Controlled Evolution of Collaborative Networks: Is it a Good Idea?

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Simplicial complex networks

Abstract symplicial complex (SC) is a collection of sets Δ with the property that if a set $F \in \Delta$, then all subsets of F belong to Δ as well. A set $F \in \Delta$ is called a **face** of the complex. And a **facet** of a complex is a maximal face that is not contained in any other faces.

Graphs are a special case of SCs containing *sets of size at most 2* (i.e. nodes and edges).
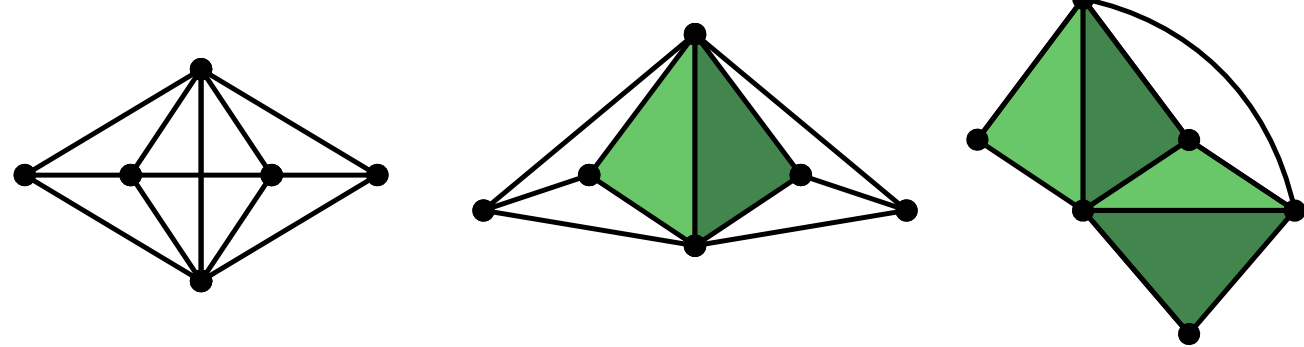
In this work, **collaborating teams are modeled as facets** of the SC. Thus collaborations of any size can be captured.

Neutral network growth model

Random mutation:
- with **25% probability**: **Add a new person** to an existing team sampled uniformly at random.
- with **25% probability**: **Make a new team** by taking a union of all people from two or more already existing teams, and sampling their subset.
- with **50% probability**: **Split an existing team** into two, assigning the team members randomly.

Neutral (not guided by a metric) network growth procedure:
- Start with a simplicial complex with one node.
- Apply Random Mutation to the network until the stopping condition is met.

Stopping conditions: (a) when the network has been "mutated" the required number of times, or (b) when the number of nodes in the network reaches the required limit.

