Discrete Structures. CSCI-150. Summer 2015.

Homework 5. Due Thr. Jun 18, 2015.

Problem 1 (Graded)

We are going to prove that the following summation formula is correct for integer $n \geq 1$:

$$1 \cdot 2 + 2 \cdot 3 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

First, check that it is correct for $n = 1$, $n = 2$, and $n = 3$.

After that, prove this formula by induction for all $n \geq 1$.

Always write inductive proofs in full. First, write what the base case is and give its proof. Then the inductive case: write the assumtion and what you have to prove, then write the proof for it.

Problem 2

Prove by induction that

$$(1 - 1/4)(1 - 1/9) \cdots (1 - 1/n^2) = \frac{n+1}{2n}$$

Problem 3

Prove by induction that $\forall n \geq 1$:

$$1^3 + 2^3 + \ldots + n^3 = (1 + 2 + \ldots + n)^2.$$

Problem 4 (Graded)

Prove by induction that for all $n \geq 0$:

$$\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = 2^n$$

In the inductive step, use Pascal's identity, $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

Problem 5

Prove by induction that $\forall n \geq 3$:

$$n^2 + 1 \geq 3n$$