Discrete Structures. CSCI-150. Summer 2015.

Homework 7. Due Mon. Jun. 29, 2015.

Problems 1, 2, and 3 are simpler than 4 and 5, so you may try proving them first, before trying to prove 4 and 5.

Problem 1

For $a, b \in \mathbb{Z}$, prove that if $a \mid b$ and $b \mid a$ then $a = b$ or $a = -b$.

Problem 2

For positive $a, b \in \mathbb{Z}$, prove that if $a \mid b$ and $a \mid (b + 2)$ then $a = 1$ or $a = 2$.

Problem 3

For positive $a, b, c \in \mathbb{Z}$, prove that if $c = \gcd(a, b)$ then $c^2 \mid ab$.

Problem 4 (Graded)

First, prove that $k(k + 1)$ is even for any $k \in \mathbb{Z}$.

Then, for positive $n \in \mathbb{Z}$, prove that if n is odd then $8 \mid (n^2 - 1)$.

Hint: An integer x is even if and only if $2 \mid x$.

Problem 5 (Graded)

Prove that for all positive $n \in \mathbb{Z}$:

$$3 \mid (n^3 + 2n).$$

It can be done either by induction, or by cases.

The proof by induction is standard. If you decide to prove it by cases, consider the remainder (n **rem** 3), it can be equal to 0, 1, or 2, so we can say that for any n: $n = 3k$, or $n = 3k + 1$, or $n = 3k + 2$.

Problem 6 (Graded)

Using Euclidean algorithm, compute

(a) $\gcd(244, 28)$ (b) $\gcd(323, 177)$

Write each step of the algorithm execution.