

# MATH 444 Problem Set Week 1

1. Prove the statement: The composition of any two isometries is an isometry.
2. Determine if each of the following map  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is a isometry or not, justify your answer. Can you recognize each map as a translation, reflection, rotation, dilation, or any composition of two or more of these basic transformations?

(a)  $(x, y) \mapsto (-x - 1, y + 2)$

(b)  $(x, y) \mapsto (-y, x)$

(c)  $(x, y) \mapsto (x - y, x + y)$

(d)  $(x, y) \mapsto \left( \frac{x - y}{\sqrt{2}}, \frac{x + y}{\sqrt{2}} \right)$

(e)  $(x, y) \mapsto (x - y, y - x)$

(f)  $(x, y) \mapsto (x^2, xy)$

3. Prove the general matrix formula for the reflection about the line  $L_\theta$  that makes an angle  $\theta$  with the  $x$ -axis at  $O$ :

$$\bar{r}_{L_\theta} \text{ or } \bar{r}_\theta : \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

(Hint: express  $\bar{r}_{L_\theta}$  as a conjugation.)

4. Let  $P = (1, 2)$ , and  $\ell$  is the line  $y = \sqrt{3}x + 4$ , find an explicit formula for the following isometries, you may write it in terms of vectors and matrices.

(a) The rotation  $R_{\pi/6, P}$ .

(b) The reflection  $\bar{r}_\ell$ .

5.