Discrete Structures. CSCI-150. Summer 2014.

Homework 5. Due Thr. June 19, 2014.

Problem 1 (Graded)

The partial sum of the cubes of natural numbers can be computed using the following formula

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}.$$

Check that it is correct for $n = 1$ and $n = 2$.

After that, prove this formula by induction for all $n \geq 1$.

Problem 2

Recall De Morgan's law for the negation of the disjuntion of two propositions,

$$\neg(p \vee q) \equiv \neg p \wedge \neg q.$$

Use mathematical induction to show that the law holds for n propositions

$$\neg(p_1 \vee p_2 \vee \ldots \vee p_n) \equiv \neg p_1 \wedge \neg p_2 \wedge \ldots \wedge \neg p_n$$

Problem 3

Prove by induction that for all $n \geq 0$:

$$\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = 2^n$$

In the inductive step, use Pascal's identity.

Problem 4 (Graded)

Prove by induction that $\forall n \geq 5$:

$$4n < 2^n$$

Problem 5

Prove by induction that $\forall n \geq 1$:

$$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$$