

Introduction

We consider the Partial Information Decomposition (PID) for neural spiking activity [1], which is commonly assumed to have a Poisson structure. Our contributions include:

- A Poisson channel model for spiking activity
- Analytical solution for the PID under our Poisson model
- Estimates of the PID for generated spiking activity and comparisons with Python DIT package estimates [2]

As an example, we take the mouse experiment in [3] (Figure 2). A single whisker is deflected either a small or large degree by a stimulus M while a population of neurons, Y , in the barrel cortex (S1) and a population Z in the superior colliculus (SC) are recorded.

Background

Bivariate PID decomposes the total information two variables (or vectors) Y and Z carry about a message M into unique (UI), redundant (RI), and synergistic (SI) partial information.

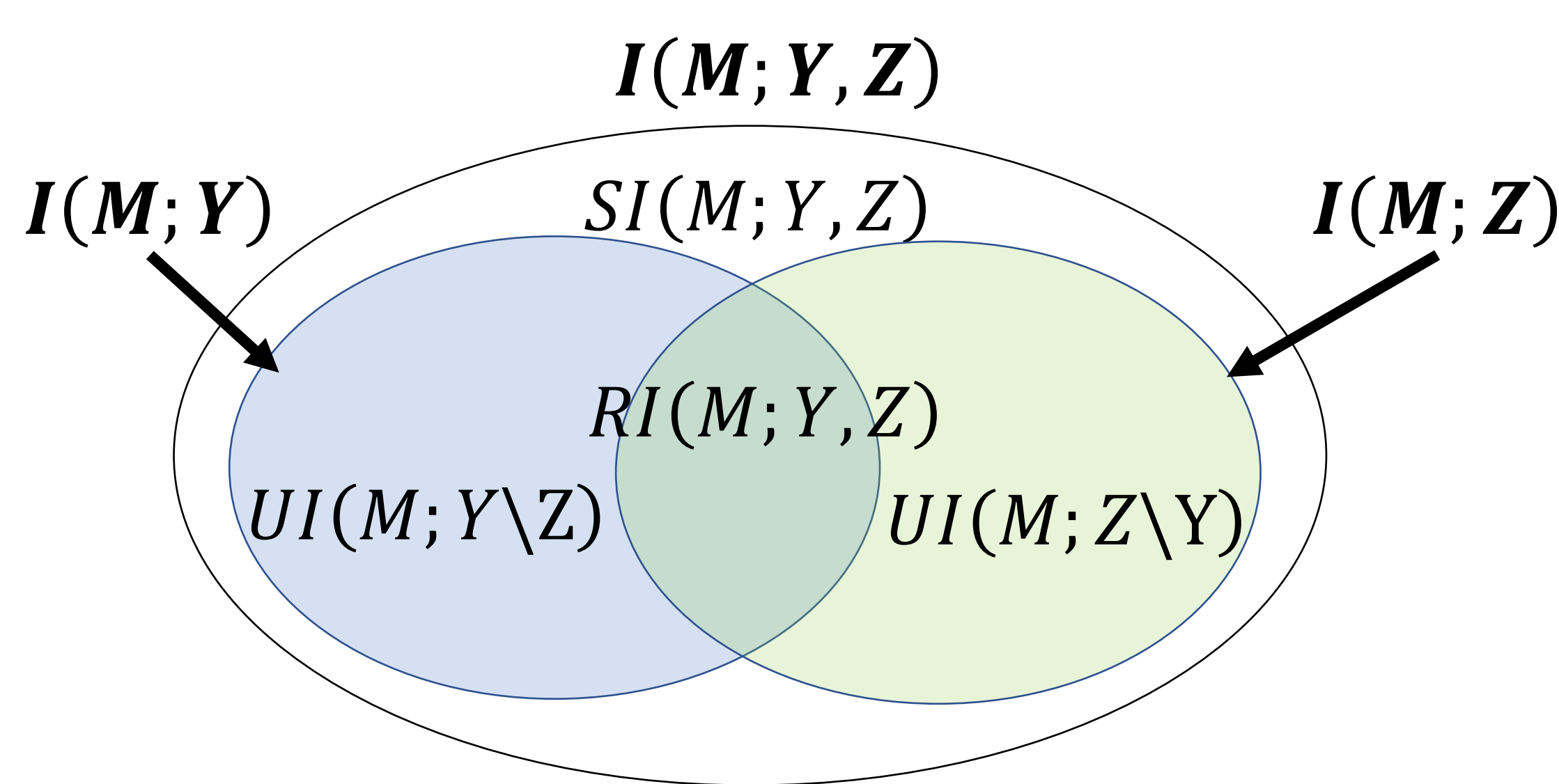


Figure 1: Decomposition of $I(M; Y, Z)$ into four partial information terms. $I(M; Y)$ and $I(M; Z)$ are also decomposed. Adapted from [4].

The four partial information terms (UI for Y and Z , RI, and SI) are related to three Shannon mutual information terms by three equalities inferred from Figure 1. But one more equation is needed to solve for PID. Of many proposed measures for the fourth equality, we select the BROJA-PID measure of unique information [5] because of its operational grounding in decision theory:

$$UI(M; Y|Z) = \min_{Q \in \Delta_P} I_Q(M; Y|Z) \quad (1)$$

Δ_P is the set of all joint distributions of M, Y, Z with marginals $Q(M, Y)$ and $Q(M, Z)$ the same as those of true distribution $P(M, Y, Z)$.

Poisson Channel Model

We assume Y and Z are vectors such that the elements of $Y|M$ and $Z|M$ follow a Poisson distribution and the elements of each vector are jointly independent:

