

## SMML: Exercise 01

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### Exercise 1-6

$$\text{Ex (1)} \quad (AB)^T_{ij} = (AB)_{ji} = \sum_{k=1}^n A_{jk} B_{ki} \rightarrow \textcircled{1}$$

$$(B^T A^T)_{ij} = \sum_{k=1}^n (B^T)_{ik} (A^T)_{kj} = \sum_{k=1}^n B_{ki} A_{jk} \rightarrow \textcircled{2}$$

$\textcircled{1} = \textcircled{2}$  hence proved.

$$\text{Ex (2)} \quad (AB)^{-1} = B^{-1} A^{-1}$$

we know that  $D^{-1} D = I$

$$\therefore (AB) (AB)^{-1} = I$$

$$(AB) (B^{-1} A^{-1}) = A B B^{-1} A^{-1} = A I A^{-1} = A A^{-1} = I$$

hence proved.

$$\text{Ex (3)} \quad \frac{\partial}{\partial x} (b^T x) = b$$

comparing the LHS and RHS row wise

$$\frac{\partial}{\partial x_i} (b^T x)_i = \frac{\partial}{\partial x_i} \sum_{j=1}^n b_j x_j$$

$$= \sum_{j=1}^n b_j \frac{\partial (x_j)}{\partial x_i} = \sum_{j=1}^n b_j \delta_{ij} = b_i$$

$$\text{Ex (4)} \quad \frac{1}{2} \frac{\partial}{\partial x} (x^T A x) = \frac{1}{2} \frac{\partial}{\partial x} \sum_{k=1}^n (x^T A)_{ik} x_k$$

$$= \frac{1}{2} \frac{\partial}{\partial x} \sum_{k=1}^n \left[ \left( \sum_{j=1}^n x_j A_{jk} \right)_{ik} x_k \right]$$

$$= \frac{1}{2} \left[ \sum_{k=1}^n \left( \sum_{j=1}^n x_j A_{jk} \right)_{ik} \delta_k + \sum_{k=1}^n \left( \frac{\partial}{\partial x} \sum_{j=1}^n x_j A_{jk} \right) x_k \right]$$

$$= \frac{1}{2} \left( \sum_{k=1}^n x_j A_{jk} + \sum_{k=1}^n A_{kj} x_j \right) \text{ as } A \text{ is symmetric}$$

