Stat238: Lab 2

September 1, 2016

Comments:

- Labs are intended to allow for hands-on work with a partner or in groups to work through problems. For problems on problem sets that were presented in lab, you may present a group answer.

Problems

1. Practice with transformation of variables (p. 21 of BDA). Recall from BDA (and from probability class) that given a continuous random variable, u, with distribution $p_u(u)$ and a one-to-one transformation, $v = f(u)$, we can derive the distribution $p_v(v)$ as

$$p_v(v) = |J| p_u(f^{-1}(v))$$

for $J = \frac{df^{-1}(v)}{dv} = \frac{du}{dv}$. I remember this derivative as "dold / dnew" because u is the old parameter and v the new one.

   (a) Let $\psi = \text{logit}(\theta) = \log \frac{\theta}{1-\theta}$ where θ might be the probability involved in a binomial likelihood. Suppose I place a flat prior on ψ, $p(\psi) \propto 1$. What prior am I implicitly placing on θ? Interpret the result in terms of distributions we've already discussed.

   (b) In the normal distribution, $X \sim N(\mu, \sigma^2)$, the standard deviation, σ, is a scale parameter. More generally if we have that the density for X has the form $\frac{1}{\theta} f(\frac{x}{\theta})$ for some function f, then θ is a scale parameter (see also BDA p. 54). Based on invariance arguments, a natural prior for a scale parameter θ is $P(\theta) \propto \frac{1}{\theta}$.

      - Derive the implied prior for $\psi = \log \theta$.
      - Derive the implied prior for $\phi = \theta^2$.

2. Conjugacy and predictive distributions (this problem is part of PS1).

   (a) Show that if $y|\theta$ is exponentially distributed with rate θ, then the gamma prior distribution is conjugate for θ given (conditionally) independent and identically distributed y values.

   (b) Derive the predictive distribution $p(y)$, including its normalizing constant. Note that the integration over θ should be easy because in part (a) you recognized the posterior for θ as a gamma distribution and you know its normalizing constant.