Karlsruhe Institute of Technology

KASTEL

An Automated Approach to Generating Card-Based Cryptographic Protocols

Final Bachelor's Thesis Presentation

Anne Hoff | March 30, 2023

Multi-Party Computation with Physical Objects

Multi-Party Computation (MPC):

Players want to correctly and securely compute a public function over their private inputs.

Physical Objects:

E.g.

(a) Borscht with Carrots and Onions (Miyahara et al., 2021)

(b) Coins (Komano and Mizuki, 2018)

(c) Playing Cards (Boer, 1990)

Advantages of MPC with Physical Objects

- Computers are not available, fail and/or are not trusted (Niemi and Renvall, 1998)
- Computations are performed by hand, so principles can be easily understood (Miyahara et al., 2021)
- Use in classrooms and lectures to illustrate MPC to nonexperts (Koch et al., 2015)

Motivation and Research Question

Current State of Research:
Protocols are mostly found "by hand"

↓

Koch et al. (2021) introduced an automated method
using formal verification to find card-based protocols

↓

Problems of the Automated Method:

- Just finds protocols for the *AND* operator
- Only works for small problems

↓

Research Question:
Can we expand upon the method of Koch et al. (2021) to make it more **universally applicable** and more **efficient**?

What are Card-Based Protocols?

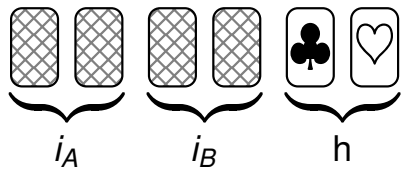
- Two-color deck: $\{\clubsuit, \heartsuit\}^n$
- Indistinguishable backsides: 🂠
- Commitment (encoding of a value):
  - $\clubsuit\ \heartsuit = 0 \mathrel{\widehat{=}}$ "No"
  - $\heartsuit\ \clubsuit = 1 \mathrel{\widehat{=}}$ "Yes"

input: Bob: $i_B =$ 🂠 🂠

Alice: $i_A =$ 🂠 🂠

A Simple Card-Based Protocol

AND protocol by Mizuki and Sone (2009)

1. 

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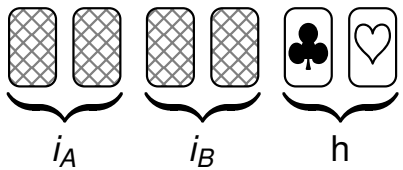
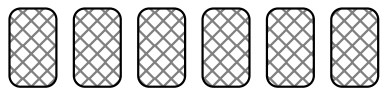
1. 

2. **Turn** all cards face-down: 

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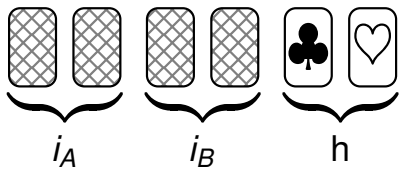
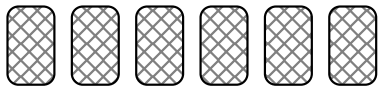
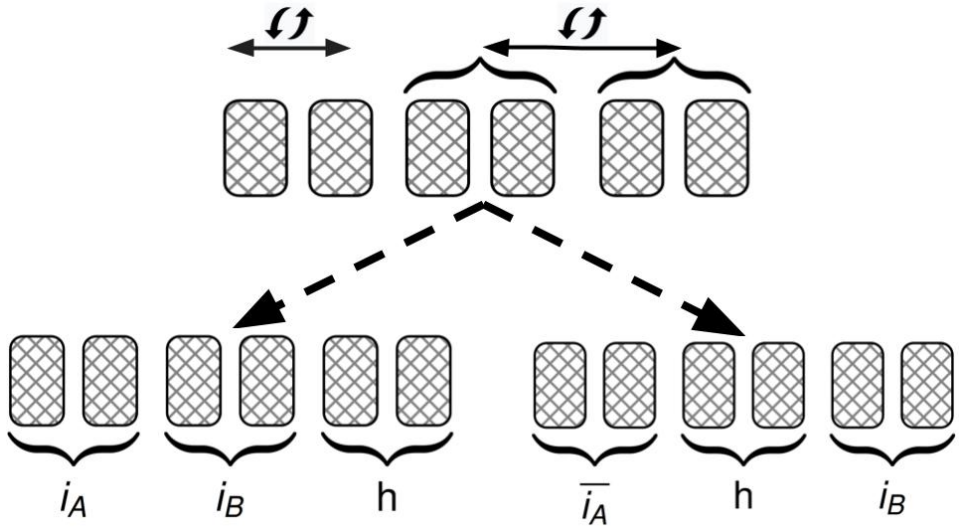
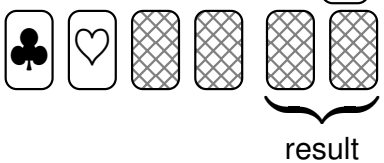
1. 

