

RAPPORT DE STAGE PFE

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DRAFT

1 Critère de Drucker-Prager

$$f(\underline{\underline{\sigma}}, p) = \sigma_{\text{eq}} + \alpha \text{tr} \underline{\underline{\sigma}} - R(p) \leq 0 \quad (1)$$

$$\underline{\underline{\dot{\varepsilon}}}^p = \dot{\lambda} \frac{\partial f}{\partial \underline{\underline{\sigma}}} = \dot{\lambda} \left(\frac{3}{2} \frac{\underline{\underline{s}}}{\sigma_{\text{eq}}} + \alpha \underline{\underline{1}} \right) \quad (2)$$

$$\dot{p} = \sqrt{\frac{2}{3} \underline{\underline{\dot{\varepsilon}}}^p : \underline{\underline{\dot{\varepsilon}}}^p} = \dot{\lambda} \sqrt{1 + 2\alpha^2} \quad (3)$$

$$\Delta \underline{\underline{\varepsilon}}_n^p = \frac{\Delta p_n}{\sqrt{1 + 2\alpha^2}} \left(\frac{3}{2} \frac{\underline{\underline{s}}_{n+1}}{\sigma_{n+1}^{\text{eq}}} + \alpha \underline{\underline{1}} \right) \quad (4)$$

$$\Delta \underline{\underline{e}}_n^p = \frac{\Delta p_n}{\sqrt{1 + 2\alpha^2}} \frac{3}{2} \frac{\underline{\underline{s}}_{n+1}}{\sigma_{n+1}^{\text{eq}}} \quad (5)$$

$$\underline{\underline{\sigma}} = (3k \underline{\underline{J}} + 2\mu \underline{\underline{K}}) : (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p) = \underline{\underline{\sigma}}_{n+1}^{\text{elas}} - (3k \underline{\underline{J}} + 2\mu \underline{\underline{K}}) : \underline{\underline{\varepsilon}}^p \quad (6)$$

$$\underline{\underline{\sigma}}_{n+1} = \underline{\underline{\sigma}}_{n+1}^{\text{elas}} - 2\mu \Delta \underline{\underline{e}}_n^p - k \text{tr}(\Delta \underline{\underline{\varepsilon}}_n^p) \underline{\underline{1}} \quad (7)$$

$$\underline{\underline{s}}_{n+1} = \underline{\underline{s}}_{n+1}^{\text{elas}} - 2\mu \Delta \underline{\underline{\varepsilon}}_n^p \quad (8)$$

$$\underline{\underline{s}}_{n+1}^{\text{elas}} = \underline{\underline{s}}_{n+1} + 3\mu \frac{\Delta p_n}{\sqrt{1 + 2\alpha^2}} \frac{\underline{\underline{s}}_{n+1}}{\sigma_{n+1}^{\text{eq}}} = \underline{\underline{s}}_{n+1} \left(1 + 3\mu \frac{\Delta p_n}{\sqrt{1 + 2\alpha^2}} \frac{1}{\sigma_{n+1}^{\text{eq}}} \right) \quad (9)$$

$$\sigma_{n+1}^{\text{elas,eq}} = \sigma_{n+1}^{\text{eq}} \left(1 + 3\mu \frac{\Delta p_n}{\sqrt{1 + 2\alpha^2}} \frac{1}{\sigma_{n+1}^{\text{eq}}} \right) \quad (10)$$

$$\frac{\underline{\underline{s}}_{n+1}^{\text{elas}}}{\sigma_{n+1}^{\text{elas,eq}}} = \frac{\underline{\underline{s}}_{n+1}}{\sigma_{n+1}^{\text{eq}}} \quad (11)$$

$$\sigma_{n+1}^{\text{eq}} = \sigma_{n+1}^{\text{elas,eq}} - \frac{3\mu}{\sqrt{1 + 2\alpha^2}} \Delta p_n \quad (12)$$

$$\text{tr} \underline{\underline{\sigma}}_{n+1} = \text{tr} \underline{\underline{\sigma}}_{n+1}^{\text{elas}} - 3\kappa \text{tr} \Delta \underline{\underline{\varepsilon}}_n^p \quad (13)$$

