Discrete Structures. CSCI-150. Fall 2015.

Problem 1
How many bit strings of length 12 contain exactly: (a) no zeroes, (b) 5 zeroes, (c) 12 zeroes?

Problem 2
Ternary strings are strings composed of at most three different symbols, $\{0, 1, 2\}$. How many ternary strings of length 12 contain exactly: (a) no zeroes, (b) 5 zeroes, (c) 12 zeroes?

Problem 3
A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?

Problem 4
How many 4-permutations of the positive integers not exceeding 100 contain three consecutive integers k, $k + 1$, $k + 2$, in the correct order

(a) where these consecutive integers can perhaps be separated by other integers in the permutation?

(b) where they are in consecutive positions in the permutation?

Problem 5
Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men?

Problem 6
In how many ways 20 coins can be selected from four large containers filled with pennies, nickels, dimes, and quarters?

Problem 7
How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 20$, where x_1, x_2, x_3, and x_4 are nonnegative integers?

Problem 8
Count the number of ways to select 5 coins from a collection of 10 consisting of 1 penny, 1 nickel, 1 dime, 1 quarter, 1 half-dollar, and 5 identical one-dollar coins.

Problem 9
In how many ways can 15 identical candy bars be distributed among five children so that the youngest gets only one or two of them?

Problem 10
How many ways is there to divide a group of 10 people into 5 pairs?

Problem 11
How many natural numbers between 100 and 1000 are not divisible by 10? How many of them are not divisible by 10 and their digits don't repeat? (in the standard decimal representation).

Problem 12
Let n be a positive integer. Show that $\binom{2n}{n+1} + \binom{2n}{n} = \binom{2n+1}{n+1} = \binom{2n+2}{n+1}/2$.

Answers

1. (a) $\binom{12}{0} = 1$

   (b) $\binom{12}{5} = 792$

   (c) $\binom{12}{12} = 1$

2. (a) $\binom{12}{0} \cdot 2^{12} = 2^{12}$

   (b) $\binom{12}{5} \cdot 2^7$

   (c) $\binom{12}{12} \cdot 2^0 = 1$

3. $2 \cdot n! \cdot n!$

4. (a) $98 \cdot 97 \cdot 4 - 97 = 37927$,

   (b) $98 \cdot 97 \cdot 2 - 97 = 18915$.

5. $\binom{15}{4}\binom{10}{2} + \binom{15}{5}\binom{10}{1} + \binom{15}{6}$

6. $r = 20$, $n = 4$.
$$\binom{n+r-1}{r} = \binom{23}{20} = 1771$$

7. Same as the previous problem.

8.
$$\binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5} = 2^5 = 32$$

9. $\binom{17}{14} + \binom{16}{13} = 1240$.

10. There are at least three (equivalent) ways to give the answer:

$$\frac{10!}{2!2!2!2!2! \cdot 5!}$$

Or

$$\frac{\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2}}{5!}$$

Or

$$9 \cdot 7 \cdot 5 \cdot 3 \cdot 1$$

11. Observations:
    1) A number is divisible by 10, if it ends with a zero.
    2) All integers have 3 digits
    Thus the first and the third digits are not zeroes.

    Question one: $9 \cdot 10 \cdot 9 = 810$

    Question two: $9 \cdot 1 \cdot 8 + 9 \cdot 8 \cdot 7 = 576$

    Notice that the second digit can be a zero. If it is indeed a zero, then there are $9 \cdot 8$ ways to select other two digits (2-permutation formula). If it is not a zero, then ther are $9 \cdot 8 \cdot 7$ ways (this is the number of 3-permutation).

    You can write a program to check the numbers.

12. Use Pascal's identity.