2. **Turn** all cards face-down: 
3. **Shuffle** the cards

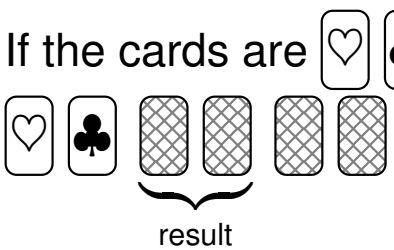


A Simple Card-Based Protocol

4. **Turn** around the first two cards
   - If the cards are ♣ ♡:

   - If the cards are ♡ ♣:

result: ♣ ♡ → "No"

♡ ♣ → "Yes"

Definitions for Card-based Protocols

Security

visible cards and output of the protocol do not reveal anything about the input

Input-possibilistic Security:
every input can produce any state of the protocol

Output-possibilistic Security:
at any state of the protocol, every output can still be possible

Shuffle Properties

Uniform Shuffle:
every permutation has the same probability

Closed Shuffle:
set of possible permutations is invariant under repetition

Finding Protocols with Software Bounded Model Checking

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- **Software Bounded Model Checking (SBMC)**:
  - Finds violations of assertions in programs (within given bound)
  - Performs static analysis without executing the programs on specific values
  - `assume()`: condition that provides more information about the program
  - `assert()`: property to check whether the program satisfies certain safety/correctness properties

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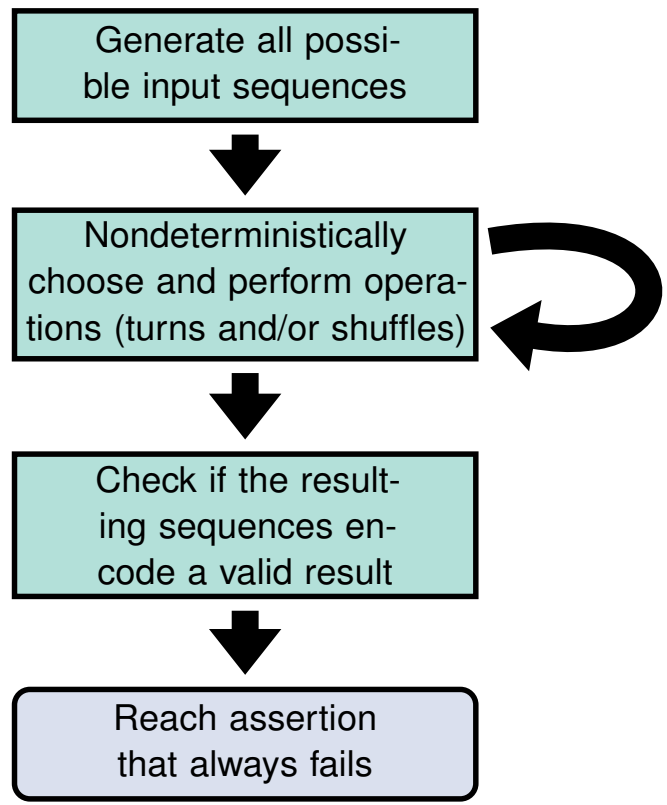
- **The symbolic program by Koch et al. (2021)**:



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3. Explore different SAT solvers (Efficiency)
4. Evaluate bit-level data structure for efficient operations (Efficiency)

