Stat238: Lab 3 DRAFT

September 9

- This lab explores the influence of priors on the posterior in more detail.

Problems

1. I call this problem "Brian's paradox" as it arose from a student question when I taught a previous version of the class. Consider $y_i, i = 1, \dots, n$ conditionally independent with $y_i|\theta \sim N(\theta, \sigma^2)$ where for the purpose of what we want to illustrate, let $\sigma^2 = n$. Consider the prior $\theta \sim N(0, \sigma_0^2)$ where $\sigma_0^2 = 1$. Now suppose we observe the ys and find $\bar{y} = 100$.

    (a) What is the posterior? Plot the prior, likelihood, and posterior on a single plot. What is the 'paradox'? Why does it happen?

    (b) Now try using a *t* distribution as the prior for θ. We don't have a closed form solution, but you can still plot the prior, likelihood, and (up to a normalizing constant) the posterior. Does that give a more sensible answer?

2. Let $y \sim N(\theta, \sigma^2)$. Suppose that there is a prior probability, q, that $\theta = 0$. With probability $1 - q$, the prior is $\theta \sim N(0, \omega^2)$. I.e., the prior is a mixture of a point mass at zero and a normal distribution centered at 0 with variance ω^2. Let's consider the posterior probability that $\theta = 0$.

    (a) Define A to be the event $\theta = 0$, namely that θ comes from the point mass part of the prior and A^c that θ comes from the continuous part of the prior. Consider Bayes theorem involving a probability and a density:
    $$P(A|y) = \frac{p(y|A)P(A)}{p(y)}$$
    Write out $p(y)$; you should see that it involves an integral that you should be able to do without any explicit integration.

    (b) Let $\omega^2 \to \infty$. What does that posterior probability converge to as $\omega^2 \to \infty$? Notice anything strange? This is basically a model selection problem in disguise (choosing between $\theta = 0$ and $\theta \neq 0$), and we'll see later in the semester that standard Bayesian model selection does not work with non-informative priors.