Discrete Structures. CSCI-150. Fall 2014.

Homework 6. Due Wed. Oct 15, 2014.

Introduction

Always explain your solutions. Answers by themselves are useless and don't prove anything.

In this homework, try to refer to the rule of summation, and the rule of product, when you are using them.

When solving a combinatorial problem, for example counting the number of certain objects (bitstrings, groups of people, etc.), always **try to think how you generate one instance** of such an object. Analyze this generation process; ask yourself: **When exactly I make a choice?**

An example. Count the number of license plates of the following format: 1 or 2 letters, followed by 1, 2, or 3 digits.

There are many ways to generate such a license plate. Consider the following two methods:

(a) Method 1. We can first, generate the letters. Then generate the digits.

We have to do both subtasks. So, once we know in how many ways we can do each of the subtasks, we can, by the rule of product, multiply the numbers and obtain the answer.

Subtask 1. To generate the letters, there must be either 1 or 2 letters. By the rule of sum,

$$L = 26 + 26^2 = 702.$$

(here, 26 is the number of ways to pick 1 letter, and $26 \cdot 26 = 26^2$ is the number of ways to pick a pair of letters)

Subtask 2. To generate the digits, there can be 1, 2, or 3 digits:

$$D = 10 + 10^2 + 10^3 = 1110.$$

Therefore, the number of ways to generate a license plate is

$$L \cdot D = 702 \cdot 1110 = 779220.$$

(b) Method 2. There are 6 possibilities for a license plate:

1 letter + 1 digit	$26 \cdot 10 = 260$
1 letter + 2 digit	$26 \cdot 10^2 = 2600$
1 letter + 3 digit	$26 \cdot 10^3 = 26000$
2 letter + 1 digit	$26^2 \cdot 10 = 6760$
2 letter + 2 digit	$26^2 \cdot 10^2 = 67600$
2 letter + 3 digit	$26^2 \cdot 10^3 = 676000$

Because all these 6 cases correspond to the disjoint sets of license plates (Do you agree? What does that mean that they are disjoint?), we add the numbers up by the rule of sum, and get

$$260 + 2600 + 26000 + 6760 + 67600 + 676000 = 779220.$$

As expected, both methods give the same answer.

Problem 1 (Graded)

(a) Count the number of bitstrings of length 7.

(b) In how many ways you can paint 7 rooms, if you have two types of paint: white and beige? (Mixing the paint is not allowed).

(c) In how many ways you can paint the same 7 rooms with 10 types of paint.

(d) What if there are N rooms and M types of paint?

Problem 2 (Graded)

Assume that at a certain university, there are 3 departments: Mathematics (with 10 faculty members), Computer Science (20 faculty members), and Economics (30 faculty members).

In how many ways can we select a committee if there should be

(a) exactly 1 representative from each department.

(b) 2 persons, and they should be from different departments.

(c) 2 persons, both from the same department.

(d) 2 persons (from any department).

Problem 3 (Graded)

A *palindrome* is a string whose reversal is identical to the string.

(a) How many bit strings of length 4 are palindromes?

(b) How many bit strings of length 5 are palindromes?

(c) How many bit strings of length 6 are palindromes?

(d) How many bit strings of length n are palindromes?

(In the last question, you can provide two formulas: One when n is even, and another when it's odd).

Problem 4 (Graded)

Count the number of bit strings that start with <u>4 zeroes</u> or end with <u>3 ones</u> if the length of the bit string is

(a) 4

(b) 7

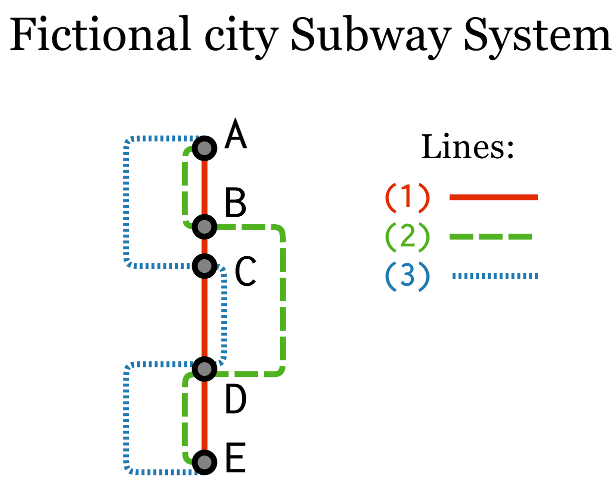
(c) 8

(d) 6

Problem 5 (Graded)

How many different sets can be made out the following 5 possible elements: a, b, c, d, e? Don't forget to count the empty set (that contains none of these elements).

Problem 6 (Graded)



In the figure above, you can see the map of a finctional subway system. There are 3 train services: (1), (2), and (3). All transfer stations are labeled with the uppercase letters.

The stations A and E are the terminals for all three trains.

If we have to count the number of ways to travel from A to E without transfers then there are, obviously, 3 ways to do so.

(a) Count the number of ways to travel from the station A to the station E, when you transfer from one train to another exactly once.

(b) The segment CD of the line (1) was closed due to construction. Repeat the task again, count the number of ways to travel from A to E with exactly one transfer.

(In case you have spare time, you may try to count the number of ways to travel from A to E transfering exactly twice).