Discrete Structures. CSCI-150. Spring 2016.

Homework 5. Due Wed. Mar 9, 2016.

Problem 1 (Graded)

In how many ways can 10 identical computers be distributed among five computer stores if

(a) there are no restrictions?

(b) each store gets at least one?

(c) the largest store gets at least three?

(d) each store gets at least two?

Problem 2

Find the number of integer solutions to the equation

$$x + y + z = 12,$$

where the variables are positive integers.

Problem 3 (Graded)

We are going to prove that the following summation formula is correct for integer $n \geq 1$:

$$1 \cdot 2 + 2 \cdot 3 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

First, check that it is correct for $n = 1$, $n = 2$, and $n = 3$.

After that, prove this formula by induction for all $n \geq 1$.

Always write inductive proofs in full:

- First, write what the base case is and give its proof.
- Then the inductive case:
  1) write the assumtion (the inductive hypothesis), then
  2) write the formula you have to prove that follows from the hypothesis,
  3) then write a proof for it.

Problem 4 (Graded)

Prove by induction that

$$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right) \cdots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$$

Problem 5

Consider n lines in the infinite plane such that no two lines are parallel, and no three lines intersect at the same point (so all intersections are only pairwise). Therefore, each line intersects with each of the other lines in exactly one distinct point. See an example below for 6 lines.

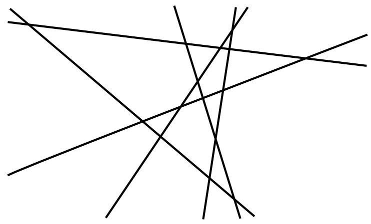

Figure 1: *An example with 6 lines*

Prove by induction that if there are n lines, then the total number of intersections is equal to $n(n-1)/2$.

Problem 6 (Graded)

Prove by induction that for all $n \geq 0$:

$$\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = 2^n$$

In the inductive step, use Pascal's identity, $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

Problem 7 (Graded)

Prove by induction that $\forall n \geq 3$:

$$n^2 + 1 \geq 3n$$