Discrete Structures. CSCI-150. Fall 2014.

Homework 8. Due Wed. Oct 29, 2014.

Problem 1

Prove that among any five points selected inside an equilateral triangle with side equal to 1, there always exists a pair at the distance not greater than 1/2.

Problem 1 (Graded)

In a room there are 10 people, none of them is younger than 1 or older than 100 years. Prove that among them, one can always find two groups of people (possibly intersecting, but different) the sums of whose ages are the same.

Hint: Use the pigeonhole principle. How many different groups of people can be picked? How large the sum of their ages can be?

Problem 2 (Graded)

In how many ways can 10 identical laptops be distributed among five computer stores if

(a) there are no restrictions?

(b) each store gets at least one?

(c) the largest store gets at least two laptops?

Problem 3

Find the number of integer solutions to the equation

$$x + y + z = 12,$$

where the variables are positive integers.

Problem 4 (Graded)

Prove by induction that for all $n \geq 0$:

$$\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = 2^n$$

In the inductive step, use Pascal's identity.

Problem 5

Consider strictly decreasing sequences of (decimal) digits such as $[9, 8, 7, 6, 5, 4, 3, 2, 1, 0]$, or $[9, 6, 4, 2, 1]$, or $[6, 3, 2, 0]$, etc.

Count the number of such sequences of length 6.

Hint: Observe that to construct a sequence, it's sufficient to pick the set of digits, e.g. $\{1, 7, 4, 6, 5, 8\}$, and then their order is already predetermined: $[8, 7, 6, 5, 4, 1]$.

Problem 6

Use double counting to prove the following identity:

$$1 \cdot \binom{n}{1} + 2 \cdot \binom{n}{2} + \ldots + n \cdot \binom{n}{n} = n \cdot 2^{n-1}$$

Hint: Suppose there is a group of n people, and they have to form a committee with a chairman.