$$A_{jk} = A_{kj}$$

$$= \sum_{k=1}^n x_j A_{jk} = Ax$$

$$\text{Ex (5)} \quad \frac{\partial}{\partial x} \|x\| = x / \|x\|$$

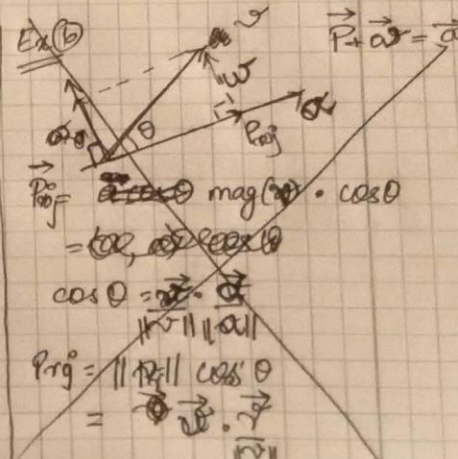
$$\|x\| = \sqrt{\sum x_i^2}$$

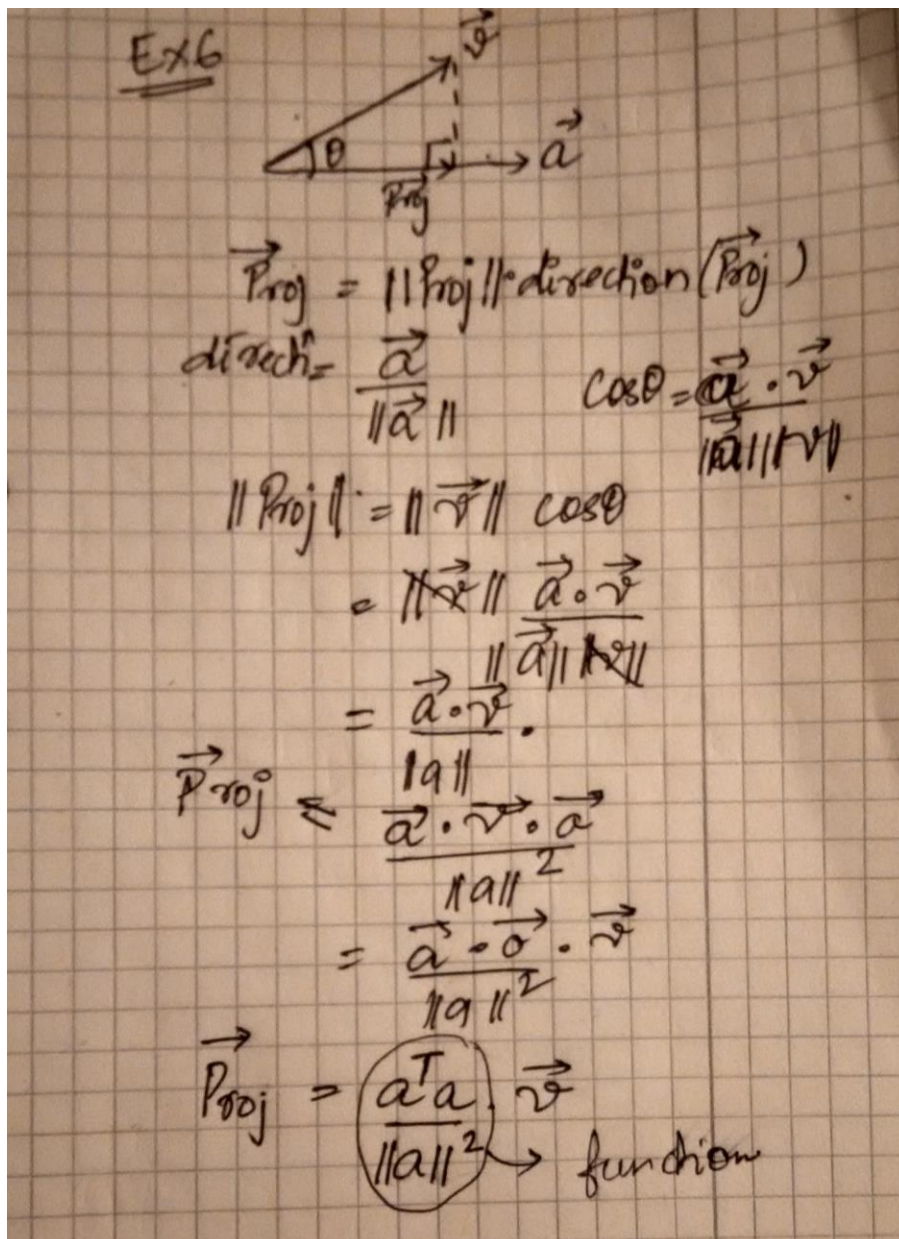
$$\frac{\partial}{\partial x} \left( \sum x_i^2 \right)^{1/2} = \frac{1}{2} \left( \sum x_i^2 \right)^{-1/2} \cdot \frac{\partial}{\partial x} \left( \sum x_i^2 \right)$$

$$= \frac{1}{2} \left( \sum x_i^2 \right)^{-1/2} \cdot \sum 2 x_i \frac{\partial x_i}{\partial x}$$

$$= \frac{1}{2} \left( \sum x_i^2 \right)^{-1/2} \cdot 2 x_i = x_i / \left( \sum x_i^2 \right)^{1/2}$$

