Change of basis: If say $A$ is a linear transformation w.r.t. the standard basis, and $\beta$ is a basis, then let $P$ be the matrix with columns as the basis vectors of $\beta$, we have $[A]_\beta = P^{-1}AP$.
Diagonalizable: if $A$ is similar to a diagonal matrix (with eigenvalues on the diagonal).

Direct Sum: $V = V_1 \oplus ... \oplus V_r$ if $v \in V$ can be uniquely written as $v = v_1 + ... + v_r$ where $v_i \in V_i$.
Sum is direct if and ony if $U_i \cap (\sum_{j \neq i} U_j) = \{0\}$ for all $i$.

$T$-invariant: $T$ is $V$-invariant if $T(V) \subseteq V$.
$p_T(z)$ is divisible by $p_{T|_W}(z)$, for $T$-invariant subspace $W$.
Diagonalizable if and only if $V$ has a basis of eigenvectors of $T$, if and only if it is a direct sum of eigenspaces of $T$.
$\ker(T) = (\ker(T) \cap V_1) \oplus ... \oplus (\ker(T) \cap V_r)$
$\operatorname{ran}(T) = (\operatorname{ran}(T) \cap V_1) \oplus ... \oplus (\operatorname{ran}(T) \cap V_r)$

$v \in V$ cyclic if $W = \operatorname{span}\{v, T(v), T^2(v), ...\} = V$.
Let $W$ be cyclic subspace generated by $v$. Exists $k$ s.t. $v, T(v), ..., T^{k-1}(v)$ is basis of $W$.

Given $p(z) = a_0 + ... + z^n$,

$$A = \begin{pmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & \cdots & 0 & -a_1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & -a_{n-1} \end{pmatrix}$$

is the companion matrix of $p(z)$.
$p_A(z) = \det(zI - A) = p(z)$.
Matrix admits cyclic vector if and only if similar to a companion matrix.

Largest $T$-cyclic subspace admits $T$-invariant complement.
Finite dim $V$ can be decomposed into direct sum of $T$-cyclic subspaces by above.

Jordan block of size $k$ with type $\lambda$:

$$J(\lambda, k) = \begin{pmatrix} \lambda & 1 & 0 & \cdots & 0 \\ 0 & \lambda & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & \lambda \end{pmatrix} \in M_{k \times k}(\mathbb{F})$$

$J(\lambda, k) = \lambda I + J(0, k)$.
Jordan Canonical Form: Matrix formed by Jordan blocks.
Admits JCF if exists basis $\beta$ s.t. $[T]_\beta$ is in JCF.
Generalized eigenspace $K(\lambda, T) = \{v \in V \mid \exists k \in \mathbb{N} : (T - \lambda I)^k v = 0\}$.
$V = U \oplus K(\lambda_1, T) \oplus ... \oplus K(\lambda_r, T)$, where $U$ is $T$-invariant with no eigenvalues.
$N$ nilpotent if $N^k = 0$ for some $k$.
Nilpotent $N$ admits JCF with $J(0, k)$.