Discrete Structures. CSCI-150. Spring 2016.

Homework 1. Due Wed. Feb 10, 2016.

Problem 1

Using the following propositions:

p : "Phyllis goes out for a walk".
r : "The Moon is out".
s : "It is snowing".

Formulate these statements in words:

(a) $(r \wedge \neg s) \rightarrow p$ (b) $r \rightarrow (\neg s \rightarrow p)$ (c) $\neg(p \leftrightarrow (s \vee r))$

Try to keep the propositions unchanged. If you really want to replace a proposition with its equivalent, first, prove that your substitution is correct.

In the question (c), you have to find a way to negate the whole sentence. I guarantee that there are ways to do that in English.

Problem 2 (Graded)

Write out the truth tables for the following propositions:

(a) $(p \wedge (\neg q)) \leftrightarrow \neg(p \vee (\neg q))$

(b) $(p \rightarrow q) \vee (\neg r)$

Compute one operation at a time, don't skip steps.

Problem 3

An interesting question is to find the correct way to negate a biconditional, $\neg(a \leftrightarrow b)$.

A naive guess could be that we can simply distribute the negation over the biconditional, obtaining $\neg a \leftrightarrow \neg b$. We are going to check if this guess is correct or not.

Write the truth tables for the following propositional formulas:
(a) $\neg(p \leftrightarrow s)$, (b) $(\neg p) \leftrightarrow (\neg s)$, (c) $p \leftrightarrow s$, (d) $(\neg p) \leftrightarrow s$, (e) $p \leftrightarrow (\neg s)$

Decide which of these formulas are equivalent, and find what is the correct way to negate a biconditional.

Problem 4 (Graded)

Prove the logical equivalence:

$$\neg((a \wedge b) \wedge c) \equiv \neg a \vee (\neg b \vee \neg c).$$

It is advised to do the proof using the equivalence formulas we already know. (Hint: apply De Morgan's Law and the associativity of $\vee$).

Problem 5 (Graded)

Using logical equivalences, prove that

$$p \leftrightarrow q \quad \equiv \quad (\neg p \wedge \neg q) \vee (p \wedge q)$$

Hint. To prove that, you can follow these steps:

(1) First, show that

$$p \leftrightarrow q \quad \equiv \quad (\neg p \vee q) \wedge (\neg q \vee p)$$

(2) Distribute $(\neg p \vee q)$ over the disjunction $(\neg q \vee p)$.

(3) Then do smething else, eventually arriving to

$$p \leftrightarrow q \quad \equiv \quad ((\neg p \wedge \neg q) \vee \text{False}) \vee (\text{False} \vee (q \wedge p))$$

(4) Then show that the right hand side in the formula above is equivalent to $(\neg p \wedge \neg q) \vee (p \wedge q)$.

Problem 6 (only part (a) is graded)

Using logical equivalences, prove that

(a) $p \to (r \to p) \quad \equiv \quad \text{True},$

(b) $(p \to r) \vee (r \to p) \quad \equiv \quad \text{True},$

(c) $r \to (p \to (r \to p)) \quad \equiv \quad \text{True},$

in other words, we want to prove that the formulas above are tautologies (they are always true, regardless of the values of the variables p and r).

Problem 7

Using logical equivalences, prove that

$$p \wedge (p \vee t) \quad \equiv \quad p$$

The task looks difficult, because, the distributivity formula does not help.
Hint: Using the identity $A \equiv A \vee \text{False}$, represent the first p as $p \vee \text{False}$.

Problem 8 (only part (c) is graded)

You are given an argument, but it's incomplete. Finish the work by specifying which inference rule was used in each step of the argument.

(a) Prove

$$\begin{array}{c} p \land q \\ q \to (r \land s) \\ \hline r \end{array}$$

Complete the argument

$$\begin{array}{lll} (1) & p \land q & \text{Given.} \\ (2) & q \to (r \land s) & \text{Given.} \\ (3) & q & \ldots \\ (4) & r \land s & \ldots \\ (5) & r & \ldots \end{array}$$

(b) Prove

$$\begin{array}{c} p \to (\neg s \land r) \\ s \lor t \\ p \\ \hline t \end{array}$$

Complete the argument

$$\begin{array}{lll} (1) & p \to (\neg s \land r) & \text{Given.} \\ (2) & s \lor t & \text{Given.} \\ (3) & p & \text{Given.} \\ (4) & \neg s \land r & \ldots \\ (5) & \neg s & \ldots \\ (6) & t & \ldots \end{array}$$

(c) Prove

$$\begin{array}{c} (\neg p \lor s) \leftrightarrow q \\ \neg q \\ \hline p \end{array}$$

Complete the argument

$$\begin{array}{lll} (1) & (\neg p \lor s) \leftrightarrow q & \text{Given.} \\ (2) & \neg q & \text{Given.} \\ (3) & ((\neg p \lor s) \to q) \land (q \to (\neg p \lor s)) & \ldots \\ (4) & (\neg p \lor s) \to q & \ldots \\ (5) & \neg(\neg p \lor s) & \ldots \\ (6) & \neg(\neg p) \land \neg s & \ldots \\ (7) & \neg(\neg p) & \ldots \\ (7) & p & \ldots \end{array}$$