

Exercise 1: Trivial Exercise

This is an example document.

Question 1

$$1 + 1 \stackrel{?}{=} 3.$$

?

Hint. Consider  $1 + 1 = 2$ .

♥

Claim 1

Yes.

◇

*Proof of Claim 1.* This is a good question.

QUOD  
ERAT  
DEM■

*Proof by Triviality.* Trivial.

QUOD  
ERAT  
DEM■

Remark 1. This is very trivial.

△

Theorem 1 – HelloWorld

This is NOT a theorem.

Definition 1 – Byebye

This is DEFINITELY a theorem.

Chapter 1: Non-Trivial Chapter

Recall that we have already explained in lecture that all rational function of a single variable can be integrated in finite terms. Starting from there, we introduce integrals of the form, known as the **binomial integrals**:

(1) 
$$J_{p,q} = \int (a + bz)^p z^q \, dz, a, b \in \mathbb{R}, p, q \in \mathbb{Q}.$$

This exercise aims at studying the rationalization of the binomial integral, as well as some of its applications.

**Question 1**

Assume that  $p \in \mathbb{Z}$ , rationalise the integrand.

?

Since  $q \in \mathbb{Q}$ , this implies there exists  $m \in \mathbb{Z}, n \in \mathbb{N}$  such that  $q = \frac{m}{n}$ . Then,  $J_{p,q}$  can be written as

$$J_{p,q} = \int (a + bz)^p z^{\frac{m}{n}} dz.$$

Let  $z = x^n$ , then  $dz = n \cdot x^{n-1} dx$ . Replace  $z$  with  $x$  we have

$$J_{p,q} = \int (a + b \cdot x^n)^p \cdot (x^n)^{\frac{m}{n}} \cdot n \cdot x^{n-1} dx.$$

Simplify it and we get

$$J_{p,q} = n \int (a + b \cdot x^n)^p \cdot x^{m+n-1} dx.$$

Now, by binomial theorem we can see the integrand is rationalised, as needed.

**Lemma 1**

This is a lemma.

This is a math environment.

**Corollary 1**

This is a corollary.