# Math 136 Homework 6

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1. 

   **Problem.** *Let $A$ be a matrix such that $A^*A$ is invertible. With $P_{Im\,A}$ given by $A(A^*A)^{-1}A^*$, find $P_{ker\,A}$, $P_{ker\,A^*}$, and $P_{Im\,A^*}$.*

2. Let $\langle \cdot , \cdot \rangle$ be a positive definite inner product on the vector space $V$. Let $L : V \longmapsto V$ be a linear transformation that satisfies the condition,

   $$\langle u, L(v)\rangle = \langle L(u), v\rangle \, \forall u, v \in V.$$

   Such an operator is said to be *self-adjoint*.

   Let $v_\lambda$ and $v_\mu$ be eigenvectors associated to the eigenvalues $\lambda$ and $\mu$ of $L$, with $\lambda \neq \mu$.

   Show that $v_\lambda \perp v_\mu$

3. Let $V$ ck the vector space of continuous functions on the close interval $[-1, 1]$, with scalar product defined by

   $$\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)\,dx.$$

   (a) Apply the Gram-Schmidt orthogonalization process to the set $\left\{1, x, x^2, x^3\right\}$ to obtain an orthogonal set of four polynomials, $\{p_0(x), p_1(x), p_2(x), p_3(x)\}$.

   (b) Verify that $p_k$ is a solution of the differential equation

   $$(1 - x^2)y'' - 2xy' + \lambda y = 0, \text{ with } \lambda = k(k + 1)$$

   for $k = 1, 2, 3, 4$.