Degree distributions

Facet degree of a node is the number of facets (teams) the node belongs to. **Edge degree** of a node is its degree in the underlying graph (=the number of neighbors).
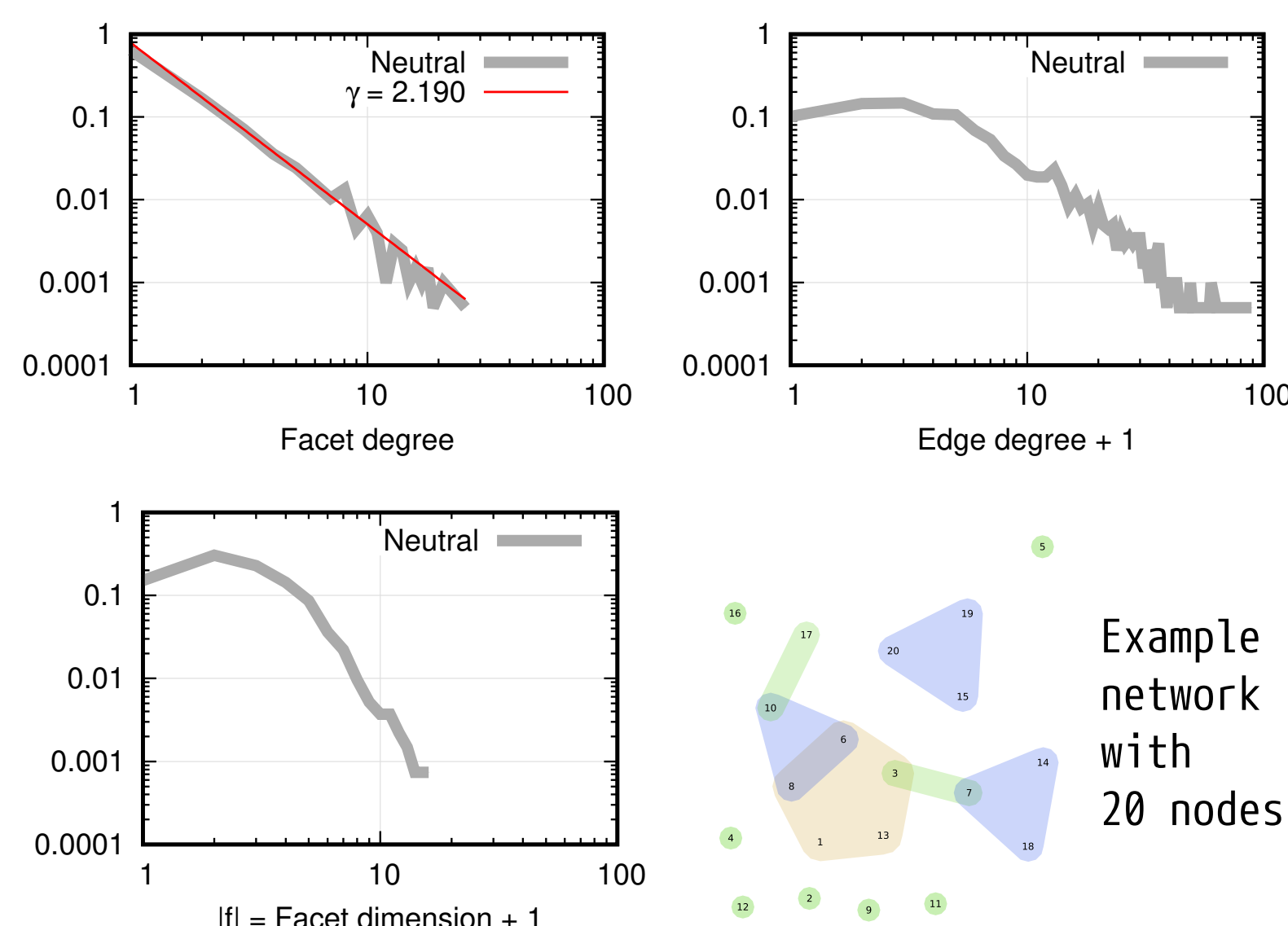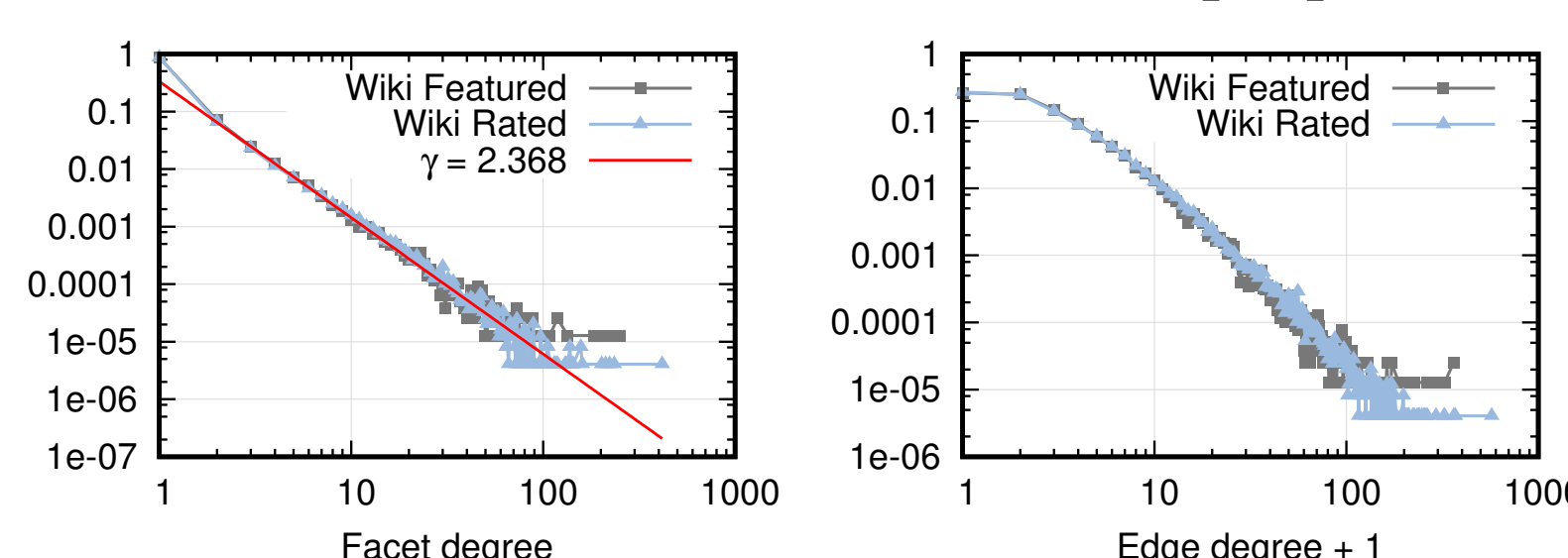

