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An analytic expression for the sheath criterion in magnetized plasmas with multi-charged ion species

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The generalized Bohm criterion in magnetized multi-component plasmas consisting of multi-charged positive and negative ion species and electrons is analytically investigated by using the hydrodynamic model. It is assumed that the electrons and negative ion density distributions are the Boltzmann distribution with different temperatures and the positive ions enter into the sheath region obliquely. Our results show that the positive and negative ion temperatures, the orientation of the applied magnetic field and the charge number of positive and negative ions strongly affect the Bohm criterion in these multi-component plasmas. To determine the validity of our derived generalized Bohm criterion, it reduced to some familiar physical condition and it is shown that monotonically reduction of the positive ion density distribution leading to the sheath formation occurs only when entrance velocity of ion into the sheath satisfies the obtained Bohm criterion. Also, as a practical application of the obtained Bohm criterion, effects of the ionic temperature and concentration as well as magnetic field on the behavior of the charged particle density distributions and so the sheath thickness of a magnetized plasma consisting of electrons and singly charged positive and negative ion species are studied numerically. © *2015 AIP Publishing LLC*.
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I. INTRODUCTION

The study of the formation of the sheath, a thin positive space charge region with a thickness of several electron Debye lengths, at the interface between a plasma and a wall has become more and more important in terms of theoretical interest, as well as in plasma applications.[1] Formation of this thin layer arises from the greater mobility of the electrons as compared to the mobility of ions because $m_i/m_e \gg 1$, where m_e and m_i are the electron and ion masses, respectively.

One of the most important data to investigate the plasma-boundary interactions is the velocity at which ions leave the plasma. This velocity is important in many plasma applications such as fusion edge plasmas, plasma-based materials processing, and space plasmas.[1] For a collisionless plasma consisting of only one species of ion, this velocity has to be greater than the ion sound velocity known as the Bohm criterion, i.e., $v_i \geq c_s = (T_e/m_i)^{1/2}$, where c_s is the ion sound velocity, T_e is the electron temperature, and m_i is the ion mass.[2] In fact, this criterion is the necessary condition for the existence of a stationary sheath.

Sheath structure and the Bohm criterion in electropositive and electronegative plasmas have been investigated by many authors, both theoretically and experimentally.[3–13] For example, Chodura introduced a hydrodynamic model for a semi-infinite plasma emerged in an oblique magnetic field where ionization and collisions were not taken into account.[4] It was shown that plasma particles pass through three regions on their way toward the wall: quasineutral plasma region, the quasineutral magnetized presheath region (the Chodura layer), and a collisionless thin space charge sheath. However, experiments do not support the existence of magnetic presheath.[5] It seems that the Chodura layer is a consequence of the specific mathematical model rather than a physical phenomenon. In this case, it was shown that the Chodura layer is eliminated by accounting for ion-neutral collisions and ionization.[6,7] In addition, Liu *et al*.[9] have investigated the sheath criterion in a collisional plasma sheath by a two-fluid model. They have shown that there are upper and lower limits for the sheath criterion when collisions between ions and neutrals are taken into account. Furthermore, Das *et al*. studied the plasma sheath formation and Bohm criterion in a collisionless thermal plasma.[10] They showed that sheath thickness decreases by increasing the positive ion temperatures. Moreover, considering the effects of ion temperature, Liu *et al*.[11] obtained the sheath criterion for a collisionless, magnetized plasma and showed that the ion velocity entering the sheath can be lower than the ion acoustic velocity, depending on the incident angle of magnetic field and the ratio of the ion temperature to electron temperature. In addition, the modified Bohm sheath criterion was investigated in a magnetized, collisional plasma consisting of electron and thermal ions in Ref. 12.

