Discrete Structures. CSCI-150. Spring 2014.

Homework 4. Due Fri. Feb 28, 2014.

Problem 1

In how many ways can 10 dimes be distributed among five children if

(a) there are no restriction?

(b) each child gets at least one dime? (Answer: 126).

(c) the oldest child gets at least two dimes? (Answer: 495).

Problem 2

Assuming that in some programming language, a variable name has to start with a lowercase letter ('a'-'z'), followed by any combination of lowercase letters, digits ('0'-'9'), or underscore symbols ('_'), count the number of valid variable names of length 12.

Problem 3

How many five-digit integers (in the conventional base-10 numeral system)

(a) start with a '9'?

(b) contain a '9'?

(c) do not contain a '9'?

Note that a five-digit number is different from a string of five digits (see Poblem 10 from the last class problem set).

Problem 4

n copies of a book have to be distributed among n libraries in the city (one copy per library). You have to plan a route to visit k of them today, and the remaining $n-k$ libraries tomorrow to deliver the books. In how many ways can you do so?

Problem 5

Prove that $\binom{n}{k} = \binom{n}{n-k}$. Explain the meaning of this result (argue why this is the case, what are the implications, etc.).

Problem 6 (Bonus)

Consider a bacterial cell constrained to a one-dimensional environment (some sort of tube, for example). This bacteria reproduce by binary fission: Every hour, each cell divides into two equal daughter cells. Immediately after the division, two daughter cells move away from each other and stop at points $x+1$ and $x-1$, where x is the original position of their parent cell before the fission.

The process starts with a single cell located at $x = 0$. We set the clock to $t = 0$ in the beginning of the experiment. After one hour, at time $t = 1$, there are two cells: at $x = -1$ and at $x = 1$, respectively. The process continues indefinitely.

Let $N(t, x)$ be the number of cells at the position x at the time t.

(a) How does the bacterial colony evolve with time? As a first step, it's recommended to compute $N(t, x)$ for small $t = 0, 1, 2, \ldots$ to see what is going on. Describe the dynamics of the system. If it's possible, try to derive a formula for $N(t, x)$.

(b) Compute $N(t, 0)$ for $t = 0, 1, \ldots 10$.

(c) Compute $N(10^{10}, 10^{10})$.

(d) Compute $N(10^{10}, 9^9)$.

(e) Find x such that $N(10^{100}, x) = N(10^{10}, 9^9)$.

(f) What is the total population of the colony as a function of time?