## Exercise 1: Trivial Exercise

This is an example document.

### Question 1

$$1 + 1 \stackrel{?}{=} 3.$$

?

**Hint.** Consider $1 + 1 = 2$.

♡

**Claim 1**

Yes. ◊

*Proof of Claim 1.* This is a good question. QUOD ERAT DEM■

*Proof by Triviality.* Trivial. QUOD ERAT DEM■

**Remark 1.** This is very trivial.

△

**Theorem 1 – HelloWorld**

This is NOT a theorem.

**Definition 1 – Byebye**

This is DEFINITELY a theorem.

## Chapter 1: Non-Trivial Chapter

Recall that we have already explained in lecture that all rational function of a single variable can be integrated in finite terms. Starting from there, we introduce integrals of the form, known as the **binomial integrals**:

$$J_{p,q} = \int (a + bz)^p z^q \, \mathrm{d}z, a, b \in \mathbb{R}, p, q \in \mathbb{Q}. \tag{1}$$

This exercise aims at studying the rationalization of the binomial integral, as well as some of its applications.

**Question 1**

Assume that $p \in \mathbb{Z}$, rationalise the integrand. ?

Since $q \in \mathbb{Q}$, this implies there exists $m \in \mathbb{Z}, n \in \mathbb{N}$ such that $q = \frac{m}{n}$. Then, $J_{p,q}$ can be written as

$$J_{p,q} = \int (a + bz)^p z^{\frac{m}{n}} \, \mathrm{d}z.$$

Let $z = x^n$, then $\mathrm{d}z = n \cdot x^{n-1} \, \mathrm{d}x$. Replace $z$ with $x$ we have

$$J_{p,q} = \int (a + b \cdot x^n)^p \cdot (x^n)^{\frac{m}{n}} \cdot n \cdot x^{n-1} \, \mathrm{d}x.$$

Simplify it and we get

$$J_{p,q} = n \int (a + b \cdot x^n)^p \cdot x^{m+n-1} \, \mathrm{d}x.$$

Now, by binomial theorem we can see the integrand is rationalised, as needed.