Discrete Structures. CSCI-150. Fall 2013.

Homework 2. Due Wed. Sep 18, 2013.

Problem 1

You are given an argument, but it's incomplete. Finish the work by giving the reasons why each step was correct.

(a) Prove

$$\begin{array}{c} p \wedge q \\ q \to (r \wedge s) \\ \hline r \end{array}$$

Complete the argument

(1)	$p \wedge q$	Given.
(2)	$q \to (r \wedge s)$	Given.
(3)	q	...
(4)	$r \wedge s$	...
(5)	r	...

(b) Prove

$$\begin{array}{c} p \to (\neg s \wedge r) \\ s \vee t \\ p \\ \hline t \end{array}$$

Complete the argument

(1)	$p \to (\neg s \wedge r)$	Given.
(2)	$s \vee t$	Given.
(3)	p	Given.
(4)	$\neg s \wedge r$	...
(5)	$\neg s$	...
(6)	t	...

(c) Prove

$$\begin{array}{c} (\neg p \vee s) \leftrightarrow q \\ \neg q \\ \hline p \end{array}$$

Complete the argument

(1)	$(\neg p \vee s) \leftrightarrow q$	Given.
(2)	$\neg q$	Given.
(3)	$((\neg p \vee s) \to q) \wedge (q \to (\neg p \vee s))$	...
(4)	$(\neg p \vee s) \to q$	...
(5)	$\neg(\neg p \vee s)$	...
(6)	$\neg(\neg p) \wedge \neg s$	...
(7)	$\neg(\neg p)$	...
(7)	p	...

Problem 2

Prove

(a)

$$\begin{array}{c} (p \vee r) \to (q \wedge s) \\ p \\ \hline s \end{array}$$

(b)

$$\begin{array}{c} p \vee s \\ \neg p \vee r \\ \neg r \\ \hline s \end{array}$$

(c) (We started this problem in the class)

$$\begin{array}{c} p \to r \\ r \to s \\ t \vee \neg s \\ \neg t \vee u \\ \neg u \\ \hline \neg p \end{array}$$

(d) Prove by contradiction

$$\begin{array}{c} (\neg p \vee s) \leftrightarrow q \\ \neg q \\ \hline p \end{array}$$

Problem 3

Using the predicates $P(x)$ to denote "x is a politician", $R(x)$ to denote "x is rich", L(x) to denote "x is a lobbyist" and $K(x, y)$ to denote "x knows y", write down quantified logical stetements to express:

(a) All lobbyists are rich.

(b) Some politicians are rich.

(c) All politicians know at least one lobbyist.

(d) All politicians know a rich lobbyist.

(e) Some lobbyists know a rich politician

(f) Everyone knows a rich politician or a rich lobbyist.

The domain of discourse are all people in the world.

Hint: The predicate "x knows politician y" can be expressed as $K(x, y) \wedge P(y)$, meaning that x knows y, and y is a politician.