Figure 1: Neutral (not guided) generation procedure. Facet degree distribution. Edge degree distribution. Facet size distribution. Obtained from a network generated in 8000 mutation operations.

How realistic this model is? Wikipedia talk pages discussions exhibit similar distribution properties:

Guided network growth model

Metric-guided network growth procedure:
- Start with a simplicial complex with one node.
- Sample three random mutations of the current state of the network, and proceed with the one that maximizes the metric.
- Repeat until the stopping condition is met.
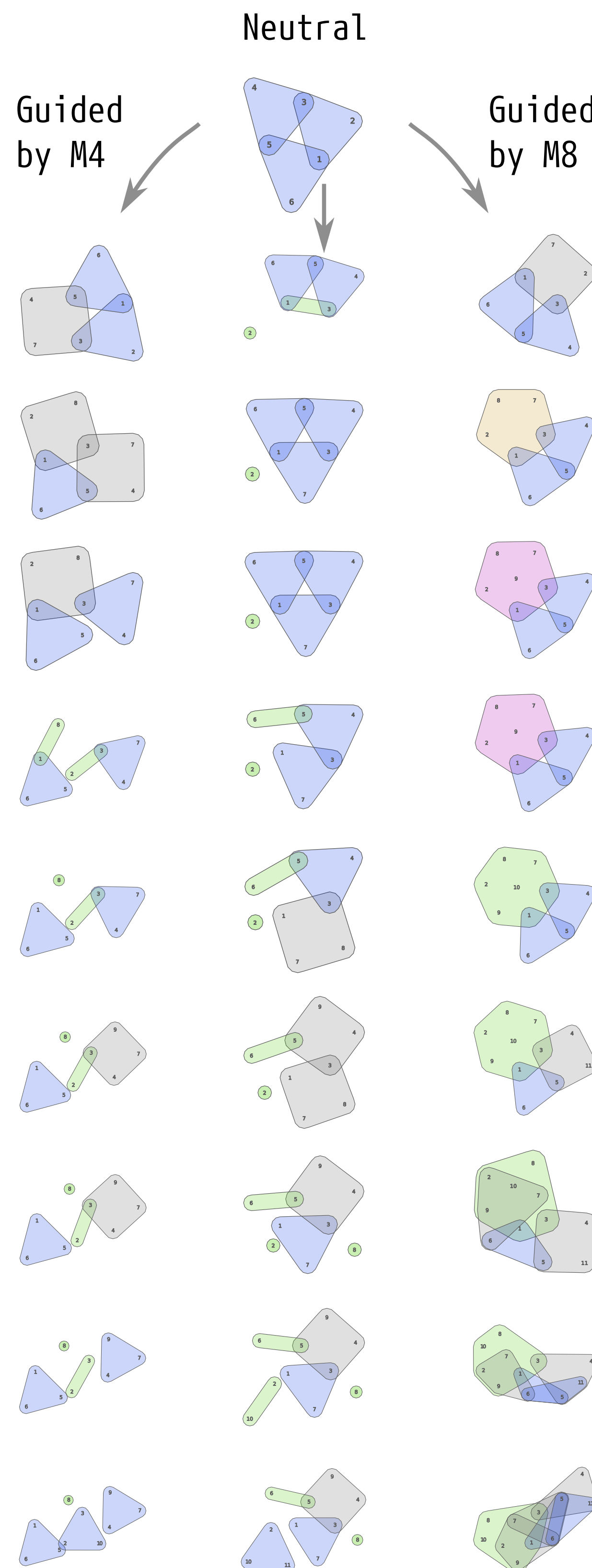
Multitasking is discouraged. All teams **became disjoint**, new team members were introduced.

