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MAT157: Analysis I
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Homework 11

Exercise 1

Let $f(x)$ be a funciton defined near 0 and $\lim_{x \to 0} f(x) = 0$.

Question 1. Prove that if $g(x) = o(\mathcal{O}(f(x)))$, then $g(x) = o(f(x))$.
[2] Prove that if $g(x) = \mathcal{O}(o((f(x)))$, then $g(x) = o(f(x))$.

EXERCISE 2

Let the angle $\angle AOB = x$. Find $n \in \mathbb{N}$ so that the following quantity $g(x)$ satisfies that $g(x) = \mathcal{O}(x^n)$ and $x^n = \mathcal{O}(g(x))$.

Question 1. The chord length $|AB|$.

Question 2. The arch height $|CD|$.

Question 3. Area of the sector AOB.

Question 4. Area of the triangle $\triangle ACB$.

<div align="center">EXERCISE 3</div>

Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = e^{x^2 + \frac{\sin(x)}{1+x^2}}$$

Question 1. Compute the approximation of the value $f(1.001)$ by using linear approximation.

Question 2. Now suppose that you need to ensure the tolerance of error is less or equal to the scale of 10^{-17}. Normally speaking, how many terms in the Taylor expansion approximation do you need, given that in our scenario $\Delta x = 0.001$?