Discrete Structures. CSCI-150. Summer 2014.

$(a \land b) \equiv (b \land a)$ commutativity of $\land$

$(a \lor b) \equiv (b \lor a)$ commutativity of $\lor$

$((a \land b) \land c) \equiv (a \land (b \land c))$ associativity of $\land$

$((a \lor b) \lor c) \equiv (a \lor (b \lor c))$ associativity of $\lor$

$\neg(\neg a) \equiv a$ double-negation elimination

$(a \to b) \equiv (\neg b \to \neg a)$ contraposition

$(a \to b) \equiv (\neg a \lor b)$ implication elimination

$(a \leftrightarrow b) \equiv (a \to b) \land (b \to a)$ biconditional elimination

$\neg(a \land b) \equiv (\neg a \lor \neg b)$ De Morgan's Law

$\neg(a \lor b) \equiv (\neg a \land \neg b)$ De Morgan's Law

$(a \land (b \lor c)) \equiv (a \land b) \lor (a \land c)$ distributivity of $\land$ over $\lor$

$(a \lor (b \land c)) \equiv (a \lor b) \land (a \lor c)$ distributivity of $\lor$ over $\land$

$a \land$ True $\equiv a$ identity

$a \lor$ False $\equiv a$ identity

$a \lor$ True $\equiv$ True domination

$a \land$ False $\equiv$ False domination

$a \lor \neg a \equiv$ True complementation (excluded middle)

$a \land \neg a \equiv$ False complementation (non-contradiction)

$$\frac{p}{p \lor q}$$ "$\lor$-Introduction"

$$\frac{\begin{array}{c}\neg q\\ p \to q\end{array}}{\neg p}$$ "Modus Tollens"

$$\frac{\begin{array}{c}p\\ q\end{array}}{p \land q}$$ "$\land$-Introduction"

$$\frac{\begin{array}{c}p \to q\\ q \to r\end{array}}{p \to r}$$ "Hypothetical Syllogism"

$$\frac{p \land q}{p}$$ "$\land$-Elimination"

$$\frac{\begin{array}{c}p \lor q\\ \neg q\end{array}}{p}$$ "Disjunctive Syllogism"

$$\frac{\begin{array}{c}p\\ p \to q\end{array}}{q}$$ "Modus Ponens"

$$\frac{\begin{array}{c}p \lor q\\ \neg p \lor r\end{array}}{q \lor r}$$ "Resolution"

$$\frac{\text{assuming } p\text{, we infer } q}{p \to q}$$ "$\to$-Introduction" (Deduction theorem)

$$\frac{\text{assuming } p\text{, we infer a contradiction}}{\neg p}$$ "Proof by contradiction"