

Discrete Structures. CSCI-150.

$(a \wedge b) \equiv (b \wedge a)$	commutativity of \wedge
$(a \vee b) \equiv (b \vee a)$	commutativity of \vee
$((a \wedge b) \wedge c) \equiv (a \wedge (b \wedge c))$	associativity of \wedge
$((a \vee b) \vee c) \equiv (a \vee (b \vee c))$	associativity of \vee
$\neg(\neg a) \equiv a$	double-negation elimination
$(a \rightarrow b) \equiv (\neg b \rightarrow \neg a)$	contraposition
$(a \rightarrow b) \equiv (\neg a \vee b)$	implication elimination
$(a \leftrightarrow b) \equiv (a \rightarrow b) \wedge (b \rightarrow a)$	biconditional elimination
$\neg(a \wedge b) \equiv (\neg a \vee \neg b)$	De Morgan's Law
$\neg(a \vee b) \equiv (\neg a \wedge \neg b)$	De Morgan's Law
$(a \wedge (b \vee c)) \equiv (a \wedge b) \vee (a \wedge c)$	distributivity of \wedge over \vee
$(a \vee (b \wedge c)) \equiv (a \vee b) \wedge (a \vee c)$	distributivity of \vee over \wedge
$a \wedge \text{True} \equiv a$	identity
$a \vee \text{False} \equiv a$	identity
$a \vee \text{True} \equiv \text{True}$	domination
$a \wedge \text{False} \equiv \text{False}$	domination
$a \vee \neg a \equiv \text{True}$	complementation (excluded middle)
$a \wedge \neg a \equiv \text{False}$	complementation (non-contradiction)

$\frac{p}{p \vee q} \quad \text{"}\vee\text{-Introduction"}$	$\frac{\neg q \quad p \rightarrow q}{\neg p} \quad \text{"Modus Tollens"}$
$\frac{p \quad q}{p \wedge q} \quad \text{"}\wedge\text{-Introduction"}$	$\frac{p \rightarrow q \quad q \rightarrow r}{p \rightarrow r} \quad \text{"Hypothetical Syllogism"}$
$\frac{p \wedge q}{p} \quad \text{"}\wedge\text{-Elimination"}$	$\frac{p \vee q \quad \neg q}{p} \quad \text{"Disjunctive Syllogism"}$
$\frac{p \quad p \rightarrow q}{q} \quad \text{"Modus Ponens"}$	$\frac{p \vee q \quad \neg p \vee r}{q \vee r} \quad \text{"Resolution"}$
$\frac{\text{assuming } p, \text{ we infer } q}{p \rightarrow q} \quad \text{"}\rightarrow\text{-Introduction" (Deduction theorem)}$	
$\frac{\text{assuming } p, \text{ we infer a contradiction}}{\neg p} \quad \text{"Proof by contradiction"}$	