

1. D'Alembert's Ratio Test:

Assume that $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = k$

- If $k > 1$, then $\sum_{n=1}^{\infty} a_n = \infty$.
- If $k < 1$, then $\sum_{n=1}^{\infty} a_n < \infty$.

2. Cauchy's Root Test:

Assume that $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = k$

- If $k > 1$, then $\sum_{n=1}^{\infty} a_n = \infty$.
- If $k < 1$, then $\sum_{n=1}^{\infty} a_n < \infty$.

3. Raabe's Test:

Let $\sum_{n=1}^{\infty} a_n$ be a positive series.

Assume that $K_n := n \left(\frac{a_n}{a_{n+1}} - 1 \right)$, and $\lim_{n \rightarrow \infty} K_n = K$.

Then

- If $K > 1$, then $\sum_{n=1}^{\infty} a_n < \infty$.
- If $K < 1$, then $\sum_{n=1}^{\infty} a_n = \infty$.

4. Kummer's Test:

Given a series $\sum_{n=1}^{\infty} a_n$.

Let c_n be positive such that $\sum_{n=1}^{\infty} \frac{1}{c_n} = \infty$.

Define $K_n := c_n \left(\frac{a_n}{a_{n+1}} \right) - c_{n+1}$.

Assume that $\lim_{n \rightarrow \infty} K_n = K$.

Then

- If $K > 0$, then $\sum_{n=1}^{\infty} a_n < \infty$.
- If $K < 0$, then $\sum_{n=1}^{\infty} a_n = \infty$.