Discrete Structures. CSCI-150. Summer 2014.

Homework 11. Due Mon. Jul 14, 2014.

Problem 1 (Graded)

A simple graph is called regular if every vertex of this graph has the same degree. A regular graph is called n-regular if every vertex in this graph has degree n.

Recall thatK_n is the complete graphs with n vertices. And $K_{m,n}$ is the complete bipartite graph (see the definition in the book).

(a) Is K_n regular?

(b) For which values of m and n graph $K_{m,n}$ is regular?

(c) How many vertices does a 4-regular graph with 10 edges have?

Problem 2 (Graded)

Assuming that friendship is always mutual, prove that in any group of $n \geq 2$ persons, there are at least 2 persons with the same number of friends in the group.

Hints: (a) Model a group of people as a graph, where the vertices are people, and there is an edge between them if they are friends. (b) What does the degree of a vertex mean in this context? (c) How to make all degrees different? What those values could be?

Problem 3

Describe an algorithm to decide whether a graph is bipartite based on the fact that a graph is bipartite if and only it is two-colorable.

Hint: You can start by picking one vertex at random, and because there is no constraints so far, mark it "red". Describe how to proceed, so that we either find that the graph is indeed two-colorable, or it is not two-colorable.

Problem 4

Show that every connected graph with n vertices has at least $n - 1$ edges.

(It can be done by induction, for example).

Problem 5 (Graded)

For which values of n, does the complete graph K_n have an Euler cycle?
For which values of n and m, does the complete bipartite graph $K_{n,m}$ have an Euler cycle?