The conditions are somewhat complicated in multi-component plasmas such as dusty plasma, electronegative plasma with two negative components and electropositive plasma with two species of positive ions. In 1995, Riemann[14] modified the Bohm criterion to the multi-component plasmas by considering a weakly collisional plasma with Maxwellian electrons as follows:

$$\sum_i \frac{n_{0i}}{n_{0e}} \left(\frac{c_{si}}{v_i}\right)^2 \leq 1, \tag{1}$$

where v_i and c_{si} are the individual drift and sound velocity of ions and n_{0i} and n_{0e} are the ion and electron densities at the

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sheath boundary, respectively. For electronegative plasmas Braithwaite and Allen[15] showed that, in the cold positive ion approximation, the Bohm criterion is modified as $v_{0i} \geq c_s[n_{0i}T_n/(n_{0e}T_n + n_{0n}T_e)]^{1/2}$, where T_n is the negative ion temperatures and n_{0n}, n_{0e}, and n_{0i} are density of negative ions, electrons, and positive ions at the sheath edge, respectively. Also, Wang *et al.*[16] studied the Bohm criterion for electronegative plasmas composed of electrons, negative, and positive ions, as well as dust grains. They found that both positive ion and dust Bohm velocities increase with the growth of dust density due to the interaction between positive ions and dust grains, while both of them decrease by increasing the negative ion density owing to the sheath edge conditions modified by negative ions. In addition, Franklin[17] showed that in an active collisionless plasma containing more than one species of ion generated by electron impact each ion species reaches its own Bohm velocity at the plasma-sheath interface when the ionization rates are constant. However, experimental results of two-ion species plasmas do not prove this prediction. The measured velocity of each ion species in these experiments was closer to the system sound velocity, c_{cs}, than their individual sound velocities[18,19]

$$c_{cs} = \sqrt{\sum_i \frac{n_{0i}}{n_{0e}} c_{si}^2}. \tag{2}$$

Using a combination of electrostatic probes and measuring the velocity of ion-acoustic wave at the sheath edge, some authors tried to investigate this common sound velocity c_{cs}.[20,21] In this case, Oksuz *et al.*[21] reported that the ion-acoustic wave velocity at the sheath edge is approximately twice of what is in the bulk plasma in a two ion species plasma. Also, the Bohm sheath criterion in two ion species plasmas was studied with laser-induced fluorescence in unmagnetized *Ar–Xe* plasmas by Lee *et al.*[22] They showed that the reports of Oksuz *et al.* can be concluded in this way that each ion species enters the sheath with the system sound velocity c_{cs}. In addition, Baalrud and Hegna[18] have shown that the ion-ion two stream instability may lead to a collisional friction between the two ion species, accelerating the slower species and decelerating the faster one. They showed that this effect establishes the solution of Bohm criterion in two ion species plasmas. They confirmed that in a cold multi-component plasma, each ion species enters the sheath at the system sound speed c_{cs} while in a multi-component plasma with finite ion temperatures the individual speeds differ from the system sound speed by an amount depending on thermal speeds, and relative densities of each species. Moreover, using a multi-fluid model, the generalized Bohm criterion in an unmagnetized plasma consisting of electrons and multi-charged ions was studied and it was shown that the ion Bohm velocity increases by increasing the charge number of positive ions ($Z = q/e$, where q and e are the electric charge of positive ion and the elementary electron charge, respectively) and decreasing the charge number of negative ions.[23] However, the transition condition for multi-component plasma sheathes in the presence of an applied magnetic field and thermal ions has not been studied yet.

In this paper, in a sequel to the earlier works,[14–16,23] the analytical derivation for the sheath formation criterion (Bohm criterion) in multi-component magnetized plasmas consisting of electrons and multi-charged negative and thermal positive ion species is presented by analyzing the Sagdeev potential for these plasmas. The obtained results show that the sheath criterion depends strongly on the temperature, concentration, and charge number of positive and negative ion species and also on the orientation of the magnetic field. In this case, it is found that an increase in the charge number of positive and negative ions leads to an increase and decrease in the positive ion Bohm velocity, respectively. Also, it is shown that by increasing the incident angle of magnetic field to the wall (more glancing) the positive ion Bohm velocity and the thickness of the sheath region decrease. In addition, it is observed that an increase in temperature of positive and negative ion species leads to an increase in the minimum entrance velocity of positive ion that is required for the Bohm criterion to satisfy. Furthermore, it is seen that the sheath width decreases by increasing the temperature of positive and negative ion species.

This work is organized in four sections including the Introduction as the first section. In Sec. II, we explain our model and basic equations. In Sec. III, we calculate the modified Bohm criterion analytically and examine it in some interesting physical conditions, and finally a brief conclusion is presented in Sec. IV.

