

Discrete Structures. CSCI-150. Spring 2014.

Homework 10.

Due Fri. Apr 25, 2014.

Problem 1

Draw the Hasse diagram for divisibility on the set:

- (a) $\{1, 2, 3, 4, 5, 6\}$, (b) $\{3, 5, 6, 9, 25, 27\}$, (c) $\{3, 5, 7, 11, 13, 16, 17\}$, (d) $\{1, 3, 9, 27, 81, 243\}$,

Problem 2

Count the number of topological sorts for each poset $(A, |)$, where

- (a) $A = \{3, 5, 7, 11, 13, 16, 17\}$, (b) $A = \{1, 3, 9, 27, 81, 243\}$, (c) $A = \{2, 3, 4, 8, 9, 16, 27, 81\}$.

That is, you have to find the number of ways to order the elements of the set A so that the partial order imposed by divisibility is preserved.

Problem 3

For each course at a university, there may be one or more other courses that are its prerequisites. How can a graph be used to model these courses and which courses are prerequisites for which courses? Should edges be directed or undirected? Looking at the graph model, how can we find courses that do not have any prerequisites and how can we find courses that are not the prerequisite for any other courses?

Problem 4

Draw these graphs: (a) K_7 , (b) $K_{2,5}$, (c) C_7 , (d) Q_4 .

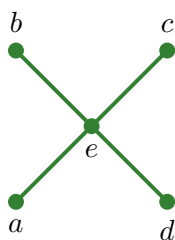
All of these special graphs are described in Rosen, K_n is the complete graph, $K_{n,m}$ is the complete bipartite graph, C_n is the cycle graph, and Q_n is the hypercube graph.

How many vertices is in K_n , $K_{n,m}$, C_n , Q_n ?

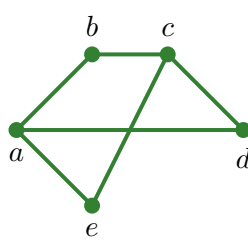
Problem 5

Determine whether the graph is bipartite

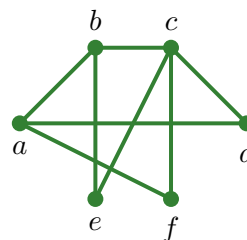
(a)



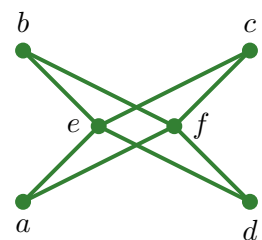
(b)



(c)



(d)



Problem 6

Assuming that friendship is always mutual, prove that in any group of $n \geq 2$ persons, there are at least 2 persons with the same number of friends in the group.

Problem 7

For which values of n , does the complete graph K_n have an Euler cycle?

For which values of n and m , does the complete bipartite graph $K_{n,m}$ have an Euler cycle?

Problem 8

Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 faces, how many vertices does this graph have?