Discrete Structures. CSCI-150. Summer 2014.

Homework 7. Due Mon. June 30, 2014.

Problem 1

For $a, b \in \mathbb{Z}$, prove that if $a \mid b$ and $b \mid a$ then $a = b$ or $a = -b$.

Problem 2

For positive $a, b \in \mathbb{Z}$, prove that if $a \mid b$ and $a \mid (b + 2)$ then $a = 1$ or $a = 2$.

Problem 3 (Graded)

First, prove that $k(k + 1)$ is even for any $k \in \mathbb{Z}$.

Then, for positive $n \in \mathbb{Z}$, prove that if n is odd then $8 \mid (n^2 - 1)$.

Problem 4 (Graded)

Prove that for all positive $n \in \mathbb{Z}$:

$$3 \mid (n^3 + 2n).$$

It can be done either by induction, or by cases.

The proof by induction is standard. If you decide to prove it by cases, consider the remainder (n **rem** 3), it can be equal to 0, 1, or 2, so we can say that for any n: $n = 3k$, or $n = 3k + 1$, or $n = 3k + 2$.

Problem 5

Write out, how Euclid's algorithm computes:

(a) $\gcd(287, 120)$

(b) $\gcd(192, 33)$

(c) $\gcd(89, 144)$