1. Generalizing the Symbolic Program

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Approach

- Function components: **domain**, **codomain** and **function behaviour**
- adapt symbolic program when components change
- E.g. *COPY* instead of *AND* $\rightarrow$ domain: one input commitment $\rightarrow$ alter generation of possible input sequences

Protocols Found with the Generalized Symbolic Program

- Secure protocols found (excerpt):

Boolean Function	Nr. Cards	Nr Steps (Best Case)	Type Shuffle
OR	4	4	uniform, not closed
	4	6	not uniform, closed
COPY	5	2	uniform, not closed

- Function without any protocols found: half adder, instead:
  - timeout or an "out-of-memory" error for five or more cards

2. Introducing a Nested Structure

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Question

Can we implement a nested approach that uses protocols as operations?

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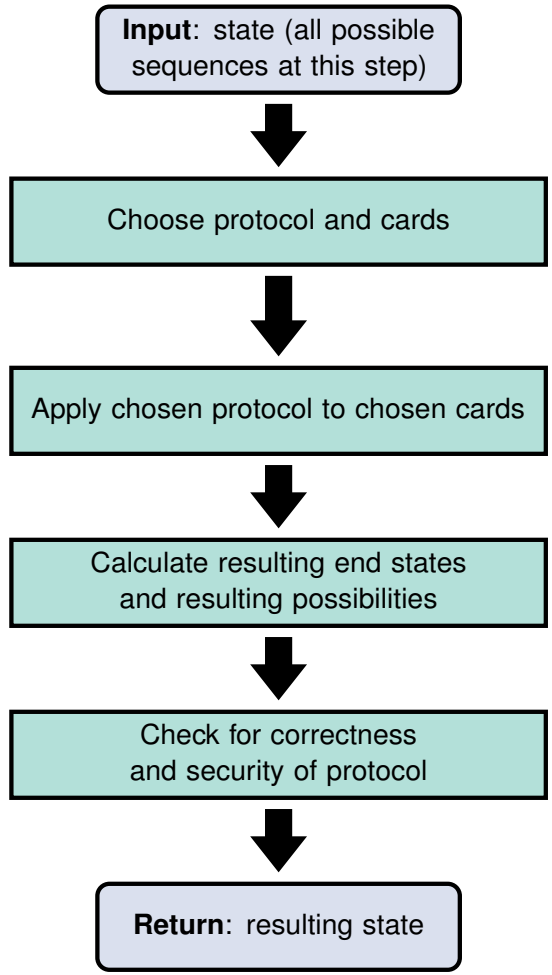
Can we implement a nested approach that uses protocols as operations?

Why use protocols as operations?

- Reduce complexity of search space
- use known protocols from the literature to compose correct and secure larger protocols



Evaluation of the Nested Structure

Implementation:

- General implementation of the protocol action
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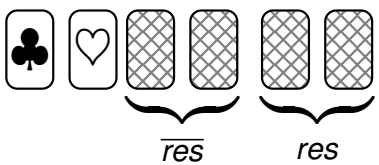
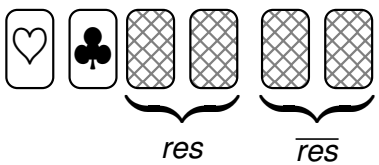
The new *COPY* protocol:

- Card we want to copy: i_C
- Apply *AND* protocol with $i_A = i_C$ and i_B = "0"
- After turning the first two cards:
  - 

  - 

3. Experimenting with SAT Solvers

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Question

Can we make our method more efficient by using a different SAT solver?

[1] https://www.labri.fr/perso/lsimon/research/glucose/
[2] http://fmv.jku.at/cadical/

3. Experimenting with SAT Solvers

Question

Can we make our method more efficient by using a different SAT solver?

