

$$F(n) = F(n-1) + F(n-2)$$

$$\frac{\varphi^n - \psi^n}{\sqrt{5}} \stackrel{?}{=} \frac{\varphi^{n-1} - \psi^{n-1}}{\sqrt{5}} + \frac{\varphi^{n-2} - \psi^{n-2}}{\sqrt{5}}$$

$$\varphi^n - \psi^n \stackrel{?}{=} \varphi^{n-1} - \psi^{n-1} + \varphi^{n-2} - \psi^{n-2}$$

$$\stackrel{?}{=} \frac{\varphi^n}{\varphi} - \frac{\psi^n}{\psi} + \frac{\varphi^n}{\varphi^2} - \frac{\psi^n}{\psi^2}$$

$$\stackrel{?}{=} \varphi^n \left(\frac{1}{\varphi} + \frac{1}{\varphi^2} \right) - \psi^n \left(\frac{1}{\psi} + \frac{1}{\psi^2} \right)$$

$$\stackrel{?}{=} \varphi^n \left(\frac{2}{\frac{1+\sqrt{5}}{2}} + \frac{4}{\frac{(1+\sqrt{5})^2}{4}} \right) - \psi^n \left(\frac{2}{\frac{1-\sqrt{5}}{2}} + \frac{4}{\frac{(1-\sqrt{5})^2}{4}} \right)$$

$$\stackrel{?}{=} \varphi^n \left(\frac{2+2\sqrt{5}}{(1+\sqrt{5})^2} + \frac{4}{(1+\sqrt{5})^2} \right) - \psi^n \left(\frac{2-2\sqrt{5}}{(1-\sqrt{5})^2} + \frac{4}{(1-\sqrt{5})^2} \right)$$

$$\stackrel{?}{=} \varphi^n \left(\frac{6+2\sqrt{5}}{5+2\sqrt{5}+1} \right) - \psi^n \left(\frac{6-2\sqrt{5}}{5-2\sqrt{5}+1} \right)$$

$$= \varphi^n - \psi^n \quad \therefore F(n) = \frac{\varphi^n - \psi^n}{\sqrt{5}}$$