II. MODEL AND BASIC EQUATIONS

In this section, the model and the basic equations are used to investigate the planar bounded plasma-wall problem and the existence condition of the electrostatic sheath is presented. Using a many-fluid model, we consider a magnetized plasma system consisting of electrons, multi-charged positive and negative ion species and neutral atoms. We assume that the ion mean-free path is larger than the sheath thickness, which can be as large as tens of Debye length. Hence, our model is collisionless. Strictly speaking, the model is valid for pressures below 50 mTorr. Also, we assume that an external constant magnetic field $\vec{B}$ is applied on the sheath region (in the x–z plane) and makes angle θ with the x direction ($\vec{B}/B = (\cos\theta, 0, \sin\theta)$). The x direction is taken as the depth direction from the plasma edge to the wall, and the boundary between the plasma ($x < 0$) and sheath ($x > 0$) is the plane of $x = 0$ (see Fig. 1). Therefore, there is a $\vec{E} \times \vec{B}$ drift in the y direction. Also, we assume that the external

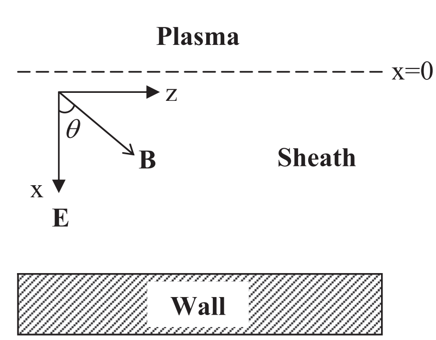

FIG. 1. Geometry of the magnetized sheath.

magnetic field is weak and the low-pressure plasma is not highly electronegative as well. Therefore, negative ion density has the Boltzmann distribution.[24,25] In addition, the wall potential is specified by a given value φ_w which can be the floating potential or a more negative value for which Boltzmann distribution holds, i.e., $|\varphi_w| > 3k_BT_e/e$.[26] Therefore, the electron and negative ion density distributions are written as follows:[15,26,27]

$$n_e = n_{0e}\exp\left(\frac{e\varphi}{T_e}\right), \tag{3}$$

$$\tilde{n}_j = \tilde{n}_{0j}\exp\left(\frac{e\tilde{Z}_j\varphi}{\tilde{T}_j}\right), \tag{4}$$

where φ is the electrostatic potential, n_{0e} and $\tilde{n}_{0j}$ are the electron and jth negative ion densities at the sheath edge, T_e is the electron temperatures, and $\tilde{T}_j$ and $\tilde{Z}_j$ are the temperature and charge number of the jth negative ion species, respectively. In this case, it should be mentioned that both electrons and negative ions experience the presence of the magnetic field; but due to the repulsion of the negative wall potential, the velocity of negative ions and electrons within the sheath slows down, and therefore, the electric forces dominate the magnetic force. Therefore, in most cases of interest, the wall potential exceeds the thermal energy of negative ions and electrons, so that the assumption of the Boltzmann distribution for electron and ions sounds quite reasonable.

In the steady state $(\partial/\partial t = 0)$, for a magnetized collisionless plasma sheath, the ion fluid equations are

$$\nabla.(\hat{n}_i\vec{v}_i) = 0, \tag{5}$$

$$(\vec{v}_i.\nabla)\vec{v}_i = \frac{e\hat{Z}_i}{m_i}\left(\vec{E} + \vec{v}_i\times\vec{B}\right) - \frac{1}{m_i\hat{n}_i}\nabla P_i, \tag{6}$$

where $\hat{n}_i, \vec{v}_i$, m_i, $\hat{Z}_i$, and P_i are the density, velocity, mass, charge number, and partial pressure of the ith positive ion, respectively, and $\vec{E} = -\nabla\varphi$.

