Discrete Structures. CSCI-150. Summer 2016.

Homework 10. Due Mon. Jul 11, 2016.

Problem 1

Let $A = \{1, 2, 3\}$, $B = \{0, 1\}$, and $C = \{x, y, z\}$.

Determine what the following sets are (list their elements):

(a) $A \cap B$, (b) $B \cup A$, (c) $A \setminus B$, (d) $(B \cap \mathbb{Z}) \setminus A$, (e) $(A \cup C) \setminus B$,

(f) $A \times B$, (g) $B \times B$, (h) $A \times B \times C$, (i) C^3,

(j) $\mathcal{P}(C)$. (k) $\mathcal{P}(B^2)$.

Problem 2 (Graded)

Let $S = \{a, b\}$.

Prove or disprove:

(a) $a \in S$, (b) $a \in \mathcal{P}(S)$, (c) $\{a\} \subseteq S$, (d) $\{a\} \in \mathcal{P}(S)$,

(e) $\varnothing \in S$, (f) $\varnothing \subseteq S$, (g) $\varnothing \in \mathcal{P}(S)$, (h) $\varnothing \subseteq \mathcal{P}(S)$

For the proofs, writing one short sentence for each question will be sufficient, if your argument is to the point and captures the main idea why the statement is true or false.

Problem 3 (Graded)

Prove the inclusion-exclusion formula (it's an extension of the subtraction rule)

$$\begin{aligned} |A \cup B \cup C| = &|A| + |B| + |C| \\ &- |A \cap B| - |A \cap C| - |B \cap C| \\ &+ |A \cap B \cap C|. \end{aligned}$$

To do the proof, let's denote $X = A \cup B$, then

$$|(A \cup B) \cup C| = |X \cup C|,$$

and we can apply the usual subtraction rule (you will have to apply it twice).

Problem 4 (Graded)

Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ that is

(a) one-to-one, but not onto,

(b) onto, but not one-to-one,

(c) neither one-to-one, nor onto.

(d) onto and one-to-one (bijection), which is not the identity function $f(x) = x$.

(e) Function $h : \mathbb{N} \to \mathbb{N} \times \{0, 1\}$ that is onto and one-to-one (bijection).

(f) Bijection $g : \mathbb{N} \to \mathbb{Z}$. (Hint: When solving this problem, if you want, you may assume that you know the function h from the previous question. Would that help to make the function g?)

When constructing the functions, try to define them by formulas. (Feel free to use such operations as *absolute value*, *floor*, *ceiling*, *remainder*, in addition to normal arithmetical operations). Definition by cases is another option.

By definition, $\mathbb{N}$ is the set of all non-negative integers: $\mathbb{N} = \{0, 1, 2, \ldots\}$.

For each function, explain (in the best way you can) why they satisfy the required conditions. (One or two sentences should be enough)