Rapport de stage PFE

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# DRAFT

# 1 Critère de Drucker-Prager

$$f(\underline{\underline{\sigma}}, p) = \sigma_{\mathrm{eq}} + \alpha \mathrm{tr}\underline{\underline{\sigma}} - R(p) \leq 0 \tag{1}$$

$$\dot{\underline{\underline{\varepsilon}}}^p = \dot{\lambda}\frac{\partial f}{\partial \underline{\underline{\sigma}}} = \dot{\lambda}\left(\frac{3}{2}\frac{\underline{\underline{s}}}{\sigma_{\mathrm{eq}}} + \alpha \underline{\underline{1}}\right) \tag{2}$$

$$\dot{p} = \sqrt{\frac{2}{3}\dot{\underline{\underline{\varepsilon}}}^p : \dot{\underline{\underline{\varepsilon}}}^p} = \dot{\lambda}\sqrt{1+2\alpha^2} \tag{3}$$

$$\Delta \underline{\underline{\varepsilon}}^p_n = \frac{\Delta p_n}{\sqrt{1+2\alpha^2}}\left(\frac{3}{2}\frac{\underline{\underline{s}}_{n+1}}{\sigma^{\mathrm{eq}}_{n+1}} + \alpha \underline{\underline{1}}\right) \tag{4}$$

$$\Delta \underline{\underline{e}}^p_n = \frac{\Delta p_n}{\sqrt{1+2\alpha^2}}\frac{3}{2}\frac{\underline{\underline{s}}_{n+1}}{\sigma^{\mathrm{eq}}_{n+1}} \tag{5}$$

$$\underline{\underline{\sigma}} = (3k\underline{\underline{\underline{\underline{J}}}} + 2\mu\underline{\underline{\underline{\underline{K}}}}) : (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p) = \underline{\underline{\sigma}}^{\mathrm{elas}}_{n+1} - (3k\underline{\underline{\underline{\underline{J}}}} + 2\mu\underline{\underline{\underline{\underline{K}}}}) : \underline{\underline{\varepsilon}}^p \tag{6}$$

$$\underline{\underline{\sigma}}_{n+1} = \underline{\underline{\sigma}}^{\mathrm{elas}}_{n+1} - 2\mu\Delta\underline{\underline{e}}^p_n - k\mathrm{tr}(\Delta\underline{\underline{\varepsilon}}^p_n)\underline{\underline{1}} \tag{7}$$

$$\underline{\underline{s}}_{n+1} = \underline{\underline{s}}^{\mathrm{elas}}_{n+1} - 2\mu\Delta\underline{\underline{e}}^p_n \tag{8}$$

$$\underline{\underline{s}}^{\mathrm{elas}}_{n+1} = \underline{\underline{s}}_{n+1} + 3\mu\frac{\Delta p_n}{\sqrt{1+2\alpha^2}}\frac{\underline{\underline{s}}_{n+1}}{\sigma^{\mathrm{eq}}_{n+1}} = \underline{\underline{s}}_{n+1}(1 + 3\mu\frac{\Delta p_n}{\sqrt{1+2\alpha^2}}\frac{1}{\sigma^{\mathrm{eq}}_{n+1}}) \tag{9}$$

$$\sigma^{\mathrm{elas,eq}}_{n+1} = \sigma^{\mathrm{eq}}_{n+1}(1 + 3\mu\frac{\Delta p_n}{\sqrt{1+2\alpha^2}}\frac{1}{\sigma^{\mathrm{eq}}_{n+1}}) \tag{10}$$

$$\frac{\underline{\underline{s}}^{\mathrm{elas}}_{n+1}}{\sigma^{\mathrm{elas,eq}}_{n+1}} = \frac{\underline{\underline{s}}_{n+1}}{\sigma^{\mathrm{eq}}_{n+1}} \tag{11}$$

$$\sigma^{\mathrm{eq}}_{n+1} = \sigma^{\mathrm{elas,eq}}_{n+1} - \frac{3\mu}{\sqrt{1+2\alpha^2}}\Delta p_n \tag{12}$$

$$\mathrm{tr}\underline{\underline{\sigma}}_{n+1} = \mathrm{tr}\underline{\underline{\sigma}}^{\mathrm{elas}}_{n+1} - 3\kappa\mathrm{tr}\Delta\underline{\underline{\varepsilon}}^p_n \tag{13}$$

