Discrete Structures. CSCI-150. Spring 2015.

Homework 9. Due Fri. Apr 24, 2015.

Problem 1 (Graded)

Let $A = \{1, 2, 3, 4\}$, and $B = \{3, 4, 5\}$. List all the elements of the set

(a) $A \cap B$, (b) $B \cup A$, (c) $A \setminus B$, (d) $B \setminus A$, (e) $A \times B$, (f) $B \times B$, (g) $\mathcal{P}(B)$.

Problem 2

First, given two not equal sets A and B, prove that there exists an element x that belongs to either A or B, but not both.

Given two non-empty sets A and B, prove that if $A \neq B$ then $A \times B \neq B \times A$.

Problem 3 (Graded)

Prove the inclusion-exclusion formula (it's an extension of the subtraction rule)

$$\begin{aligned}|A \cup B \cup C| =& |A| + |B| + |C| \\ & - |A \cap B| - |A \cap C| - |B \cap C| \\ & + |A \cap B \cap C|.\end{aligned}$$

To do the proof, let's denote $X = A \cup B$, then

$$|(A \cup B) \cup C| = |X \cup C|,$$

and we can apply the usual subtraction rule (you will have to apply it twice).

Problem 4

Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ that is

(a) one-to-one, but not onto,

(b) onto, but not one-to-one,

(c) neither one-to-one, nor onto.

(d) Function $f : \mathbb{N} \to \mathbb{N} \times \{0, 1\}$ that is onto and one-to-one (bijection).

When constructing the functions, try to define them by formulas. (Feel free to use such operations as *absolute value*, *floor*, *ceiling*, *remainder*, in addition to normal arithmetical operations).

By definition, $\mathbb{N}$ is the set of all non-negative integers: $\mathbb{N} = \{0, 1, 2, \ldots\}$.

For each function, explain (in the best way you can) why they satisfy the required conditions.

Problem 5 (Graded)

Draw the diagrams (as we did in class) for all bijections $f : A \to A$ when the set A is

(a) $A = \{1\}$

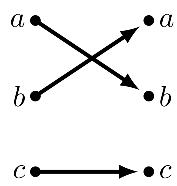
(b) $A = \{1, 2\}$

(c) $A = \{1, 2, 3\}$

(d) For this question, either repeat the task for $A = \{1, 2, 3, 4\}$, or derive a formula for the total number of bijections from A to A, when $|A| = n$. (Explain your answer).

Example of the diagram for a function

$f : \{a, b, c\} \to \{a, b, c\}$



Problem 6 (Graded)

(a) Please count how many functions

$$f \; : \; D \to \{0, 1\}$$

can be defined if the domain D is a finite set with the cardinality $|D| = n$.

(b) Can you find a bijection between the set of all such functions and the powerset $\mathcal{P}(D)$?

Bonus (extra credit).

There will be a bonus problem for this homework, posted separately.