Discrete Structures. CSCI-150. Fall 2013.

Homework 11.   Due Wed. Dec 4, 2013.

Problem 1

For each course at a university, there may be one or more other courses that are its prerequisites. How can a graph be used to model these courses and which courses are prerequisites for which courses? Should edges be directed or undirected? Looking at the graph model, how can we find courses that do not have any prerequisites and how can we find courses that are not the prerequisite for any other courses?

Problem 2

Assuming that friendship is always mutual, prove that in any group of $n \geq 2$ persons, there are at least 2 persons with the same number of friends in the group.

Problem 3

For which values of n, does the complete graph K_n have an Euler cycle?
For which values of n and m, does the complete bipartite graph $K_{n,m}$ have an Euler cycle?

Problem 4

Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 faces, how many vertices does this graph have?

Problem 5

How many edges does a full binary tree with 10000 internal vertices have?

Problem 6

Suppose 1000 people enter a chess tournament. Use a rooted tree model of the tournament to determine how many games must be played to determine a champion, if a player is eliminated after one loss and games are played until only one entrant has not lost. (Assume there are no ties.)

Problem 7

Show that a simple graph is a tree if and only if it is connected but the deletion of any of its edges produces a graph that is not connected.

Problem 8

Use Huffman coding to encode these symbols with given frequencies:
A: 0.05, B: 0.07, C: 0.08, D: 0.10, E: 0.15, F: 0.25, G: 0.30.

What is the average number of bits required to encode a symbol?