- Built-in SAT solver: MiniSat
- Alternative SAT solvers:
  - CBMC interface for various SAT and SMT solvers
  - Chosen solvers: Glucose [1] and Cadical [2]
    - Fast and efficient, award-winning SAT-Solvers

[1] https://www.labri.fr/perso/lsimon/research/glucose/
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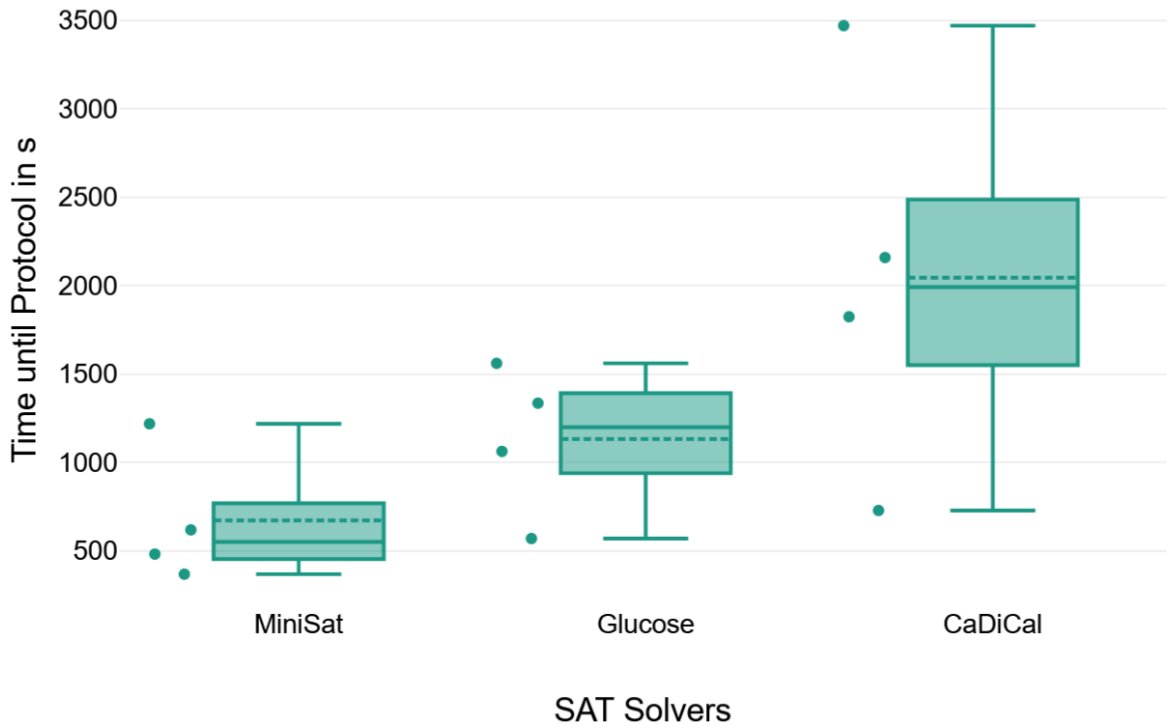
Results of SAT Solver Experiments



- 4 experiments
  - 2x with *XOR* function that has a protocol and different security definitions
  - 2x with *OR* function where no protocol exists and different security definitions

4. Evaluating an Alternative Data Structure

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Is there a data structure that is more efficient than the one by Koch et al. (2021)?

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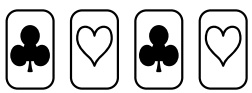
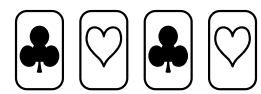
Koch et al. (2021):

- Arrays
- E.g.  → `[1, 2, 1, 2]`

Alternative data structure:

- Cards are bits in a single variable
- E.g.  → `0101`

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Advantage of the Alternative Data Structure

- **Hypothesis**: Bitwise operations are **faster** than array accesses

Turn Operation

Using Array Representation

```
turnedCardNumber
    = seq.val[turnPosition];
```

Using Bit Representation

```
turnedCardNumber =
    (sequence.val &
    (1 << turnPosition));
```

Shuffle Operation

Applying permutation j to sequence i:

