Discrete Structures. CSCI-150. Spring 2015.

Homework 1. Due Fri. Feb 6, 2015.

Problem 1 (Graded)

Using the following propositions:

$$\begin{aligned} p &: \text{"Phyllis goes out for a walk".} \\ r &: \text{"The Moon is out".} \\ s &: \text{"It is snowing".} \end{aligned}$$

Formulate these statements in words:

(a) $(r \wedge \neg s) \to p$ (b) $r \to (\neg s \to p)$ (c) $\neg(p \leftrightarrow (s \vee r))$

Try to keep the propositions unchanged. If you really want to replace a proposition with its equivalent, first, prove that your substitution is correct.

In the question (c), you have to find a way to negate the whole sentence. I guarantee that there are ways to do that in English.

Problem 2

Consider the proposition "When it's cold, I put on a sweater".

Does this proposition have the form of an impilcation ($p \to q$), or the form of a biconditional ($p \leftrightarrow q$)? Explain your answer (don't forget to mention, what's the meaning of the propositional variables p and q in this case).

As any other proposition, it can be true or it can be false. Describe a situation when this proposition is false. Alternatively, if the statement cannot be false in any situation, explain why.

Problem 3 (Graded)

Write out the truth table for the following propositions:

(a) $p \to (\neg q)$

(b) $(p \wedge (\neg q)) \leftrightarrow \neg(p \vee (\neg q))$

(c) $r \to ((\neg s) \to p)$

Compute one operation at a time, don't skip steps.

Problem 4 (Graded)

Check if the given propositions are equivalent or not:

(a) $\neg(p \leftrightarrow s)$ and $(\neg p) \leftrightarrow (\neg s)$

(b) $p \leftrightarrow s$ and $(\neg p) \leftrightarrow (\neg s)$

(c) $(\neg p) \leftrightarrow s$ and $\neg(p \leftrightarrow s)$

Can you make any conclusions from this problem? For example, about the negation of a biconditional (if-and-only-if) proposition.

Problem 5

Prove the logical equivalence:

$$\neg((a \wedge b) \wedge c) \equiv \neg a \vee (\neg b \vee \neg c).$$

It is advised to do the proof using the known equivalences. (Hint: using De Morgan's Law and the associativity of $\vee$).

Problem 6 (Graded)

First, prove the following logical equivalence using the known equivalences:

$$p \leftrightarrow \neg q \quad \equiv \quad \neg(p \wedge q) \wedge (q \vee p)$$

After that, using the same technique, prove that

$$(p \leftrightarrow \neg q) \vee (p \wedge q) \quad \equiv \quad (p \vee q) \vee (p \wedge q)$$

Is it possible to simplify the right-hand-side expression even further?

Note that you are not allowed to use truth tables in this problem!

Problem 7

Using logical equivalences, prove that

$$p \leftrightarrow q \quad \equiv \quad (\neg p \wedge \neg q) \vee (p \wedge q)$$

Problem 8

You are given an argument, but it's incomplete. Finish the work by giving the reasons why each step was correct.

(a) Prove

$$\begin{array}{c} p \wedge q \\ q \rightarrow (r \wedge s) \\ \hline r \end{array}$$

Complete the argument

(1)	$p \wedge q$	Given.
(2)	$q \rightarrow (r \wedge s)$	Given.
(3)	q	...
(4)	$r \wedge s$	...
(5)	r	...

(b) Prove

$$\begin{array}{c} p \rightarrow (\neg s \wedge r) \\ s \vee t \\ p \\ \hline t \end{array}$$

Complete the argument

(1)	$p \rightarrow (\neg s \wedge r)$	Given.
(2)	$s \vee t$	Given.
(3)	p	Given.
(4)	$\neg s \wedge r$	...
(5)	$\neg s$	...
(6)	t	...

(c) Prove

$$\begin{array}{c} (\neg p \vee s) \leftrightarrow q \\ \neg q \\ \hline p \end{array}$$

Complete the argument

(1)	$(\neg p \vee s) \leftrightarrow q$	Given.
(2)	$\neg q$	Given.
(3)	$((\neg p \vee s) \rightarrow q) \wedge (q \rightarrow (\neg p \vee s))$	...
(4)	$(\neg p \vee s) \rightarrow q$	...
(5)	$\neg(\neg p \vee s)$	...
(6)	$\neg(\neg p) \wedge \neg s$	...
(7)	$\neg(\neg p)$	...
(7)	p	...