CSC 240 Winter 2024 Quiz 4

Give a well-structured informal proof by induction that, for all natural numbers n and for all integers $m \geq 2$,

$$\sum_{k=0}^{n} m^k = \frac{m^{n+1}-1}{m-1}.$$

Proof by induction.

Let $P(n) =$ " $\displaystyle\sum_{k=0}^{n} m^k = \frac{m^{n+1}-1}{m-1}$ ".

Consider the base case where $n = 0$. Then, we can see $\displaystyle\sum_{k=0}^{n} m^k = m^n = \sum_{k=0}^{0} m^k = m^0 = 1 = \frac{m^{0+1}-1}{m-1} = \frac{m^{n+1}-1}{m-1}$, this shows $P(0)$ holds;

Now, pick an arbitrary $n \in \mathbb{N}$;

Assume $P(n)$ holds, i.e., $\displaystyle\sum_{k=0}^{n} m^k = \frac{m^{n+1}-1}{m-1}$;

Then, we can see $\displaystyle\sum_{k=0}^{n+1} m^k = \sum_{k=0}^{n} m^k + m^{n+1} = \frac{m^{n+1}-1}{m-1} + m^{n+1} = \frac{m^{n+1}-1+m^{n+1}(m-1)}{m-1} = \frac{m^{n+1}-1+m^{n+2}-m^{n+1}}{m-1} = \frac{m^{n+2}-1}{m-1} = \frac{m^{(n+1)+1}-1}{m-1}$, we can see $P(n+1)$ also holds;

By induction, we can conclude that $P(n)$ holds for all $n \in \mathbb{N}$, this completes the proof. $\square$