Discrete Structures. CSCI-150. Spring 2014. Review.

Problem 1

First, prove that $k(k+1)$ is even for any $k \in \mathbb{Z}$.
Then, for positive $n \in \mathbb{Z}$, prove that if n is odd then $8 \mid (n^2 - 1)$.

Problem 2

Modular exponentiation. Prove that $3^{23} \equiv 5 \pmod{11}$.
You are allowed to use a calculator only for computing multiplication, division, addition, and subtraction. Particularly, not allowed to use the power function.

Problem 3

Given two numbers,

$$a_0 = 133, \quad a_1 = 129,$$

Find $a_k = \gcd(a_0, a_1)$ and Bezout's coefficients x_k and y_k, i.e. the numbers such that the following equation is satisfied:

$$a_k = \gcd(a_0, a_1) = x_k a_0 + y_k a_1$$

If it's possible, find the multiplicative inverse of a_1 modulo a_0.

(Note that the multiplicative iniverse exists if and only if a_1 and a_0 are relative primes)

Problem 4

Given three sets

$$A = \{1, 2, 3\}, \quad B = \{1, 2, 3, 4\}, \quad C = \{1, 2, 3, 4, 5\},$$

construct the following functions (or prove that they don't exist):

1. one-to-one function from A to B
2. one-to-one function from C to B
3. onto function from C to B
4. bijection from A to B

Problem 5

Draw the Hasse diagram for divisibility on the set $\{2, 4, 5, 6, 10, 12\}$.

Find the maximal elements. Find the minimal elements.

Construct a topological sort of this poset.

Problem 6

Find two incomparable elements in these posets.
(a) $(\mathcal{P}(\{0, 1, 2\}), \subseteq)$, where $\mathcal{P}(X)$ denotes the powerset of X.

(b) $(\{1, 2, 4, 6, 8\}, |)$

Problem 7

Given a multigraph G, does it have an Eulerian cycle? Does it have an Eulerian path? (A drawing of the graph will be supplied).

Problem 8

We define the complement G^c of a graph G as the graph on the same vertex set in which two vertices are joined by an edge if and only if they are not joined by an edge in G. Prove that it cannot happen that both G and G^c are disconnected.

Problem 9

How many leaves does a full 3-ary tree with 100 vertices have?

Problem 10

Use Huffman coding to encode these symbols with given frequencies:
A: 0.05, B: 0.07, C: 0.08, D: 0.10, E: 0.15, F : 0.25, G: 0.30.

What is the average number of bits required to encode a symbol?

Problem 11

By rolling a six-sided die 6 times, a strictly increasing sequence of numbers was obtained, what is the probability of such an event?

Problem 12

A project was implemented by three developers: Alice, Bob, and Carol. They used four languages: C, C++, Python, and JavaScript. The table summarizes what fraction of the code was written by each person in each language.

	C	C++	Python	JavaScript
Alice	5/24	1/8	1/6	0
Bob	1/24	1/8	1/12	0
Carol	0	0	1/12	1/6

You pick a piece of code at random.

(a) Who is most likely to be the author of that piece of code?

(b) Who is most likely to be the author given that it was written in JS?

(c) Who is most likely to be the author given that it was written in C or C++?

(d) What is the probability that it was written by Bob? Does the probability change if we know that the code is in Python? Are the events $Python$ and Bob independent or not?

(e) Are the events $Alice$ and C independent?

(f) The same question for $Carol$ and JS.