Teams are formed and split, the process is ambivalent, **no specific goal or direction**.

Accumulation of new teams and members. **Not worried by overloading** with too many tasks.

Guiding metrics:

Previously, the authors considered [1] the following performance-measuring functions (all sums over f go over all *facets* of the complex and $d(v)$ is the facet degree of the node v). Now we use them to guide the network growth:

$$\mathbf{M1}(\Delta) = \prod_v \left(1 + \frac{1}{d(v)}\right)^{d(v)}$$

$$\mathbf{M2}(\Delta) = \prod_f \frac{1}{|f|} \cdot \sum_{v \in f}\left(1 + \frac{1}{d(v)}\right)^{|f|}$$

$$\mathbf{M3}(\Delta) = \prod_f \left(1 + \frac{1}{\sum_{v \in f} d(v)}\right)^{\sum_{v \in f} d(v)}$$

$$\mathbf{M4}(\Delta) = \sum_f \left((|f|-1) \cdot \prod_{v \in f}\frac{1}{d(v)}\right)$$

$$\mathbf{M5}(\Delta) = \sum_f \left(H_{|f|} \sum_{v \in f}\frac{1}{d(v)}\right)$$

$$\mathbf{M6}(\Delta) = \sum_f \left(\sum_{v \in f}\frac{1}{\sqrt{d(v)}}\right)^2$$

