MATH 444 Problem Set Week 1

1. Prove the statement: The composition of any two isometries is an isometry.

2. Determine if each of the following map $f : \mathbb{R}^2 \to \mathbb{R}^2$ is a isometry or not, justify your answer. Can you recognize each map as a translation, reflection, rotation, dilation, or any composition of two or more of these basic transformations?

   (a) $(x, y) \mapsto (-x - 1, y + 2)$

   (b) $(x, y) \mapsto (-y, x)$

   (c) $(x, y) \mapsto (x - y, x + y)$

   (d) $(x, y) \mapsto \left(\dfrac{x - y}{\sqrt{2}}, \dfrac{x + y}{\sqrt{2}}\right)$

   (e) $(x, y) \mapsto (x - y, y - x)$

   (f) $(x, y) \mapsto (x^2, xy)$

3. Prove the general matrix formula for the reflection about the line L_θ that makes an angle θ with the x-axis at O:

$$\bar{r}_{L_\theta} \text{ or } \bar{r}_\theta : \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

   (Hint: express $\bar{r}_{L_\theta}$ as a conjugation.)

4. Let $P = (1, 2)$, and ℓ is the line $y = \sqrt{3}x + 4$, find an explicit formula for the following isometries, you may write it in terms of vectors and matrices.

   (a) The rotation $R_{\pi/6,P}$.

   (b) The reflection $\bar{r}_\ell$.

5.