Discrete Structures. CSCI-150. Summer 2016.

Homework 13. Due Mon. Jul 25, 2016.

A bit of theory first

The complement of an event A, is the event $\overline{A} = \Omega \setminus A$, thus the following properties hold: $A \cap \overline{A} = \varnothing$ and $A \cup \overline{A} = \Omega$.

$$P(\overline{A}) \equiv P(\Omega \setminus A) = 1 - P(A)$$

The formula can be very useful, because sometimes it is much easier to compute $P(A)$ rather than $P(\overline{A})$, or the other way around.

Example of A and $\overline{A}$: Five cards are drawn from a standard deck. What is the probability that there is at least one ace among them?

A: there is at least one ace. $\overline{A}$: there are no aces.

(This is not needed for the problem 1, by the way).

Problem 1 (Graded)

Given six cards:

$$A\spadesuit, J\spadesuit, 2\spadesuit, A\heartsuit, 2\heartsuit, 2\diamondsuit,$$

you pick one card at random.

Consider two events:

$$\begin{aligned} A: &\quad \text{the chosen card is an ace} \\ S: &\quad \text{the chosen card is a spade} \end{aligned}$$

(a) What is the sample space Ω?

(b) Compute the probabilities $P(A)$ and $P(S)$.

(c) Are the events A and S independent?

(d) Can you find any (other?) pair of independent events for the given set of cards?

Problem 2 (Graded)

There is a 10-volume encyclopedia on your bookshelf. The volumes are arranged in the order of increasing number of pages (from the thinnest to the thickest):

5 pages, 10 pages, 20 pages, 40 pages, 80 pages, ... 2560 pages

(so, every subsequent book is twice as thick as the previous).

Your grandmother left an important note on one of the pages of those books, but you don't know the book and the page.

(a) Assuming she could choose any page with equal probability, what is the probability that the note is in the volume number 4?

(b) What's the probability that the note is in one of the thinner books (volumes 1 through 5)?

(c) In one the the thicker books (volumes 6 through 10)?

Problem 3

Three cards are drawn from a standard 52-card deck. Each combination of three cards was equally likely.

Find the probability that the drawn hand is

(a) $\{K\spadesuit, Q\heartsuit, J\diamondsuit\}$ (a hand is a set, that is, the card order does not matter).

(b) King, Queen, and Jack of any suit.

(c) At least one Ace.

Problem 4 (Graded)

A project was implemented by three developers: Alice, Bob, and Carol. They used four languages: C, C++, Python, and JavaScript. The table summarizes what fraction of the code was written by each person in each language.

	C	C++	Python	JavaScript
Alice	5/24	1/8	1/6	0
Bob	1/24	1/8	1/12	0
Carol	0	0	1/12	1/6

You pick a piece of code at random.

(a) Who is most likely to be the author of that piece of code?

(b) Who is most likely to be the author given that it was written in JS?

(c) Who is most likely to be the author given that it was written in C or C++?

(d) What is the probability that it was written by Bob? Does the probability change if we know that the code is in Python? Are the events *Python* and *Bob* independent or not?

(e) Are the events *Alice* and C independent?

(f) The same question for $Carol$ and JS.

Problem 5

A fair six-sided die is rolled twice. What is the probability that the outcome of the second roll is the same as the outcome of the first roll?

Problem 6 (Graded)

Given a complete bipartite graph $K_{n,m}$, you paint its nodes **black** or **white** choosing both colors with equal probability.

Find the probability that the result is a correct node coloring (that is, no two adjacent nodes have the same color).

Problem 7 (Graded)

Find each of the following probabilities when n independent Bernoulli trials are carried out with probability of success p.

(a) the probability of no successes

(b) the probability of at least one success

(c) the probability of at most one success

(d) the probability of at least two successes