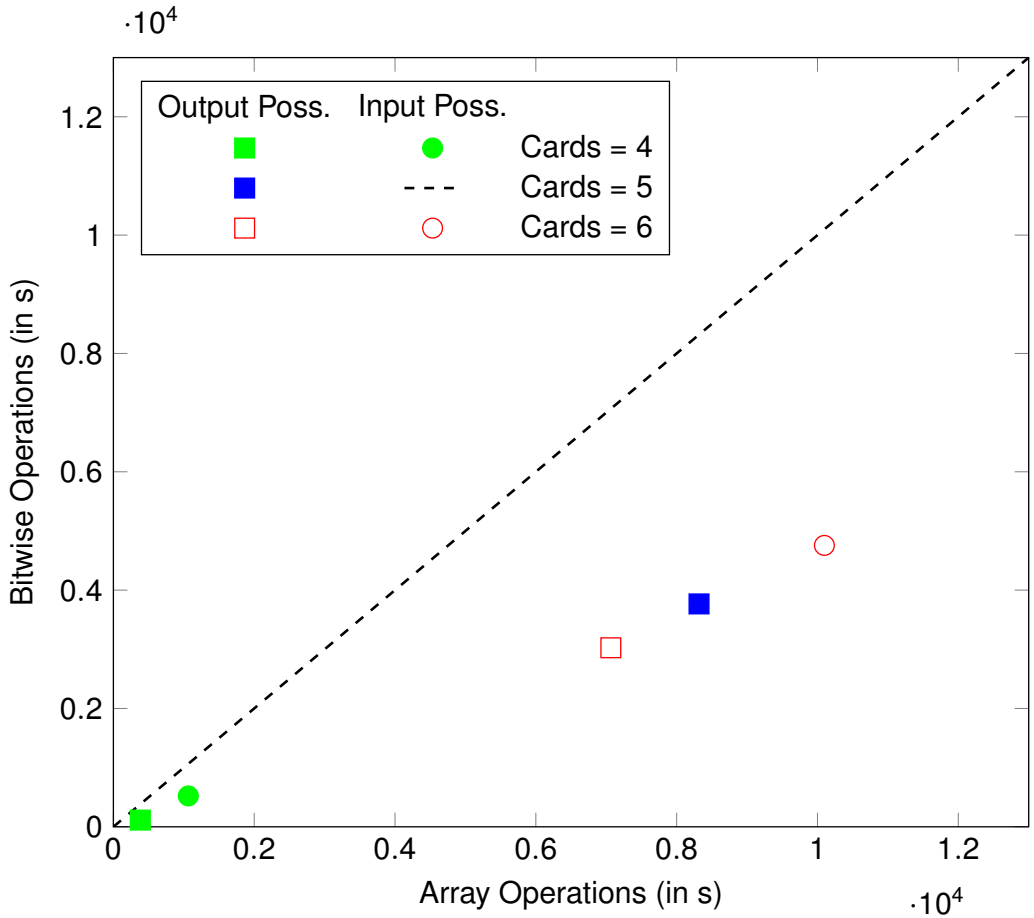
Using Array Representation

```
for (k = 0; k < N; k++) {
    resultingSeq.arr[
        permutationSet[j][k]]
    = s.seq[i].val[k];
}
```

Using Bit Representation

```
resultingSeq = 0;
for (k = 0; k < N; k++) {
    temp = seq.val & (1 << k);
    shift =
        permutationSet[j][k]
        - k;
    resultingSeq =
        resultingSeq |
        temp << (shift);
}
```

Experiments with the Alternative Data Structure



Conclusion

Results

- Generalized the symbolic program by Koch et al. (2021)
  - Discovered new protocols
- Introduced and implemented technique to integrate arbitrary protocols as actions
  - Discovered a *COPY* protocol using boolean operators
- Evaluated the efficiency of using different SAT solvers
- Introduced a bitwise data structure for sequence representation
  - Improved the runtime of the bounded model checker in an experiment setting

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Future Work

- Standardized program using the bitwise data structure
- Test with further SAT or SMT solvers

Bibliography I


Boer, Bert den (1990). “More Efficient Match-Making and Satisfiability The Five Card Trick”. en. In: *Advances in Cryptology — EUROCRYPT ’89*. Ed. by Jean-Jacques Quisquater and Joos Vandewalle. Lecture Notes in Computer Science. Berlin, Heidelberg: Springer, pp. 208–217. ISBN: 978-3-540-46885-1. DOI: 10.1007/3-540-46885-4_23.