$$\mathrm{tr}\Delta\underline{\underline{\varepsilon}}^p_n = \frac{3\alpha}{\sqrt{1+2\alpha^2}}\Delta p_n \tag{14}$$

$$\sigma^{\mathrm{eq}}_{n+1} + \alpha \mathrm{tr} \underline{\underline{\sigma}}_{n+1} - R(p_n + \Delta p_n) = 0 \tag{15}$$

$$R(p) = \sigma_0 + hp \tag{16}$$

$$\sigma^{\mathrm{elas,eq}}_{n+1} - \frac{3\mu}{\sqrt{1+2\alpha^2}}\Delta p_n + \alpha \mathrm{tr}\underline{\underline{\sigma}}^{\mathrm{elas}}_{n+1} - 3\kappa\frac{3\alpha^2}{\sqrt{1+2\alpha^2}}\Delta p_n - \sigma_0 - hp_n - h\Delta p_n = 0 \tag{17}$$

$$\Delta p_n = \frac{\sigma^{\mathrm{elas,eq}}_{n+1} + \alpha \mathrm{tr}\underline{\underline{\sigma}}^{\mathrm{elas}}_{n+1} - \sigma_0 - hp_n}{\frac{3\mu+9\alpha^2\kappa}{\sqrt{1+2\alpha^2}} + h} = \frac{\sigma^{\mathrm{elas,eq}}_{n+1} + \alpha \mathrm{tr}\underline{\underline{\sigma}}^{\mathrm{elas}}_{n+1} - \sigma_0 - hp_n}{\gamma} \tag{18}$$

$$\gamma = \frac{3\mu + 9\alpha^2\kappa}{\sqrt{1+2\alpha^2}} + h \tag{19}$$

$$\underline{\underline{\sigma}}_{n+1} = \underline{\underline{\sigma}}^{\mathrm{elas}}_{n+1} - 2\mu\Delta\underline{\underline{e}}^p_n - \kappa \mathrm{tr}(\Delta\underline{\underline{\varepsilon}}^p_n)\underline{\underline{1}} \tag{20}$$

$$\underline{\underline{\sigma}}_{n+1} = \underline{\underline{\sigma}}^{\mathrm{elas}}_{n+1} - \frac{1}{\sqrt{1+2\alpha^2}}\left(\beta\underline{\underline{s}}^{\mathrm{elas}}_{n+1} + 3k\alpha\Delta p_n \underline{\underline{1}}\right) \tag{21}$$

$$\beta = 3\mu\frac{\Delta p_n}{\sigma^{\mathrm{elas,eq}}_{n+1}} \tag{22}$$

$$\frac{\partial \underline{\underline{s}}^{\mathrm{elas}}_{n+1}}{\partial \Delta\underline{\underline{\varepsilon}}_n} = 2\mu\underline{\underline{\underline{\underline{K}}}} \tag{23}$$

$$\frac{\partial \sigma^{\mathrm{elas,eq}}_{n+1}}{\partial \Delta\underline{\underline{\varepsilon}}_n} = \frac{3\mu}{\sigma^{\mathrm{elas,eq}}_{n+1}}\underline{\underline{s}}^{\mathrm{elas}}_{n+1} \tag{24}$$

$$\frac{\partial \mathrm{tr}\sigma^{\mathrm{elas,eq}}_{n+1}}{\partial \Delta\underline{\underline{\varepsilon}}_n} = 3k\underline{\underline{1}} \tag{25}$$

$$2\mu\Delta\underline{\underline{e}}^p_n + k \mathrm{tr}\Delta\underline{\underline{\varepsilon}}^p_n\underline{\underline{1}} = \frac{\Delta p_n}{\sqrt{1+2\alpha^2}}\left(3\mu\frac{\underline{\underline{s}}^{\mathrm{elas}}_{n+1}}{\sigma^{\mathrm{elas,eq}}_{n+1}} + 3k\alpha\underline{\underline{1}}\right) \tag{26}$$

$$\frac{\partial \Delta p_n}{\partial \Delta\underline{\underline{\varepsilon}}_n} = \frac{1}{\gamma}\frac{\partial(\sigma^{\mathrm{elas,eq}}_{n+1} + \alpha \mathrm{tr}\underline{\underline{\sigma}}^{\mathrm{elas}}_{n+1})}{\partial \Delta\underline{\underline{\varepsilon}}_n} = \frac{1}{\gamma}\left(\frac{3\mu}{\sigma^{\mathrm{elas,eq}}_{n+1}}\underline{\underline{s}}^{\mathrm{elas}}_{n+1} + 3k\alpha\underline{\underline{1}}\right) = \frac{1}{\gamma}(3\mu\underline{\underline{n}}_{\mathrm{elas}} + 3k\underline{\underline{1}}) \tag{27}$$