$$= \frac{x}{\|x\|}$$



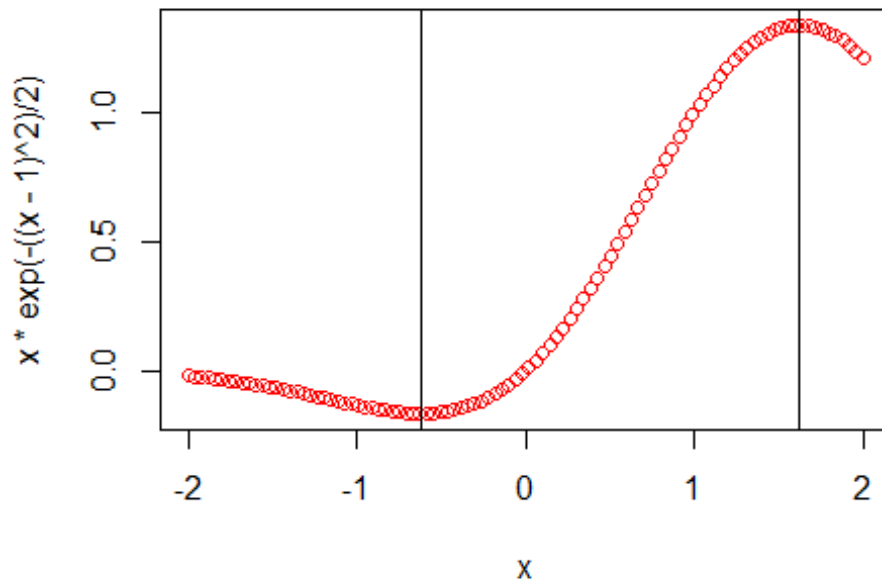


## Exercise 7

```

x=seq(-2,2,length.out = 100)
v_max=(1+sqrt(5))/2
v_min=(1-sqrt(5))/2
m=c(v_max,v_min)
plot(x,x*exp(-((x-1)**2)/2), col='red')
abline(v=m)

```



## Exercise 8

```
data("iris")
summary(iris$Sepal.Length)

##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
##   4.300   5.100   5.800   5.843   6.400   7.900

summary(iris$Petal.Length)

##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
##   1.000   1.600   4.350   3.758   5.100   6.900

summary(iris$Sepal.Width)

##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
##   2.000   2.800   3.000   3.057   3.300   4.400

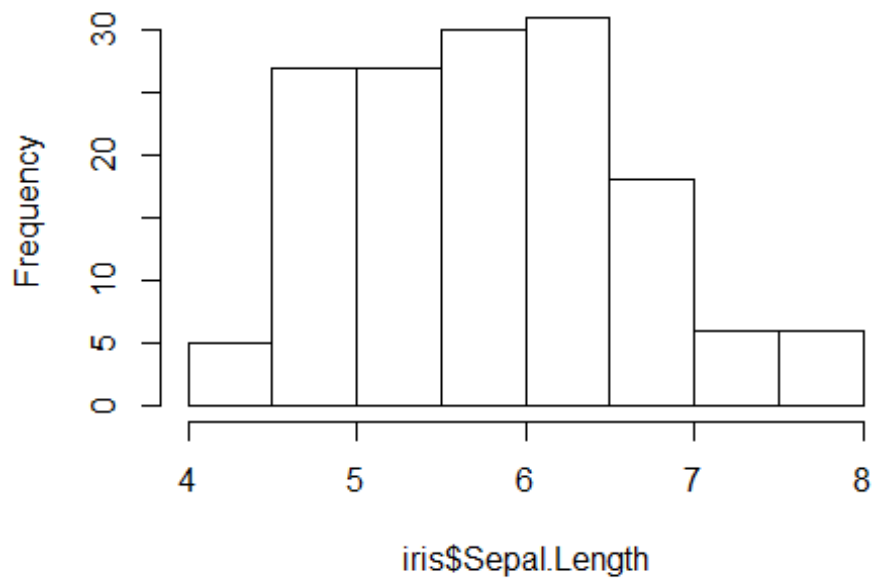
summary(iris$Petal.Width)

##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
##   0.100   0.300   1.300   1.199   1.800   2.500
```

