Lecture 4 Activity: Atmospheric Escape and Retention

Alexandre Lipson

January 30, 2025

The average speed of a molecule of gas in m/s is given by

$$v_{\text{gas}} = 157\sqrt{\frac{\text{Temperature}}{\text{molecular mass}}}.$$

A planet can hold onto a gas for the age of the solar system if

$$v_{\text{gas}} \leq \frac{1}{6} v_{\text{esc}}.$$

Problem 1. What is the average speed in m/s of a hydrogen molecule in the atmosphere of a planet at 1 AU?

A hydrogen molecule has a molecular mass of 2. The temperature of a planet at 1 AU distance from the Sun would be about 400 K. So, the average velocity of the hydrogen gas molecules would be

$$v_{\text{H}_2} = 157\sqrt{\frac{400}{2}} = 157 \cdot \sqrt{2} \cdot 10 \approx 2220 \text{ m/s}.$$

Problem 2. As calculated above, molecules in the Earth's atmosphere must achieve an average speed of 1,867 m/s for some to escape the atmosphere. Considering your answer to question 1, can the Earth hold an atmosphere of hydrogen? Explain your answer.

Since the average velocity of hydrogen gas at earth's distance of 1 AU, 2220 m/s, is greater than 1867 m/s, then the earth cannot hold an atmosphere of hydrogen.

Problem 3. What is the speed in m/s of a nitrogen molecule in the atmosphere of a planet at 1 AU?

A nitrogen molecule has a molecular mass of 28 amu. The temperature of a planet 1 AU away from the Sun is about 400 K. So, the average velocity of nitrogen gas molecules would be,

$$v_{\text{N}_2} = 157\sqrt{\frac{400}{28}} = 157 \cdot \frac{10}{\sqrt{7}} \approx 593 \text{ m/s}.$$

Problem 4. As calculated above, molecules in the Earth's atmosphere must achieve an average speed of 1,867 m/s for some to escape the atmosphere. Considering your answer to question 3, can the Earth hold an atmosphere of nitrogen? Explain your answer.

Since the average velocity of nitrogen gas at earth's distance of 1 AU, 593 m/s, is less than 1867 m/s, then the earth can, and in fact does, hold an atmosphere of nitrogen.

Problem 5. Calculate how far in AU the Moon would have to be from the Sun for it to retain a nitrogen atmosphere.

Hint: set both v_{gas} equations equal to each other and solve for the missing variable. Does your result make sense? Think about velocities and temperatures to explain.

Since we are trying to determine what distance the moon would have to be to hold a nitrogen atmosphere and temperature relates to distance, then we will fix all other variables and solve for temperature in our equation.

The moon has an escape velocity of 2300 m/s, so gas must achieve a speed of at least $\frac{1}{6}2300 \approx 383$ m/s to escape...

Nitrogen gas has a molecular weight of 28 amu.

Then, solving for a temperature T such that the nitrogen gas will not exceed the escape velocity of the moon for geologic time, we have

$$157\sqrt{\frac{T}{28}} = 383$$
$$T = \left(\frac{383\sqrt{28}}{157}\right)^2$$
$$T \approx 167 \text{ K.}$$

For a temperature of 167 K, the moon would be located at around 6 AU instead of its current 1 AU.

This answer makes sense because the only variable we are changing is the distance, and therefore temperature, of the moon. Since the moon keeps its same gravity, the only way to increase its atmosphere holding capabilities is by reducing the kinetic energy of such atmospheric gasses. This is accomplished by lowering their temperature through increasing the distance from the sun.

Problem 6. Which gases from Table 3 could Mars retain?

Mars has an escape velocity of 5000 m/s, so it can contain gases moving at less than $\frac{1}{6}5000 \approx 833$ m/s.

Mars is located 1.5 AU away from the Sun, this temperature is not directly given on Table 2, but we expect it to be between 283 and 400 K (the temps for 2 and 1 AU respectively).

We can research the average temperature on mars to see that is is slightly colder, at around 250 K. [1]

However, we used a value of 400 K for the temperature of earth for this exercise, that's equivalent to around 260 F, which is much hotter than the actual earth.

So, we should attempt to determine what temperature Table 2 would give at a distance of 1.5 AU, noting that our value ought to be within the range [283,400].

We can create an regression given the data in Table 2 of the form $T\ \frac{a}{\sqrt{r}} + b$, where T is temperature, r is distance from the world to the sun, and a and b are constants.

We arrive at this form given that the energy received per area is inversely proportional to the square

[1] `https://www.britannica.com/question/What-is-the-temperature-on-Mars`.

of distance.

But, our form has $\frac{1}{\sqrt{r}}$ and not $\frac{1}{r^2}$; we also must consider the Stefano-Boltzmann law that energy is proportional to the fourth power of temperature.

Thus, $\frac{1}{r^2} \propto T^4 \implies T \propto \frac{1}{r^{1/2}}$.

Our model now produces a graph with an R^2 of about 0.9998, coefficients $a = 392.41373$ and $b = 3.83379$, and a predicted temperature of 324 K at 1.5 AU. [2]

We will compute the average velocity for each gas in Table 3 as if it were in the Martian atmosphere at 324 K.

gas	average velocity (m/s)
H_2	$157\sqrt{\frac{324}{2}} \approx 1998$
CH_4	$157\sqrt{\frac{324}{16}} \approx 707$
NH_3	$157\sqrt{\frac{324}{17}} \approx 685$
H_20	$157\sqrt{\frac{324}{18}} \approx 666$
N_2	$157\sqrt{\frac{324}{28}} \approx 534$
CO_2	$157\sqrt{\frac{324}{44}} \approx 426$

Since Mars has an escape velocity of 833 m/s, it can retain all gases within the table moving slower, which include all but hydrogen.