Koch, Alexander, Michael Schrempp, and Michael Kirsten (2021). “Card-Based Cryptography Meets Formal Verification”. In: *New Gener. Comput.* 39.1, pp. 115–158. DOI: 10.1007/s00354-020-00120-0.

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Bibliography III


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Mizuki, Takaaki and Hideaki Sone (2009). “Six-Card Secure AND and Four-Card Secure XOR”. In: *Frontiers in Algorithmics, Third International Workshop, FAW 2009, Hefei, China, June 20-23, 2009. Proceedings*. Ed. by Xiaotie Deng, John E. Hopcroft, and Jinyun Xue. Vol. 5598. Lecture Notes in Computer Science. Springer, pp. 358–369. DOI: 10.1007/978-3-642-02270-8_36.

Bibliography IV

 Niemi, Valtteri and Ari Renvall (Jan. 1998). “Secure multiparty computations without computers”. en. In: *Theoretical Computer Science* 191.1, pp. 173–183. ISSN: 0304-3975. DOI: 10.1016/S0304-3975(97)00107-2. URL: https://www.sciencedirect.com/science/article/pii/S0304397597001072.

Picture Sources

- Soup: https://images.emojiterra.com/google/noto-emoji/v2.034/512px/1f372.png
- Cards: http://clipart-library.com/clipart/kTMbeg59c.htm
- Coin: https://images.emojiterra.com/google/android-11/512px/1fa99.png
- Coffee Date: https://c8.alamy.com/comp/2DA676P/single-continuous-line-drawing-of-young-happy-male-and-female-couple-doing-romantic-date-and-dinner-together-at-coffee-shop-marriage-life-concept-2DA676P.jpg
- Title Image: https://encrypted-tbn1.gstatic.com/images?q=tbn:ANd9GcSV7IX0PlyXBbN8PXqQH-xWOwZhK5PP9BZM0RO_TkUFXZzH_SbO

