Discrete Structures. CSCI-150. Fall 2013.

Homework 4. Due Wed. Oct 2, 2013.

Problem 1

In how many ways can 10 dimes be distributed among five children if

(a) there are no restriction?

(b) each child gets at least one dime? (Answer: 126).

(c) the oldest child gets at least two dimes? (Answer: 495).

Problem 2

Ellen draws 5 cards from a standard deck of 52 cards.

(a) In how many ways can her selection result in a hand with no clubs?

(b) A hand with at least one club?

Problem 3

Consider the following game: Each of the two players is tossing a coin five times. To win, a player has to get more "heads" than the opponent.

(a) How many game scenatios are possible?

(b) How many games result in a tie? (The answer is between 249 and 261).

Problem 4

Show that $\binom{n}{k} = \binom{n}{n-k}$. Explain the meaning of this result.

Problem 5

Consider a bacterial cell constrained to a one-dimensional environment (some sort of tube, for example). This bacteria reproduce by binary fission: Every hour, each cell divides into two equal daughter cells. Immediately after the division, two daughter cells move away from each other and stop at points $x + 1$ and $x - 1$, where x is the original position of their parent cell before the fission.

The process starts with a single cell located at $x = 0$. We set the clock to $t = 0$ in the beginning of the experiment. After one hour, at time $t = 1$, there are two cells: at $x = -1$ and at $x = 1$, respectively. The process continues indefinitely.

Let $N(t, x)$ be the number of cells at the position x at the time t.

(a) Please, describe the dynamics of the bacterial colony.
It's recommended to derive a formula for $N(t, x)$.

(b) Compute $N(t, 0)$ for $t = 0, 1, \ldots 12$.

(c) Compute $N(10^{10}, 7^7)$.

(d) Find x such that $N(10^{100}, x) = N(10^{10}, 7^7)$.

(e) What is the total population of the colony as a function of time?