Discrete Structures. CSCI-150. Fall 2015.

Homework 1. Due Wed. Sep 9, 2015.

Problem 1

Using the following propositions:

p : "Phyllis goes out for a walk".
r : "The Moon is out".
s : "It is snowing".

Formulate these statements in words:

(a) $(r \wedge \neg s) \to p$ (b) $r \to (\neg s \to p)$ (c) $\neg(p \leftrightarrow (s \vee r))$

Try to keep the propositions unchanged. If you really want to replace a proposition with its equivalent, first, prove that your substitution is correct.

In the question (c), you have to find a way to negate the whole sentence. I guarantee that there are ways to do that in English.

Problem 2 (Graded)

Write out the truth tables for the following propositions:

(a) $p \wedge \neg(p \to q)$

(b) $(p \leftrightarrow \neg(q \vee r)) \wedge (r \to q)$

Compute one operation at a time, don't skip steps.

Problem 3 (Graded)

An interesting question is to find the correct way to negate a biconditional, $\neg(a \leftrightarrow b)$.

A naive guess could be that we can simply distribute the negation over the biconditional, obtaining $\neg a \leftrightarrow \neg b$. We are going to check if this guess is correct or not.

Write the truth tables for the following propositional formulas:
(a) $\neg(p \leftrightarrow s)$, (b) $(\neg p) \leftrightarrow (\neg s)$, (c) $p \leftrightarrow s$, (d) $(\neg p) \leftrightarrow s$, (e) $p \leftrightarrow (\neg s)$

Decide which of these formulas are equivalent, and find what is the correct way to negate a biconditional.

Problem 4

Prove the logical equivalence:

$$\neg((a \wedge b) \wedge c) \equiv \neg a \vee (\neg b \vee \neg c).$$

It is advised to do the proof using the equivalence formulas we already know. (Hint: apply De Morgan's Law and the associativity of $\vee$).

Problem 5 (Graded)

Using logical equivalences, prove that

(a) $p \to (r \to p) \quad \equiv \quad \text{True},$

(b) $(p \to r) \vee (r \to p) \quad \equiv \quad \text{True},$

(c) $r \to (p \to (r \to p)) \quad \equiv \quad \text{True},$

in other words, we want to prove that the formulas above are tautologies (they are always true, regardless of the values of the variables p and r).

Problem 6 (Graded)

Using logical equivalences, prove that

$$p \wedge (p \vee t) \quad \equiv \quad p$$

The task looks difficult, because, the distributivity formula does not help.
Hint: Using the identity $A \equiv A \vee \text{False}$, represent the first p as $p \vee \text{False}$.

Problem 7

Using logical equivalences, prove that

$$p \leftrightarrow q \quad \equiv \quad (\neg p \wedge \neg q) \vee (p \wedge q)$$

Hint. To prove that, you can follow these steps:

(1) First, show that

$$p \leftrightarrow q \quad \equiv \quad (\neg p \vee q) \wedge (\neg q \vee p)$$

(2) Distribute $(\neg p \vee q)$ over the disjunction $(\neg q \vee p)$.

(3) Then do smething else, eventually arriving to

$$p \leftrightarrow q \quad \equiv \quad ((\neg p \wedge \neg q) \vee \text{False}) \vee (\text{False} \vee (q \wedge p))$$

(4) Then show that the right hand side in the formula above is equivalent to $(\neg p \wedge \neg q) \vee (p \wedge q)$.

Problem 8

You are given an argument, but it's incomplete. Finish the work by specifying which inference rule was used in each step of the argument.

(a) Prove

$$\begin{array}{c} p \wedge q \\ q \rightarrow (r \wedge s) \\ \hline r \end{array}$$

Complete the argument

$$\begin{array}{lll} (1) & p \wedge q & \text{Given.} \\ (2) & q \rightarrow (r \wedge s) & \text{Given.} \\ (3) & q & \ldots \\ (4) & r \wedge s & \ldots \\ (5) & r & \ldots \end{array}$$

(b) Prove

$$\begin{array}{c} p \rightarrow (\neg s \wedge r) \\ s \vee t \\ p \\ \hline t \end{array}$$

Complete the argument

$$\begin{array}{lll} (1) & p \rightarrow (\neg s \wedge r) & \text{Given.} \\ (2) & s \vee t & \text{Given.} \\ (3) & p & \text{Given.} \\ (4) & \neg s \wedge r & \ldots \\ (5) & \neg s & \ldots \\ (6) & t & \ldots \end{array}$$

(c) Prove

$$\begin{array}{c} (\neg p \vee s) \leftrightarrow q \\ \neg q \\ \hline p \end{array}$$

Complete the argument

$$\begin{array}{lll} (1) & (\neg p \vee s) \leftrightarrow q & \text{Given.} \\ (2) & \neg q & \text{Given.} \\ (3) & ((\neg p \vee s) \rightarrow q) \wedge (q \rightarrow (\neg p \vee s)) & \ldots \\ (4) & (\neg p \vee s) \rightarrow q & \ldots \\ (5) & \neg(\neg p \vee s) & \ldots \\ (6) & \neg(\neg p) \wedge \neg s & \ldots \\ (7) & \neg(\neg p) & \ldots \\ (7) & p & \ldots \end{array}$$