Discrete Structures. CSCI-150. Fall 2013. Midterm - Example.

No books, no notes allowed.

Problem 1 (3 pts)

Write out the truth table for the proposition $(p \vee q) \rightarrow (\neg r)$

Problem 2 (4 pts)

Prove

$$\begin{array}{c} p \rightarrow q \\ (q \wedge s) \leftrightarrow (\neg q \vee t) \\ p \wedge s \\ \hline t \end{array}$$

Problem 3 (2 pts)

On Saturday morning, you have to hand in 7 assignments for 7 different classes. Today is Monday morning, so there are 5 full days left. You don't spend more than one day on an assignment, and you can make more than one assigment a day. In how many ways can you schedule your work?

What if you want to do at least 4 assignments on Friday?

Problem 4 (2 pts)

How many solutions are there to the equation

$$n_1 + n_2 + n_3 + n_4 + n_5 = 7$$

if n_1, n_2, n_3, n_4, and n_5 are non-negative integers?

Problem 5 (4 pts)

You invited four families to a theater. There are 3 persons in the first family, 4 persons in the second family, 4 in the third, and 5 in the fourth.

The theater is relatively small, and there are only 16 seats in the first row. You bought all 16 of them for your guests, and now have to distribute the tickets. There is only one condition: All members of the same family should be able to sit together.

In how many ways can you distribute the tickets between four families?

What is the total number of acceptable seatings for these 16 people?

(It can be *interesting*[1] to think about an alternative problem, with 17 seats and the same 16 people)

[1] if you have time

Problem 6 (3 pts)

Prove by induction that

$$1 + 2 + 4 + 8 + \ldots + 2^n = 2^{n+1} - 1 \qquad (\text{for } n \geq 0).$$

Problem 7 (2 pts)

Solve linear recurrence

$$f(0) = 0, \quad f(1) = 1,$$
$$f(n) = 2f(n-1) - f(n-2).$$

$$(a \wedge b) \equiv (b \wedge a) \qquad \text{commutativity of } \wedge$$
$$(a \vee b) \equiv (b \vee a) \qquad \text{commutativity of } \vee$$
$$((a \wedge b) \wedge c) \equiv (a \wedge (b \wedge c)) \qquad \text{associativity of } \wedge$$
$$((a \vee b) \vee c) \equiv (a \vee (b \vee c)) \qquad \text{associativity of } \vee$$
$$\neg(\neg a) \equiv a \qquad \text{double-negation elimination}$$
$$(a \to b) \equiv (\neg b \to \neg a) \qquad \text{contraposition}$$
$$(a \to b) \equiv (\neg a \vee b) \qquad \text{implication elimination}$$
$$(a \leftrightarrow b) \equiv (a \to b) \wedge (b \to a) \qquad \text{biconditional elimination}$$
$$\neg(a \wedge b) \equiv (\neg a \vee \neg b) \qquad \text{De Morgan's Law}$$
$$\neg(a \vee b) \equiv (\neg a \wedge \neg b) \qquad \text{De Morgan's Law}$$
$$(a \wedge (b \vee c)) \equiv (a \wedge b) \vee (a \wedge c) \qquad \text{distributivity of } \wedge \text{ over } \vee$$
$$(a \vee (b \wedge c)) \equiv (a \vee b) \wedge (a \vee c) \qquad \text{distributivity of } \vee \text{ over } \wedge$$

$$\frac{p}{p \vee q} \quad \text{“}\vee\text{-Introduction”}$$

$$\frac{\begin{array}{c}\neg q\\ p \to q\end{array}}{\neg p} \quad \text{“Modus Tollens”}$$

$$\frac{\begin{array}{c}p\\ q\end{array}}{p \wedge q} \quad \text{“}\wedge\text{-Introduction”}$$

$$\frac{\begin{array}{c}p \to q\\ q \to r\end{array}}{p \to r} \quad \text{“Hypothetical Syllogism”}$$

$$\frac{p \wedge q}{p} \quad \text{“}\wedge\text{-Elimination”}$$

$$\frac{\begin{array}{c}p \vee q\\ \neg q\end{array}}{p} \quad \text{“Disjunctive Syllogism”}$$

$$\frac{\begin{array}{c}p\\ p \to q\end{array}}{q} \quad \text{“Modus Ponens”}$$

$$\frac{\begin{array}{c}p \vee q\\ \neg p \vee r\end{array}}{q \vee r} \quad \text{“Resolution”}$$

$$\frac{\text{assuming } p \text{, we infer } q}{p \to q} \quad \text{“}\to\text{-Introduction” (Deduction theorem)}$$

$$\frac{\text{assuming } p \text{, we infer a contradiction}}{\neg p} \quad \text{“Proof by contradiction”}$$