AY: 2022-2023
Exam | AI-ECUE122
27/01/23 (09:00→10:30)

M1-S1: Dept. of Electrical Engineering
Teacher: A. Mhamdi
Time Limit: $1\frac{1}{2}$ h

This document contains **5** pages numbered from **1/5** to **5/5**. As soon as it is handed over to you, make sure that it is complete. The **4** tasks are independent and can be treated in the order that suits you.

The following rules apply:

❶ **No document** is allowed in the examination room.

❷ **Any electronic material**, except basic calculator, is prohibited.

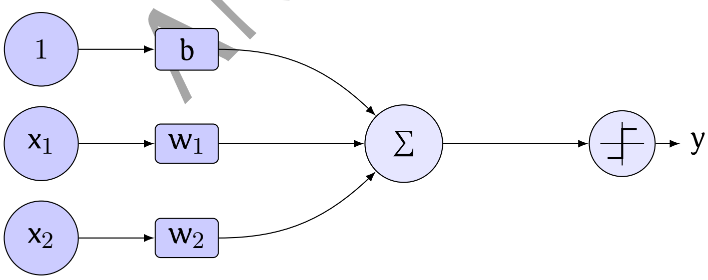
❸ **Round results** to the nearest thousandth *(i.e., third digit after the decimal point)*.

❹ **Mysterious or unsupported answers** will not receive full credit.

Task N°1

⌛ 20mn | (4 points)

We consider the vastly simplified model of real neuron, also known as **Threshold Logic Unit**. The processing element sums the weighted inputs $w_1x_1 + w_2x_2$, add a bias b and then applies a non linear activation function. The output transmits $+1$ *if and only if* the input is positive. Otherwise, it transmits -1.



Use bipolar data instead of binary data for the inputs x_1 and x_2, *i.e.* ± 1. Weights and bias are all set initially to zero: $w_1 = w_2 = b = 0$.

Consider the problem approximating an $\vee$ **(OR)** gate, according to **Hebbian** learning rule. Reproduce and fill in the following table on your answer sheet.

x_1	x_2	b	y	Δw_1	Δw_2	Δb	w_1	w_2	b
-1	-1	1	-1	1	1	-1	1	1	-1
-1	1	1	1	-1	1	1	0	2	0
1	-1	1	1	1	-1	1	1	1	1
1	1	1	1	1	1	1	2	2	2

Task №2

⌛ 15mn | (4 points)

Perform the following arithmetic operations.

(a) (1 point) $[0,\ 1] + [\text{-}6,\ 5]\ =$ ___[-6, 6]___

(b) (1 point) $[0,\ 1]\ \text{-}\ [\text{-}6,\ 5]\ =$ ___[-5, 7]___

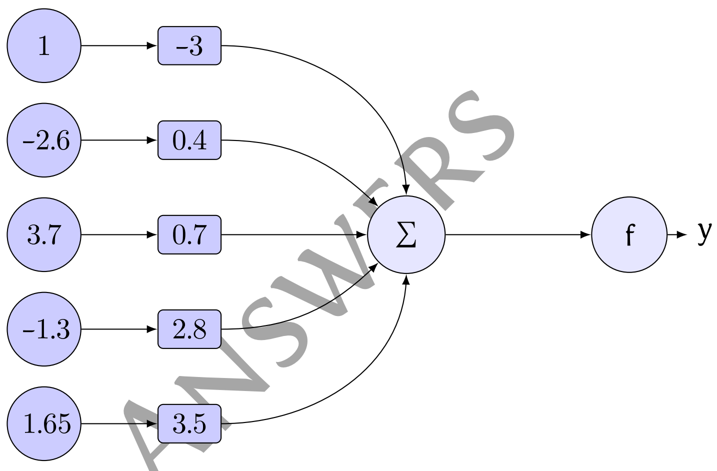
(c) (1 point) $[3,\ 4] \times [2,\ 2]\ =$ ___[6, 8]___

(d) (1 point) $[4,\ 10] \div [1,\ 2]\ =$ ___[2, 10]___

Task №3

⌛ 25mn | (6 points)

(a) (2 points) Compute the output being fired by the following neuron. f designates the tanh function.



The output is given by:

$$y\ =\ \tanh\left(\text{-}3 \times 1\ \text{-}\ 2.6 \times 0.4 + 3.7 \times 0.7\ \text{-}\ 1.3 \times 2.8 + 1.65 \times 3.5\right)\ =\ \tanh\left(0.685\right)$$

It yields:

$$y\ =\ 0.594760307$$

(b) (1 point) What is the machine learning library we used in `Julia` to train artificial neural networks.

