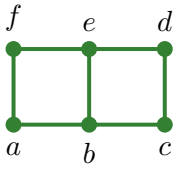
Discrete Structures. CSCI-150. Fall 2014.

Homework 13. Due Wed. Dec 10, 2014.

Problem 1

What are the adjacency matrix and the adjacency list of a graph?
Find the adjacency matrix of the graph shown in the figure.
Find the adjacency list of the graph.



Problem 2 (Graded)

For which values of n, does the complete graph K_n have an Euler cycle?
For which values of n and m, does the complete bipartite graph $K_{n,m}$ have an Euler cycle?

Problem 3 (Graded)

Given a graph with n vertices, prove that if the degree of each vertex is at least $(n-1)/2$ then the graph is connected.

Hint: First, you may consider a small graph, for example a graph with 5 vertices, can you make it disconnected?

Problem 4

Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 faces, how many vertices does this graph have?

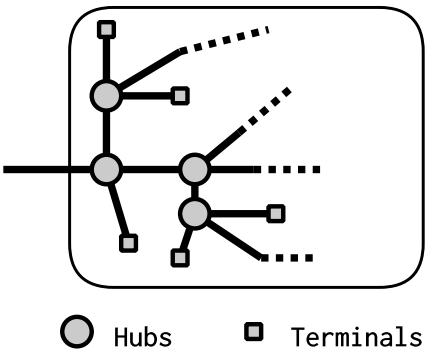
Problem 5 (Graded)

How many edges does a full binary tree with 10000 internal vertices have?

Problem 6 (Graded)

Amtrak plans to extend their railroad network to a big island, which is connected to the continent by a bridge.

According to the plans, there will be M stations on the island. There are two types of stations: the first type are hubs that connect 4 railroads, the second type are dead-end (terminal) stations, with only one railway line. To reduce the costs, the railroads don't make loops, that is, there are *no simple cycles* in the network, so the system is cheaper, although all stations are connected. Only one of the hubs is directly connected to the outside world.



How many hubs, and how many terminals will be built? (The total number of stations is M).

Problem 7 (Graded)

Use Huffman coding to encode these symbols with given frequencies:
A: 0.05, B: 0.07, C: 0.08, D: 0.10, E: 0.15, F: 0.25, G: 0.30.

Show all intermediate steps.

What is the average number of bits required to encode a symbol?

Problem 8

A simple graph is called n-regular if every vertex of the graph has degree n.

Show that if a bipartite graph $G = (V, E)$ with a bipartition of the vertex set (V_1, V_2) is n-regular for some positive integer n then $|V_1| = |V_2|$.