

Homework 14.

Due Wed. Dec 9, 2015.

Problem 1 (Graded)

For which values of n , does the complete graph K_n have an Euler cycle?

For which values of n and m , does the complete bipartite graph $K_{n,m}$ have an Euler cycle?

Problem 2

Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 faces, how many vertices does this graph have?

Problem 3 (Graded)

How many edges does a full binary tree with 10000 internal vertices have?

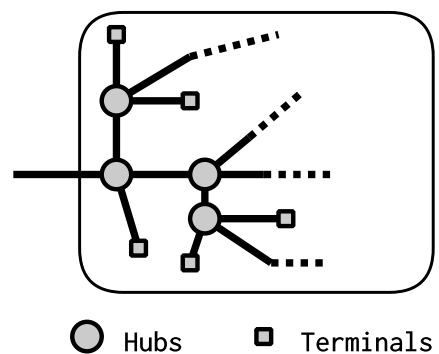
Problem 4

Suppose 1000 people enter a chess tournament. Use a rooted tree model of the tournament to determine how many games must be played to determine a champion, if a player is eliminated after one loss and games are played until only one entrant has not lost. (Assume there are no ties.)

Problem 5 (Graded)

Amtrak plans to extend their railroad network to a big island, which is connected to the continent by a bridge.

According to the plans, there will be M stations on the island. There are two types of stations: the first type are hubs that connect 4 railroads, the second type are dead-end (terminal) stations, with only one railway line. To reduce the costs, the railroads don't make loops, that is, there are *no simple cycles* in the network, so the system is cheaper, although all stations are connected. Only one of the hubs is directly connected to the outside world.



How many hubs, and how many terminals will be built? (The total number of stations is M).

Problem 6

Show that a simple graph is a tree if and only if it is connected but the deletion of any of its edges produces a graph that is not connected.

Problem 7 (Graded)

Use Huffman coding to encode these symbols with given frequencies:

A : 0.05, B : 0.07, C : 0.08, D : 0.10, E : 0.15, F : 0.25, G : 0.30.

Show all intermediate steps. What is the average number of bits required to encode a symbol?

Problem 8 (Graded)

Three cards are drawn from a standard 52-card deck. Each combination of three cards was equally likely.

Find the probability that the drawn hand is

- (a) $\{K\spadesuit, Q\heartsuit, J\diamondsuit\}$ (a hand is a set, that is, the card order does not matter).
- (b) King, Queen, and Jack of any suit.
- (c) At least one Ace.