Discrete Structures. CSCI-150. Fall 2015.

Homework 3. Due Mon. Sep 21, 2015.

Problem 1 (Graded)

We are going to prove that the following summation formula is correct for integer $n \geq 1$:

$$1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}.$$

First, check that it is correct for $n = 1$, $n = 2$, and $n = 3$.

After that, prove this formula by induction for all $n \geq 1$.

Always write inductive proofs in full. First, write what the base case is and give its proof. Then the inductive case: write the assumtion and what you have to prove, then write the proof for it.

Problem 2 (Graded)

Prove by induction that

$$(1 - 1/4)(1 - 1/9) \cdots (1 - 1/n^2) = \frac{n+1}{2n}$$

Problem 3 (Graded)

Consider n lines in the infinite plane such that no two lines are parallel, and no three lines intersect at the same point (so all intersections are only pairwise). Therefore, each line intersects with each of the other lines in exactly one distinct point. See an example below for 6 lines.

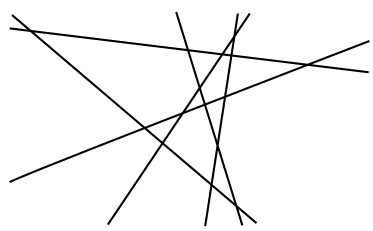

Figure 1: *An example with 6 lines*

Prove by induction that if there are n lines, then the total number of intersections is equal to $n(n-1)/2$.

Problem 4

Prove by induction that $\forall n \geq 3$:

$$n^2 + 1 \geq 3n$$