Discrete Structures. CSCI-150. Fall 2013.

Homework 12. Due Wed. Dec 11, 2013.

A bit of theory first

The complement of an event A, is the event $\overline{A} = \Omega \setminus A$, thus the following properties hold: $A \cap \overline{A} = \varnothing$ and $A \cup \overline{A} = \Omega$.

$$P(\overline{A}) \equiv P(\Omega \setminus A) = 1 - P(A)$$

The formula can be very useful, because sometimes it is much easier to compute $P(A)$ rather than $P(\overline{A})$, or the other way around.

Example of A and $\overline{A}$: Five cards are drawn from a standard deck. What is the probability that there is at least one ace among them?

A: there is at least one ace. $\qquad \overline{A}$: there are no aces.

(This is not needed for the problems 1 and 2, by the way...)

Problem 1

Given six cards:

$$A\spadesuit, J\spadesuit, 2\spadesuit, A\heartsuit, 2\heartsuit, 2\diamondsuit,$$

you pick one card at random.

Consider two events:

$$\begin{aligned} A: &\quad \text{the chosen card is an ace} \\ S: &\quad \text{the chosen card is a spade} \end{aligned}$$

(a) What is the sample space Ω?

(b) Compute the probabilities $P(A)$ and $P(S)$.

(c) Are the events A and S independent?

(d) Can you find any (other?) pair of independent events for the given set of cards?

Problem 2

Three cards are drawn from a standard 52-card deck.

Each combination of three cards was equally likely, find the probability that the following hand is obtained: $\{K\spadesuit, Q\heartsuit, J\diamondsuit\}$ (this is a set, the order does not matter).

Problem 3

A fair six-sided die is rolled twice. What is the probability that the outcome of the second roll is the same as the outcome of the first roll?

Problem 4 (Birthdays)

(See the discussion in Rosen and LL).

What is the minimum number of people who need to be in a room so that the probability that at least two of them have the same birthday is greater than $1/2$?

Assume that there are $n = 366$ days in a year, and all birthdays are independent and equally likely.

Problem 5

Find each of the following probabilities when n independent Bernoulli trials are carried out with probability of success p.

(a) the probability of no successes

(b) the probability of at least one success

(c) the probability of at most one success

(d) the probability of at least two successes

Problem 6

You are playing a game, in which at every stage you can either win a dollar or lose one, with probabilities p and $1 - p$, respectively. The game is going until you don't have any money. You start with $N_0 = \$1$ in the beginning. What is the probability that after the stage n you have again $N_n = \$1$ in your bank?

Problem 7

Assume that it's observed that in each episode of The Simpsons, the probability that Homer will say "D'oh!" k times is $\frac{1}{2^{k+1}}$.

Today, you are going to watch a new episode:

(a) What is the probability that Homer will express his annoyance at least twice?

(b) What is the expected number of times he will do that during the episode?