Discrete Structures. CSCI-150. Spring 2016.

Homework 12. Due Wed. May 18, 2016.

Problem 1 (Graded)

How many edges does a full binary tree with 10000 internal vertices have?

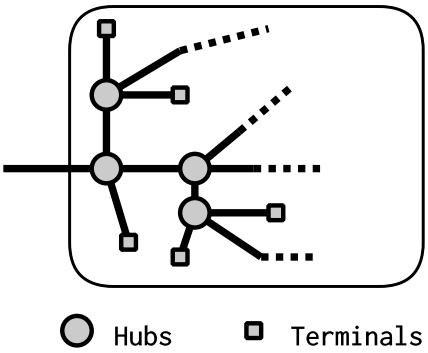
Problem 2

Suppose 1000 people enter a chess tournament. Use a rooted tree model of the tournament to determine how many games must be played to determine a champion, if a player is eliminated after one loss and games are played until only one entrant has not lost. (Assume there are no ties.)

Problem 3 (Graded)

Amtrak plans to extend their railroad network to a big island, which is connected to the continent by a bridge.

According to the plans, there will be M stations on the island. There are two types of stations: the first type are hubs that connect 4 railroads, the second type are dead-end (terminal) stations, with only one railway line. To reduce the costs, the railroads don't make loops, that is, there are *no simple cycles* in the network, so the system is cheaper, although all stations are connected. Only one of the hubs is directly connected to the outside world.



How many hubs, and how many terminals will be built? (The total number of stations is M).

Problem 4 (Graded)

Use Huffman coding to encode these symbols with given frequencies:
A: 0.05, B: 0.07, C: 0.08, D: 0.10, E: 0.15, F: 0.25, G: 0.30.

Show all intermediate steps. What is the average number of bits required to encode a symbol?

Problem 5 (Graded)

You've got infinitely-many dollar bills, all numbered by natural numbers: 0, 1, 2, 3, ... No wonder, the Devil shows up, offering you to play a game: According to the rules, you will have to guess the infinite bit-string the Devil has in mind.

(Every time you guess, you provide an entire bit-string and the Devil tells you is it correct or not, you cannot guess bits one by one. Everything is instantaneous, so no problem with time.)

For every failed attempt you pay with your lowest-numbered bill. However, if you succeed, he promises to return you all the money he won, plus give you the philosopher's stone, the

mysterious substance that grants immortality and is capable of turning any metal into gold. Also, if you played but lost, the Devil promises to reveal the bit-string he had in mind (so you could verify that he did not cheat).

What strategy the Devil can follow to get all your money without breaking the rules?

Alternatively, if you think that there is a way to outsmart the Devil, describe your strategy.