$$P(Y|M) = \begin{bmatrix} \text{Poisson}(\gamma_1^Y f(M)) \\ \vdots \\ \text{Poisson}(\gamma_{d_1}^Y f(M)) \end{bmatrix} \quad f: \mathbb{R} \rightarrow \mathbb{R}^+ \text{ is a well-defined function of } M \text{ and } \gamma_i^Y, \gamma_i^Z \text{ are positive constants. Note that } f(M) \text{ is the same for all elements. Also, bolded is for vectors and unbolded is for variables.}$$

$$P(Z|M) = \begin{bmatrix} \text{Poisson}(\gamma_1^Z f(M)) \\ \vdots \\ \text{Poisson}(\gamma_{d_2}^Z f(M)) \end{bmatrix}$$

Theoretical Result

Theorem: Let M , Y , and Z be defined as in our Poisson channel. If

$$\gamma^Y = \sum_{i=1}^{d_1} \gamma_i^Y \geq \sum_{i=1}^{d_2} \gamma_i^Z = \gamma^Z$$

then $Q^*(M, Y, Z)$, the minimizing distribution of equation (1), is the Markov chain $M - Y - Z$. Thus, $UI(M; Z|Y) = 0$.

In [1], we construct the Markov chain into $Q^*(M, Y, Z)$ so that

$$Q^*(M, Y, Z) = P(M)P(Y|M)Q(Z|Y)$$

Here, $Q(Z|Y)$ is a multinomial distribution. The probability of seeing z_i spikes from neuron Z_i is the ratio γ_i^Z / γ^Z . Typically, $P(M)$ is known from the experiment design. Thus, to obtain the PID, only the rates need to be estimated for the parameters of $P(Y|M)$ and $Q(Z|Y)$, rather than estimating the whole distribution to solve equation (1).

Simulation Result

Inspired by [3], we simulate spiking in two neurons: Y in SC and Z in S1. The spikes are modeled as inhomogeneous Poisson processes but the data are discretized so that each bin is modeled as a Poisson variable, providing a ground truth. We find that our estimates have less error than the DIT package [2] for several configurations of γ^Z / γ^Y .

$$P(M) = \text{Uniform}(1/2)$$

$$f(M, t) = M \cdot \exp(-|t - T_M|^d) + b$$

$$T_M = 100 \text{ ms (stimulus onset)}$$

$$d = 0.25 \text{ (decay rate)}$$

$$b = 0.02 \text{ spikes/ms (baseline)}$$

$$\frac{\gamma^Z}{\gamma^Y} \in \left\{ \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10} \right\}$$

