Calculus with Matlab | Octave[1]

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[1]Available @ https://github.com/a-mhamdi/isetbz/

Honor Code

(THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL: Department of Physics and Astronomy[a])

[a]See: http://physics.unc.edu/undergraduate-program/labs/general-info/

During this course, you will be working with one or more partners with whom you may discuss any points concerning laboratory work. However, you must write your lab report, in your own words. Lab reports that contain identical language are not acceptable, so do not copy your lab partner's writing.

If there is a problem with your data, include an explanation in your report. Recognition of a mistake and a well-reasoned explanation is more important than having high-quality data, and will be rewarded accordingly by your instructor. A lab report containing data that is inconsistent with the original data sheet will be considered a violation of the Honor Code.

Falsification of data or plagiarism of a report will result in prosecution of the offender(s) under the University Honor Code.

On your first lab report you must write out the entire honor pledge:

"The work presented in this report is my own, and the data was obtained by my lab partner and me during the lab period."

On future reports, you may simply write "*Laboratory Honor Pledge*" and sign your name.

What is Matlab?

Matlab (Matrix Laboratory), developed by Mathworks, is a numerical computing environment. It allows data manipulations, implementation of algorithms, plotting functions and creating GUI. It is an easy to use environment, a fourth-generation programming language (4GL).

Key Features

- Interfacing capabilities with programs written in other languages (C, C^{++}, Java & Fortran)
- Third-party products (LabVIEW, XILINX, PSIM, etc.)
- Dozens of toolboxes: image acquisition, signal processing...
- Matlab users come from various domains of engineering, economics and science

Matlab is widely used in industry, academic institutions as well as research structures, click here to see a full list of Matlab's users stories.

Outline

Linear Algebra I

Task #1

Comment the following code:

```
clear, clc
A=10*rand(4,4)
R1=A*A
R2=A.*A
R3=A\A
R4=A./A
R5=det(A)
R6=inv(A)
```

Linear Algebra II

Task #2

What does the code below do?

```
clear, clc
grades = input('Please enter the grades as a vector [x x x]: ');
number = length(grades);
aver = mean(grades);
standard_dev = std(grades);
middle = median(grades);
fprintf('\nThere are %i grades.\n',number)
fprintf('The average grade is %.1f.\n',aver)
fprintf('The standard deviation is %.1f.\n',standard_dev)
fprintf('The median grade is %.1f.\n',middle)
```

Task #3

Given the matrices

$$\mathcal{A} = \begin{pmatrix} 1 & -1 \\ 0 & 2 \\ 3 & 2 \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} 2 & -1 \\ -1 & 0 \\ 3 & 2 \end{pmatrix}, \quad \mathcal{C} = \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}.$$

Calculate where possible $\mathcal{A}+\mathcal{B},\ \mathcal{A}\mathcal{C},\ \mathcal{C}\mathcal{B},\ (\mathcal{A}-\mathcal{B})\,\mathcal{C}$ and $\mathcal{A}\mathcal{C}-\mathcal{B}\mathcal{C}$.

Linear Algebra III

Task #4

Calculate the quantities

$$\begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 5 & -2 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 4 & 0 & 1 & -1 \\ 2 & 1 & -2 & -1 \end{pmatrix}$$

Task #5

Given $X = [4,\ 1,\ 6]$ and $Y = [6,\ 2,\ 7]$, compute the following arrays:

1. The matrix A whose elements are $a_{ij} = x_i y_j$.
2. The matrix B whose elements are $b_{ij} = \dfrac{x_i}{y_j}$.
3. The vector C whose elements are $c_i = x_i y_{4-i}$.

Task #6

Show that the calculation $\mathcal{X}\mathcal{X}^\mathsf{T}$, where $\mathcal{X}$ is a row vector with real entries, always gives a positive scalar.

Linear Algebra IV

Task #7

Using the vector $r = 1:4$, construct the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Hint!

Use these two commands in MATLAB: *fliplr* (flip left–right) and *flipud* (flip up–down).

Linear Algebra V

Task #8

Expand the matrix equation and write it as two simultaneous equations

$$\begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Write the set of three simultaneous equation in matrix form:

$$\left\{ \begin{array}{lcr} x + y + z & = & 0 \\ x - 2y - z & = & 2 \\ -x + 3y - z & = & -1. \end{array} \right.$$

Linear Algebra VI

Task #9

Given $X = \begin{pmatrix} 5 & 0.35 & -3.5 & 5.47 & -2 \end{pmatrix}$, what are the commands that will execute the following operations:

1. Set the negative values of X to zero.
2. Extract the values of X greater than 3 in a vector Y.
3. Add 3 to the values of X that are even.
4. Set the values of X that are less than the mean to zero.
5. Set the values of X that are greater than the mean to their difference with the mean.

