Discrete Structures. CSCI-150. Fall 2015.

Homework 2. Due Wed. Sep 16, 2015.

Problem 1 ((b), (d), (e), and (g) are graded)

Prove

(a)
$$\begin{array}{c} r \to p \\ p \to q \\ r \wedge t \\ \hline q \wedge t \end{array}$$

(b)
$$\begin{array}{c} (p \vee r) \to (q \wedge s) \\ p \\ \hline s \end{array}$$

(c)
$$\begin{array}{c} \neg q \wedge (r \vee s) \\ q \vee \neg s \\ (\neg q \wedge r) \to (t \vee s) \\ \hline \neg q \wedge t \end{array}$$

(d)
$$\begin{array}{c} (p \wedge \neg q) \vee (r \to p) \\ p \leftrightarrow q \\ \hline r \to q \end{array}$$

(e)
$$\begin{array}{c} (q \vee r) \to (\neg q \wedge t) \\ (\neg q \vee u) \to (s \to r) \\ s \\ \hline t \end{array}$$

(f)
$$\begin{array}{c} p \vee (r \wedge t) \\ \neg p \vee \neg(q \wedge u) \\ (q \wedge u) \vee s \\ \neg s \\ \hline r \end{array}$$

(g)
$$\begin{array}{c} q \\ t \leftrightarrow r \\ p \wedge (q \to r) \\ \hline p \wedge t \end{array}$$

(h)
$$\begin{array}{c} s \wedge t \\ \neg p \vee \neg(r \to s) \\ \hline \neg p \end{array}$$

(i)
$$\begin{array}{c} p \vee s \\ \neg p \vee r \\ \neg r \\ \hline s \end{array}$$

(j)
$$\begin{array}{c} p \to r \\ r \to s \\ t \vee \neg s \\ \neg t \vee u \\ \neg u \\ \hline \neg p \end{array}$$

(k)
$$\begin{array}{c} (\neg p \vee q) \to r \\ r \to (s \vee t) \\ \neg s \wedge \neg u \\ \neg u \to \neg t \\ \hline p \wedge \neg q \end{array}$$

If you have trouble with this problem, consider solving the last problem from HW 1 first.

Also, don't try to be very smart, this task is all about simple application of the rules. Found a matching rule? Try to apply it.

Hint for question (e): Try to apply the equivalence formulas to the first formula.

Problem 2 (Graded)

Using the predicates $P(x)$ to denote "x is a politician", $R(x)$ to denote "x is rich", L(x) to denote "x is a lobbyist" and $K(x, y)$ to denote "x knows y", write down quantified logical stetements to express:

(a) Some people are politicians.

(b) All lobbyists are rich.

(c) Not everyone is rich.

(d) Not every rich person is a politician or a lobbyist.

(e) All politicians know at least one lobbyist.

(f) Some lobbyists know a rich politician

(g) Everyone knows a rich politician or a rich lobbyist.

The domain of discourse are all people in the world.

Hint: The predicate "x knows politician y" can be expressed as $K(x, y) \wedge P(y)$, meaning that x knows y, and y is a politician.

Problem 3

Construct a contrapositive proof that for all real numbers x, if $2x - x^2 \neq 1$ then $x \neq 1$.

Problem 4 (Graded)

Prove by cointradiction that there are no positive integer solutions to the equation $x^2 - y^2 = 1$.

Problem 5. Knights and Knaves

A similar problem is discussed as an example in Rosen (ed.6: p. 14; ed.7: p. 19). However, the solution in Rosen is rather cumbersome.

Imagine an island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie.

(a) Is it possible for any inhabitant of this island to claim that he is a knave?

(b) Is it possible for an inhabitant of the island to claim that he and his brother are both knaves?

(c) Suppose an inhabitant A says about himself and his brother B: "At least one of us is a knave." What type is A and what type is B?

(d) Suppose A instead says: "Exactly one of us is a knave." What can be deduced about A and what can be deduced about B?

(e) On the first day you arrived to the island, you met an inhabitant and asked him: "Are you a knight or a knave?" He angrily replied: "I refuse to tell you!" and walked away. That's the last you ever saw or heard of him.

    Was he a knight or a knave?

These knights and knaves puzzles (as well as many other) were created by Raymond Smullyan, an extraordinary mathematician and philosopher. Try to find his books if you like this kind of puzzles. Smullyan was born in Far Rockaway in 1919, received his PhD from Princeton, and was teaching at New York City colleges, particularly at Lehman College.