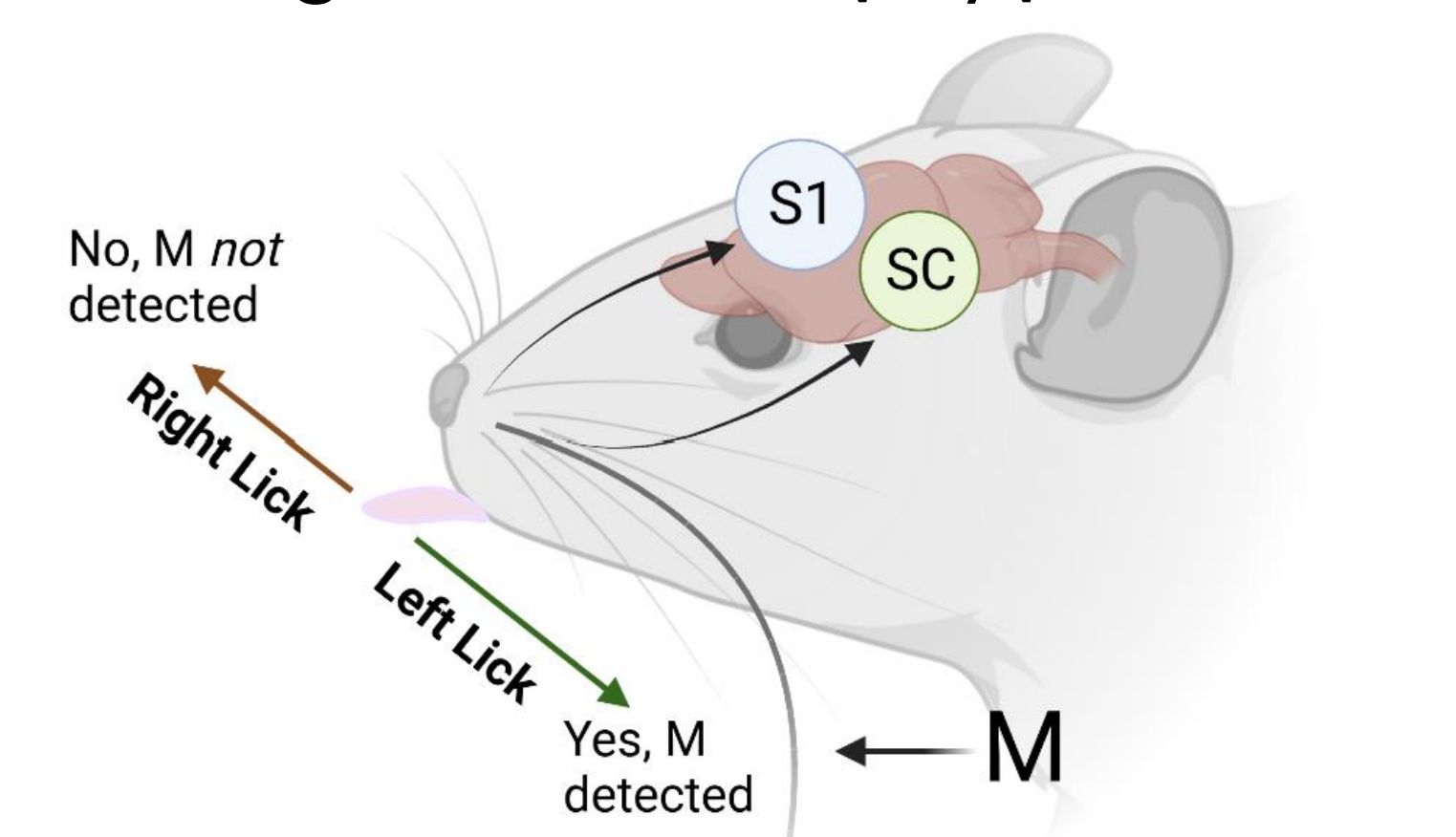
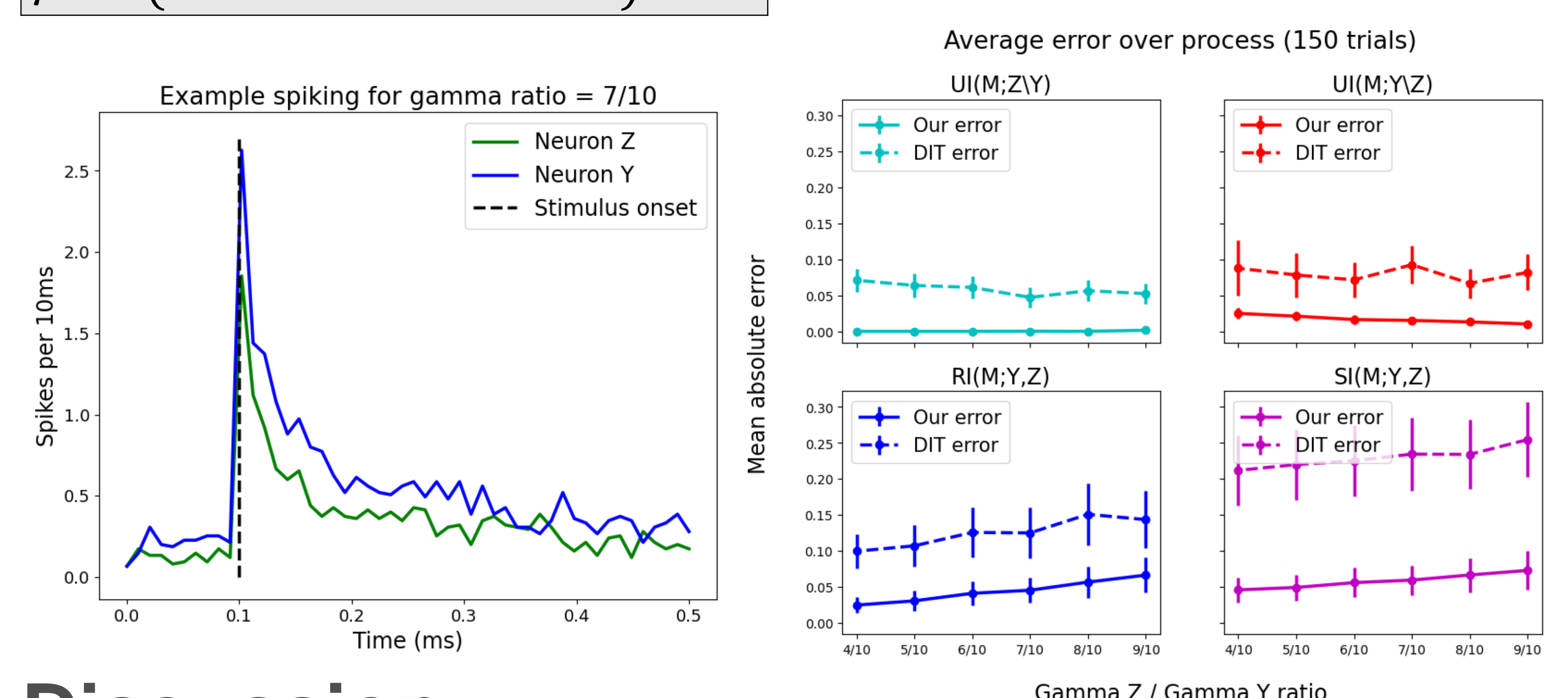


Figure 2: Whisker stimulation experiment [3].



Discussion

Our theoretical results are extended in [1] to dependent multivariate Poisson distributions as well as for a multinomial channel. While our results perform better than [2] for simulated data, the main limitation currently is that $f(M)$, the function of the firing rate on the stimulus, must be the same for both neurons/neural populations. Future work will consider estimation for departures from our assumptions.

References

- [1] "Computing unique information for Poisson and multinomial systems", Arxiv Preprint, C. Goswami, A. Merkle, P. Grover, 2023.
- [2] Discrete Information Theory, Python package, <https://dit.readthedocs.io/en/latest/>
- [3] "Deep and superficial layers of the primary somatosensory cortex are critical for whisker-based texture discrimination in mice", J. M. Park et al., 2022.
- [4] "Nonnegative decomposition of multivariate information", P. L. Williams et al., 2010.
- [5] "Quantifying unique information", N. Bertschinger et al., 2014.

Acknowledgements

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