## Exercise 9

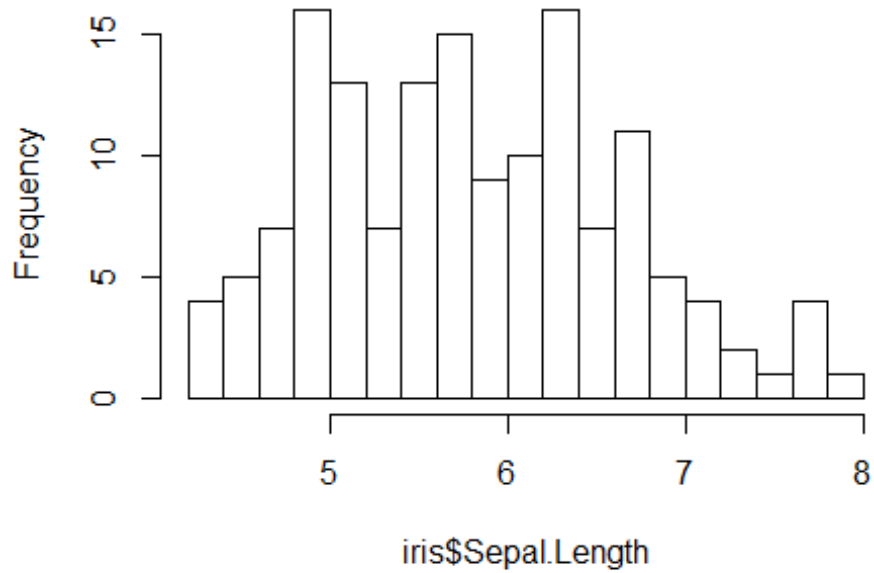
```
## Histogram of Sepal Length default with break
hist(iris$Sepal.Length)
```

**Histogram of iris\$Sepal.Length**



```
## Histogram of Sepal Length with break 20  
hist(iris$Sepal.Length, breaks = 20)
```

**Histogram of iris\$Sepal.Length**



## Exercise 10

```
sertosa=iris$Species==levels(iris$Species)[1]
sertosa_no=iris[sertosa,]$Sepal.Length<5.5
sprintf("%d %s flowers have Sepal Length <5.5, rest %d do
not",sum(sertosa_no),levels(iris$Species)[1],50-sum(sertosa_no))

## [1] "45 setosa flowers have Sepal Length <5.5, rest 5 do not"

virginica=iris$Species==levels(iris$Species)[3]
virginica_no=iris[virginica,]$Sepal.Length<5.5
sprintf("%d %s flowers have Sepal Length <5.5, rest %d do
not",sum(virginica_no),levels(iris$Species)[3],50-sum(virginica_no))

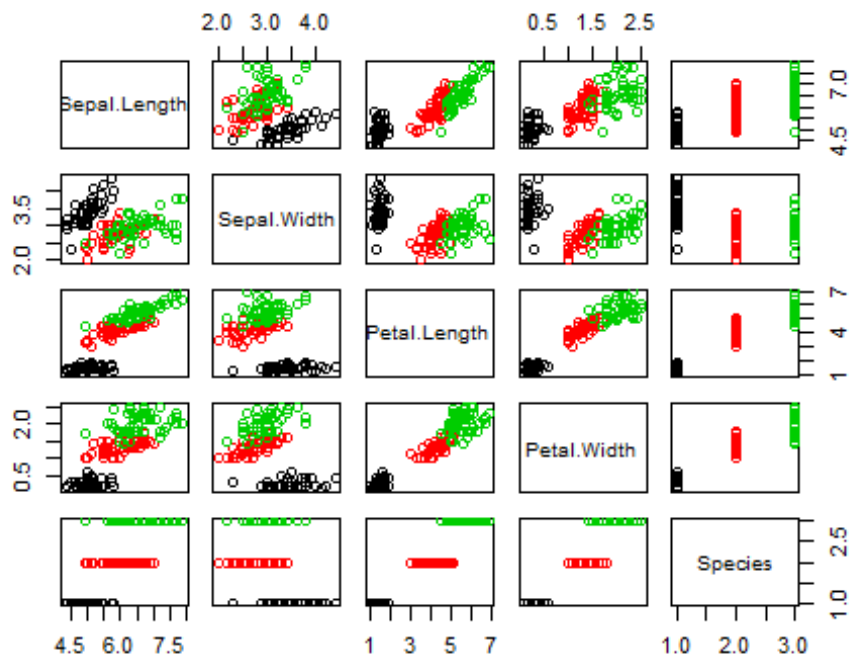
## [1] "1 virginica flowers have Sepal Length <5.5, rest 49 do not"

versicolor=iris$Species==levels(iris$Species)[2]
versicolor_no=iris[versicolor,]$Sepal.Length<5.5
sprintf("%d %s flowers have Sepal Length <5.5, rest %d do
not",sum(versicolor_no),levels(iris$Species)[2],50-sum(versicolor_no))

## [1] "6 versicolor flowers have Sepal Length <5.5, rest 44 do not"
```

