# Discrete Structures. CSCI-150. Spring 2016.

## Homework 11. Due Mon. May 2, 2016.

### Problem 1 (Graded)

Count the number of topological sorts for each poset $(A, |)$, where

(a) $A = \{3, 5, 7, 11, 13, 16, 17\}$, (b) $A = \{1, 3, 9, 27, 81, 243\}$, (c) $A = \{2, 3, 4, 8, 9, 16, 27, 81\}$.

That is, you have to find the number of ways to order the elements of the set $A$ so that the partial order imposed by divisibility is preserved.

### Problem 2

Prove that the "divides" relation on $\mathbb{N} \times \mathbb{N}$ is a partial order relation.

Prove that the "subset" relation ($\subseteq$) is a partial order relation, and the "proper subset" relation ($\subsetneq$) is not.

### Problem 3 (Graded)

Draw these graphs: (a) $K_7$, (b) $K_{2,5}$, (c) $C_7$, (d) $Q_4$.
All of these special graphs are described in Rosen, $K_n$ is the complete graph, $K_{n,m}$ is the complete bipartite graph, $C_n$ is the cycle graph, and $Q_n$ is the hypercube graph.

How many vertices is in $K_n$, $K_{n,m}$, $C_n$, $Q_n$?

### Problem 4 (Graded)

A simple graph is called regular if every vertex of this graph has the same degree. A regular graph is called $n$-regular if every vertex in this graph has degree $n$.

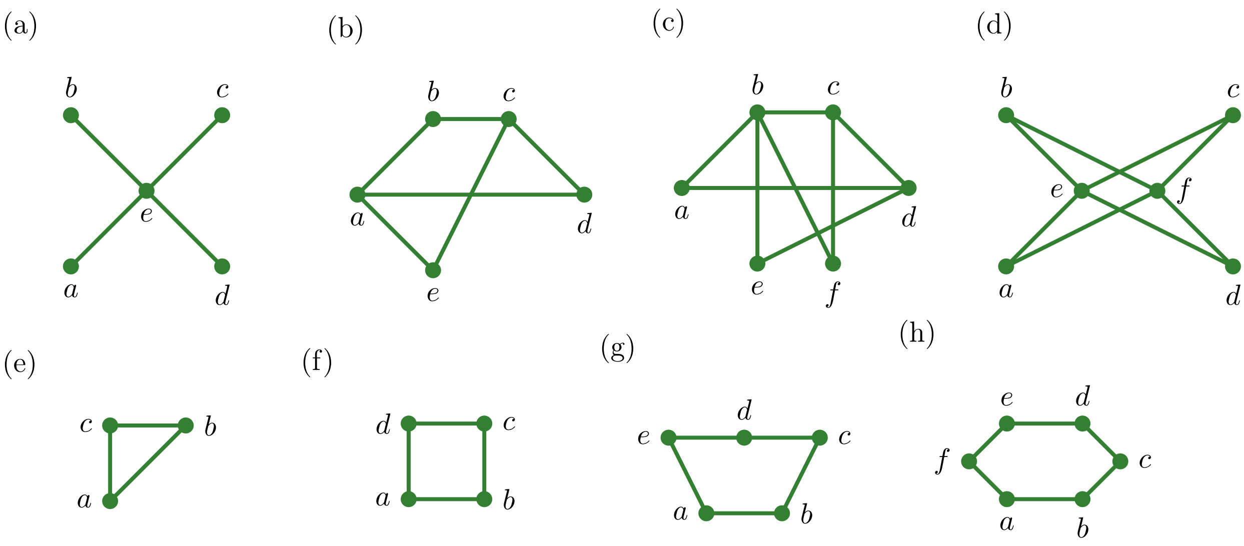
Recall that$K_n$ is the complete graphs with $n$ vertices. And $K_{m,n}$ is the complete bipartite graph (see the definition in the book).

(a) Is $K_n$ regular?

(b) For which values of $m$ and $n$ graph $K_{m,n}$ is regular?

(c) How many vertices does a 4-regular graph with 10 edges have?

## Problem 5

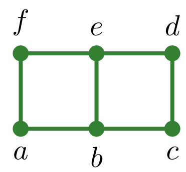



We know that a graph is bipartite if and only if it is 2-colorable.

For the graphs given above, either prove that they are bipartite by showing that they are 2-colorable, or prove that they are not 2-colorable, and so they are not bipartite.

## Problem 6

What are the adjacency matrix and the adjacency list of a graph?
Find the adjacency matrix of the graph shown in the figure.
Find the adjacency list of the graph.




## Problem 7 (Graded)

Given a graph with $n$ vertices, prove that if the degree of each vertex is at least $(n-1)/2$ then the graph is connected.

Hint: First, you may consider a small graph, for example a graph with 5 vertices, can you make it disconnected?

## Problem 8 (Graded)

A simple graph is called $n$-regular if every vertex of the graph has degree $n$.

Show that if a bipartite graph $G = (V, E)$ with a bipartition of the vertex set $(V_1, V_2)$ is $n$-regular for some positive integer $n$ then $|V_1| = |V_2|$.

## Problem 9

For which values of $n$, does the complete graph $K_n$ have an Euler cycle?
For which values of $n$ and $m$, does the complete bipartite graph $K_{n,m}$ have an Euler cycle?