Discrete Structures. CSCI-150. Fall 2015.

Homework 6. Due Wed. Oct. 14, 2015.

Problem 1

For $a, b \in \mathbb{Z}$, prove that if $a \mid b$ and $b \mid a$ then $a = b$ or $a = -b$.

Problem 2

For positive $a, b \in \mathbb{Z}$, prove that if $a \mid b$ and $a \mid (b+2)$ then $a = 1$ or $a = 2$.

Problem 3 (Graded)

Prove that if positive integers a and b are odd then $2 \mid (a^2 + b^2)$, but $4 \nmid (a^2 + b^2)$.

Problem 4 (Graded)

First, prove that $k(k+1)$ is even for any $k \in \mathbb{Z}$.

Then, for positive $n \in \mathbb{Z}$, prove that if n is odd then $8 \mid (n^2 - 1)$.

Hint 1: An integer x is even if and only if $2 \mid x$.
Hint 2: In my opinion, using induction in the first part of the problem is an unnecessary heavy-lifting. However, if you really want an inductive proof there, please make sure that your argument covers the cases when k is positive, equal to zero, and negative.

Problem 5 (Graded)

Prove that for all positive $n \in \mathbb{Z}$:

$$3 \mid (n^3 + 2n).$$

It can be done either by induction, or by cases.

The proof by induction is standard. If you decide to prove it by cases, consider the remainder (n **rem** 3), it can be equal to 0, 1, or 2, so we can say that for any n: $n = 3k$, or $n = 3k + 1$, or $n = 3k + 2$.

Problem 6 (Graded)

Using Euclidean algorithm, compute

(a) $\gcd(234, 54)$, (b) $\gcd(416, 175)$

Write each step of the algorithm execution.