Discrete Structures. CSCI-150. Fall 2015.

Homework 4. Due Wed. Sep 30, 2015.

Problem 1

In this problem, we would like to prove by induction that the number of diagonals in a convex polygon is equal to $\dfrac{n(n-3)}{2}$.

Let $D(n)$ be the number of diagonals of a convex n-sided polygon.

(a) First, demonstrate that $D(3) = 0$. In addition, you also can show that $D(4) = 2$ and $D(5) = 5$.

(b) Then show that for for convex polygons with $n \geq 3$ sides: $D(n+1) = D(n) + n - 1$.

(c) Using these results, prove by induction that $\forall n \geq 3$:

$$D(n) = \frac{n(n-3)}{2}$$

Problem 2 (Graded)

Given the recurrence

$$\begin{aligned} &S(1) = 1, \\ &S(n) = 2S(n-1) + 3 \qquad \text{(for } n > 1) \end{aligned}$$

prove by induction that for all $n \geq 1$:

$$S(n) = 2^{n+1} - 3.$$

Problem 3 (Graded)

Given the recurrence

$$\begin{aligned} &T(1) = 2, \\ &T(n) = T(n-1) + 2n \qquad \text{(for } n > 1), \end{aligned}$$

first, find the closed form expression for $T(n)$. You may apply the method we used in class, where we repeatedly substitute $T(n)$ in terms of $T(n-1)$, then $T(n-1)$ in terms of $T(n-2)$, and so on, eventually identifying the pattern. This method is also described in Lehman and Leighton's book (p.147), where it is called "Plug-and-Chug" method.

After that, prove by induction that the closed form expression you've found is correct.

Problem 4

Solve another recurrence (do the same steps as in the previous problem):

$$\begin{aligned} R(1) &= 1, \\ R(n) &= 2R(n/2) + n^2 \qquad \text{(for } n > 1), \end{aligned}$$

(You can assume that n is a power of 2, that is, $n = 2^k$).

Hint: The closed-form formula for the recurrence will be **[** **]**

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Problem 5

This is just another problem similar to the Problem 2 if you are willing to do more exercises.

Given the recurrence

$$\begin{aligned} S(0) &= 0, \\ S(n+1) &= 3S(n) + 1, \end{aligned}$$

prove by induction that for all $n \geq 0$:

$$S(n) = \frac{3^n - 1}{2}.$$