UW Math 134 Theorem Compendium

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Numbers

Definition 1 (Rational Numbers)**.** $\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z}\}$.

Algebra

Definition 2 (Absolute Value Function)**.**

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}, \quad |a| = \sqrt{a^2}.$$

a. $|x| < \delta \iff -\delta < x < \delta$.

b. $|x| > \delta \iff x > \delta \lor x < -\delta$.

c. $|x - c| < \delta \iff c - \delta < x < c + \delta$.

d. $0 < |x - c| < \delta \iff c - \delta < x < c \lor c < x < c + \delta$.

Definition 3.

Theorem 1 (Triangle Inequalities)**.**

$$|a + b| \leq |a| + |b|, \quad \big||a| - |b|\big| \leq |a - b|.$$

Theorem 2.

Definition 4 (Functions)**.** *$dom(f) = D$, $range(f) = \{f(x) : x \in D\} = R$.*

$$f : D \longrightarrow R$$

Definition 5 (Graphs)**.** *The graph of $f = (x, y) : x \in D, y = f(x)$.*

Definition 6 (Even & Odd Functions)**.** $\forall x \in dom(f)$,

a. f is an even function $\iff f(x) = f(-x)$.

b. f is an odd function $\iff -f(x) = f(-x)$.

Definition 7 (Polynomial Functions)**.**

$$\forall x, a_k \in \mathbb{R}, n > 0 \in \mathbb{Z}. \quad \sum_{k=0}^{n} a_k x^k, \quad a_n \neq 0.$$

Definition 8 (Rational Functions)**.** *For any polynomials P and Q,*

$$\frac{P(x)}{Q(x)}.$$

Limits

Theorem 3. *For f defined on $(c-p) \cup (c+p)$,*

$$\forall \epsilon > 0, \exists \delta > 0. \quad 0 < |x-c| < \delta \implies |f(x) - L| < \epsilon \iff \lim_{x \to c} f(x) = L.$$

a. $\lim_{x \to c} x = c.$

b. $\lim_{x \to c} |x| = |c|.$

c. $\lim_{x \to c} k = k.$

d. $\lim_{x \to c} f(x) = 0 \iff \lim_{x \to c} |f(x)| = 0$

Theorem 4.

$$\lim_{x \to c} f(x) = L \iff \lim_{x \to c^-} f(x) = L \wedge \lim_{x \to c^+} f(x) = L.$$

Theorem 5. $\lim_{x \to c} f(x) = L \wedge \lim_{x \to c} g(x) = M \implies,$

(i) $\lim_{x \to c} [f(x) + g(x)] = L + M.$

(ii) $\lim_{x \to c} [\alpha f(x)] = \alpha L, \quad \alpha \in \mathbb{R}.$

(iii) $\lim_{x \to c} [f(x) g(x)] = LM.$

(iv) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}, \quad M \neq 0.$