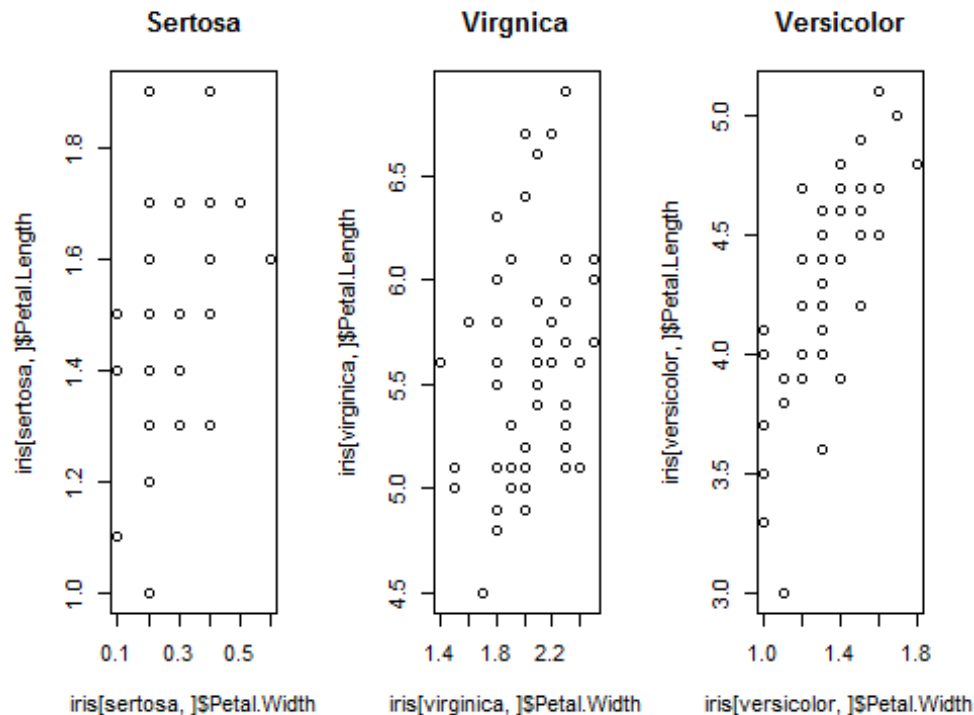
## Exercise 11

```
pairs(iris, col=iris$Species)
```



## Exercise 12

```
par(mfrow=c(1,3))
plot(iris[setosa,]$Petal.Width, iris[setosa,]$Petal.Length, main =
'Setosa')
plot(iris[virginica,]$Petal.Width, iris[virginica,]$Petal.Length, main =
'Virginica')
plot(iris[versicolor,]$Petal.Width, iris[versicolor,]$Petal.Length, main =
'Versicolor')
```



