Unit 1: Predicate and propositional logic

Terminologies

- **Conjunctive Normal Form (CNF)** = a *conjunction* of *disjunctions* of *literals*
- **Disjunctive Normal Form (DNF)** = a *disjunction* of *conjunctions* of *literals*
- **Interpretation** = a case with non-empty domain; free variables with assigned domain elements; function from relevant domain to range
- **Predicate** = *proposition* whose truth depends on *variables* / function with range {*T*,*F*}
- **Predicate Logic Formula** = *predicates* (fixed number of arguments) + *connectives* + *quantifiers*
- **Prenex Normal Form** = [some quantifications]+[formula without quantifiers]
- **Proposition** = statement either *true* or *false*
- **Propositional Formula** = expression built from *Boolean variables* using *connectives* with **no** *predicates* or *quantifiers*
- **Satisfiability Problem (SAT)** = output *YES* if the *propositional formula* is *satisfiable*, vice versa
- **Truth Assignment** = function from a set of *propositional variables* to {*T*,*F*}; a row of the truth table

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Trivial terms

- Connectives = negation, conjunction, disjunction, exclusive-or, implication, equivalence
- Universal / Existential quantification
- Boolean variable = variables that are either True of False (does not depend on other variables like predicate does)
- Truth table = n variables truth table has 2^n rows

- Tautology / Valid = **propositional formula** which all entries are *True* / **predicate logic formula** which all *interpretations* are true
- Unsatisfiable / Contradiction = **propositional formula** which all entries are *False* / **predicate logic formula** is false for all *interpretation*
- Satisfiable = *propositional formula* which at least one entry is *True* / **predicate logic formula** is true for some *interpretation*

- **P** = all decision problems can be solved in polynomial time
- **NP** = all decision problems can be verified in polynomial time; SAT $\in$ NP
- Literal = variable or the negation of it
- Clause = disjunction of literals
- CNF-SAT = *SAT* but *propositional formula* in *CNF*

- Constant = a particular element in a domain
- Variable = any element in a domain
- quantified = variable with quantifier
- unquantified / free = variables that are not quantified
- valuation = maps free variable to domain element
- logically implies (equivalent) = P's interpretation is True =¿ Q is also True

Hint.

1) Quantifiers are flipped with Negation / Hypothesis of implication
2) Order of quantifiers does matter
3) Every propositional formula is equivalent to a *DNF / CNF*
4) To construct a DNF form, for each line with output T, conjuct the variables with appropriate negations, then disjunct the lines.
5) To construct a CNF form, for each line with output F, conjuct the variables with appropriate negations, disjunt them, negate the entire statement.
6) Do not repeat symbols for variables and constants.

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Unit 2: Proof

Terminologies

- **Proposition** = statement either true or false
- **Axiom** = *proposition* that we agree is true
- **Proof** = convincing argument that a *proposition* is true. [Sequence of axioms] + [Proved propositions] + [logical deductions]
- **Logical Deduction** = use inference rules to prove new propositions from *axioms* and *proved propositions*.

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Substitution

1) R is a tautology containing variable P
   R' := replace **every** P in R by (Q)
   Then, R' is a tautology.
2) S' is a formula logically equivalent to S
   S is a subformula of R
   R' := replace **some** occurences of S in R by S'
   Then, R' is still a tautology.

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[Insert Proof Outline here] ☺

To write a formal proof:

1) number each line
2) Write one proposition each line
3) justify each line