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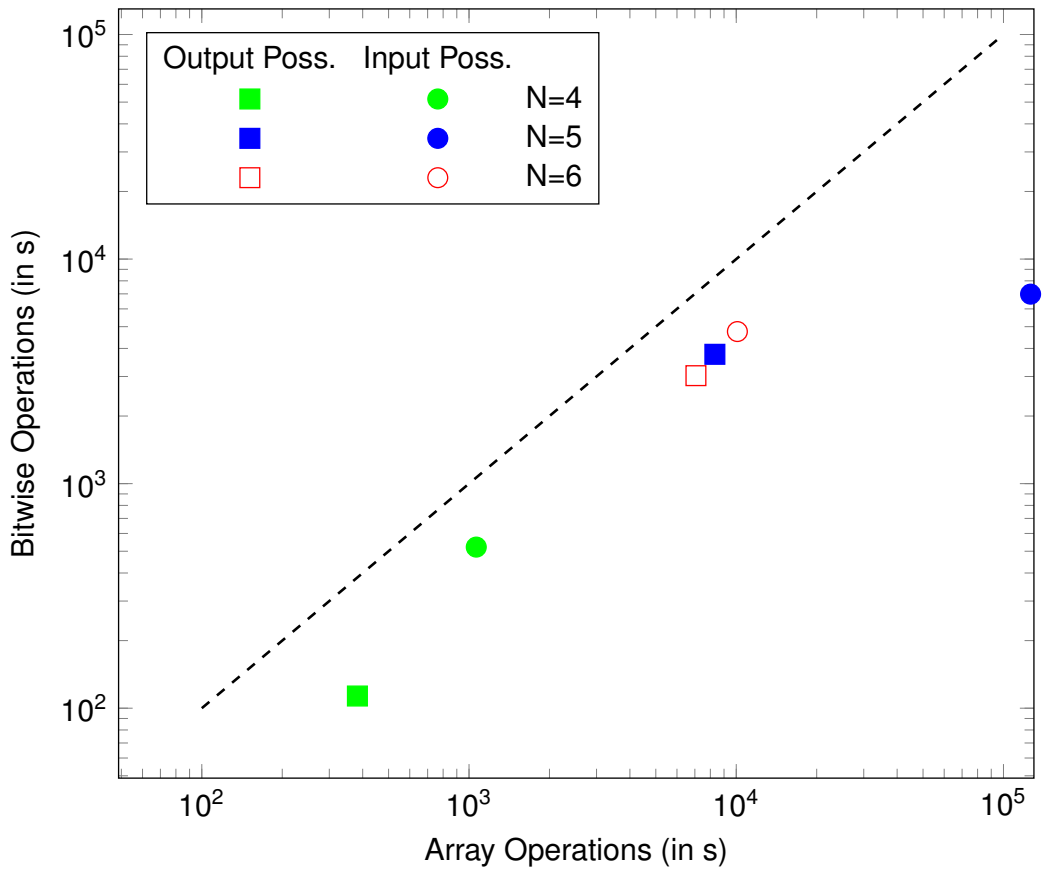
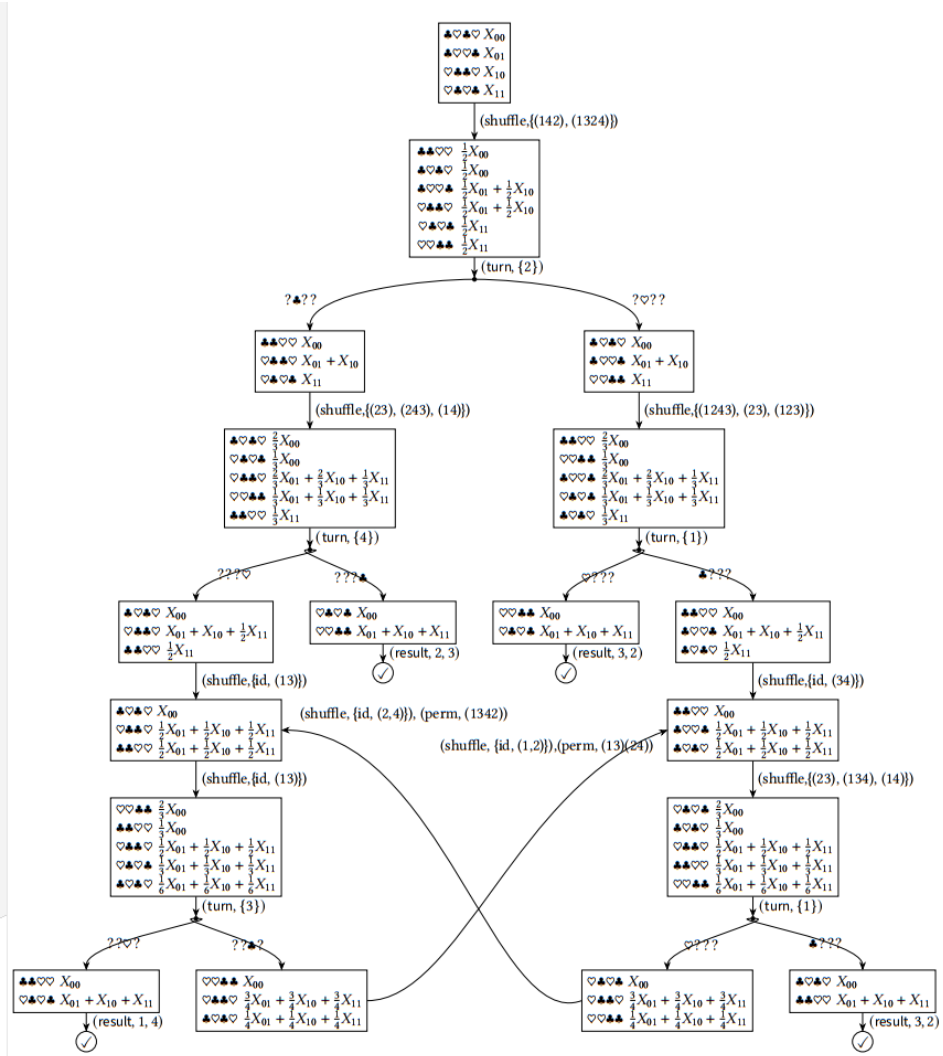
Anne Hoff | March 30, 2023

Experiments with the Alternative Data Structure - Logarithmic Scale



OR Protocol with Uniform, Non-closed Shuffles



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