## Exercise 13

```
p1= c(-1,-1,0)
p2= c(-1,1,0)
p3= c(1,1,0)
p4= c(1,-1,0)
p5= c(0,0,2)

v = seq(0,1,0.05)

e1 = v %%% (p2-p1) + rep(1,21) %% p1
e2 = v %%% (p3-p2) + rep(1,21) %% p2
e3 = v %%% (p4-p3) + rep(1,21) %% p3
e4 = v %%% (p1-p4) + rep(1,21) %% p4
e5 = v %%% (p5-p1) + rep(1,21) %% p1
e6 = v %%% (p5-p2) + rep(1,21) %% p2
e7 = v %%% (p5-p3) + rep(1,21) %% p3
```



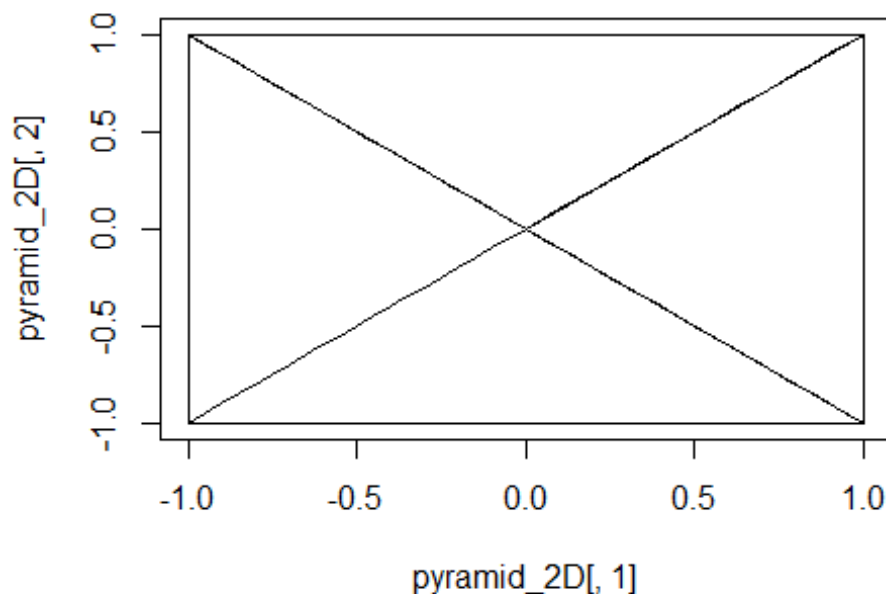
```

e8 = v %o% (p5-p4) + rep(1,21) %o% p4

pyramid = rbind(e1,e2,e3,e4,e5,e6,e7,e8)

projection_matrix <- t(matrix(c(1,0,0,0,1,0), nrow =2, ncol= 3,byrow= TRUE))
pyramid_2D <- pyramid %*% projection_matrix
plot(pyramid_2D[,1],pyramid_2D[,2], type="l")

```



## Exercise 14

```

phi=0.05*pi
theta=0.3*pi

rotate_z<-matrix(c(cos(phi),-sin(phi),0,sin(phi),cos(phi),0,0,0,1), nrow =3,
ncol= 3,
                byrow= TRUE)

rotate_y<-matrix(c(cos(theta), 0, sin(theta), 0, 1, 0, -sin(theta), 0,
cos(theta)),
                nrow =3, ncol= 3, byrow= TRUE)

post_rot_z = pyramid %*% rotate_z %*% projection_matrix
post_rot_y = pyramid %*% rotate_y %*% projection_matrix
par(mfrow=c(1,2))
plot(post_rot_z[,1], post_rot_z[,2],type = "l")
plot(post_rot_y[,1], post_rot_y[,2], type = "l")

```

