AY: 2022-2023
EXAM | **AI-ECUE122**
27/01/23 (09:00⟶10:30)

M1-S1: Dept. of Electrical Engineering
Teacher: A. Mhamdi
Time Limit: $1\frac{1}{2}$ **h**

This document contains **5** pages numbered from **1** to **5**. Upon receiving it, verify completeness. The **4** tasks are independent and can be solved in any order you prefer. The following rules apply:

❶ **A handwritten double-sided A4** sheet is permitted.

❷ **Any electronic material**, except basic calculator, is prohibited.

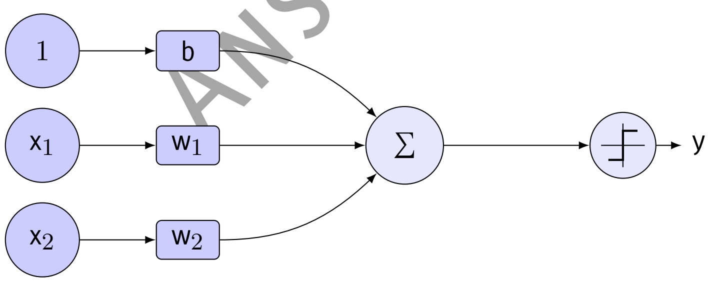
❸ **Mysterious or unsupported answers** will not receive full credit.

❹ **Round results** to the nearest thousandth *(i.e., third digit after the decimal point).*

Task N°1

⌛ 20mn | (4 points)

We consider the vastly simplified model of real neuron, also known as **Threshold Logic Unit**. The processing element sums the weighted inputs $w_1x_1 + w_2x_2$, add a bias b and then applies a non linear activation function. The output transmits $+1$ *if and only if* the input is positive. Otherwise, it transmits -1.



Use bipolar data instead of binary data for the inputs x_1 and x_2, *i.e.* ± 1. Weights and bias are all set initially to zero: $w_1 = w_2 = b = 0$.

Consider the problem approximating an $\vee$ **(OR)** gate, according to **Hebbian** learning rule. The learning rate η is set to 1. Reproduce and fill in the following table on your answer sheet.

x_1	x_2	b	y	Δw_1	Δw_2	Δb	w_1	w_2	b
-1	-1	1	-1	1	1	-1	1	1	-1
-1	1	1	1	-1	1	1	0	2	0
1	-1	1	1	1	-1	1	1	1	1
1	1	1	1	1	1	1	2	2	2

Task N$^{\underline{o}}$2

⌛ 15mn | (4 points)

Perform the following arithmetic operations.

(a) (1 point) $[0,\ 1] + [\text{-}6,\ 5] =$ ___ $[\text{-}6,\ 6]$ ___

(b) (1 point) $[0,\ 1] - [\text{-}6,\ 5] =$ ___ $[\text{-}5,\ 7]$ ___

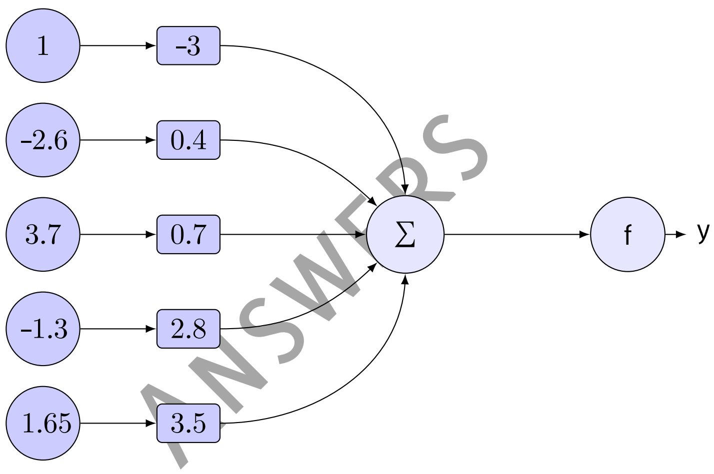
(c) (1 point) $[3,\ 4] \times [2,\ 2] =$ ___ $[6,\ 8]$ ___

(d) (1 point) $[4,\ 10] \div [1,\ 2] =$ ___ $[2,\ 10]$ ___

Task N$^{\underline{o}}$3

⌛ 25mn | (6 points)

(a) (2 points) Compute the output being fired by the following neuron. f designates the tanh function.



The output is given by:

$$y = \tanh(\text{-}3 \times 1 - 2.6 \times 0.4 + 3.7 \times 0.7 - 1.3 \times 2.8 + 1.65 \times 3.5) = \tanh(0.685)$$

It yields:

$$y = 0.594760307$$

(b) (1 point) What is the machine learning library we used in `Julia` to train artificial neural networks.