$$\text{tr} \Delta \underline{\underline{\varepsilon}}_n^p = \frac{3\alpha}{\sqrt{1 + 2\alpha^2}} \Delta p_n \quad (14)$$

$$\sigma_{n+1}^{\text{eq}} + \alpha \text{tr} \sigma_{\equiv n+1} - R(p_n + \Delta p_n) = 0 \quad (15)$$

$$R(p) = \sigma_0 + hp \quad (16)$$

$$\sigma_{n+1}^{\text{elas,eq}} - \frac{3\mu}{\sqrt{1+2\alpha^2}} \Delta p_n + \alpha \text{tr} \sigma_{\equiv n+1}^{\text{elas}} - 3\kappa \frac{3\alpha^2}{\sqrt{1+2\alpha^2}} \Delta p_n - \sigma_0 - hp_n - h\Delta p_n = 0 \quad (17)$$

$$\Delta p_n = \frac{\sigma_{n+1}^{\text{elas,eq}} + \alpha \text{tr} \sigma_{\equiv n+1}^{\text{elas}} - \sigma_0 - hp_n}{\frac{3\mu+9\alpha^2\kappa}{\sqrt{1+2\alpha^2}} + h} = \frac{\sigma_{n+1}^{\text{elas,eq}} + \alpha \text{tr} \sigma_{\equiv n+1}^{\text{elas}} - \sigma_0 - hp_n}{\gamma} \quad (18)$$

$$\gamma = \frac{3\mu + 9\alpha^2\kappa}{\sqrt{1+2\alpha^2}} + h \quad (19)$$

$$\sigma_{n+1} = \sigma_{\equiv n+1}^{\text{elas}} - 2\mu \Delta e_n^p - \kappa \text{tr}(\Delta \varepsilon_n^p) \underline{1} \quad (20)$$

$$\sigma_{\equiv n+1} = \sigma_{\equiv n+1}^{\text{elas}} - \frac{1}{\sqrt{1+2\alpha^2}} \left(\beta s_{\equiv n+1}^{\text{elas}} + 3k\alpha \Delta p_n \underline{1} \right) \quad (21)$$

$$\beta = 3\mu \frac{\Delta p_n}{\sigma_{n+1}^{\text{elas,eq}}} \quad (22)$$

$$\frac{\partial s_{\equiv n+1}^{\text{elas}}}{\partial \Delta \varepsilon_n} = 2\mu K_{\equiv} \quad (23)$$

$$\frac{\partial \sigma_{n+1}^{\text{elas,eq}}}{\partial \Delta \varepsilon_n} = \frac{3\mu}{\sigma_{n+1}^{\text{elas,eq}}} s_{\equiv n+1}^{\text{elas}} \quad (24)$$

$$\frac{\partial \text{tr} \sigma_{n+1}^{\text{elas,eq}}}{\partial \Delta \varepsilon_n} = 3k \underline{1} \quad (25)$$

$$2\mu \Delta e_n^p + k \text{tr} \Delta \varepsilon_n^p \underline{1} = \frac{\Delta p_n}{\sqrt{1+2\alpha^2}} \left(3\mu \frac{s_{\equiv n+1}^{\text{elas}}}{\sigma_{n+1}^{\text{elas,eq}}} + 3k\alpha \underline{1} \right) \quad (26)$$

$$\frac{\partial \Delta p_n}{\partial \Delta \varepsilon_n} = \frac{1}{\gamma} \frac{\partial (\sigma_{n+1}^{\text{elas,eq}} + \alpha \text{tr} \sigma_{\equiv n+1}^{\text{elas}})}{\partial \Delta \varepsilon_n} = \frac{1}{\gamma} \left(\frac{3\mu}{\sigma_{n+1}^{\text{elas,eq}}} s_{\equiv n+1}^{\text{elas}} + 3k\alpha \underline{1} \right) = \frac{1}{\gamma} (3\mu n_{\equiv \text{elas}} + 3k \underline{1}) \quad (27)$$

$$\frac{\partial \sigma_{\underline{\underline{n}}+1}}{\partial \Delta_{\underline{\underline{\varepsilon}}_n}} = \underline{\underline{C}} - \frac{\partial(2\mu\Delta_{\underline{\underline{\varepsilon}}_n} e^p + k \text{tr} \Delta_{\underline{\underline{\varepsilon}}_n}^p \underline{\underline{1}})}{\partial \Delta_{\underline{\underline{\varepsilon}}_n}} = \underline{\underline{C}} - \underline{\underline{D}} \quad (28)$$

