Discrete Structures. CSCI-150. Fall 2014.

Homework 3. Due Wed. Sep 24, 2014.

Problem 1 (Graded)

Construct a contrapositive proof that for all real numbers x, if $2x - x^2 \neq 1$ then $x \neq 1$.

Problem 2 (Graded)

Prove by contradiction that there are no positive integer solutions to the equation $x^2 - y^2 = 1$.

Problem 3

Find a mistake in the "inductive proof" that all horses are the same color. The proof was given at the very end of the lecture 5.

Problem 4

The sum of the cubes of the first n positive integers can be computed by the following formula

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}.$$

Prove this formula by induction for all $n \geq 1$.

Problem 5 (Graded)

Prove by induction that for all $n \geq 1$.

$$\sum_{k=1}^{n} \frac{2}{3^k} = \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots + \frac{2}{3^n} = 1 - \frac{1}{3^n}.$$

Before doing the proof, you may check whether the formula works or not for small values of n: $n = 1$, $n = 2$, etc.

Problem 6 (Graded)

Prove by induction that $\forall n \geq 5$:

$$2^n > 4n$$

Problem 7 (Graded)

Please, prove by induction that the recurrence

$$\begin{aligned} f(1) &= 1, \\ f(n) &= 2f(n-1) + 3 \qquad (\forall n > 1) \end{aligned}$$

has the following closed-form solution

$$f(n) = 2^{n+1} - 3.$$