Flux

(c) (1 point) Given the code snippet as shown in FIG. 1, p. 4. Explain why are there 75 trainable parameters in `model`.

Total number of trainable parameters is

$$5 \times 8 + 8 + 8 \times 3 + 3 = 75$$

(d) (1 point) What does the factor $w_{3,1}^{(1)}$ refer to, and what is its value.

$w_{3,1}^{(1)}$ designates the synaptic weight that connects the first input to the third neuron of the hidden layer. Its values is:

$$w_{3,1}^{(1)} = 0.503822$$

(e) (1 point) What is the value of synaptic weight between neuron #7 of the hidden layer and neuron #2 of the output layer.

The synaptic weight is denoted by $w_{2,7}^{(2)}$, and is given by:

$$w_{2,7}^{(2)} = -0.568683$$

```
 _/ |\__'_|_|_|\__'_|  |
|__/                   |

julia> using Flux

julia> model = Chain(
           Dense(5 => 8, relu),
           Dense(8 => 3, σ)
           )
Chain(
  Dense(5 => 8, relu),                  # 48 parameters
  Dense(8 => 3, σ),                     # 27 parameters
)                   # Total: 4 arrays, 75 parameters, 556 bytes.

julia> model.layers[2].weight
3×8 Matrix{Float32}:
 -0.313452  -0.20532    0.0164611   0.40596   …  -0.205025   -0.486443    0.6834
 -0.565272  -0.422708  -0.409977    0.716418     -0.0062125  -0.568683    0.0565138
  0.201109  -0.372519   0.0448279  -0.493097     -0.611313    0.0418072  -0.476514

julia> model.layers[1].weight
8×5 Matrix{Float32}:
 -0.675119   0.581365   0.510767  -0.0869667  -0.226824
 -0.543659  -0.553984  -0.111358  -0.0142636  -0.117261
  0.503822   0.432929   0.535884  -0.536289    0.509142
 -0.296016  -0.551115   0.562242  -0.500802   -0.640758
 -0.431213  -0.358273   0.502375  -0.0316917   0.394754
  0.597894   0.651843   0.254814  -0.211738    0.211448
 -0.525809  -0.504301   0.631113   0.113009   -0.510733
  0.132268   0.08442    0.0403786  -0.184404  -0.552655

julia> 
```

FIG. 1. Julia REPL

Task N°4

⌛ 30mn | (6 points)

Suppose we have three fuzzy predicates: $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ described by these trapezoidal fuzzy sets:

$\mathcal{A}$ $\Pi(1,\ 2,\ 5,\ 7)$

$\mathcal{B}$ $\Pi(4,\ 12,\ 15,\ 16)$

$\mathcal{C}$ $\Pi(7,\ 8,\ 9,\ 12)$

x and y are fuzzy variables, each one ranges between 0 and 16. Given the following three rules:

$\Re_1$ (x is $\mathcal{A}$ $\vee$ x is $\mathcal{B}$) $\wedge$!(y is $\mathcal{C}$) $\rightarrow$ u $= 9$

$\Re_2$!(x is $\mathcal{B}$) $\wedge$ (y is $\mathcal{C}$) $\rightarrow$ u $= 3$

$\Re_3$ (x is $\mathcal{B}$ $\vee$ x is $\mathcal{C}$) $\wedge$!(y is $\mathcal{A}$) $\rightarrow$ u $= 16$

Compute the degree of satisfaction for x $= 6$ & y $= 10$.

$\mathfrak{R}_1$ $(\mu_{\mathcal{A}}(6)$ max $\mu_{\mathcal{B}}(6))$ min $(1 - \mu_{C}(10))$ = $(0.5$ max $0.12)$ min $(1 - 2/3) \;=\; 1/3$

$\mathfrak{R}_2$ $(1 - \mu_{\mathcal{B}}(6))$ min $\mu_{C}(10)$ = $(1 - 1/4)$ min $2/3 \;=\; 2/3$

$\mathfrak{R}_3$ $(\mu_{\mathcal{B}}(6)$ max $\mu_{C}(6))$ min $(1 - \mu_{\mathcal{A}}(y))$ = $(1/4$ max $0)$ min $(1 - 0) \;=\; 1/4$

The final result is:

$$u^{\star} \;=\; \frac{1/3 \times 9 + 2/3 \times 3 + 1/4 \times 16}{1/3 + 2/3 + 1/4} \;=\; 7.2$$