Some Visualizations

Homotopy Type Theory

pictures are good

Types vs. Sets

Sets

Sets contain elements.

Elements belong to sets via membership ($\in$).

Sets may mix different kinds of elements.

Sets are built from other sets.

Types vs. Sets

Sets	Types
Sets contain elements.	Types contain terms.
Elements belong to sets via membership (∈).	Terms inhabit types via typing (:).
Sets may mix different kinds of elements.	Terms have exactly one type.
Sets are built from other sets.	Terms are built with constructors.

Common Types

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empty	$\mathbb{0}$	
unit	$\mathbb{1}$	$\star$
naturals	$\mathbb{N}$	$0, 1, 2, \ldots$

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Identity types may have no terms, one term, or even many!

Curry Howard

$$\text{Types} \longleftrightarrow \text{Propositions}$$

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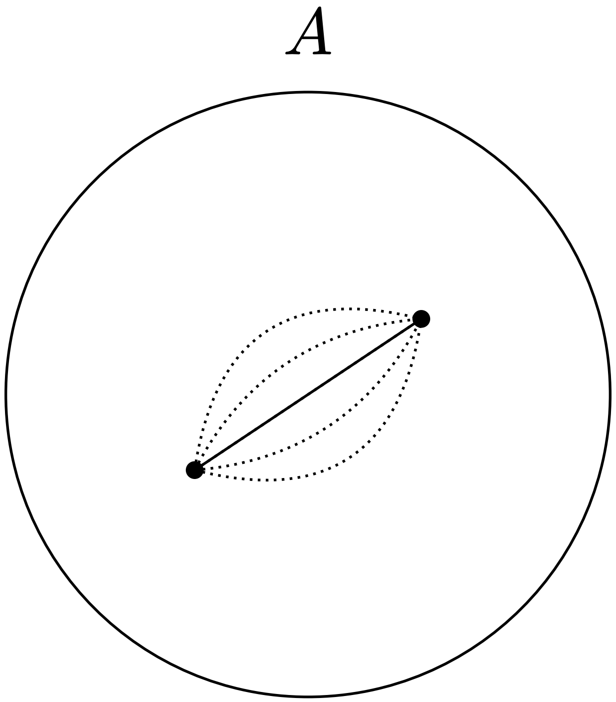
$\therefore$ Constructing a term of a type is the same as proving a proposition.

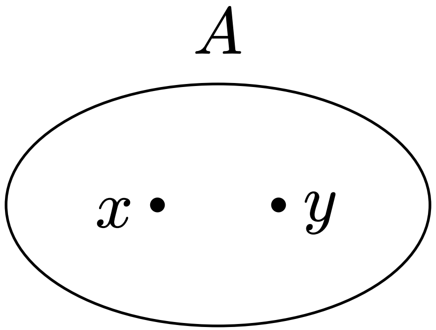
Homotopy



Homotopy Type Theory

$$\text{Types} \sim \text{Spaces}$$

$$\text{Terms} \sim \text{Points in space}$$



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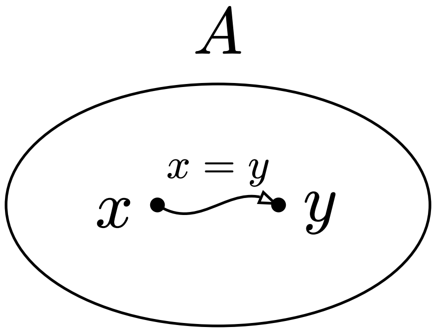
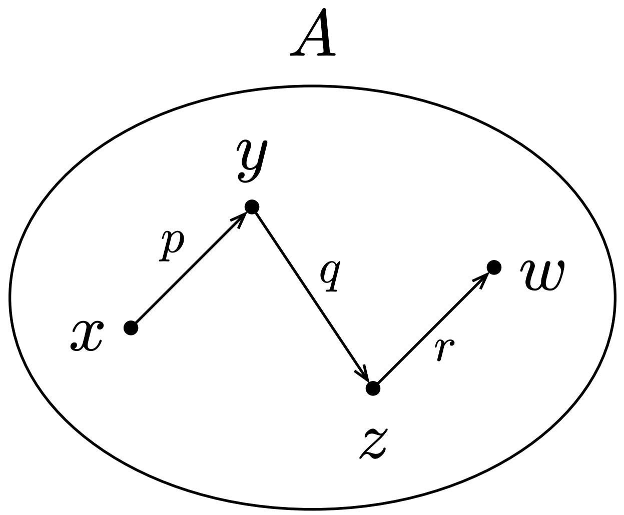


Terms of the identity type $x = y$ are paths from x to y.

