ESERCIZIO 1: PROBLEMA GUARDAROBIERE DISTRATTO

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Svolgimento

N = 4 persone, Casi: $N^N = 4^4 = 256$, dei quali considero solo $N! = 4! = 24$ che hanno probabilità non nulla

Persona -> Cappello   (persone che hanno ricevuto indietro il loro cappello)

1->1 , 2->2 , 3->3 , 4->4 (4 persone)

1->1 , 2->2 , 3->4 , 4->3 (2 persone)

1->1 , 2->4 , 3->3 , 4->2 (2 persone)

1->1 , 2->3 , 3->2 , 4->4 (2 persone)

1->4 , 2->2 , 3->3 , 4->1 (2 persone)

1->3 , 2->2 , 3->1 , 4->4 (2 persone)

1->2 , 2->1 , 3->3 , 4->4 (2 persone)

1->1 , 2->3 , 3->4 , 4->2 (1 persone)

1->1 , 2->4 , 3->2 , 4->3 (1 persone)

1->4 , 2->2 , 3->1 , 4->3 (1 persone)

1->3 , 2->2 , 3->4 , 4->1 (1 persone)

1->4 , 2->1 , 3->3 , 4->2 (1 persone)

1->2 , 2->4 , 3->3 , 4->1 (1 persone)

1->2 , 2->3 , 3->1 , 4->4 (1 persone)

1->3 , 2->1 , 3->2 , 4->4 (1 persone)

1->2 , 2->1 , 3->4 , 4->3 (0 persone)

1->2 , 2->3 , 3->4 , 4->1 (0 persone)

1->2 , 2->4 , 3->1 , 4->3 (0 persone)

1->3 , 2->1 , 3->4 , 4->2 (0 persone)

1->3 , 2->4 , 3->1, 4->2 (0 persone)

1->3 , 2->4 , 3->2 , 4->1 (0 persone)

1->4 , 2->3 , 3->2 , 4->1 (0 persone)

1->4 , 2->3 , 3->1 , 4->2 (0 persone)

1->4 , 2->1 , 3->2 , 4->3 (0 persone)

Probabilità congiunta

Poiché

p(1,2,3,4) + p(1,2,4,3) + p(1,4,3,2) + p(1,3,2,4) + p(4,2,3,1) + p(3,2,1,4) + p(2,1,3,4) + p(1,3,4,2) +
+ p(1,4,2,3) + p(4,2,1,3) + p(3,2,4,1) + p(4,1,3,2) + p(2,4,3,1) + p(2,3,1,4) + p(3,1,2,4) + p(2,1,4,3)+
+ p(2,3,4,1) + p(2,4,1,3) + p(3,1,4,2) + p(3,4,1,2) + p(3,4,2,1) + p(4,3,2,1) + p(4,3,1,2) + p(4,1,2,3)=
=1

Calcolo valor medio utilizzando probabilità congiunta

4 * p(1,2,3,4) + 2 * p(1,2,4,3) + 2 * p(1,4,3,2) + 2 * p(1,3,2,4) + 2 * p(4,2,3,1) + 2 * p(3,2,1,4) +
+ 2 * p(2,1,3,4) + 1 * p(1,3,4,2) + 1 * p(1,4,2,3) + 1 * p(4,2,1,3) + 1 * p(3,2,4,1) + 1 * p(4,1,3,2) +
+ 1 * p(2,4,3,1) + 1 * p(2,3,1,4) + 1 * p(3,1,2,4) + 0 * p(2,1,4,3) + 0 * p(2,3,4,1) + 0 * p(2,4,1,3) +
+ 0 * p(3,1,4,2) + 0 * p(3,4,1,2) + 0 * p(3,4,2,1) + 0 * p(4,3,2,1) + 0 * p(4,3,1,2) + 0 * p(4,1,2,3) =
= 4*1/24 + 2*1/24 + 2*1/24 + 2*1/24 + 2*1/24 + 2*1/24 + 2*1/24 + 1*1/24 + 1*1/24 + 1*1/24 +
+ 1*1/24 + 1*1/24 + 1*1/24 + 1*1/24 + 1*1/24 = 1/6 + 1/12 + 1/12 + 1/12 + 1/12 + 1/12 + 1/12 +
+ 1/24 + 1/24 + 1/24 + 1/24 + 1/24 + 1/24 + 1/24 +1/24 = 1/6 + 1/2 + 1/3 = 1

Probabilità marginali

$p_1(1)$ = p(1,2,3,4) + p(1,2,4,3) + p(1,4,3,2) + p(1,3,2,4) + p(1,3,4,2) + p(1,4,2,3) = 1/24 + 1/24 +
+ 1/24 + 1/24 + 1/24 + 1/24 = 1/4

$p_2(2)$ = p(1,2,3,4) + p(1,2,4,3) + p(4,2,3,1) + p(3,2,4,1) + p(4,2,1,3) + p(3,2,4,1) = 1/24 + 1/24 +
+ 1/24 + 1/24 + 1/24 + 1/24 = 1/4

$p_3(3)$ = p(1,2,3,4) + p(1,4,3,2) + p(4,2,3,1) + p(2,1,3,4) + p(4,1,3,2) + p(2,4,3,1) = 1/24 + 1/24 +
+ 1/24 + 1/24 + 1/24 + 1/24 = 1/4

$p_4(4)$ = p(1,2,3,4) + p(1,3,2,4) + p(3,2,1,4) + p(2,1,3,4) + p(2,3,1,4) + p(3,1,2,4) = 1/24 + 1/24 +
+ 1/24 + 1/24 + 1/24 + 1/24 = 1/4

Pertanto, posso fare la somma delle medie marginali

1 * $p_1(1)$ + 0 * $p_1(2)$ + 0 * $p_1(3)$ + 0 * $p_1(4)$ + 0 * $p_2(1)$ + 1 * $p_2(2)$ + 0 * $p_2(3)$ + 0 * $p_2(4)$ + 0 * $p_3(1)$ +
+ 0 * $p_3(2)$ + 1 * $p_3(3)$ + 0 * $p_3(4)$ + 0 * $p_4(1)$ + 0 * $p_4(2)$ + 0 * $p_4(3)$ + 1 * $p_4(4)$ = 1