Discrete Structures. CSCI-150. Fall 2014.

Homework 12. Due Wed. Dec 3, 2014.

Problem 1 (Graded)

Let Σ be the set of letters $\{a, b, \dots z\}$. Also, let Σ^* (notice the superscript star in its name) be the set of all finite string made of the letters Σ, for example

$$\text{“cat”} \in \Sigma^*, \qquad \text{“dog”} \in \Sigma^*, \qquad \text{“mathematics”} \in \Sigma^*, \qquad \text{“”} \in \Sigma^* \text{ (the empty string).}$$

We define a "permutation" relation P as follows

$$P = \{(x, y) \in \Sigma^* \times \Sigma^* \mid x \text{ is a permutation of } y\}.$$

It contains all the pairs of strings that are permutations of each other, for example

$$(\text{“flow”}, \text{“wolf”}) \in P, \qquad (\text{“teenager”}, \text{“generate”}) \in P, \qquad (\text{“player”}, \text{“replay”}) \in P.$$

Question: Is this relation P an equivalence relation or a partial order relation? (An equivalence relation is reflexive, symmetric, and transitive, and a partial order relation is reflexive, antisymmetric, and transitive).

Prove your claim. Your argument does not have to be very formal, a good explanation in English should be sufficient.

For those who want to do a more formal mathematical proof, observe that a string of length n is a permutation of another string of the same length if and only if there exists a bijection that specifies how to reorder the letters.

By the way, although it's not important for the problem, we could define the set Σ^* as the union of Cartiesian products $\bigcup_{k \in \mathbb{N}} \Sigma^k$, so it's the union of the strings of length 0, 1, 2, and so on.

Problem 2 (Graded)

Draw the Hasse diagram for divisibility on the set:

(a) $\{1, 2, 3, 4, 5, 6\}$, (b) $\{3, 5, 6, 9, 25, 27\}$, (c) $\{3, 5, 7, 11, 13, 16, 17\}$, (d) $\{1, 3, 9, 27, 81, 243\}$.

For the question (a), find two incomparable elements and explain why they are incomparable.

Problem 3

Count the number of topological sorts for each poset $(A, |)$, where

(a) $A = \{3, 5, 7, 11, 13, 16, 17\}$, (b) $A = \{1, 3, 9, 27, 81, 243\}$, (c) $A = \{2, 3, 4, 8, 9, 16, 27, 81\}$.

That is, you have to find the number of ways to order the elements of the set A so that the partial order imposed by divisibility is preserved.

Problem 4

For each course at a university, there may be one or more other courses that are its prerequisites. How can a graph be used to model these courses and which courses are prerequisites for which courses? Should edges be directed or undirected? Looking at the graph model, how can we find courses that do not have any prerequisites and how can we find courses that are not the prerequisite for any other courses?

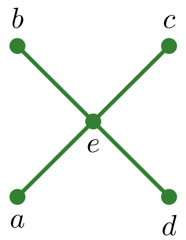
Problem 5 (Graded)

Draw these graphs: (a) K_7, (b) $K_{2,5}$, (c) C_7, (d) Q_4.
All of these special graphs are described in Rosen, K_n is the complete graph, $K_{n,m}$ is the complete bipartite graph, C_n is the cycle graph, and Q_n is the hypercube graph.

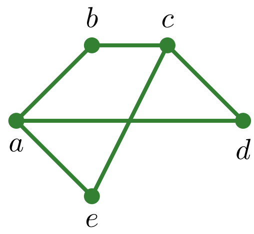
How many vertices is in K_n, $K_{n,m}$, C_n, Q_n?

Problem 6 (Graded)

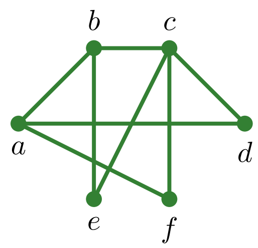
(a)



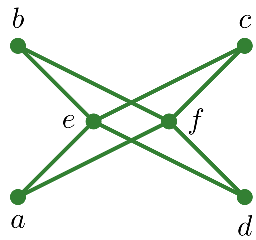
(b)



(c)



(d)



We know that a graph is bipartite if and only if it is 2-colorable.

For the graphs given above, either prove that they are bipartite by showing that they are 2-colorable, or prove that they are not 2-colorable, and so they are not bipartite.

Problem 7 (Graded)

A simple graph is called regular if every vertex of this graph has the same degree. A regular graph is called n-regular if every vertex in this graph has degree n.

(a) Is K_n regular?

(b) For which values of m and n graph $K_{m,n}$ is regular?

(c) How many vertices does a 4-regular graph with 10 edges have?