Discrete Structures. CSCI-150. Spring 2015.

Homework 5. Due Fri. Mar 6, 2015.

Problem 1 (Graded)

In how many ways can 10 identical computers be distributed among five computer stores if

(a) there are no restrictions?

(b) each store gets at least one?

(c) the largest store gets at least three?

(d) each store gets at least two?

Problem 2

Find the number of integer solutions to the equation

$$x + y + z = 12,$$

where the variables are positive integers.

Problem 3 (Graded)

The sum of the cubes of the first n positive integers can be computed by the following formula

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}.$$

First, check that it is correct for $n = 1$ and $n = 2$.

After that, prove this formula by induction for all $n \geq 1$.

Always write inductive proofs in full. First, write what the base case is and give its proof. Then the inductive case: write the assumtion and what you have to prove, then write the proof for it.

Problem 4

Prove by induction that $\forall n \geq 1$:

$$\sum_{k=1}^{n} k(k+1) = 1 \cdot 2 + 2 \cdot 3 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

Problem 5 (Graded)

Prove by induction that for all $n \geq 0$:

$$\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = 2^n$$

In the inductive step, use Pascal's identity.

Problem 6 (Graded)

Prove by induction that $\forall n \geq 3$:

$$n^2 + 1 \geq 3n$$