Discrete Structures. CSCI-150. Spring 2016.

Homework 6. Due Wed. Mar 16, 2016.

Problem 1 (Graded)

Given the recurrence

$$S(0) = 0,$$
$$S(n+1) = 3S(n) + 1,$$

prove by induction that for all $n \geq 0$:

$$S(n) = \frac{3^n - 1}{2}.$$

Problem 2 (Graded)

Given the recurrence

$$T(0) = 1,$$
$$T(n) = n! + n \cdot T(n-1) \qquad \text{(for } n > 0\text{)},$$

first, find the closed form expression for $T(n)$. Apply the method we used in class, where we repeatedly substitute $T(n)$ in terms of $T(n-1)$, then $T(n-1)$ in terms of $T(n-2)$, and so on, eventually identifying the pattern. This method is also described in Lehman and Leighton's book (p.147), where it is called "Plug-and-Chug" method.

After that, prove by induction that the closed form expression you've found is correct.

Hint: The closed-form solution should be [ ]
(select the line above to see the spoiler; If your PDF viewer cannot do so, feel free to ask me)

Problem 3

Solve another recurrence (do the same steps as in the previous problem):

$$R(1) = 1,$$
$$R(n) = 2R(n/2) + n^2 \qquad \text{(for } n > 1\text{)},$$

(You can assume that n is a power of 2, that is, $n = 2^k$).

Hint: The closed-form formula for the recurrence will be [ ]
(select the line above to see the spoiler; If your PDF viewer cannot do so, feel free to ask me)

Linear recurrences (we will learn it on Monday, March 14)

Problem 4

Solve the linear recurrence (for $n \geq 0$)

$$f(0) = 1, \quad f(1) = -1,$$
$$f(n) = f(n-2).$$

Although this problem is not graded, it's easier than the other linear recurrences in this homework, so you are advised to do it first, before solving problems 6 and 7.

Problem 5

Solve the linear recurrence (for $n \geq 1$)

$$f(1) = 10, \quad f(2) = -2,$$
$$f(n) = f(n-1) + 12f(n-2).$$

Problem 6 (Graded)

Solve linear recurrence

$$f(0) = 3, \quad f(1) = 1,$$
$$f(n) = 4f(n-1) + 21f(n-2).$$

Problem 7 (Graded)

First, verify that $x^3 - 3x^2 + 4 = (x^2 - 4x + 4)(x+1)$.

Then, solve the linear recurrence

$$f(0) = 1, \quad f(1) = 0, \quad f(2) = 14,$$
$$f(n) = 3f(n-1) - 4f(n-3).$$

Problem 8 (Graded)

Consider a rewriting operation that transforms a given bit string into a new bit string according the following two rules:

each **1** is replaced by **100**

each **0** is replaced by **1**

For example, if we apply this rewriting operation several times to the string "**1**", it will be transforming as follows:

$$\mathbf{1} \mapsto \mathbf{100} \mapsto \mathbf{10011} \mapsto \mathbf{10011100100} \mapsto \cdots$$

(a) Assume that the starting string is "**1**" as in the example above. Find a recurrent formula for **the number of 1s in the string after n rewrites**.

(b) Prove that the recurrence has the following closed-form solution: $\dfrac{2^{n+1} + (-1)^n}{3}$.