Mathematical Methods I

Assignment 6: Probability, Statistics & Data II

Total marks: 25

All questions are compulsory. Please hand in your answer sheets and submit your codes to one of the tutors no later than **5pm Monday 25 September 2017**. You are encouraged to use online help in writing the code for generating the plots below.

1 Random number generators (RNG) *(25 marks)*

Consider the linear congruential generators defined, for $n \geq 0$, by

$$\begin{aligned} \text{PiE}: &\quad Y_{n+1} = (3141592653\, Y_n + 2718281829) \mod 2^{35} \\ \text{RandU}: &\quad Y_{n+1} = (65539\, Y_n) \mod 2^{31} \end{aligned}$$

where the seed Y_0 can be an arbitrary integer for PiE while it must be an odd integer for RandU. The generator PiE is due to Knuth while RandU was a popular RNG in the 1960's and early 1970's.

(Caution!): *The numbers appearing above are exact integers. Do not modify their values!*

In the following, use the seed value $Y_0 = 1$ in all calculations.

1. Set up a numerical code for generating the integer sequence $\{Y_1, Y_2, \ldots, Y_N\}$ for arbitrary $N \geq 1$, for each of the generators. Having computed these integers, the code should output the floating point numbers $U_n \equiv Y_n/m$, $1 \leq n \leq N$, where the modulus $m = 2^{35}$ for PiE and $m = 2^{31}$ for RandU. Demonstrate that your code reproduces the values (up to 5 significant digits) $U_{10} = 5.5224 \times 10^{-1}$ for PiE and $U_{10} = 6.8024 \times 10^{-3}$ for RandU. *(Make sure you get these values, before proceeding.)*

2. For each of PiE and RandU, generate the sequence $\{U_n\}_{n=1}^N$ with $N = 10^4$ and construct a histogram $p(u)$ of these values using 10 equally spaced bins in the range $0 \leq u < 1$. The histograms should be normalised so that they cover unit area underneath themselves: $\int_0^1 \mathrm{d}u\, p(u) = 1$. Plot and compare the resulting histograms with the expected distribution $p(u) = 1$ if $0 \leq u < 1$ and zero otherwise.

3. For each of PiE and RandU, generate the sequence of *doublets* $\{(U_{2n-1}, U_{2n})\}_{n=1}^N$ with $N = 10^4$. Plot each of these sequences (separately) as scatter plots in 2 dimensions (i.e., interpret these sequences as $\{(x_n, y_n)\}_{n=1}^N$). Do the 2-d distributions appear uniform in each case?

4. For each of PIE and RANDU, generate the sequence of *triplets* $\{(U_{3n-2}, U_{3n-1}, U_{3n})\}_{n=1}^{N}$ with $N = 10^4$. Plot each of these sequences (separately) as scatter plots in 3 dimensions (i.e., interpret these sequences as $\{(x_n, y_n, z_n)\}_{n=1}^{N}$). Do the 3-d distributions appear uniform in each case?

   [*Hint:* To answer this question properly you must explore a wide range of viewing angles ("camera positions") for the plot. You only need to report the most interesting of these.]

5. Based on the results above, which RNG is better, PIE or RANDU? Why?