For any language $S \subseteq \Sigma^*$, define $C(S) = \{x \in \Sigma^* \mid \exists w \in \Sigma^*.y \in \Sigma^*.(xwxy \in S)\}$.

For example, if $S = \{ababc, aabaab\}$, then $C(S) = \{\lambda, a, aa, ab, aab\}$.

Question 1. Describe the language $S = L(((01)^* + 1^*)^*) = \{z \in \{0,1\}^* \mid \dots\}$ by replacing the $\dots$ with at most 10 words. (z counts as one word.) Briefly justify your answer.

Question 2. Describe the language $T = L\left(\overline{\overline{\phi} \cdot 00 \cdot \overline{\phi}}\right) = \{x \in \{0,1\}^* \mid \dots\}$ by replacing the $\dots$ with at most 10 words. (x counts as one word.) Briefly justify your answer.

Question 3. Explain why $C(S) = T$.

Question 4. Give any deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$, construct a finite automaton $M' = (Q', \Sigma, \delta', q_0', F')$ such that $L(M') = C(L(M))$.

Question 5. briefly describe how M' works.

Question 6. Prove that $L(M') = C(L(M))$.