Discrete Structures. CSCI-150. Summer 2015.

Homework 1. Due Thr. Jun. 4, 2015.

Problem 1

Using the following propositions:

p : "Phyllis goes out for a walk".
r : "The Moon is out".
s : "It is snowing".

Formulate these statements in words:

(a) $(r \wedge \neg s) \to p$ (b) $r \to (\neg s \to p)$ (c) $\neg(p \leftrightarrow (s \vee r))$

Try to keep the propositions unchanged. If you really want to replace a proposition with its equivalent, first, prove that your substitution is correct.

In the question (c), you have to find a way to negate the whole sentence. I guarantee that there are ways to do that in English.

Problem 2 (Graded)

Write out the truth tables for the following propositions:

(a) $(\neg p) \to (\neg q)$

(b) $(p \wedge (\neg q)) \leftrightarrow \neg(p \vee (\neg q))$

(c) $(p \to q) \vee (\neg r)$

Compute one operation at a time, don't skip steps.

Problem 3 (Graded)

Check if the given propositions are equivalent or not:

(a) $\neg(p \leftrightarrow s)$ and $(\neg p) \leftrightarrow (\neg s)$

(b) $p \leftrightarrow s$ and $(\neg p) \leftrightarrow (\neg s)$

(c) $(\neg p) \leftrightarrow s$ and $\neg(p \leftrightarrow s)$

(For this problem, you can use either the equivalence formulas or the truth tables method).

Can you make any conclusions from this problem? For example, about the negation of a biconditional (if-and-only-if) proposition.

Problem 4 (Graded)

Prove the logical equivalence:

$$\neg((a \wedge b) \wedge c) \equiv \neg a \vee (\neg b \vee \neg c).$$

It is advised to do the proof using the known equivalences. (Hint: using De Morgan's Law and the associativity of $\vee$).

Problem 5 (Graded)

Using logical equivalences, prove that

(a) $p \to (r \to p) \quad \equiv \quad \text{True},$

(b) $r \to (p \to (r \to p)) \quad \equiv \quad \text{True},$

in other words, we want to prove that the formulas above are tautologies (they are always true, regardless of the values of the variables p and r).

Problem 6 (Graded)

Using logical equivalences, prove that

$$p \leftrightarrow q \quad \equiv \quad (\neg p \wedge \neg q) \vee (p \wedge q)$$

Hint. To prove that, you can follow these steps:

(1) First, show that
$$p \leftrightarrow q \quad \equiv \quad (\neg p \vee q) \wedge (\neg q \vee p)$$

(2) Distribute $(\neg p \vee q)$ over the disjunction $(\neg q \vee p)$.

(3) Then do smething else, eventually arriving to

$$p \leftrightarrow q \quad \equiv \quad ((\neg p \wedge \neg q) \vee \text{False}) \vee (\text{False} \vee (q \wedge p))$$

(4) Then show that the right hand side in the formula above is equivalent to $(\neg p \wedge \neg q) \vee (p \wedge q)$.

Problem 7

You are given an argument, but it's incomplete. Finish the work by giving the reasons why each step was correct.

(a) Prove

$$\begin{array}{c} p \wedge q \\ q \rightarrow (r \wedge s) \\ \hline r \end{array}$$

Complete the argument

$$\begin{array}{lll} (1) & p \wedge q & \text{Given.} \\ (2) & q \rightarrow (r \wedge s) & \text{Given.} \\ (3) & q & \ldots \\ (4) & r \wedge s & \ldots \\ (5) & r & \ldots \end{array}$$

(b) Prove

$$\begin{array}{c} p \rightarrow (\neg s \wedge r) \\ s \vee t \\ p \\ \hline t \end{array}$$

Complete the argument

$$\begin{array}{lll} (1) & p \rightarrow (\neg s \wedge r) & \text{Given.} \\ (2) & s \vee t & \text{Given.} \\ (3) & p & \text{Given.} \\ (4) & \neg s \wedge r & \ldots \\ (5) & \neg s & \ldots \\ (6) & t & \ldots \end{array}$$

(c) Prove

$$\begin{array}{c} (\neg p \vee s) \leftrightarrow q \\ \neg q \\ \hline p \end{array}$$

Complete the argument

$$\begin{array}{lll} (1) & (\neg p \vee s) \leftrightarrow q & \text{Given.} \\ (2) & \neg q & \text{Given.} \\ (3) & ((\neg p \vee s) \rightarrow q) \wedge (q \rightarrow (\neg p \vee s)) & \ldots \\ (4) & (\neg p \vee s) \rightarrow q & \ldots \\ (5) & \neg(\neg p \vee s) & \ldots \\ (6) & \neg(\neg p) \wedge \neg s & \ldots \\ (7) & \neg(\neg p) & \ldots \\ (7) & p & \ldots \end{array}$$