Discrete Structures. CSCI-150. Fall 2013.

Homework 10. Due Wed. Nov 20, 2013.

Problem 1

Is the set $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ countable or uncountable?

Problem 2

Show that the set of real numbers that are the roots of quadratic equations $ax^2 + bx + c = 0$ with integer coefficients (i.e. $a, b, c \in \mathbb{Z}$) is countable.

Problem 3

Use the Schröder-Bernstein theorem to show that the $(0, 1) \subseteq \mathbb{R}$ and $[0, 1] \subseteq \mathbb{R}$ have the same cardinality.

Problem 4

Recall the question about the Devil and infinitely many dollar bills (the end of the Lecture 18).

You have all positive odd-numbered dollar bills 1, 3, 5, ... The Devil has all positive even-numbered bills. He is willing to pay you 2 dollars for each of your 1 dollar bills. There is one condition: He always buys from you your lowest-numbered bill, and pays with two higher-numbered bills.

The first sub-transaction takes 1/2 hour, then 1/4 hour, 1/8, and so on, so that after one hour the entire exchange will be complete.

How could the deal harm you?

Problem 5

Draw the Hasse diagram for divisibility on the set:

(a) $\{1, 2, 3, 4, 5, 6\}$, (b) $\{3, 5, 6, 9, 25, 27\}$, (c) $\{3, 5, 7, 11, 13, 16, 17\}$, (d) $\{1, 3, 9, 27, 81, 243\}$,

Problem 6

Count the number of topological sorts for each poset $(A, |)$, where

(a) $A = \{3, 5, 7, 11, 13, 16, 17\}$, (b) $A = \{1, 3, 9, 27, 81, 243\}$, (c) $A = \{2, 3, 4, 8, 9, 16, 27, 81\}$.

That is, you have to find the number of ways to order the elements of the set A so that the partial order imposed by divisibility is preserved.