$$\frac{\partial \underline{\underline{\sigma}}_{n+1}}{\partial \Delta \underline{\underline{\varepsilon}}_n} = \underline{\underline{\underline{\underline{C}}}} - \frac{\partial(2\mu\Delta \underline{\underline{e}}^p_n + k\,\mathrm{tr}\Delta\underline{\underline{\varepsilon}}^p_n \underline{\underline{1}})}{\partial\Delta\underline{\underline{\varepsilon}}_n} = \underline{\underline{\underline{\underline{C}}}} - \underline{\underline{\underline{\underline{D}}}} \tag{28}$$

$$\underline{\underline{\underline{\underline{D}}}} = \frac{1}{\sqrt{1+2\alpha^2}} \frac{\partial}{\partial \Delta\underline{\underline{\varepsilon}}_n} \left( \Delta p_n \left( 3\mu \frac{\underline{\underline{s}}^{\mathrm{elas}}_{n+1}}{\sigma^{\mathrm{elas,eq}}_{n+1}} + 3k\alpha\underline{\underline{1}} \right)\right) = \tag{29}$$

$$= \frac{1}{\sqrt{1+2\alpha^2}} \left( \frac{\partial \Delta p_n}{\partial \Delta\underline{\underline{\varepsilon}}_n} \otimes \left( 3\mu \frac{\underline{\underline{s}}^{\mathrm{elas}}_{n+1}}{\sigma^{\mathrm{elas,eq}}_{n+1}} + 3k\alpha\underline{\underline{1}} \right) + \Delta p_n \left( 3\mu \frac{\partial \underline{\underline{s}}^{\mathrm{elas}}_{n+1}}{\partial \Delta\underline{\underline{\varepsilon}}_n} \frac{1}{\sigma^{\mathrm{elas,eq}}_{n+1}} \right) - \Delta p_n 3\mu \frac{\underline{\underline{s}}^{\mathrm{elas}}_{n+1}}{(\sigma^{\mathrm{elas,eq}}_{n+1})^2} \otimes \frac{\partial \sigma^{\mathrm{elas,eq}}_{n+1}}{\partial \Delta\underline{\underline{\varepsilon}}_n} \right) = \tag{30}$$

$$= \frac{1}{\sqrt{1+2\alpha^2}} \left( \left( 3\mu \underline{\underline{n}}_{\mathrm{elas}} + 3\alpha k \underline{\underline{1}} \right) \frac{1}{\gamma} \otimes \left( 3\mu \underline{\underline{n}}_{\mathrm{elas}} + 3\alpha k \underline{\underline{1}} \right) + 2\mu\beta \underline{\underline{\underline{\underline{K}}}} - 3\mu\beta \underline{\underline{n}}_{\mathrm{elas}} \otimes \underline{\underline{n}}_{\mathrm{elas}} \right) \tag{31}$$

$$\underline{\underline{\underline{\underline{D}}}} = \frac{1}{\sqrt{1+2\alpha^2}} \left( 3\mu \left( \frac{3\mu}{\gamma} - \beta \right) \underline{\underline{n}}_{\mathrm{elas}} \otimes \underline{\underline{n}}_{\mathrm{elas}} + \frac{9\alpha\mu k}{\gamma} (\underline{\underline{1}} \otimes \underline{\underline{n}}_{\mathrm{elas}} + \underline{\underline{n}}_{\mathrm{elas}} \otimes \underline{\underline{1}}) + \frac{9\alpha^2 k^2}{\gamma} \underline{\underline{1}} \otimes \underline{\underline{1}} + 2\mu\beta \underline{\underline{\underline{\underline{K}}}} \right) \tag{32}$$

$$\underline{\underline{\underline{\underline{D}}}} : \Delta\underline{\underline{\varepsilon}}_n = \tag{33}$$

