

# DESIGN DOCUMENT FOR FUNCTION FIELD PROJECT

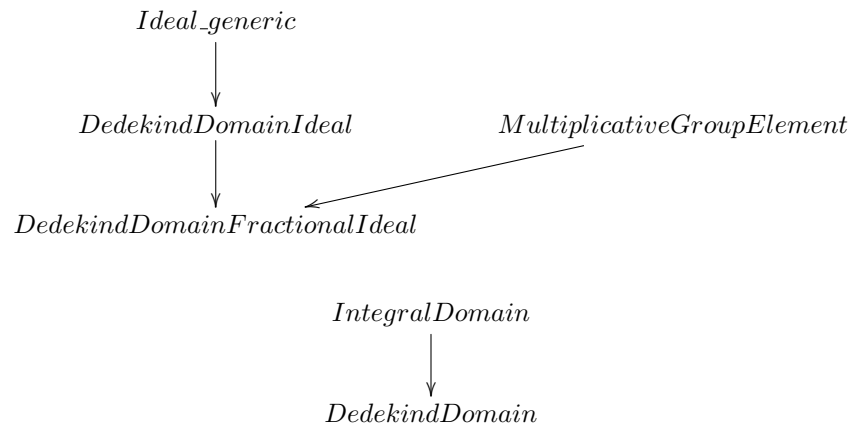
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## 1. CLASS HIERARCHY

An arrow  $A \longrightarrow B$  indicates that the class  $B$  inherits from the class  $A$ .

**Remark.** The `DedekindDomain` class is depreciated, but already exists in sage. However, the implementation there seems to be unsubstantial.

The documentation also recommends the creation of a `DedekindDomain` *category*. We should probably ask about this. Regardless, it seems like we need to code the class anyways.



## 2. CLASSES

### 2.1. Class/Category: `DedekindDomain`. :

Attributes:

- Maximal Order

Init:

- `NumberFields`  $\rightarrow$  Invoke sage's routines
- `FunctionFields`  $\rightarrow$  do our things
- Else  $\rightarrow$  `NotImplementedError`

Methods:

- `FractionalIdealGroup`

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*Date:* August 18, 2017.

- `krull_dimension`
- `is_commutative`
  - Force this to be true. We may even consider changing the class to `CommutativeDedekindDomain`.
- `valuation`

## 2.2. Class/Category: `DedekindDomainElt.` :

Attributes:

- `denominator`

Init:

- `NumberFields` → Invoke sage's routines
- `FunctionFields` → do our things
- Else → `NotImplementedError`

Methods:

- `FractionalIdealGroup`
- `krull_dimension`
- `is_commutative`
  - Force this to be true. We may even consider changing the class to `CommutativeDedekindDomain`.

## 2.3. Class: `DedekindDomainIdeal.` :

**Do we want to make the 2-element representation/ OM representation different classes, or do we want the same object to record both representations?**

Attributes:

- `Basis`

Methods:

- `is_prime`
- `denominator`
- comparison method
- `mult`
- `norm`
- `index`
- `inertia_degree`
- `ramification_degree`
- `maximal_order_basis`
- `convert_to_OM`
  - if self is prime, do conversion
  - else raise `NotImplementedError`
- `convert_to_hnf-representation`
  - Only for number fields
- $\mathbb{Z}$ -basis to 2-elt representation

#### 2.4. Class: `DedekindDomainFractionalIdeal` :

Attributes:

- Basis

Methods:

- `is_integral`
- `denominator`
- `comparison method`
- `mult`
- `inver`
- `norm`
- `index`
- `inertia_degree`
- `ramification_degree`
- `convert_to_OM`
  - if self is prime, do conversion
  - else raise `NotImplementedError`
- `convert_to_hnf-representation`
  - Only for number fields

### 3. ACCESSORY FUNCTIONS

#### 4. TODO

- (i) Set up Git repository
- (ii) Decide if 2-elt/OM representations should be distinct classes or unified in a single class
- (iii) Decide on Category versus Object approach
- (iv) Determine useful information to store as attributes