Discrete Structures. CSCI-150. Fall 2015.

Homework 13. Due Wed. Dec 2, 2015.

Problem 1 (Graded)

Draw the Hasse diagram for divisibility on the set:

(a) $\{1, 2, 3, 4, 5, 6\}$, (b) $\{3, 4, 7, 12, 28, 42\}$, (c) $\{3, 5, 6, 9, 25, 27\}$, (d) $\{3, 5, 7, 11, 13, 16, 17\}$,

(e) $\{6, 10, 14, 15, 21, 22, 26, 33, 35, 39, 55, 65, 77, 91, 143\}$, (f) $\{1, 3, 9, 27, 81, 243\}$.

For the question (a), find two incomparable elements and explain why they are incomparable.

Problem 2 (Graded)

Count the number of topological sorts for each poset $(A, |)$, where

(a) $A = \{3, 5, 7, 11, 13, 16, 17\}$, (b) $A = \{1, 3, 9, 27, 81, 243\}$, (c) $A = \{2, 3, 4, 8, 9, 16, 27, 81\}$.

That is, you have to find the number of ways to order the elements of the set A so that the partial order imposed by divisibility is preserved.

Problem 3

Prove that the "divides" relation on $\mathbb{N} \times \mathbb{N}$ is a partial order relation.

Prove that the "subset" relation ($\subseteq$) is a partial order relation, and the "proper subset" relation ($\subsetneq$) is not.

Problem 4

Draw these graphs: (a) K_7, (b) $K_{2,5}$, (c) C_7, (d) Q_4.
All of these special graphs are described in Rosen, K_n is the complete graph, $K_{n,m}$ is the complete bipartite graph, C_n is the cycle graph, and Q_n is the hypercube graph.

How many vertices is in K_n, $K_{n,m}$, C_n, Q_n?

Problem 5 (Graded)

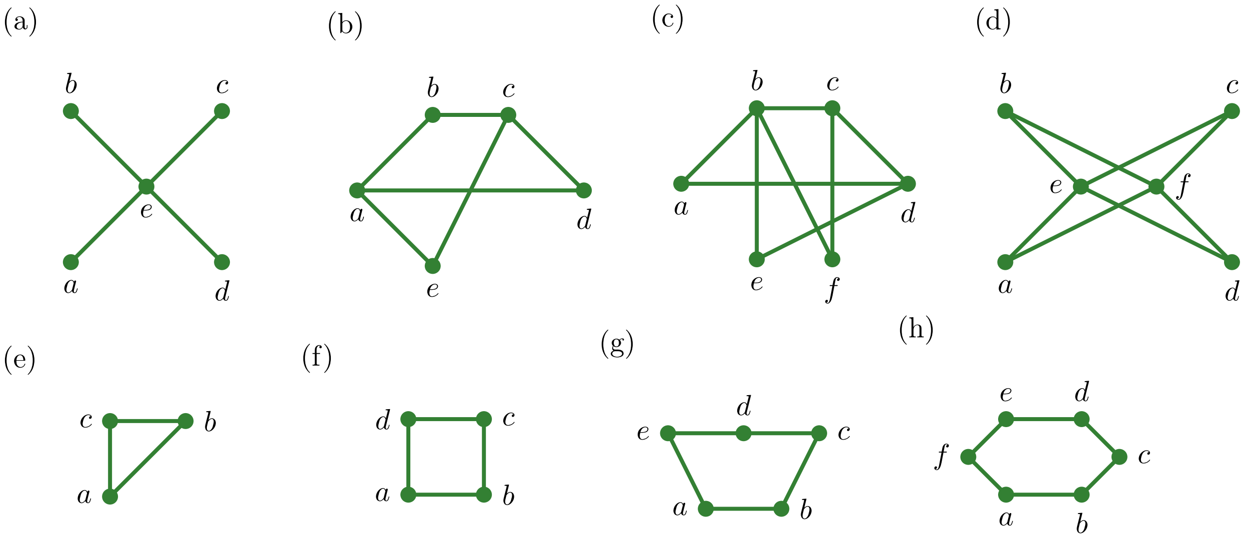
A simple graph is called regular if every vertex of this graph has the same degree. A regular graph is called n-regular if every vertex in this graph has degree n.

(a) Is K_n regular?

(b) For which values of m and n graph $K_{m,n}$ is regular?

(c) How many vertices does a 4-regular graph with 10 edges have?

Problem 6 (Graded)

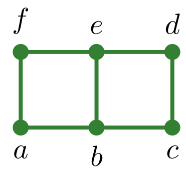


We know that a graph is bipartite if and only if it is 2-colorable.

For the graphs given above, either prove that they are bipartite by showing that they are 2-colorable, or prove that they are not 2-colorable, and so they are not bipartite.

Problem 7

What are the adjacency matrix and the adjacency list of a graph?
Find the adjacency matrix of the graph shown in the figure.
Find the adjacency list of the graph.



Problem 8 (Graded)

Given a graph with n vertices, prove that <u>if</u> the degree of each vertex is at least $(n-1)/2$ <u>then</u> the graph is connected.

Hint: First, you may consider a small graph, for example a graph with 5 vertices, can you make it disconnected?

Problem 9

A simple graph is called n-regular if every vertex of the graph has degree n.

Show that <u>if</u> a bipartite graph $G = (V, E)$ with a bipartition of the vertex set (V_1, V_2) <u>is n-regular</u> for some positive integer n <u>then</u> $|V_1| = |V_2|$.

Problem 10

For which values of n, does the complete graph K_n have an Euler cycle?
For which values of n and m, does the complete bipartite graph $K_{n,m}$ have an Euler cycle?