Discrete Structures. CSCI-150. Spring 2014.

Problem 1

How many binary string of length 12 contain exactly: (a) 2 zeroes, (b) 6 zeroes, (c) 12 zeroes?

Problem 2

Ternary strings are strings composed of three symbols: $\{0,\ 1,\ 2\}$. How many ternary string of length 12 contain exactly: (a) 2 zeroes, (b) 6 zeroes, (c) 12 zeroes?

Problem 3

In how many ways 20 coins can be selected from four large containers filled with pennies, nickels, dimes, and quarters?

Problem 4

How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 20$, where x_1, x_2, x_3, and x_4 are nonnegative integers?

Problem 5

Count the number of ways to select 5 coins from a collection of 10 consisting of 1 penny, 1 nickel, 1 dime, 1 quarter, 1 half-dollar, and 5 identical one-dollar coins.

Problem 6

In how many ways can 15 identical candy bars be distributed among five children so that the youngest gets only one or two of them?

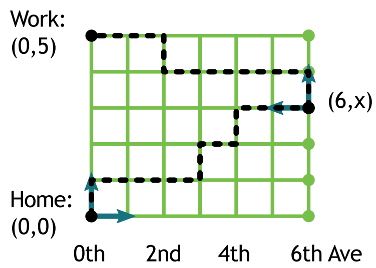
Problem 7

In how many ways can we distribute eight identical white balls into four distinct containers so that (a) no container is left empty, (b) the fourth container has an odd number of balls in it?

Problem 8

Your home is at $(0, 0)$, and you work at $(0, 5)$, five blocks to the north. However, before going to the work, you want to get a cup of coffee, but all coffee shops are located at the east-most 6th Avenue (where there is exactly one coffee shop at each intersection).

Which routes are optimal, assuming you can move in all 4 directions (north, south, west, east)? Count the number of optimal routes.



Problem 9

Alice has an infinite supply of beads of n different colors. What is the value of n, if she can select 20 beads (with repetition of colors) in 230,230 ways.

Problem 10

How many natural numbers between 100 and 1000 are not divisible by 10? What if we also want all digits in the numbers to be different?

Problem 11

Let n be a positive integer. Show that

$$\binom{2n}{n+1} + \binom{2n}{n} = \binom{2n+1}{n+1} = \binom{2n+2}{n+1}/2$$