Discrete Structures. CSCI-150. Fall 2013.

Homework 2.   Due Wed. Sep 18, 2013.

Problem 1

You are given an argument, but it's incomplete. Finish the work by giving the reasons why each step was correct.

(a) Prove

$$p \wedge q$$
$$\underline{q \rightarrow (r \wedge s)}$$
$$r$$

Complete the argument

(1)	$p \wedge q$	Given.
(2)	$q \rightarrow (r \wedge s)$	Given.
(3)	q	...
(4)	$r \wedge s$	...
(5)	r	...

(b) Prove

$$p \rightarrow (\neg s \wedge r)$$
$$s \vee t$$
$$\underline{p}$$
$$t$$

Complete the argument

(1)	$p \rightarrow (\neg s \wedge r)$	Given.
(2)	$q \vee t$	Given.
(3)	p	Given.
(4)	$\neg s \wedge r$	...
(5)	$\neg s$	...
(6)	t	...

(c) Prove

$$(\neg p \vee s) \leftrightarrow q$$
$$\underline{\neg q}$$
$$p$$

Complete the argument

(1)	$(\neg p \vee s) \leftrightarrow q$	Given.
(2)	$\neg q$	Given.
(3)	$((\neg p \vee s) \rightarrow q) \wedge (q \rightarrow (\neg p \vee s))$	...
(4)	$(\neg p \vee s) \rightarrow q$	...
(5)	$\neg(\neg p \vee s)$	...
(6)	$\neg(\neg p) \wedge \neg s$	...
(7)	$\neg(\neg p)$	...
(7)	p	...

Problem 2

Prove

(a)
$$(p \lor r) \rightarrow (q \land s)$$
$$\frac{p}{s}$$

(b)
$$p \lor s$$
$$\neg p \lor r$$
$$\frac{\neg r}{s}$$

(c) (We started this problem in the class)
$$p \rightarrow r$$
$$r \rightarrow s$$
$$t \lor \neg s$$
$$\neg t \lor u$$
$$\frac{\neg u}{\neg p}$$

(d) Prove by contradiction
$$(\neg p \lor s) \leftrightarrow q$$
$$\frac{\neg q}{p}$$

Problem 3

Using the predicates $P(x)$ to denote "x is a politician", $R(x)$ to denote "x is rich", L(x) to denote "x is a lobbyist" and $K(x, y)$ to denote "x knows y", write down quantified logical stetements to express:

(a) All lobbyists are rich.

(b) Some politicians are rich.

(c) All politicians know at least one lobbyist.

(d) All politicians know a rich lobbyist.

(e) Some lobbyists know a rich politician

(f) Everyone knows a rich politician or a rich lobbyist.

The domain of discourse are all people in the world.

Hint: The predicate "x knows politician y" can be expressed as $K(x, y) \land P(y)$, meaning that x knows y, and y is a politician.