Discrete Structures. CSCI-150. Fall 2014.

Homework 1. Due Wed. Sep 10, 2014.

Problem 1

Using the following propositions:

p : "Phyllis goes out for a walk".
r : "The Moon is out".
s : "It is snowing".

Formulate these statements in words (try to keep the propositions unchanged, if you replace a proposition with its equivalent, prove that your substitution is correct):

(a) $(r \wedge \neg s) \rightarrow p$ (b) $r \rightarrow (\neg s \rightarrow p)$ (c) $\neg(p \leftrightarrow (s \vee r))$

Problem 2 (Graded)

Consider a compound proposition "If the sky is red, we are on Mars".

Show that it has the form of implication $p \rightarrow q$.

Come up with realistic situations (a) when this statement is true, and (b) when this statement is false. What are the values p and q for those situations? Use a truth table to support your answer.

Problem 3 (Graded)

Write out the truth table for the following propositions:

(a) $(\neg p) \leftrightarrow q$

(b) $(p \wedge q) \rightarrow \neg(p \vee q)$

(c) $(r \wedge \neg s) \rightarrow p$

Compute one operation at a time, don't skip steps.

Problem 4

Prove logical equivalence:

$$\neg((a \wedge b) \wedge c) \equiv \neg a \vee (\neg b \vee \neg c).$$

It is advised to do the proof by constructing a chain of equivalences. (Hint: using De Morgan's Law and associativity of $\vee$).

Problem 5 (Graded)

Check if the given propositions are equivalent or not:

(a) $\neg(p \leftrightarrow s)$ and $(\neg p) \leftrightarrow (\neg s)$

(b) $p \leftrightarrow s$ and $(\neg p) \leftrightarrow (\neg s)$

(c) $(p \wedge \neg r) \wedge \neg s$ and $\neg(p \rightarrow (r \vee s))$

(You are free to do the proof either by the truth table method, or using known equivalences).

Problem 6

You are given an argument, but it's incomplete. Finish the work by giving the reasons why each step was correct.

(a) Prove

$$\frac{\begin{array}{c} p \wedge q \\ q \rightarrow (r \wedge s) \end{array}}{r}$$

Complete the argument

(1)	$p \wedge q$	Given.
(2)	$q \rightarrow (r \wedge s)$	Given.
(3)	q	...
(4)	$r \wedge s$	...
(5)	r	...

(b) Prove

$$\frac{\begin{array}{c} p \rightarrow (\neg s \wedge r) \\ s \vee t \\ p \end{array}}{t}$$

Complete the argument

(1)	$p \rightarrow (\neg s \wedge r)$	Given.
(2)	$s \vee t$	Given.
(3)	p	Given.
(4)	$\neg s \wedge r$	...
(5)	$\neg s$	...
(6)	t	...

(c) Prove

$$\frac{\begin{array}{c} (\neg p \vee s) \leftrightarrow q \\ \neg q \end{array}}{p}$$

Complete the argument

(1)	$(\neg p \vee s) \leftrightarrow q$	Given.
(2)	$\neg q$	Given.
(3)	$((\neg p \vee s) \rightarrow q) \wedge (q \rightarrow (\neg p \vee s))$	...
(4)	$(\neg p \vee s) \rightarrow q$	...
(5)	$\neg(\neg p \vee s)$	...
(6)	$\neg(\neg p) \wedge \neg s$	...
(7)	$\neg(\neg p)$	...
(7)	p	...