Discrete Structures. CSCI-150. Spring 2016.

Homework 2. Due Wed. Feb 17, 2016.

Problem 1 ((b), (c), and (p) are graded)

Prove some of these

(a)
$$\begin{array}{c} r \to p \\ p \to q \\ r \land t \\ \hline q \land t \end{array}$$

(b)
$$\begin{array}{c} (p \lor r) \to (q \land s) \\ p \\ \hline s \end{array}$$

(c)
$$\begin{array}{c} \neg q \land (r \lor s) \\ q \lor \neg s \\ (\neg q \land r) \to (t \lor s) \\ \hline \neg q \land t \end{array}$$

(d)
$$\begin{array}{c} (p \land \neg q) \lor (r \to p) \\ p \leftrightarrow q \\ \hline r \to q \end{array}$$

(e)
$$\begin{array}{c} (q \lor r) \to (\neg q \land t) \\ (\neg q \lor u) \to (s \to r) \\ s \\ \hline t \end{array}$$

(f)
$$\begin{array}{c} p \lor (r \land t) \\ \neg p \lor \neg(q \land u) \\ (q \land u) \lor s \\ \neg s \\ \hline r \end{array}$$

(g)
$$\begin{array}{c} q \\ t \leftrightarrow r \\ p \land (q \to r) \\ \hline p \land t \end{array}$$

(h)
$$\begin{array}{c} s \land t \\ \neg p \lor \neg(r \to s) \\ \hline \neg p \end{array}$$

(i)
$$\begin{array}{c} t \land s \\ (p \land s) \leftrightarrow r \\ p \lor \neg t \\ \hline r \end{array}$$

(j)
$$\begin{array}{c} p \lor (r \land t) \\ \neg p \lor \neg(q \land u) \\ (q \land u) \lor s \\ \neg s \\ \hline r \end{array}$$

(k)
$$\begin{array}{c} (\neg q \land p) \lor s \\ p \to (q \lor r) \\ \hline r \lor s \end{array}$$

(l)
$$\begin{array}{c} q \\ t \leftrightarrow r \\ p \land (q \to r) \\ \hline p \land t \end{array}$$

(m)
$$\begin{array}{c} s \land t \\ \neg p \lor \neg(r \to s) \\ \hline \neg p \end{array}$$

(n)
$$\begin{array}{c} p \lor s \\ \neg p \lor r \\ \neg r \\ \hline s \end{array}$$

(o)
$$\begin{array}{c} p \to r \\ r \to s \\ t \lor \neg s \\ \neg t \lor u \\ \neg u \\ \hline \neg p \end{array}$$

(p)
$$\begin{array}{c} (\neg p \lor q) \to r \\ r \to (s \lor t) \\ \neg s \land \neg u \\ \neg u \to \neg t \\ \hline p \land \neg q \end{array}$$

If you have trouble with questions like these, consider going back to the last problem from HW 1, it should help.

Also, don't try to be very smart, this task is all about simple application of the rules. Found a matching rule? Try to apply it.

Problem 2 (Graded)

Using predicates

$$\begin{aligned} B(x): &\quad x \text{ likes reading books,} \\ M(x): &\quad x \text{ is a mathematician,} \\ L(x): &\quad x \text{ is a linguist,} \\ K(x,y): &\quad x \text{ knows person } y, \end{aligned}$$

write the following sentences as quantified logical formulas:

(a) Some people like reading books.

(b) Some mathematicians like reading books.

(c) All linguists like reading books.

(d) Among people who like reading book, some are neither mathematicians nor linguists.

(e) Everyone knows at least one mathematician.

(f) Every linguist knows a mathematician who does not like reading books.

Hint: The predicate "x knows linguist y" can be expressed as $K(x,y) \wedge L(y)$, meaning that x knows y, and y is a linguist.

Problem 3

Construct a contrapositive proof that for all real numbers x, if $2x - x^2 \neq 1$ then $x \neq 1$.

Problem 4 (Graded)

Prove by contradiction that there are no positive integer solutions to the equation $x^2 - y^2 = 1$.

Hint 1: How do we do a proof by contradiction? We make an assumption that the statement we want to prove is not true (that is, we have to assume that there exist positive integer solutions to the equation), and then show that this assumption leads to some absurd result, therefore the assumption was false (and so, the original statement we wanted to prove must be true).

Hint 2: Consider a product of two integers $A \cdot B$. Find all possible integer values of A and B that make this product equal to 1.

Problem 5. Knights and Knaves ((a), (b), and (c) are graded)

A similar problem is discussed as an example in Rosen (ed.6: p. 14; ed.7: p. 19). However, the solution in Rosen is rather cumbersome.

Imagine an island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie.

(a) Is it possible for any inhabitant of this island to claim that he is a knave?

(b) Is it possible for an inhabitant of the island to claim that he and his brother are both knaves?

(c) Suppose an inhabitant A says about himself and his brother B: “At least one of us is a knave.” What type is A and what type is B?

(d) Suppose A instead says: “Exactly one of us is a knave.” What can be deduced about A and what can be deduced about B?

(e) On the first day you arrived to the island, you met an inhabitant and asked him: “Are you a knight or a knave?” He angrily replied: “I refuse to tell you!” and walked away. That’s the last you ever saw or heard of him.

    Was he a knight or a knave?

These knights and knaves puzzles (as well as many other) were created by Raymond Smullyan, an extraordinary mathematician and philosopher. Try to find his books if you like this kind of puzzles. Smullyan was born in Far Rockaway in 1919, received his PhD from Princeton, and was teaching at New York City colleges, particularly at Lehman College.