Discrete Structures. CSCI-150. Summer 2016.

Homework 11. Due Thr. Jul 14, 2016.

Problem 1 (Graded)

Prove that the divisibility relation is a partial order on $\mathbb{N}$, but not on $\mathbb{Z}$.

Problem 2 (Graded)

Let Σ be the set of letters $\{a, b, \dots z\}$. Also, let Σ^* (notice the superscript star in its name) be the set of all finite string made of the letters Σ, for example

$$\text{“cat”} \in \Sigma^*, \qquad \text{“dog”} \in \Sigma^*, \qquad \text{“mathematics”} \in \Sigma^*, \qquad \text{“”} \in \Sigma^* \text{ (the empty string).}$$

We define a “permutation” relation P as follows

$$P = \{(x, y) \in \Sigma^* \times \Sigma^* \mid x \text{ is a permutation of } y\}.$$

It contains all the pairs of strings that are permutations of each other, for example

$$(\text{“flow”}, \text{“wolf”}) \in P, \qquad (\text{“teenager”}, \text{“generate”}) \in P, \qquad (\text{“player”}, \text{“replay”}) \in P.$$

Question: Is this relation P an equivalence relation, a partial order relation, or neither? (An equivalence relation is reflexive, symmetric, not antisymmetric, and transitive, and a partial order relation is reflexive, antisymmetric, not symmetric, and transitive).

Prove your claim (explain the best way you can). Your argument does not have to be very formal, a good explanation in English should be sufficient. Think of the examples of “connectivity” and “downhill” relations we had in class.

Problem 3 (Graded)

Draw the Hasse diagram for divisibility on the set:

(a) $\{1, 2, 3, 4, 5, 6\}$, (b) $\{3, 4, 7, 12, 28, 42\}$, (c) $\{3, 5, 6, 9, 25, 27\}$, (d) $\{3, 5, 7, 11, 13, 16, 17\}$,

(e) $\{6, 10, 14, 15, 21, 22, 26, 33, 35, 39, 55, 65, 77, 91, 143\}$, (f) $\{1, 3, 9, 27, 81, 243\}$.

Problem 4

Count the number of topological sorts for each poset $(A, |)$, where

(a) $A = \{3, 5, 7, 11, 13, 16, 17\}$, (b) $A = \{1, 3, 9, 27, 81, 243\}$, (c) $A = \{2, 3, 4, 8, 9, 16, 27, 81\}$.

That is, you have to find the number of ways to order the elements of the set A so that the partial order imposed by divisibility is preserved.

Problem 5

Prove that the "subset" relation ($\subseteq$) is a partial order relation, and the "proper subset" relation ($\subsetneq$) is not.

Problem 6 (Graded)

Draw these graphs: (a) K_7, (b) $K_{2,5}$, (c) C_7, (d) Q_4.
All of these special graphs are described in Rosen, K_n is the complete graph, $K_{n,m}$ is the complete bipartite graph, C_n is the cycle graph, and Q_n is the hypercube graph.

How many vertices is in K_n, $K_{n,m}$, C_n, Q_n?

Problem 7

A simple graph is called regular if every vertex of this graph has the same degree. A regular graph is called n-regular if every vertex in this graph has degree n.

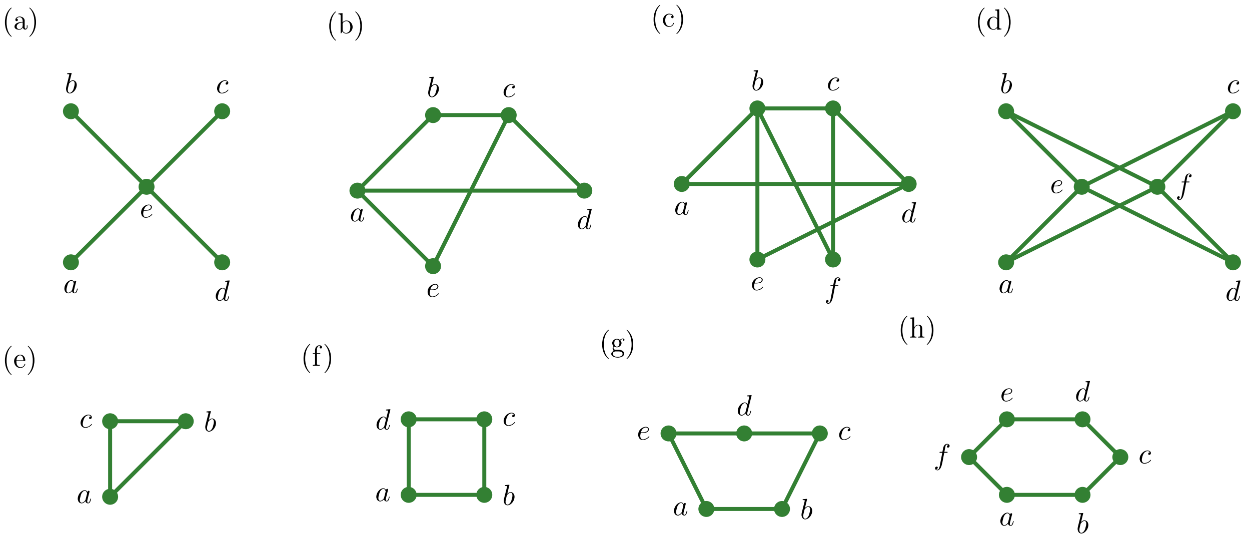
Recall thatK_n is the complete graphs with n vertices. And $K_{m,n}$ is the complete bipartite graph (see the definition in the book).

(a) Is K_n regular?

(b) For which values of m and n graph $K_{m,n}$ is regular?

(c) How many vertices does a 4-regular graph with 10 edges have?

Problem 8 (Graded)



We know that a graph is bipartite if and only if it is 2-colorable.

For the graphs given above, either prove that they are bipartite by showing that they are 2-colorable, or prove that they are not 2-colorable, and so they are not bipartite.

Problem 9 (Graded)

Given a simple graph with n vertices, prove that <u>if</u> the degree of each vertex is at least $(n-1)/2$ <u>then</u> the graph is connected.

Hint: First, you may consider a small graph, for example a graph with 5 vertices, can you make it disconnected? Then generalize to the general case with n vertices.

Problem 10

A simple graph is called n-regular if every vertex of the graph has degree n.

Show that <u>if</u> a bipartite graph $G = (V, E)$ with a bipartition of the vertex set (V_1, V_2) <u>is n-regular</u> for some positive integer n <u>then</u> $|V_1| = |V_2|$.