$$= \frac{1}{\sqrt{1+2\alpha^2}} \left( \underline{\underline{n}}_{\mathrm{elas}} : \Delta\underline{\underline{\varepsilon}}_n 3\mu \left( \frac{3\mu}{\gamma} - \beta \right) \underline{\underline{n}}_{\mathrm{elas}} + \frac{9\alpha\mu k}{\gamma} (\underline{\underline{n}}_{\mathrm{elas}} : \Delta\underline{\underline{\varepsilon}}_n \underline{\underline{1}} + \mathrm{tr}\Delta\underline{\underline{\varepsilon}}_n \underline{\underline{n}}_{\mathrm{elas}}) + \frac{9\alpha^2 k^2}{\gamma} \mathrm{tr}\Delta\underline{\underline{\varepsilon}}_n \underline{\underline{1}} + 2\mu\beta \Delta \underline{\underline{e}}^p_n \right) \tag{34}$$

$$\tag{35}$$

$$F = \int\limits_{\Omega} \underline{\underline{\sigma}}_{n+1} : \underline{\underline{\varepsilon}}(\underline{v})\, \mathrm{d}\Omega - \underline{F}_{\mathrm{ext}} = \tag{36}$$

$$= \int\limits_{\Omega} \left( \underline{\underline{\sigma}}_n + \underline{\underline{\underline{\underline{C}}}} : (\Delta\underline{\underline{\varepsilon}}_n - \Delta\underline{\underline{\varepsilon}}^p_n) \right) : \underline{\underline{\varepsilon}}(\underline{v})\, \mathrm{d}\Omega - \underline{F}_{\mathrm{ext}} \tag{37}$$

where $\underline{F}_{\mathrm{ext}} = q \int\limits_{\partial\Omega_{\mathrm{inside}}} \underline{n} \cdot \underline{v}\, \mathrm{d}s$, $\Delta\underline{\underline{\varepsilon}}_n = \underline{\underline{\varepsilon}}(\Delta\underline{u}_n)$ and $\Delta\underline{\underline{\varepsilon}}_n = \underline{\underline{\varepsilon}}^p(\Delta\underline{u}_n)$

$$\underline{\underline{\varepsilon}}(\Delta\underline{u}_n) = \frac{1}{2}\left( \nabla\Delta\underline{u} + \nabla\Delta\underline{u}^T \right) \tag{38}$$

$$\underline{\underline{\varepsilon}}^p(\Delta\underline{u}_n) = \begin{cases} \Delta p_n \left( \frac{3}{2} \frac{\underline{\underline{s}}^{\mathrm{elas}}_{n+1}}{\sigma^{\mathrm{elas,eq}}_{n+1}} + \alpha\underline{\underline{1}} \right), & \text{if } f(\underline{\underline{\sigma}}^{\mathrm{elas}}, p_n) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{39}$$

where $\Delta p_n = p_{n+1} - p_n$

$$\underline{\underline{\varepsilon}}^p(\Delta\underline{u}) = \underline{\underline{\varepsilon}}^p(\Delta\underline{u}, p_n, p_{n+1}, \underline{\underline{\sigma}}_n) \tag{40}$$

$$F(\Delta\underline{u}, \underline{v}) = \int\limits_\Omega \left(\underline{\underline{\sigma}}_n + \underline{\underline{\underline{\underline{C}}}} : (\underline{\underline{\varepsilon}}(\Delta\underline{u}) - \underline{\underline{\varepsilon}}^p(\Delta\underline{u}))\right) : \underline{\underline{\varepsilon}}(\underline{v}) \,\mathrm{d}\Omega - \underline{F}_{\text{ext}} \tag{41}$$

$$J(\underline{u}, \underline{v}) = \frac{\partial F(\Delta\underline{u}, \underline{v})}{\partial \Delta\underline{u}}(\underline{u}) \tag{42}$$

Plan

1. Introduction
2. Main part
   (a) Theory
      i. Plasticity and return-mapping algorithm
      ii. Plasticity using conic optimization
      iii. Plasticity problem description
   (b) Development
      i. Conventional return-mapping algorithm
      ii. Custom assembler implementation in FEniCsX (motivs : convex + loi de comport.)
      iii. Return-mapping via conic optimization
   (c) Results
      i. Qualitative yield criterions comparison
      ii. Calculation time for different approaches and solvers
      iii. Patch size influence for vectorized convex optimization problem
      iv. Different convex solvers comparison (+ patches)
   (d) Perspectives*
      i. Convex approach integration into the custom assembler
      ii. Adaptation of diffcp library
3. Conclusion