Discrete Structures. CSCI-150. Fall 2014.

Homework 7. Due Wed. Oct 22, 2014.

Problem 1

In some programming language, a variable name starts with a lowercase letter ('a'-'z') followed by any combination of lowercase letters, digits ('0'-'9'), or underscore symbols ('_'). Count the number of valid variable names of length 12.
(Answer: 4 625 858 166 265 970 738)

Problem 2 (Only (d) is graded)

How many permutations of the letters ABCDEFGH contain

(a) the string AB

(b) the string FGH

(c) the strings AB and FGH

(d) the string AB or the string FGH

Think carefully when solving the last question. A hint: $|A \cup B|$.

Problem 3 (Graded)

How many bit strings contain exactly eight 0s and ten 1s if

(a) every 0 is immediately followed by a 1?

(b) every 0 is either the last bit in the string or it is immediately followed by a 1?
So, for instance, the following string is valid: 011010111010101010.

Problem 4 (Graded)

Ellen draws 5 cards from a standard deck of 52 cards.

(a) In how many ways can her selection result in a hand with no clubs?

(b) A hand with at least one club?

Problem 5 (Graded)

A computer science professor has seven different programming books on a bookshelf. Three of the books deal with C++, the other four with Java. In how many ways can the professor arrange these books on the shelf

(a) if there are no restrictions?

(b) if the languages should alternate?

(c) if all the C++ books must be next to each other?

(d) if all the C++ books must be next to each other and all the Java books must be next to each other?

Note that all the books are distinct. Not for grade, you may consider the case when two of the C++ books are identical copies. Then, what if all the C++ books and all the Java books are identical copies?

Problem 6 (Graded)

In this problem, you have to find two proofs for the following identity (for $1 \leq k \leq n$):

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$

(a) The first proof is algebraic. Using the fact that $\binom{n}{r} = \dfrac{n!}{(n-r)!\, r!}$, show that the equation is always true. It involves some factorial manipulation and summation of fractions.

Remember: when proving the identity (or anything else), don't prove it "backwards", it's a logically inconsistent faulty technique.

(b) The second proof is called "Double counting". It's a combinatorial proof technique for showing that two expressions are equal by demonstrating that they are two ways of counting the size of one set.

In this particular case, show that two formulas: $\binom{n}{k}$ and $\binom{n-1}{k-1} + \binom{n-1}{k}$ describe two counting procedures that count the same set.

A hint: We know that the first formula, $\binom{n}{k}$, counts the number of ways to choose k objects from a set of n objects. Show that the second formula, $\binom{n-1}{k-1} + \binom{n-1}{k}$, counts the same thing.

Problem 7

How many bit strings of length 10 contain at least three 1s and at least three 0s? (Answer: 912).