Math 135 Homework 5

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1. The equation $y' + P(x)y = Q(x)y^n$ is called the Bernoulli equation. It becomes a linear equation after the change of variable $y^{1-n} = z$. Soylve the equation $y(6y^2 - x - 1)\,dx + 2x\,dy = 0$ using this idea.

2. Let $I = [0,1]$ and $Y_n(t) = t^n$. Show that the sequence (Y_n) is not Cauchy by computing $||Y_n - Y_m||$.

3. Let $I = [-\pi, \pi]$, and considen the function $f_0 : I \to \mathbb{R}$ defined by $f_0(t) = e^t$. Let $f_n, n \in \mathbb{Z}^+$ be the sequence of functions on I defined inductively by the formula

$$f_{n+1}(t) = \cos(t) + \frac{1}{2}\sin(t)f_n(t).$$

So $f_1(t) = \cos(t) + \frac{1}{2}\sin(t)e^t, \quad f_2(t) = \cos(t) + \frac{1}{2}\sin(t)\left(\cos(t) + \frac{1}{2}\sin(t)e^t\right)$, etc.

Show that f_n converges uniformly to a continuous function and find the limit $\lim_{n\to\infty} f_n$.