

**THE COLLISIONAL, CAPACITIVE RF SHEATH:
MODELS WITH AND WITHOUT THE
APPROXIMATION OF A SHARP ELECTRON EDGE**

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The transition from quasi-neutrality to charge depletion is one of the characteristic features of the plasma boundary sheath. For modeling purposes, this transition is often described in terms of the so-called step model which assumes a sharp transition point s (electron step) where the electron density n_e drops from a value equal to the ion density n_i (in the bulk, $x > s$) to a value of zero (in the sheath, $x < s$). Inserted into Poisson's equation, the step model yields an expression for the field which is realistic deep in the sheath but fails to merge correctly into the ambipolar field of the bulk. This work studies the consequences of that approximation for the example of the collision-dominated, capacitive RF sheath by Lieberman¹. The equations of the model combine an equation of continuity and a mobility ansatz for the ion fluid with the assumption of quasi-static Boltzmann equilibrium for the electrons and with Poisson's equation for the field. The modulation is controlled by the amplitude of the harmonic RF current. First, the model is solved exactly, using a relaxation scheme on a suitable spatio-temporal discretization. Then, the step approximation is applied which recovers Lieberman's semi-analytical solution. It is demonstrated that the step approximation induces a spurious divergence of the ion density at the sheath edge and prevents a matching of the sheath model to a bulk model. Integral sheath quantities, on the other hand, like the capacitance or the overall voltage drop, are faithfully reproduced, provided that the applied RF voltage is large compared to the thermal voltage T_e/e . The consequences of this finding for the modeling of RF plasmas are discussed. Also studied is the performance of a recently proposed improved approximation of the Boltzmann-Poisson equation² which treats the transition from quasi-neutrality to charge depletion more rigorously. It is shown that this approximation avoids the spurious divergence of the step model and leads to better agreement with the numerical solution.

1. M. A. Lieberman, "Analytical Solution for a Capacitive Sheath", IEEE Trans. Plasma Sci. 16, pp. 638-644 (1988).
2. R. P. Brinkmann, "Beyond the step model: Approximate expressions for the field in the plasma boundary sheath" J. Appl. Phys. 102, 093303 (2007).