Flux

(c) (1 point) Given the code snippet as shown in FIG. 1, p. 4. Explain why are there 75 trainable parameters in `model`.

Total number of trainable parameters is

$$5 \times 8 + 8 + 8 \times 3 + 3 \; = \; 75$$

(d) (1 point) What does the factor $w^{(1)}_{3,1}$ refer to, and what is its value.

$w^{(1)}_{3,1}$ designates the synaptic weight that connects the first input to the third neuron of the hidden layer. Its values is:

$$w^{(1)}_{3,1} \; = \; 0.503822$$

(e) (1 point) What is the value of synaptic weight between neuron #7 of the hidden layer and neuron #2 of the output layer.

The synaptic weight is denoted by $w^{(2)}_{2,7}$, and is given by:

$$w^{(2)}_{2,7} \; = \; -0.568683$$

```
~/appware/julia/julia-1.8/julia

 _/ |\__'_|_|_|\__'_|  |
|__/                   |

julia> using Flux

julia> model = Chain(
           Dense(5 => 8, relu),
           Dense(8 => 3, σ)
           )
Chain(
  Dense(5 => 8, relu),                  # 48 parameters
  Dense(8 => 3, σ),                     # 27 parameters
)                    # Total: 4 arrays, 75 parameters, 556 bytes.

julia> model.layers[2].weight
3×8 Matrix{Float32}:
 -0.313452  -0.20532    0.0164611   0.40596   …  -0.205025   -0.486443    0.6834
 -0.565272  -0.422708  -0.409977    0.716418     -0.0062125  -0.568683    0.0565138
  0.201109  -0.372519   0.0448279  -0.493097     -0.611313    0.0418072  -0.476514

julia> model.layers[1].weight
8×5 Matrix{Float32}:
 -0.675119   0.581365   0.510767   -0.0869667  -0.226824
 -0.543659  -0.553984  -0.111358   -0.0142636  -0.117261
  0.503822   0.432929   0.535884   -0.536289    0.509142
 -0.296016  -0.551115   0.562242   -0.500802   -0.640758
 -0.431213  -0.358273   0.502375   -0.0316917   0.394754
  0.597894   0.651843   0.254814   -0.211738    0.211448
 -0.525809  -0.504301   0.631113    0.113009   -0.510733
  0.132268   0.08442    0.0403786  -0.184404   -0.552655

julia> 
```

FIG. 1. Julia REPL

Task N°4 ⌛ 30mn | (6 points)

Suppose we have three fuzzy predicates: $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ described by these trapezoidal fuzzy sets:

$\mathcal{A}$ $\Pi\,(1,\ 2,\ 5,\ 7)$

$\mathcal{B}$ $\Pi\,(4,\ 12,\ 15,\ 16)$

$\mathcal{C}$ $\Pi\,(7,\ 8,\ 9,\ 12)$

x and y are fuzzy variables, each one ranges between 0 and 16. Given the following three rules:

$\mathfrak{R}_1$ $(\text{x is } \mathcal{A} \lor \text{x is } \mathcal{B}) \land !(\text{y is } \mathcal{C}) \rightarrow \text{u} = 9$

$\mathfrak{R}_2$ $!(\text{x is } \mathcal{B}) \land (\text{y is } \mathcal{C}) \rightarrow \text{u} = 3$

$\mathfrak{R}_3$ $(\text{x is } \mathcal{B} \lor \text{x is } \mathcal{C}) \land !(\text{y is } \mathcal{A}) \rightarrow \text{u} = 16$

Compute the degree of satisfaction for $\text{x} = 6$ & $\text{y} = 10$.

$\Re_1$ $(\mu_{\mathcal{A}}(6)$ max $\mu_{\mathcal{B}}(6))$ min $(1 - \mu_C(10))$ = $(0.5$ max $0.12)$ min $(1 - 2/3) = 1/3$

$\Re_2$ $(1 - \mu_{\mathcal{B}}(6))$ min $\mu_C(10)$ = $(1 - 1/4)$ min $2/3 = 2/3$

$\Re_3$ $(\mu_{\mathcal{B}}(6)$ max $\mu_C(6))$ min $(1 - \mu_{\mathcal{A}}(y))$ = $(1/4$ max $0)$ min $(1 - 0) = 1/4$

The final result is:

$$u^\star = \frac{1/3 \times 9 + 2/3 \times 3 + 1/4 \times 16}{1/3 + 2/3 + 1/4} = 7.2$$