$$\mathbf{M7}(\Delta) = \sum_f \sqrt{\sum_{v \in f}\frac{1}{d(v)^2}}$$

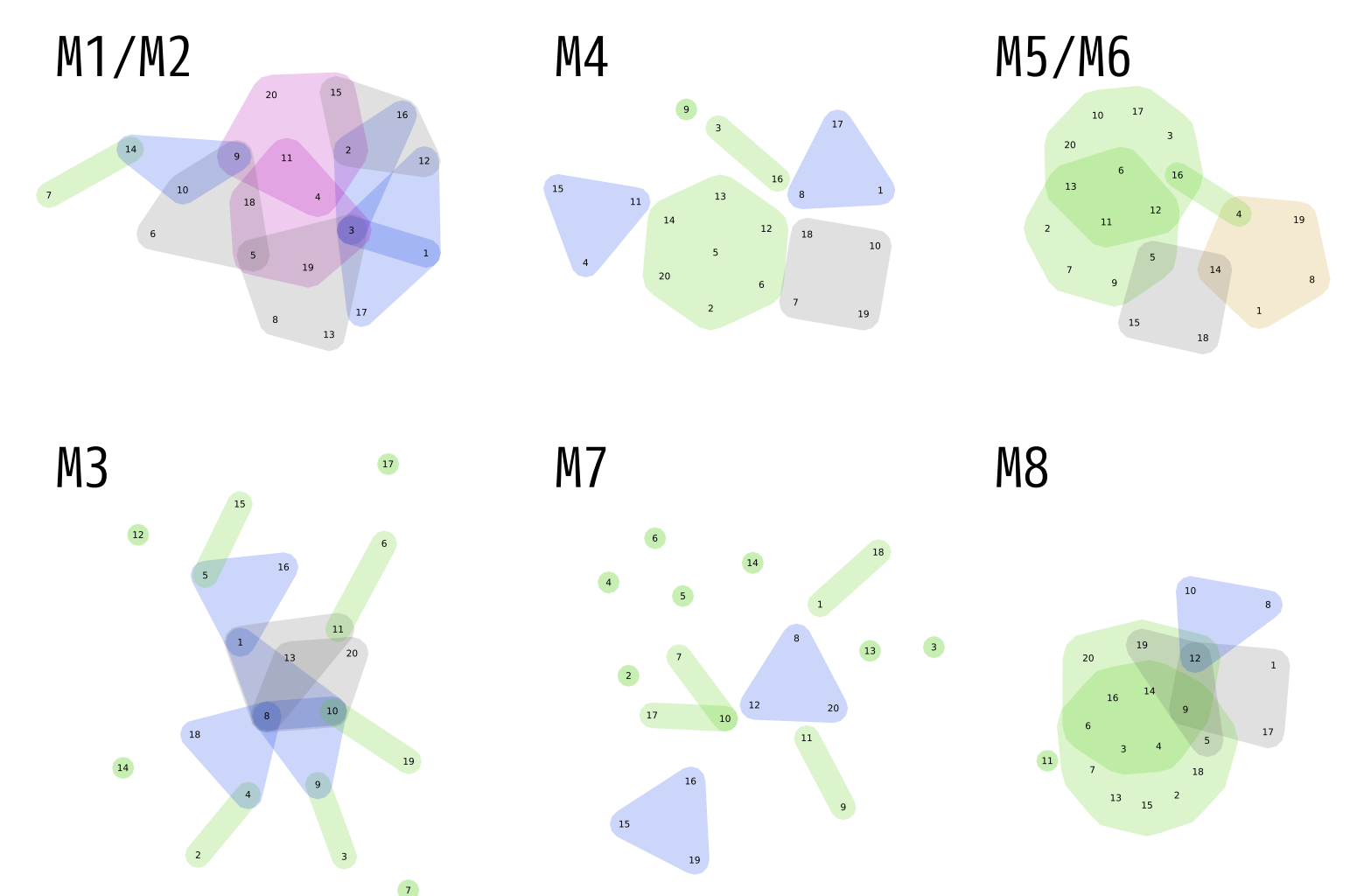$$\mathbf{M8}(\Delta) = \sum_f |f|!$$

Figure 2: Some typical networks with 20 nodes generated with the metric-guided generation processes.

Degree distributions - Guided

Observed properties

The following table summarizes the properties of the metrics M1-M8 observed from statistics on large networks and from visual inspection of small networks of size 20.

Metric	Overlapping teams (and avg. facet degree*)		# of teams (w.r.t. Neutral)	Team size (w.r.t. Neutral)	# of connect. components	Allows single big team
M1	yes	(**1.66**)	—	+	few (1–3)	—
M2	yes	(**1.73**)	—	+	few	—
M3	yes	(**1.71**)	+	0—**	some (2–6)	—
M4	no	(*1.00*)	———	+	some	yes
M5	okay	(1.31)	———	+	few	yes
M6	okay	(1.23)	———	+	few	yes
M7	no	(*1.03*)	+	—	many (> 10)	—
M8	yes!	(**1.81**)	—	+	few	yes

Table 1: (*) Average facet degree of a node is reported for small networks of size 20. For larger networks, M5 and M6 eventually catch up with M1-M3, but M8 still surpasses them all by the factor of 1.5–2. (**) The metric M3 slightly decreases the team sizes with respect to the neutral generation process.

References

[1] A. Assarpour et al., "Measuring the strength of networks of teams: Metrics and properties", *2015 IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS)*, pp. 414–419, IEEE, 2015.