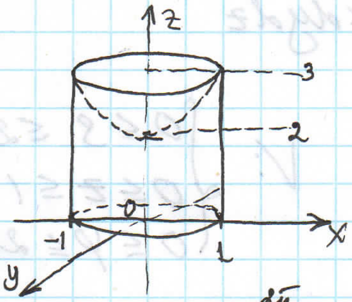
№8

Найти поток векторного поля a через замкнутую поверхность S (нормаль внешняя). Вычислить 2-мя сп-ми.

$$a = zi + (3y-x)j - zk, \quad S = \begin{cases} x^2+y^2=1, \\ z = x^2+y^2+2, \ z=0 \end{cases}$$

① $a_x = z$; $a_y = 3y - x$; $a_z = -z$

$$\operatorname{div}\bar a = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z} = 0 + 3 - 1 = 2$$



$$\begin{cases} 0 \le \varphi \le 2\pi \\ 0 \le \rho \le 1 \\ 0 \le z \le \rho^2+2 \end{cases}$$

$$V = \iiint\limits_V \operatorname{div}\bar a\,dx\,dy\,dz = 2\iiint\limits_V dx\,dy\,dz = 2\int_0^{2\pi} d\varphi \int_0^1 \rho\,d\rho \int_0^{\rho^2+2} dz =$$

$$= 2\int_0^{2\pi} d\varphi \int_0^1 (\rho^3 + 2\rho)\,d\rho = 2\int_0^{2\pi} \left.\frac{\rho^4}{4} + \rho^2\right|_0^1 d\varphi = \frac{2\cdot 5}{4}\int_0^{2\pi} d\varphi = 5\pi$$

② 1. $x^2+y^2=1 \Rightarrow n_1^\circ = \frac{1}{2\sqrt{x^2+y^2}}(2x, 2y, 0) = \frac{1}{\sqrt{x^2+y^2}}(x, y, 0)$

$$\Pi_1 = \iint\limits_S (a, n_1^\circ)\,dS = \iint\limits_S \frac{1}{\sqrt{x^2+y^2}}\left(zx + y(3y-x)\right)\sqrt{(z'_x)^2+(z'_y)^2+1}\,dx\,dy =$$

$$= 2\int_{-1}^1 dx \int_0^{\sqrt{1-x^2}} (zx + 3y^2 - xy)\,dy = 2\cdot\frac{3\pi}{8} = \frac{3\pi}{4}$$

2. $x^2+y^2+2-z=0 \Rightarrow n_2^\circ = \frac{1}{\sqrt{4x^2+4y^2+1}}(2x, 2y, -1)$ $\quad \sqrt{(z'_x)^2+(z'_y)^2+1}$

$$\Pi_2 = \iint\limits_S (a, n_2^\circ)\,dS = \iint\limits_S \frac{1}{\sqrt{4x^2+4y^2+1}}\left(2xz + 2y(3y-x) + z\right)\sqrt{4x^2+4y^2+1}\,dx\,dy =$$

$$= 2\int_{-1}^1 dx \int_0^{\sqrt{1-x^2}} (2x+1)(x^2+y^2+2) + 2y(3y-x)\,dy = 2\cdot 2\pi = 4\pi$$

3. $z = 0 \Rightarrow n_3^\circ = (0, 0, -1)$

$$\iint\limits_S z\,dS = \int_{-1}^1 dx \int_0^{\sqrt{1-x^2}} (x^2+y^2)\,dy = \frac{\pi}{4}$$

$$\Pi = \frac{3\pi}{4} + 4\pi + \frac{\pi}{4} = 5\pi$$

Ответ: 5π