

## Terminologies

- **Conjunctive Normal Form (CNF)** = a *conjunction* of *disjunctions* of *literals*
- **Disjunctive Normal Form (DNF)** = a *disjunction* of *conjunctions* of *literals*
- **Interpretation** = a case with non-empty domain; free variables with assigned domain elements; function from relevant domain to range
- **Predicate** = *proposition* whose truth depends on *variables* / function with range  $\{T, F\}$
- **Predicate Logic Formula** = *predicates* (fixed number of arguments) + *connectives* + *quantifiers*
- **Prenex Normal Form** = [some quantifications]+[formula without quantifiers]
- **Proposition** = statement either *true* or *false*
- **Propositional Formula** = expression built from *Boolean variables* using *connectives* with **no predicates** or *quantifiers*
- **Satisfiability Problem (SAT)** = output *YES* if the *propositional formula* is *satisfiable*, vice versa
- **Truth Assignment** = function from a set of *propositional variables* to  $\{T, F\}$ ; a row of the truth table



## Trivial terms

- Connectives = negation, conjunction, disjunction, exclusive-or, implication, equivalence
- Universal / Existential quantification
- Boolean variable = variables that are either True or False (does not depend on other variables like predicate does)
- Truth table =  $n$  variables truth table has  $2^n$  rows
- Tautology / Valid = **propositional formula** which all entries are *True* / **predicate logic formula** which all *interpretations* are true
- Unsatisfiable / Contradiction = **propositional formula** which all entries are *False* / **predicate logic formula** is false for all *interpretation*
- Satisfiable = *propositional formula* which at least one entry is *True* / **predicate logic formula** is true for some *interpretation*
- **P** = all decision problems can be solved in polynomial time
- **NP** = all decision problems can be verified in polynomial time;  $SAT \in NP$
- Literal = variable or the negation of it
- Clause = disjunction of literals
- CNF-SAT = SAT but *propositional formula* in CNF
- Constant = a particular element in a domain
- Variable = any element in a domain
- quantified = variable with quantifier
- unquantified / free = variables that are not quantified
- valuation = maps free variable to domain element
- logically implies (equivalent) = P's interpretation is True  $\Rightarrow$  Q is also True

### Subtle Details.

- 1) Quantifiers are flipped with Negation / Hypothesis of implication
- 2) Order of quantifiers does matter
- 3) Every propositional formula is equivalent to a *DNF* / *CNF*
- 4) To construct a DNF form, for each line with output T, conjunct the variables with appropriate negations, then disjunct the lines.
- 5) To construct a CNF form, for each line with output F, conjunct the variables with appropriate negations, disjunct them, negate the entire statement.
- 6) Do not repeat symbols for variables and constants.



## Unit 2: Proof

### Terminologies

- **Proposition** = statement either true or false
- **Axiom** = *proposition* that we agree is true
- **Proof** = convincing argument that a *proposition* is true. [Sequence of axioms] + [Proved propositions] + [logical deductions]
- **Logical Deduction** = use inference rules to prove new propositions from *axioms* and *proved propositions*.



### Substitution

- 1) R is a tautology containing variable P  
 $R' := \text{replace every } P \text{ in } R \text{ by } (Q)$   
 Then,  $R'$  is a tautology.
- 2)  $S'$  is a formula logically equivalent to S  
 $S$  is a subformula of R  
 $R' := \text{replace some occurrences of } S \text{ in } R \text{ by } S'$   
 Then,  $R'$  is still a tautology.



[Insert Proof Outline here]



### To write a formal proof:

- 1) number each line
- 2) Write one proposition each line
- 3) justify each line

Subtle Details.