$$\underline{\underline{D}} = \frac{1}{\sqrt{1+2\alpha^2}} \frac{\partial}{\partial \Delta_{\underline{\underline{\varepsilon}}_n}} \left(\Delta p_n \left(3\mu \frac{s_{\underline{\underline{n}}+1}^{\text{elas}}}{\sigma_{\underline{\underline{n}}+1}^{\text{elas,eq}}} + 3k\alpha \underline{\underline{1}} \right) \right) = \quad (29)$$

$$= \frac{1}{\sqrt{1+2\alpha^2}} \left(\frac{\partial \Delta p_n}{\partial \Delta_{\underline{\underline{\varepsilon}}_n}} \otimes \left(3\mu \frac{s_{\underline{\underline{n}}+1}^{\text{elas}}}{\sigma_{\underline{\underline{n}}+1}^{\text{elas,eq}}} + 3k\alpha \underline{\underline{1}} \right) + \Delta p_n \left(3\mu \frac{\partial s_{\underline{\underline{n}}+1}^{\text{elas}}}{\partial \Delta_{\underline{\underline{\varepsilon}}_n}} \frac{1}{\sigma_{\underline{\underline{n}}+1}^{\text{elas,eq}}} \right) - \Delta p_n 3\mu \frac{s_{\underline{\underline{n}}+1}^{\text{elas}}}{(\sigma_{\underline{\underline{n}}+1}^{\text{elas,eq}})^2} \otimes \frac{\partial \sigma_{\underline{\underline{n}}+1}^{\text{elas,eq}}}{\partial \Delta_{\underline{\underline{\varepsilon}}_n}} \right) = \quad (30)$$

$$= \frac{1}{\sqrt{1+2\alpha^2}} \left(\left(3\mu \underline{\underline{n}}_{\text{elas}} + 3k\alpha \underline{\underline{1}} \right) \frac{1}{\gamma} \otimes \left(3\mu \underline{\underline{n}}_{\text{elas}} + 3k\alpha \underline{\underline{1}} \right) + 2\mu\beta \underline{\underline{K}} - 3\mu\beta \underline{\underline{n}}_{\text{elas}} \otimes \underline{\underline{n}}_{\text{elas}} \right) \quad (31)$$

$$\underline{\underline{D}} = \frac{1}{\sqrt{1+2\alpha^2}} \left(3\mu \left(\frac{3\mu}{\gamma} - \beta \right) \underline{\underline{n}}_{\text{elas}} \otimes \underline{\underline{n}}_{\text{elas}} + \frac{9\alpha\mu k}{\gamma} (\underline{\underline{1}} \otimes \underline{\underline{n}}_{\text{elas}} + \underline{\underline{n}}_{\text{elas}} \otimes \underline{\underline{1}}) + \frac{9\alpha^2 k^2}{\gamma} \underline{\underline{1}} \otimes \underline{\underline{1}} + 2\mu\beta \underline{\underline{K}} \right) \quad (32)$$

$$\underline{\underline{D}} : \Delta_{\underline{\underline{\varepsilon}}_n} = \quad (33)$$

$$= \frac{1}{\sqrt{1+2\alpha^2}} \left(\underline{\underline{n}}_{\text{elas}} : \Delta_{\underline{\underline{\varepsilon}}_n} 3\mu \left(\frac{3\mu}{\gamma} - \beta \right) \underline{\underline{n}}_{\text{elas}} + \frac{9\alpha\mu k}{\gamma} (\underline{\underline{n}}_{\text{elas}} : \Delta_{\underline{\underline{\varepsilon}}_n} \underline{\underline{1}} + \text{tr} \Delta_{\underline{\underline{\varepsilon}}_n} \underline{\underline{n}}_{\text{elas}}) + \frac{9\alpha^2 k^2}{\gamma} \text{tr} \Delta_{\underline{\underline{\varepsilon}}_n} \underline{\underline{1}} + 2\mu\beta \Delta_{\underline{\underline{\varepsilon}}_n}^p \right) \quad (34)$$

$$(35)$$