Discrete Structures. CSCI-150. Summer 2015.

Homework 3. Due Thr. Jun. 11, 2015.

Introduction

Always explain your solutions. Answers by themselves are useless and don't prove anything.

In this homework, try to refer to the rule of summation, and the rule of product, when you are using them.

When solving a combinatorial problem, for example counting the number of certain objects (bitstrings, groups of people, etc.), always **try to think how you generate one instance** of such an object. Analyze this generation process; ask yourself: **When exactly I make a choice?**

An example. Count the number of license plates of the following format: 1 or 2 letters, followed by 1, 2, or 3 digits.

There are many ways to generate such a license plate. Consider the following two methods:

(a) Method 1. We can first, generate the letters. Then generate the digits.

We have to do both subtasks. So, once we know in how many ways we can do each of the subtasks, we can, by the rule of product, multiply the numbers and obtain the answer.

Subtask 1. To generate the letters, there must be either 1 or 2 letters. By the rule of sum,

$$L = 26 + 26^2 = 702.$$

(here, 26 is the number of ways to pick 1 letter, and $26 \cdot 26 = 26^2$ is the number of ways to pick a pair of letters)

Subtask 2. To generate the digits, there can be 1, 2, or 3 digits:

$$D = 10 + 10^2 + 10^3 = 1110.$$

Therefore, the number of ways to generate a license plate is

$$L \cdot D = 702 \cdot 1110 = 779220.$$

(b) Method 2. There are 6 possibilities for a license plate:

1 letter + 1 digit	$26 \cdot 10 = 260$
1 letter + 2 digit	$26 \cdot 10^2 = 2600$
1 letter + 3 digit	$26 \cdot 10^3 = 26000$
2 letter + 1 digit	$26^2 \cdot 10 = 6760$
2 letter + 2 digit	$26^2 \cdot 10^2 = 67600$
2 letter + 3 digit	$26^2 \cdot 10^3 = 676000$

Because all these 6 cases correspond to the disjoint sets of license plates (Do you agree? What does that mean that they are disjoint?), we add the numbers up by the rule of sum, and get

$$260 + 2600 + 26000 + 6760 + 67600 + 676000 = 779220.$$

As expected, both methods give the same answer.

Problem 1 (Graded)

(a) Count the number of bitstrings of length 7.

(b) In how many ways you can paint 7 rooms, if you have two types of paint: white and beige? (Mixing the paint is not allowed).

(c) In how many ways you can paint the same 11 rooms with 13 types of paint.

(d) What if there are R rooms and N types of paint?

Problem 2 (Graded)

A certain company has 5 departments, with 100, 200, 300, 400, and 500 employees respectively. In how many ways you can select:

(a) a committee of 5 persons, so that no two are from the same department,

(b) a committee of 4 persons, so that no two are from the same department,

(c) a committee of 2 persons who are not from the same department,

(d) one person from any department.

Problem 3 (Graded)

In some programming language, a variable name has to start with a lowercase letter ('a'-'z'), followed by any combination of lowercase letters, digits ('0'-'9'), or underscore symbols ('_').

Count the number of valid variable names

(a) of length n,

(b) of length at most 5.

Problem 4 (Graded)

There are 8 professors at a certain CS department. According the tentative course schedule, there are 6 distinct courses that should be taught next semester.

Please count in how many ways the teaching assignments can be distributed among the department faculty if:

(a) each professor should teach at most one class?

(b) all six classes must be taught by Professor Lamport and Professor Papadimitriou.

Problem 5 (Graded)

How many five-digit integers (in the conventional base-10 numeral system)

(a) start with a '9'?

(b) contain a '9'?

(c) do not contain a '9'?

Note that a five-digit number is different from a string of five digits.

Problem 6 (Graded)

A *palindrome* is a string whose reversal is identical to the string.

(a) How many bit strings of length 4 are palindromes?

(b) How many bit strings of length 5 are palindromes?

(c) How many bit strings of length 6 are palindromes?

(d) How many bit strings of length n are palindromes?
(In the last question, you can provide two formulas: One when n is even, and another when it's odd).

Problem 7 (Graded)

Count the number of bit strings that start with <u>4 zeroes</u> or end with <u>3 ones</u> if the length of the bit string is

(a) 4

(b) 7

(c) 8

(d) 6

Don't forget that, if you have to count the number of objects that belong to at least one of the two given sets, then the simple summation rule does not work if the sets are <u>not disjoint</u>.

In that case, you have to also subtract the cardinality of the intersection of the two sets.