Discrete Structures. CSCI-150. Fall 2015.

Homework 11. Due Wed. Nov. 18, 2015.

Problem 1 (Graded)

Let $S = \{a, b\}$.

Prove or disprove:

(a) $a \in S$, (b) $a \in \mathcal{P}(S)$, (c) $\{a\} \subseteq S$, (d) $\{a\} \in \mathcal{P}(S)$,

(e) $\varnothing \in S$, (f) $\varnothing \subseteq S$, (g) $\varnothing \in \mathcal{P}(S)$, (h) $\varnothing \subseteq \mathcal{P}(S)$

For the proofs, writing one short sentence for each question will be sufficient, if your argument is to the point and captures the main idea why the statement is true or false.

Problem 2

(We will introduce the notion of Cartesian product in the Monday lecture, it is necessary to solve the questions (f), (g), (h), and (i)).

Let $A = \{1, 2, 3\}$, $B = \{0, 1\}$, and $C = \{x, y, z\}$.

Determine what the following sets are (list their elements):

(a) $A \cap B$, (b) $B \cup A$, (c) $A \setminus B$, (d) $(B \cap \mathbb{Z}) \setminus A$, (e) $(A \cup C) \setminus B$,

(f) $A \times B$, (g) $B \times B$, (h) $A \times B \times C$, (i) C^3,

(j) $\mathcal{P}(C)$. (k) $\mathcal{P}(B^2)$.

Problem 3 (Graded)

Prove the inclusion-exclusion formula (it's an extension of the subtraction rule)

$$\begin{aligned} |A \cup B \cup C| = &|A| + |B| + |C| \\ &- |A \cap B| - |A \cap C| - |B \cap C| \\ &+ |A \cap B \cap C|. \end{aligned}$$

To do the proof, let's denote $X = A \cup B$, then

$$|(A \cup B) \cup C| = |X \cup C|,$$

and we can apply the usual subtraction rule (you will have to apply it twice).

Problem 4

Prove that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

You may use logical equivalences to do the proof. Observe that this identity mimics the logical law of the distributivity of $\wedge$ over $\vee$.

You may also use Venn diagrams.