

# Discrete Structures. CSCI-150. Fall 2014.

## Homework 11.

Due Wed. Nov 26, 2014.

### Problem 1 (Graded)

Let  $A = \{1, 2, 3, 4\}$ , and  $B = \{3, 4, 5\}$ . List all the elements of the set

- (a)  $A \cap B$ , (b)  $B \cup A$ , (c)  $A \setminus B$ , (d)  $B \setminus A$ , (e)  $A \times B$ , (f)  $B \times B$ , (g)  $\mathcal{P}(B)$ .

### Problem 2

First, given two not equal sets  $A$  and  $B$ , prove that there exists an element  $x$  that belongs to either  $A$  or  $B$ , but not both.

Given two non-empty sets  $A$  and  $B$ , prove that if  $A \neq B$  then  $A \times B \neq B \times A$ .

### Problem 3 (Graded)

Prove the inclusion-exclusion formula (it's an extension of the subtraction rule)

$$\begin{aligned} |A \cup B \cup C| = & |A| + |B| + |C| \\ & - |A \cap B| - |A \cap C| - |B \cap C| \\ & + |A \cap B \cap C|. \end{aligned}$$

To do the proof, let's denote  $X = A \cup B$ , then

$$|(A \cup B) \cup C| = |X \cup C|,$$

and we can apply the usual subtraction rule (you will have to apply it twice).

### Problem 4 (Graded)

Give an example of a function  $f : \mathbb{N} \rightarrow \mathbb{N}$  that is

- (a) one-to-one, but not onto,  
(b) onto, but not one-to-one,  
(c) neither one-to-one, nor onto.  
(d) Function  $f : \mathbb{N} \rightarrow \mathbb{N} \times \{0, 1\}$  that is onto and one-to-one (bijection).

When constructing the functions, try to define them by formulas. (Feel free to use such operations as *absolute value*, *floor*, *ceiling*, *remainder*, in addition to normal arithmetical operations).

By definition,  $\mathbb{N}$  is the set of all non-negative integers:  $\mathbb{N} = \{0, 1, 2, \dots\}$ .

For each function, explain (in the best way you can) why they satisfy the required conditions.

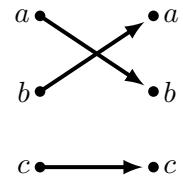
### Problem 5

Draw the diagrams (as we did in class) for all bijections  $f : A \rightarrow A$  when the set  $A$  is

- (a)  $A = \{1\}$
- (b)  $A = \{1, 2\}$
- (c)  $A = \{1, 2, 3\}$
- (d) For this question, either repeat the task for  $A = \{1, 2, 3, 4\}$ , or derive a formula for the total number of bijections from  $A$  to  $A$ , when  $|A| = n$ . (Explain your answer).

Example of the diagram for a function

$$f : \{a, b, c\} \rightarrow \{a, b, c\}$$



**Bonus (extra credit).**

There will be a bonus problem, it will be posted later, by Saturday or Sunday.