Problem 7. Which gases from Table 3 could Ceres retain?

We will repeat the procedure used in problem 6.

First, we will compute the maximum speed that gas molecules can have in order to remain in Ceres' atmosphere for geologic time.

$$\frac{1}{6}510 = 85 \text{ m/s}.$$

We will use the same temperature distance model to determine the temperature of Ceres at a distance of 2.8 AU, this is about 238 K.

Next, we will fill in the table,

gas	average velocity (m/s)
H_2	$157\sqrt{\frac{238}{2}} \approx 1713$
CH_4	$157\sqrt{\frac{238}{16}} \approx 606$
NH_3	$157\sqrt{\frac{238}{17}} \approx 587$
H_20	$157\sqrt{\frac{238}{18}} \approx 571$
N_2	$157\sqrt{\frac{238}{28}} \approx 458$
CO_2	$157\sqrt{\frac{238}{44}} \approx 365$

Since all of these gasses would have an average velocity greater than Ceres' atmospheric capture velocity of 85 m/s, then Ceres could retain any of the gasses from Table 3.

[2]Using Desmos graphing calculator.

Problem 8. What is the main physical driver for the difference between the molecules that Mars can retain and those that Ceres can retain?

Hint: if you could move Ceres to Mars' orbit, could it retain the same molecules as Mars?

The difference in temperature was small, $324 - 238 = 86 \approx 0.36 * 238$, about a 36 percent increase from Ceres to Mars.

The more important factor was the escape speed for each of these bodies; Mars has nearly 10 times the escape velocity of Ceres as it is much larger and has a greater gravitational force.

So, if we moved Ceres to the distance of Mars' orbit at 1.5 AU, it would still probably not retain most, if not all, of the gasses.

Problem 9. Jupiter currently orbits the Sun at 5 AU. Could Jupiter retain an atmosphere of hydrogen molecules if you could magically move Jupiter to 0.5 AU from the Sun? Why or why not? Explain your answer.

Currently, Jupiter can retain hydrogen since it is very massive and has a strong gravity; but, at the same time, its large distance from the Sun also cools the hydrogen gas molecules, giving them less kinetic energy and velocity.

So, following the procedure of problems 6 and 7, we will first compute the maximum velocity for a gas retained through geologic time,

$$\frac{1}{6}59500 \approx 9917 \text{ m/s}.$$

Now, we will determine the temperature that Jupiter at 0.5 AU from the Sun would have. This is given directly by the table as 566 K.

So, we will build out the same table as before,

gas	average velocity (m/s)
H_2	$157\sqrt{\frac{566}{2}} \approx 2641$
$\vdots$	$\vdots$

Since even hydrogen, the lightest gas, would have insufficient velocity to escape Jupiter, even at the increased temperature at 0.5 AU from the Sun, then all the heavier gasses would be retained as well.

Thus, Jupiter would retain at atmosphere with hydrogen molecules at a distance of 0.5 AU.

In order to force Jupiter to be too hot to retain its hydrogen, we can consider the following,

$$9917 \leq 157\sqrt{\frac{T}{2}} \implies \left(\frac{9917\sqrt{2}}{157}\right)^2 = T.$$

Recalling the regression for temperature T K in terms of distance r AU, $T = \frac{392.41373}{\sqrt{r}} + 3.83379$, we can combine with the above equation to solve for the distance r.

$$\begin{aligned}\left(\frac{9917\sqrt{2}}{157}\right)^2 &= \frac{392.41373}{\sqrt{r}} + 3.83379 \\ r &= \left(\frac{\left(\frac{9917\sqrt{2}}{157}\right)^2 - 3.83379}{392.41373}\right)^{-2} \\ r &\approx 2.42 \cdot 10^{-3}.\end{aligned}$$

The radius of the Sun is about $4.65 \cdot 10^{-6}$ AU. Jupiter would need to be very very close to the sun in order to lose its hydrogen.

Problem 10. Mars and Venus are at very different distances from the Sun and Venus is twice the size of Mars, yet their atmospheric compositions are nearly identical.

How can this be given their dramatic physical differences?

Explain using the data and equations above. (Note: you may need to do some calculations here to backup your reasoning).

Since Mars is further from the Sun than Venus, it is colder. Since Mars is colder, gasses in its atmosphere will have a lower average velocity.

However, since Venus is twice the size of Mars, then it will have a stronger gravitational force. Since Venus has stronger gravity, then its gas retention velocity will be greater.

These two effects are equally strong enough to render the similar atmospheric compositions of Mars and Venus.

We have previously seen that Mars, with an gas retention velocity of 833 m/s, can retain all gasses in Table 3 except for hydrogen.

First, We will find the distance from Venus to the Sun, 0.72 AU. [3]

Then, we will convert this into surface temperature using the model as before. This gives a temperature of around 466 K.

Next, we will compute an abridged average gas velocity table for Venus in order to place bounds on its escape velocity.

gas	average velocity (m/s)
H_2	$157\sqrt{\frac{466}{2}} \approx 2397$
CH_4	$157\sqrt{\frac{466}{16}} \approx 847$
$\vdots$	$\vdots$

So, the gas retention velocity of Venus must be between 847 and 2397 m/s.

We can verify our guess by research; Venus' escape velocity is about 10360 m/s. [4] So, we have that $\frac{1}{6}10360 \approx 1727$ m/s, which does indeed fall into the range [847,2397].

[3] `https://science.nasa.gov/venus/venus-facts/#h-size-and-distance`.
[4] `https://nssdc.gsfc.nasa.gov/planetary/factsheet/venusfact.html`.