Discrete Structures. CSCI-150. Spring 2015.

Homework 4. Due Fri. Feb 27, 2015.

Problem 1

In some programming language, a variable name starts with a lowercase letter ('a'-'z') followed by any combination of lowercase letters, digits ('0'-'9'), or underscore symbols ('_'). Count the number of valid variable names of length 12.
(Answer: 4 625 858 166 265 970 738)

Problem 2 (Graded)

Ellen draws 5 cards from a standard deck of 52 cards.

(a) In how many ways can her selection result in a hand with no clubs?

(b) A hand with at least one club?

Problem 3 (Graded)

A computer science professor has seven different programming books on a bookshelf. Three of the books deal with C++, the other four with Java. In how many ways can the professor arrange these books on the shelf

(a) if there are no restrictions?

(b) if the languages should alternate?

(c) if all the C++ books must be next to each other?

(d) if all the C++ books must be next to each other and all the Java books must be next to each other?

Note that all the books are distinct. Not for grade, you may consider the case when two of the C++ books are identical copies. Then, what if all the C++ books and all the Java books are identical copies?

Problem 4

How many bit strings of length 10 contain at least three 1s and at least three 0s?
(Answer: 912).

Problem 5 (Graded)

(a) There are 50 white socks and 50 black socks in a drawer. How many socks do you have to take to be sure that you have at least one matching pair?

(b) At least one mismatching pair?

Problem 6 (Graded)

Prove that among any five points selected inside an equilateral triangle with side equal to 1, there always exists a pair at the distance not greater than 1/2.

Problem 7

In a room there are 10 people, none of them is younger than 1 or older than 100 years. Prove that among them, one can always find two groups of people (possibly intersecting, but different) the sums of whose ages are the same.

Hint: Use the pigeonhole principle. How many different groups of people can be picked? How large the sum of their ages can be?

Problem 8 (Graded)

In this problem, you have to find two proofs for the following identity:

$$\binom{2n}{2} = 2\binom{n}{2} + n^2.$$

(a) The first proof is algebraic. Using the fact that $\binom{n}{r} = \dfrac{n!}{(n-r)!\,r!}$, show that the equation is always true. It involves some factorial manipulation, but almost everything should cancel out.

   Remember: when proving the identity (or anything else in general), don't prove it "backwards", it's a logically inconsistent and faulty technique.

   You may consider the left-hand side and the right-hand side separately, showing that they are equal to the same formula. *However, don't make it look like a "backwards" proof!*

(b) For the second part, prove the same identity using the technique called "Double counting". It's a combinatorial proof technique for showing that two expressions are equal by demonstrating that they are two ways of counting the size of one set.

   In this particular case, show that two formulas: $\binom{2n}{2}$ and $2\binom{n}{2} + n^2$ describe two counting procedures that count the same set.

   A hint: We know that the first formula, $\binom{2n}{2}$, counts the number of ways of choosing 2 objects from a set of $2n$ objects. Show that the second formula, $2\binom{n}{2} + n^2$, counts the same thing.