Discrete Structures. CSCI-150. Fall 2015.

Homework 9. Due Wed. Nov 4, 2015.

Introduction

Always explain your solutions. Answers by themselves are useless and don't prove anything.

In this homework, try to refer to the rule of summation, and the rule of product, when you are using them.

When solving a combinatorial problem, for example counting the number of certain objects (bitstrings, groups of people, etc.), always **try to think how you generate one instance** of such an object. Analyze this generation process; ask yourself: **When exactly I make a choice?**

An example. Count the number of license plates of the following format: 1 or 2 letters, followed by 1, 2, or 3 digits.

There are many ways to generate such a license plate. Consider the following two methods:

(a) Method 1. We can first, generate the letters. Then generate the digits.

We have to do both subtasks. So, once we know in how many ways we can do each of the subtasks, we can, by the rule of product, multiply the numbers and obtain the answer.

Subtask 1. To generate the letters, there must be either 1 or 2 letters. By the rule of sum,

$$L = 26 + 26^2 = 702.$$

(here, 26 is the number of ways to pick 1 letter, and $26 \cdot 26 = 26^2$ is the number of ways to pick a pair of letters)

Subtask 2. To generate the digits, there can be 1, 2, or 3 digits:

$$D = 10 + 10^2 + 10^3 = 1110.$$

Therefore, the number of ways to generate a license plate is

$$L \cdot D = 702 \cdot 1110 = 779220.$$

(b) Method 2. There are 6 possibilities for a license plate:

1 letter + 1 digit	$26 \cdot 10 = 260$
1 letter + 2 digit	$26 \cdot 10^2 = 2600$
1 letter + 3 digit	$26 \cdot 10^3 = 26000$
2 letter + 1 digit	$26^2 \cdot 10 = 6760$
2 letter + 2 digit	$26^2 \cdot 10^2 = 67600$
2 letter + 3 digit	$26^2 \cdot 10^3 = 676000$

Because all these 6 cases correspond to the disjoint sets of license plates (Do you agree? What does that mean that they are disjoint?), we add the numbers up by the rule of sum, and get

$$260 + 2600 + 26000 + 6760 + 67600 + 676000 = 779220.$$

As expected, both methods give the same answer.

Problem 1 (Graded)

(a) Count the number of bitstrings of length 11.

(b) In how many ways you can paint 11 rooms, if you have two types of paint: white and beige? (Mixing the paint is not allowed).

(c) In how many ways you can paint the same 11 rooms with 13 types of paint.

(d) What if there are R rooms and N types of paint?

Problem 2 (Graded)

A certain company has 4 departments, with 100, 200, 300, and 400 employees respectively. In how many ways you can select:

(a) a committee of 4 persons, so that no two are from the same department,

(b) a committee of 3 persons, so that no two are from the same department,

(c) one person from any department.

Problem 3

In some programming language, a variable name has to start with a lowercase letter ('a'-'z'), followed by any combination of lowercase letters, digits ('0'-'9'), or underscore symbols ('_').

Count the number of valid variable names

(a) of length n,

(b) of length at most 5.

Problem 4 (Graded)

How many five-digit integers (in the conventional base-10 numeral system)

(a) start with the digit '7'?

(b) start with a '7' or with a '9'?

(c) contain a '9'?

(d) do not contain a '9'?

Note that a five-digit number is different from a string of five digits.

Problem 5

Count the number of bit strings that start with 4 zeroes or end with 3 ones if the length of the bit string is

(a) 4, (b) 7, (c) 8, (d) 6.

Don't forget that, if you have to count the number of objects that belong to at least one of the two given sets, then the simple summation rule does not work if the sets are not disjoint.

In that case, you have to also subtract the cardinality of the intersection of the two sets.

Problem 6 (Graded)

How many bit strings of length 10 contain at least three 1s and at least three 0s?

Problem 7 (Graded)

How many bit strings contain exactly seven 0s and nine 1s such that every 0 is immediately followed by a 1?

Hint: Observe that "0" can be found in a string only as a combination "01". So, in some sense it's similar to the problem with the blocks of letters, but it's not exactly like that problem.

Problem 8 (Graded)

How many different sets can be made out the following 5 possible elements: a, b, c, d, e? Don't forget to count the empty set (that contains none of these elements).

Problem 9 (Graded)

A computer science professor has nine different programming books on a bookshelf. Four of the books deal with the programming language C++, the other five with LISP. In how many ways can the professor arrange these books on the shelf

(a) if there are no restrictions?

(b) if the languages should alternate?

(c) if all the C++ books must be next to each other?

(d) if all the C++ books must be next to each other and all the LISP books must be next to each other?

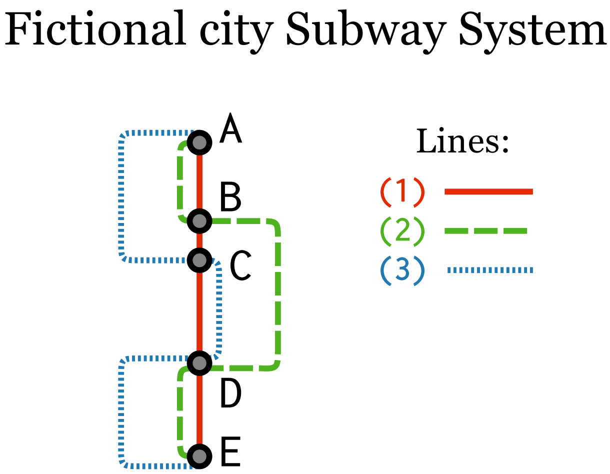
Note that all the books are distinct. Not for grade, you may consider the case when two of the C++ books are identical copies. Then, what if all the C++ books and all the LISP books are identical copies?

Problem 10 (Graded)

A pizzeria offers 777 types of pizza and 3 types of soda. Mary goes there everyday for lunch, always buying one slice of pizza and one soda. However, she never gets exactly the same thing on two consecutive days (that is, each time, either the drink or the pizza (or both) is different from what she had yesterday).

In how many ways can she plan her lunch for the next 15 days if today she tried a different pizzeria and did not like that place at all?

Problem 11



In the figure above, you can see the map of a finctional subway system. There are 3 train services: (1), (2), and (3). All transfer stations are labeled with the uppercase letters.

The stations A and E are the terminals for all three trains.

If we have to count the number of ways to travel from A to E without transfers then there are, obviously, 3 ways to do so.

(a) Count the number of ways to travel from the station A to the station E, when you transfer from one train to another exactly once.

(b) The segment CD of the line (1) was closed due to construction. Repeat the task again, count the number of ways to travel from A to E with exactly one transfer.

(In case you have spare time, you may try to count the number of ways to travel from A to E transfering exactly twice).