Unit 1: Predicate and propositional logic

Terminologies

- Conjunctive Normal Form (CNF) = a conjunction of disjunctions of literals
- Disjunctive Normal Form (DNF) = a disjunction of conjunctions of literals
- **Interpretation** = a case with non-empty domain; free variables with assigned domain elements; function from relevant domain to range
- **Predicate** = proposition whose truth depends on variables / function with range $\{T,F\}$
- **Predicate Logic Formula** = predicates (fixed number of arguments) + connectives + quantifiers
- Prenex Normal Form = [some quantifications]+[formula without quantifiers]
- **Proposition** = statement either *true* or *false*
- Propositional Formula = expression built from Boolean variables using connectives with no predicates or quantifiers
- Satisfiability Problem (SAT) = output YES if the propositional formula is satisfiable, vice versa
- Truth Assignment = function from a set of propositional variables to $\{T,F\}$; a row of the truth table

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Trivial terms

- Connectives = negation, conjunction, disjunction, exclusive-or, implication, equivalence
- Universal / Existential quantification
- Boolean variable = variables that are either True of False (does not depend on other variables like predicate does)
- Truth table = n variables truth table has 2^n rows
- Tautology / Valid = **propositional formula** which all entries are *True* / **predicate logic formula** which all *interpretations* are true
- Unsatisfiable / Contradiction = **propositional formula** which all entries are False / **predicate logic formula** is false for all interpretation
- Satisfiable = propositional formula which at least one entry is True / predicate logic formula is true for some interpretation
- \bullet **P** = all decision problems can be solved in polynomial time
- \bullet ${\bf NP}$ = all decision problems can be verified in polynomial time; SAT \in NP
- Literal = variable or the negation of it
- ullet Clause = disjunction of literals
- \bullet CNF-SAT = SAT but propositional formula in CNF
- Constant = a particular element in a domain
- Variable = any element in a domain
- quantified = variable with quantifier
- \bullet unquantified / free = variables that are not quantified
- valuation = maps free variable to domain element
- \bullet logically implies (equivalent) = P's interpretation is True \Rightarrow Q is also True

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🧐 Definition Sheet 🤒

Hint.

- 1) Quantifiers are flipped with Negation / Hypothesis of implication
- 2) Order of quantifiers does matter
- 3) Every propositional formula is equivalent to a DNF / CNF
- 4) To construct a DNF form, for each line with output T, conjuct the variables with appropriate negations, then disjunct the lines.
- 5) To construct a CNF form, for each line with output F, conjuct the variables with appropriate negations, disjunt them, negate the entire statement.
- 6) Do not repeat symbols for variables and constants.



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Unit 2: Proof

Terminologies

- **Proposition** = statement either true or false
- Axiom = proposition that we agree is true
- **Proof** = convincing argument that a *proposition* is true. [Sequence of axioms] + [Proved propositions] + [logical deductions]
- **Logical Deduction** = use inference rules to prove new propositions from *axioms* and *proved propositions*.

Substitution

- 1) R is a tautology containing variable P
 - $R' := \text{replace } \mathbf{every} P \text{ in } R \text{ by } (Q)$

Then, R' is a tautology.

2) S' is a formula logically equivalent to S

S is a subformula of R

R' := replace some occurrences of S in R by S'

Then, R' is still a tautology.

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[Insert Proof Outline here]



To write a formal proof:

- 1) number each line
- 2) Write one proposition each line
- 3) justify each line