Discrete Structures. CSCI-150. Summer 2014.

Homework 10. Due Thr. Jul 10, 2014.

Problem 1 (Graded) (not a hard problem, but question (d) is non-trivial)

Draw the diagrams (as we did in class) for all bijections from A to A when

(a) $A = \{1\}$

(b) $A = \{1, 2\}$

(c) $A = \{1, 2, 3\}$

(d) For this question, either repeat the task for $A = \{1, 2, 3, 4\}$, or derive a formula for the total number of bijections from A to A, when $|A| = n$.

Problem 2

Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ that is

(a) one-to-one, but not onto,

(b) onto, but not one-to-one,

(c) neither one-to-one, nor onto,

(d) onto and one-to-one (bijection), which is not the identity function $f(x) = x$.

When constructing the functions, try to define them by formulas. (Feel free to use such operations as *absolute value*, *floor*, *ceiling*, *remainder*, in addition to normal arithmetical operations).

By definition, $\mathbb{N}$ is the set of all non-negative integers: $\mathbb{N} = \{0, 1, 2, \ldots\}$.

For each function, explain why they satisfy the required conditions.

Problem 3 (Graded)

Draw the Hasse diagram for divisibility on the set:

(a) $\{1, 2, 3, 4, 5, 6\}$, (b) $\{3, 5, 6, 9, 25, 27\}$, (c) $\{3, 5, 7, 11, 13, 16, 17\}$, (d) $\{1, 3, 9, 27, 81, 243\}$,

Problem 4 (Graded) (question (c) is not easy)

Count the number of topological sorts for each poset $(A, |)$, where

(a) $A = \{3, 5, 7, 11, 13, 16, 17\}$, (b) $A = \{1, 3, 9, 27, 81, 243\}$, (c) $A = \{2, 3, 4, 8, 9, 16, 27, 81\}$.

That is, you have to find the number of ways to order the elements of the set A so that the partial order imposed by divisibility is preserved.

Problem 5 (Graded) (rather easy question)

For each course at a university, there may be one or more other courses that are its prerequisites. How can a graph be used to model these courses and which courses are prerequisites for which courses? Should edges be directed or undirected? Looking at the graph model, how can we find courses that do not have any prerequisites and how can we find courses that are not the prerequisite for any other courses?

Problem 6 (Graded)

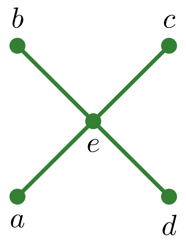
Draw these graphs: (a) K_7, (b) $K_{2,5}$, (c) C_7, (d) Q_4.
All of these special graphs are described in Rosen, K_n is the complete graph, $K_{n,m}$ is the complete bipartite graph, C_n is the cycle graph, and Q_n is the hypercube graph.

How many vertices is in K_n, $K_{n,m}$, C_n, Q_n?

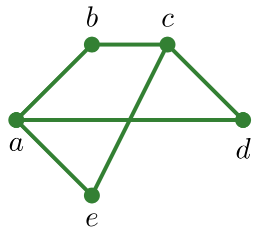
Problem 7 (Graded)

Determine whether the graph is bipartite

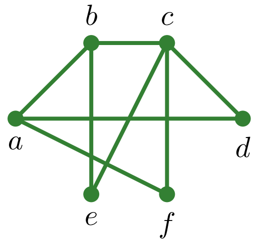
(a)



(b)



(c)



(d)