Discrete Structures. CSCI-150. Summer 2016.

Homework 1. Due Mon. Jun 6, 2016.

Problem 1

Write out the truth tables for the following propositions:

(a) $p \wedge \neg(p \rightarrow q)$

(b) $(p \leftrightarrow \neg(q \vee r)) \wedge (r \rightarrow q)$

Compute one operation at a time, don't skip steps.

Problem 2 (Graded)

An interesting question is to find the correct way to negate a biconditional, $\neg(p \leftrightarrow t)$.

A naive guess could be that we can simply distribute the negation over the biconditional, obtaining $\neg p \leftrightarrow \neg t$. We are going to check if this guess is correct or not.

Write the truth tables for the following propositional formulas:

(a) $\neg(p \leftrightarrow t)$, (b) $(\neg p) \leftrightarrow (\neg t)$, (c) $p \leftrightarrow t$, (d) $(\neg p) \leftrightarrow t$, (e) $p \leftrightarrow (\neg t)$

Decide which of these formulas are equivalent, and find what is the correct way to negate a biconditional.

Problem 3 (Graded)

Using logical equivalences, prove that

$$p \leftrightarrow q \quad \equiv \quad (\neg p \wedge \neg q) \vee (p \wedge q)$$

<u>Hint</u>. To prove that, you can follow these steps:

(1) First, show that

$$p \leftrightarrow q \quad \equiv \quad (\neg p \vee q) \wedge (\neg q \vee p)$$

(2) Distribute $(\neg p \vee q)$ over the disjunction $(\neg q \vee p)$.

(3) Then do smething else, eventually arriving to

$$p \leftrightarrow q \quad \equiv \quad ((\neg p \wedge \neg q) \vee \text{False}) \vee (\text{False} \vee (q \wedge p))$$

(4) Then show that the right hand side in the formula above is equivalent to $(\neg p \wedge \neg q) \vee (p \wedge q)$.

Problem 4

Using logical equivalences, prove that

(a) $p \to (r \to p) \quad \equiv \quad \text{True}$,

(b) $(p \to r) \vee (r \to p) \quad \equiv \quad \text{True}$,

(c) $r \to (p \to (r \to p)) \quad \equiv \quad \text{True}$,

in other words, we want to prove that the formulas above are tautologies (they are always true, regardless of the values of the variables p and r).

Problem 5 (Graded)

Using logical equivalences, prove that the following three formulas are equivalent:

$$a \to (b \to c), \qquad (a \wedge b) \to c, \qquad (a \to b) \to (a \to c)$$

Problem 6

You are given an argument, but it's incomplete. Finish the work by specifying which inference rule was used in each step of the argument.

(a) Prove

$$\begin{array}{c} p \wedge q \\ q \to (r \wedge s) \\ \hline r \end{array}$$

Complete the argument

$$\begin{array}{lll} (1) & p \wedge q & \text{Given.} \\ (2) & q \to (r \wedge s) & \text{Given.} \\ (3) & q & \dots \\ (4) & r \wedge s & \dots \\ (5) & r & \dots \end{array}$$

(b) Prove

$$\begin{array}{c} p \to (\neg s \wedge r) \\ s \vee t \\ p \\ \hline t \end{array}$$

Complete the argument

$$\begin{array}{lll} (1) & p \to (\neg s \wedge r) & \text{Given.} \\ (2) & s \vee t & \text{Given.} \\ (3) & p & \text{Given.} \\ (4) & \neg s \wedge r & \dots \\ (5) & \neg s & \dots \\ (6) & t & \dots \end{array}$$

(c) Prove

$$\begin{array}{c} (\neg p \vee s) \leftrightarrow q \\ \neg q \\ \hline p \end{array}$$

Complete the argument

(1)	$(\neg p \vee s) \leftrightarrow q$	Given.
(2)	$\neg q$	Given.
(3)	$((\neg p \vee s) \to q) \wedge (q \to (\neg p \vee s))$	$\ldots$
(4)	$(\neg p \vee s) \to q$	$\ldots$
(5)	$\neg(\neg p \vee s)$	$\ldots$
(6)	$\neg(\neg p) \wedge \neg s$	$\ldots$
(7)	$\neg(\neg p)$	$\ldots$
(7)	p	$\ldots$