Discrete Structures. CSCI-150. Fall 2013.

Homework 5. Due Wed. Oct 9, 2013.

Problem 1

The partial sum of the cubes of natural numbers can be computed using the following formula

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

Check that it is correct for $n = 1$ and $n = 2$.

After that, prove this formula by induction for all $n \geq 1$.

Problem 2

Let $D(n)$ be the number of diagonals of a convex n-sided polygon.

(a) Demonstrate that $D(3) = 0$, $D(4) = 2$, and $D(5) = 5$.

(b) Show that for $n \geq 3$: $D(n+1) = D(n) + n - 1$

(c) By induction, prove that $\forall n \geq 3$:

$$D(n) = \frac{n(n-3)}{2}$$

Problem 3

Prove by induction that for all $n \geq 0$:

$$\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = 2^n$$

In the inductive step, use Pascal's identity.

Problem 4

Prove by induction that $\forall n \geq 3$:

$$n^2 + 1 \geq 3n$$