

Discrete Structures. CSCI-150. Fall 2013.

Review

Problem

First, prove that $k(k+1)$ is even for any $k \in \mathbb{Z}$.

Then, for positive $n \in \mathbb{Z}$, prove that if n is odd then $8 \mid (n^2 - 1)$.

Problem

Modular exponentiation. Prove that $3^{23} \equiv 5 \pmod{11}$.

(RSA encryption/decryption is done similarly)

Problem

Given two numbers,

$$a_0 = 135, \quad a_1 = 129,$$

Find $a_k = \gcd(a_0, a_1)$ and Bezout's coefficients x_k and y_k , i.e. the numbers such that the following equation is satisfied:

$$a_k = \gcd(a_0, a_1) = x_k a_0 + y_k a_1$$

If it's possible, find the multiplicative inverse of a_1 modulo a_0 .

(Note that the multiplicative inverse exists if and only if a_1 and a_0 are relative primes)

Problem

Given three sets

$$A = \{1, 2, 3\}, \quad B = \{1, 2, 3, 4\}, \quad C = \{1, 2, 3, 4, 5\},$$

construct the following functions (or prove that they don't exist):

1. one-to-one function from A to B
2. one-to-one function from C to B
3. onto function from C to B
4. bijection from A to B

Problem

Draw the Hasse diagram for divisibility on the set $\{2, 4, 5, 6, 10, 12\}$.

Find the maximal elements. Find the minimal elements.

Construct a topological sort of this poset.

Problem

Find two incomparable elements in these posets.

- (a) $(\mathcal{P}(0, 1, 2), \subseteq)$, where $\mathcal{P}(X)$ denotes the powerset of X .
- (b) $(\{1, 2, 4, 6, 8\}, |)$

Problem

Use the Schröder-Bernstein theorem to show that there is a bijection between two intervals $[0, 1] \subseteq \mathbb{R}$ and $[1, \infty) \subseteq \mathbb{R}$, thus they have the same cardinality.

What about the sets $(0, 5)$ and $(10, 20)$? Is there a bijection between them? (Don't need the Schröder-Bernstein theorem here). Similarly, consider $[0, \infty)$ and $[1, \infty)$.

Problem

Prove by diagonalization that the interval $[0, 1] \subseteq \mathbb{R}$ is uncountable.

Problem

Given a multigraph G , does it have an Eulerian cycle? Does it have an Eulerian path? (A drawing of the graph will be supplied).

Problem

We define the complement G^c of a graph G as the graph on the same vertex set in which two vertices are joined by an edge if and only if they are not joined by an edge in G . Prove that it cannot happen that both G and G^c are disconnected.

Problem

How many leaves does a full 3-ary tree with 100 vertices have?

Problem

Use Huffman coding to encode these symbols with given frequencies:
A: 0.05, B: 0.07, C: 0.08, D: 0.10, E: 0.15, F : 0.25, G: 0.30.

What is the average number of bits required to encode a symbol?

Problem

By rolling a six-sided die 6 times, a strictly increasing sequence of numbers was obtained, what is the probability of such an event?

Problem

Find each of the following probabilities when n independent Bernoulli trials are carried out with probability of success p .

- (a) the probability of at least two successes
- (b) the probability of at least two failure