Discrete Structures. CSCI-150. Summer 2014.

Homework 9. Due Mon. Jul 7, 2014.

Problem 1

Prove that if x is a multiplicative inverse of a modulo n, that is

$$x \cdot a \equiv 1 \pmod{n},$$

then $x + n$ is also a multiplicative inverse.

Then, prove that there are infinitely many multiplicative inverses of a modulo n.

Problem 2 (Graded)

Let $A = \{1, 2, 3, 4\}$, and $B = \{2, 4, 6\}$. List all the elements of the set

(a) $A \cap B$, (b) $B \cup A$, (c) $A \setminus B$, (d) $B \setminus A$, (e) $A \times B$, (f) $B \times B$, (g) $\mathcal{P}(B)$.

Problem 3 (Graded)

First, given two not equal sets A and B, prove that there exists an element x that belongs to either A or B, but not both.

Given two non-empty sets A and B, prove that if $A \neq B$ then $A \times B \neq B \times A$.

Problem 4

This is an advanced problem.

The defining property of an ordered pair is that two ordered pairs are equal if and only if their first elements are equal and their second elements are equal. Surprisingly, instead of taking the ordered pair as a primitive concept, we can construct ordered pairs using basic notions from set theory.

Show that if we define the ordered pair (a, b) to be $\{\{a\}, \{a, b\}\}$, then $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$.

Hint: Show that $\{\{a\}, \{a, b\}\} = \{\{c\}, \{c, d\}\}$ if and only if $a = c$ and $b = d$.