Answers

1. (a) $\binom{12}{2}$
   (b) $\binom{12}{6}$
   (c) $\binom{12}{12}$
2. (a) $\binom{12}{2} \cdot 2^{10}$
   (b) $\binom{12}{6} \cdot 2^{6}$
   (c) $\binom{12}{12} \cdot 2^{0}$
3. $r = 20$, $n = 4$.
$$\binom{n+r-1}{r} = \binom{23}{20} = 1771$$
4. Same as problem 4.
5. $$\binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5} = 2^5 = 32$$
6. Case 1. The youngest gets one candy bar: $r = 14$, $n = 4$. There are $\binom{17}{14}$ ways to distribute.

   Case 2. The youngest gets two cand bars: $r = 13$, $n = 4$. $\binom{16}{13}$ ways.

   The two cases are disjoint, so by the rule of sum $\binom{17}{14} + \binom{16}{13} = 1600$.
7. (a) $r = 4$, $n = 4$. $\binom{7}{4} = 35$.
   (b) Four disjoint cases:
   $r = 7$, $n = 3$. $\binom{9}{7}$ ways.
   $r = 5$, $n = 3$. $\binom{7}{5}$ ways.
   $r = 3$, $n = 3$. $\binom{5}{3}$ ways.
   $r = 1$, $n = 3$. $\binom{3}{1}$ ways.
   In total, $\binom{9}{7} + \binom{7}{5} + \binom{5}{3} + \binom{3}{1} = 70$.
8. We assume that the paths are different, if they have different stopping points at the 6th Avenue.

   There are 6 ways to select a stopping point $(6, k)$. This makes six disjoint cases ($k = 0, 1, 2, \ldots 5$).

   For each stopping point, we apply the product rule: The first subtask is to get to that point, and the second subtask is to get to the north-western corner. Therefore, there answer is
$$\sum_{k=0}^{5} \binom{6+k}{k}\binom{6+(5-k)}{5-k} = \binom{6}{0}\binom{11}{5} + \binom{7}{1}\binom{10}{4} + \binom{8}{2}\binom{9}{3} + \binom{9}{3}\binom{8}{2} + \binom{10}{4}\binom{7}{1} + \binom{11}{5}\binom{6}{0}$$
   In principle, this is just a combination of a sum rule and product rule.
9. We know that $r = 20$, and
$$\binom{n+r-1}{20} = \binom{n+19}{20} = 230230$$
   You can try all different n, one by one, until you get the answer.

Eventually, you get that $n = 7$ is the correct answer.

There is one simplification to the solution. Because $230230 = 230 \cdot 1001 = 23 \cdot 10 \cdot 7 \cdot 7 \cdot 20$, we could deduce that $n \geq 4$, so we did not really have to check all the numbers, and could start with $n = 4$, and go up.

10. Observations:
    1) A number is divisible by 10, if it ends with a zero.
    2) All integers have 3 digits
    Thus the first and the third digits are not zeroes.

    Question one: $9 \cdot 10 \cdot 9 = 810$

    Question two: $9 \cdot 1 \cdot 8 + 9 \cdot 8 \cdot 7 = 576$

    Notice that the second digit can be a zero. If it is indeed a zero, then there are $9 \cdot 8$ ways to select other two digits (2-permutation formula). If it is not a zero, then ther are $9 \cdot 8 \cdot 7$ ways (this is the number of 3-permutation).

    You can write a program to check the numbers (it can be interesting to see if we are correct or not, let me know if you try that).

11. Use Pascal's identity.