\implies \tilde{U}_3(\theta,\phi,\lambda) = Z_{\lambda+\phi+\theta} \cdot \left[\tilde{U}(\lambda+\theta+\pi) \cdot \tilde{U}(\lambda)\right] \tag{13}$$

$$(\implies = Z_{\lambda+\phi-\theta} \cdot \left[\tilde{U}(\lambda-\theta) \cdot \tilde{U}(\lambda+\pi)\right]) \tag{14}$$

Note carefully how there is no $\delta\gamma$ error in the middle $Z_{\pi-\theta}$ due to cancellation. This means angle error is self-correcting for 'sandwiched' Z gates under our assumptions.

1.3 X_θ

$$\begin{aligned} X_\theta = \exp\left(-i\frac{\theta}{2}\sigma_x\right) &= \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \text{ by Euler's Formula.} \\ &= U_3(\theta,-\pi/2,\pi/2) \\ \implies \tilde{X}_\theta &= Z_\theta \cdot \left[\tilde{U}(\theta-\pi/2) \cdot \tilde{U}(\pi/2)\right] \\ (\implies \tilde{X}_\theta &= Z_{-\theta} \cdot \left[\tilde{U}(\pi/2-\theta) \cdot \tilde{U}(-\pi/2)\right]) \end{aligned} \tag{15}$$

We compute for $\theta = \pi$; further, due to phase freedom we have that σ_x is implemented as $\tilde{U}_3(\pi,0,\pi)$ in IBM systems:

$$\tilde{X}_\pi^2 = \tilde{U}(-\pi/2)\tilde{U}(-\pi/2)\tilde{U}(\pi/2)\tilde{U}(\pi/2) \tag{16}$$

$$\tilde{\sigma}_x^2 = \tilde{U}(\pi)\tilde{U}(\pi)\tilde{U}(\pi)\tilde{U}(\pi) \tag{17}$$

$$\text{ampcal} = \tilde{U}(0)\tilde{U}(-\pi/2)\tilde{U}(\pi)\tilde{U}(\pi/2) \tag{18}$$

$$\tag{19}$$

$$(\tilde{X}_\pi^2 = \tilde{U}(\pi/2)\tilde{U}(\pi/2)\tilde{U}(-\pi/2)\tilde{U}(-\pi/2)) \tag{20}$$

$$(\tilde{\sigma}_x^2 = \tilde{U}(0)\tilde{U}(0)\tilde{U}(0)\tilde{U}(0)) \tag{21}$$

$$(\text{ampcal} = \tilde{U}(0)\tilde{U}(-\pi/2)\tilde{U}(\pi)\tilde{U}(\pi/2)) \tag{22}$$

1.3.1 Experiment Probabilities

Assuming always that we start with our system initialised in the $|0\rangle$ state, we calculate the probability of measuring $|0\rangle$ after $\tilde{X}_\theta$:

$$2\tilde{\mathcal{P}}(0) - 1 = \sin^2(\delta\theta) + \cos^2(\delta\theta)\cos(\theta) \tag{23}$$

1.3.2 Repeated X_θ gates

Now we compute the probability $\mathcal{P}^{|1\rangle}_{\tilde{X}_{\pi/2}}(n)$ of measuring $|1\rangle$ after n applications of $\tilde{X}_{\pi/2} = \tilde{U}(0)$

$$\mathcal{P}^{|1\rangle}_{\tilde{X}_{\pi/2}}(n) = \frac{1}{2} - \frac{1}{2}\cos\left[\left(\delta\theta + \frac{\pi}{2}\right)n\right] \tag{24}$$

$$\implies \text{F.T.}\left(\mathcal{P}^{|1\rangle}_{\tilde{X}_{\pi/2}}\right)(\omega) = \frac{1}{2}\delta(\omega) - \frac{\pi}{2}\left[\delta(\omega - (\pi/2 + \delta\theta)) + \delta(\omega + \pi/2 + \delta\theta)\right] \tag{25}$$

So $\delta\theta$ can empirically be determined via FFT.

References