Discrete Structures. CSCI-150. Fall 2013.

Problem 1
How many binary string of length 12 contain exactly: (a) 2 zeroes, (b) 6 zeroes, (c) 12 zeroes?

Problem 2
Ternary strings are strings composed of three symbols: $\{0,\ 1,\ 2\}$. How many ternary string of length 12 contain exactly: (a) 2 zeroes, (b) 6 zeroes, (c) 12 zeroes?

Problem 3
How many bit strings contain exactly eight 0s and 10 1s if every 0 must be immediately followed by a 1?

Problem 4
In how many ways 20 coins can be selected from four large containers filled with pennies, nickels, dimes, and quarters?

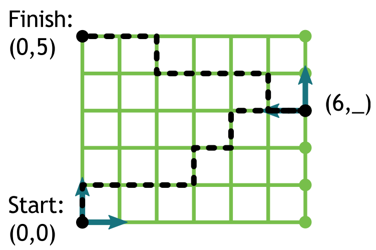
Problem 5
How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 20$, where x_1, x_2, x_3, and x_4 are nonnegative integers?

Problem 6
Count the number of ways to select 5 coins from a collection of 10 consisting of 1 penny, 1 nickel, 1 dime, 1 quarter, 1 half-collar, and 5 identical one-dollar coins.

Problem 7
In how many ways can 15 identical candy bars be distributed among five children so that the youngest gets only one or two of them?

Problem 8
In how many ways can we distribute eight identical while balls into four distinct containers so that (a) no container is left empty, (b) the fourth container has an odd number of balls in it?

Problem 9
You are in the south-western corner of a city, $(0, 0)$.
1) First, by moving **east** and **north**, you have to get to the eastern boundary of the city, $(6, y)$, for any $0 \le y \le 5$.
2) After that, by moving **west** and **north**, you need to go to the north-western corner, $(0, 5)$.
How many routes are possible.



Problem 10
Alice has an infinite supply of beads of n different colors. What is the value of n, if she can select 20 beads (with repetition of colors) in 230,230 ways.

Problem 11
How many natural numbers between 100 and 1000 are not divisible by 10? What changes if we also want all digits in the numbers to be different?

Problem 12
Let n be a positive integer. Show that

$$\binom{2n}{n+1} + \binom{2n}{n} = \binom{2n+1}{n+1} = \binom{2n+2}{n+1}/2$$