Linear Algebra VII

Task #10

Determine the solution of the systems

$$\left\{\begin{array}{lcl} x_1 - x_4 & = & 0 \\ -x_1 + 2x_2 - x_3 & = & 0 \\ -x_2 + 2x_3 - x_4 & = & 0 \\ x_4 & = & 1; \end{array}\right.$$

and

$$\left\{\begin{array}{lcl} x_1 - x_4 & = & 1 \\ -x_1 + 2x_2 - x_3 & = & 0 \\ -x_2 + 2x_3 - x_4 & = & 0 \\ x_4 & = & 0. \end{array}\right.$$

Linear Algebra VIII

Task #11

Determine the eigenvalues of the matrix

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}.$$

Task #12

Prove that $\mathcal{A}^n = \mathcal{P}\mathcal{D}^n\mathcal{P}^{-1}$ using induction where $n \in \mathbb{N}$ and $\mathcal{P}$ and $\mathcal{D}$ are comprised of the eigenvectors (as columns) and eigenvalues of $\mathcal{A}$ respectively.

Linear Algebra IX

Task #13

Calculate and plot the two functions:

$$
\begin{aligned}
x_1(t) &= \frac{2+\sin(t)}{2-\cos\left(\frac{1}{4}t\right)}\mathrm{e}^{-0.05t}, & 0 \leq t \leq 30, \\
x_2(t) &= \frac{2+\sin(t)}{2-\cos\left(\frac{1}{4}t\right)}\mathrm{e}^{-0.2t}, & 0 \leq t \leq 30.
\end{aligned}
$$

Linear Algebra X

Task #14

Create two matrices A and B: $A = \begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -2 \\ -6 & 3 \end{pmatrix}$.

1. Compute $C_1 = A + B$ and $C_2 = A - B$.
2. Compute the matrix products $D_1 = AB$ and $D_2 = BA$.
3. Using element by element operations, compute the matrix F whose elements are obtained as follows: $f_{ij} = b_{ij} + a_{ij} b_{ij}^{\frac{1}{3}}$.
4. Are A and B singular? If no, compute their inverse.
5. Compute the eigenvalues of B.
6. In A, substract to the second row, the first row multiplied by 3.

Scripts & Functions I

Task #15

What does this code do?

```
Num = 0; Myeps = 1;
while (1+Myeps) > 1
Myeps = Myeps/2;
Num = Num + 1;
end
Num
Myeps = 2*Myeps
eps
```

Task #16

Develop in Matlab a script that generates a 4-bit Fibonacci LFSR.

Scripts & Functions II

Task #17

The Legendre polynomials, P_n, are defined by the following recurrence relation:

$$(n+1)P_{n+1}(x) - (2n+1)P_n(x) + nP_{n-1}(x) = 0,$$

with $P_0(x) = 1$, $P_1(x) = x$ and $P_2(x) = \dfrac{3x^2 - 1}{2}$. Compute the next three Legendre polynomials and plot all 6 over the interval $[-1,\ 1]$ on the same figure.

Scripts & Functions III

Task #18

The Fibonacci numbers are computed according to the following equation:

$$F_n = F_{n-1} + F_{n-2},$$

with $F_0 = F_1 = 1$.

1. Compute the first 10 Fibonacci numbers.
2. For the first 50 Fibonacci numbers, compute the ratio:

$$\frac{F_n}{F_{n-1}}.$$

3. Compare this ratio to the golden mean $\dfrac{1+\sqrt{5}}{2}$.

Ordinary Differential Equations (ODE) I

Task #19

Use the Trapezoidal rule to evaluate $\int_0^4 x^2 dx$, using a step length of 1 sec.

Task #20

Numerically approximate the solution of the first order differential equation

$$\frac{dy}{dx} = xy^2 + y; \quad y_0 = 1,$$

on the interval $x \in [0,\ 5]$.

Ordinary Differential Equations (ODE) II

Task #21

Solve the differential equation, given by:

$$\frac{dy}{dt} = 0.5\frac{\cos(t)}{y-1},$$

for an arbitrary initial condition of your choice.

Task #22

Solve the differential equation:

$$\frac{dy}{dt} + 4y = e^{-t},$$

with initial condition $y_0 = 1$.

Ordinary Differential Equations (ODE) III

Task #23

Solve the system of Lorenz equations,

$$\left\{\begin{array}{rcl} \dfrac{dx}{dt} & = & -\sigma x + \sigma y \\ \dfrac{dy}{dt} & = & \rho x - y - xz \\ \dfrac{dz}{dt} & = & -\beta z + xy, \end{array}\right.$$

where $\sigma == 10,\ \beta = 8/3$ and $\rho = 28$.For the purpose of this example, we will take IVs as $x_0 = -8,\ y_0 = 8$ and $z_0 = 27$.

Task #24

Though `MATLAB` is primarily a numerics package, it can certainly solve straightforward differential equations symbolically. For instance, solve the first ODE

$$\dot{y}(x) \;=\; xy,$$

using the built-in function *dsolve*.

Optimization I

Task #25

Solve the two equations:

$$\begin{aligned} u^2 v^2 &= 0, \\ u - \frac{v}{2} - \alpha &= 0. \end{aligned}$$

Extrapolation & Interpolation I

Further Reading... I

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