Concatenate Paths

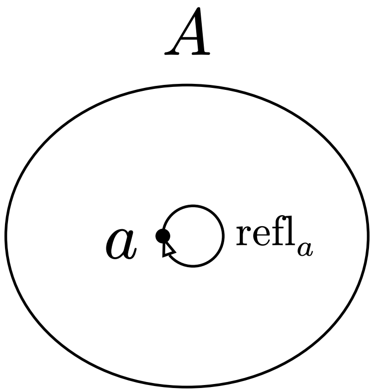


$$x \overset{p}{=\!=\!=} y \overset{q}{=\!=\!=} z \overset{r}{=\!=\!=} w$$

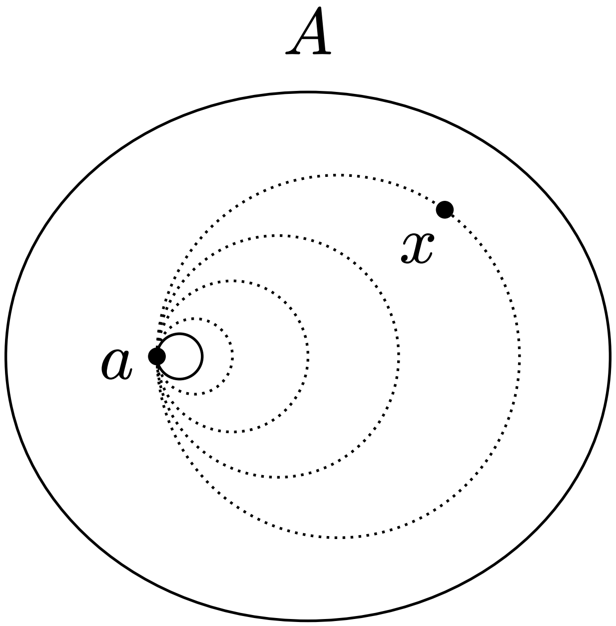
Identity Type

The identity type has one constructor:

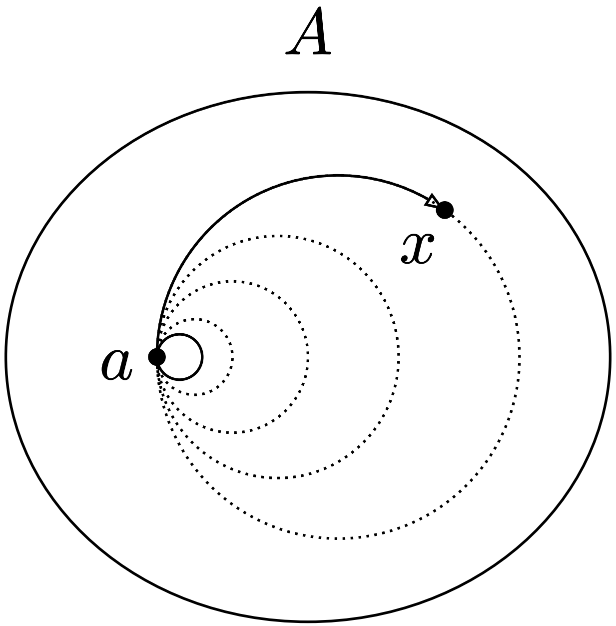
$$\mathrm{refl}_a : a =_A a \quad \text{where} \quad a : A.$$



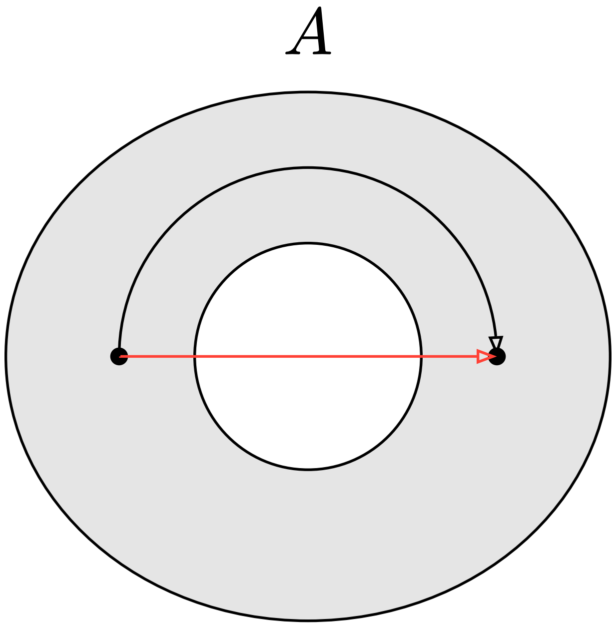
Path Induction



Path Induction

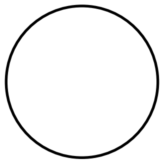


Simply Connectedness



Action on Paths

Sections & Retractions



Fibers

Transports

Truncation Levels

Separate types by their equalities.

-2. Contractible $\cong \mathbb{1}$

-1. Propositions $\cong \mathrm{Eq}_{\mathbb{N}}$

0. Sets $\cong \mathbb{N}$

$\vdots$