As shown by Riemann,[28] the fluid approximation for a plane sheath is consistent if the positive ion flow is adiabatic. Therefore, the positive ion partial pressure P_i is related to the ion density as follows:

$$P_i = k_B\hat{T}_i\frac{\hat{n}_i^{\gamma_i}}{\hat{n}_{0i}^{\gamma_{i-1}}}, \tag{7}$$

where k_B is Boltzmann constant, $\hat{T}_i$ and $\hat{n}_{0i}$ are the ith positive ion temperature and density at the sheath edge, and $\gamma_i = 3, 2, 5/3$ for unidimensional, bidimensional, or tridimensional adiabatic flow, respectively.[14,28]

The Poisson's equation for such a plasma consisting of multi-charged positive and negative ion species and electrons is written as follows:

$$\nabla^2\varphi = \frac{e}{\varepsilon_0}\left[n_e + \sum_j\tilde{Z}_j\tilde{n}_j - \sum_i\hat{Z}_i\hat{n}_i\right], \tag{8}$$

where ε_0 is the electric permittivity of free space.

Finally, for such a plasma the quasineutrality condition at the plasma-sheath edge can be written as follows:

$$n_{0e} + \sum_j\tilde{Z}_j\tilde{n}_{0j} = \sum_i\hat{Z}_i\hat{n}_{0i}. \tag{9}$$

Assuming the wall is infinitely long in y and z directions, the quantities change only in x direction normal to the wall, i.e., $\nabla\to\hat{x}\partial/\partial x$. Therefore, Eqs. (5), (6), and (8) take the following forms:

$$\hat{n}_iv_{ix} = \hat{n}_{0i}v_{0ix}, \tag{10}$$

$$v_{ix}\frac{\partial v_{ix}}{\partial x} = \frac{e\hat{Z}_i}{m_i}\left(-\frac{\partial\varphi}{\partial x} + Bv_{iy}\sin\theta\right) - \frac{\hat{T}_i}{m_i\hat{n}_i}\frac{\partial}{\partial x}\left(\frac{\hat{n}_i^{\gamma_i}}{\hat{n}_{0i}^{\gamma_{i-1}}}\right), \tag{11}$$

$$v_{ix}\frac{\partial v_{iy}}{\partial x} = \frac{e\hat{Z}_i}{m_i}B(v_{iz}\cos\theta - v_{ix}\sin\theta), \tag{12}$$

$$v_{ix}\frac{\partial v_{iz}}{\partial x} = \frac{-e\hat{Z}_i}{m_i}Bv_{iy}\cos\theta, \tag{13}$$

$$\frac{\partial^2\varphi}{\partial x^2} = \frac{e}{\varepsilon_0}\left(n_e + \sum_j\tilde{Z}_j\tilde{n}_j - \sum_i\hat{Z}_i\hat{n}_i\right). \tag{14}$$

The following normalizations have been introduced in order to express the model equations, i.e., Eqs. (3), (4), and (10)–(14), in terms of the fundamental plasma quantities

$$\vec{u}_i = \vec{v}_i/c_{si},\quad \hat{N}_i = \hat{n}_i/n_{0e},\quad \tilde{N}_j = \tilde{n}_j/n_{0e},\quad N_e = n_e/n_{0e},$$
$$\hat{\zeta}_i = \hat{T}_i/T_e,\quad \tilde{\zeta}_j = \tilde{T}_j/T_e,\quad \xi = x/\lambda_{De},\quad \alpha_i = \omega_{ci}/\omega_{pi},$$
$$\phi = e\varphi/T_e,\quad \hat{\delta}_i = \hat{n}_{0i}/n_{0e},\quad \tilde{\delta}_j = \tilde{n}_{0j}/n_{0e},$$

where $\lambda_{De} = (\varepsilon_0T_e/n_{0e}e^2)^{1/2}$ is the electron Debye length, and $c_{si} = (T_e/m_i)^{1/2}$, $\omega_{ci} = eB/m_i$, and $\omega_{pi} = (n_{0i}e^2/\varepsilon_0m_i)^{1/2}$ are the cold-ion-acoustic speed, gyrofrequency, and plasma frequency for the ith positive ion, respectively.

With these dimensionless quantities the normalized set of the model equations can be written as follows:

$$\frac{\partial u_{ix}}{\partial\xi} = -\hat{Z}_iu_{ix}^{\gamma_i}\left(\frac{\frac{\partial\phi}{\partial\xi} - \alpha_i\hat{\delta}_i^{1/2}u_{iy}\sin\theta}{u_{ix}^{\gamma_i+1} - \gamma_i\hat{\zeta}_iu_{0ix}^{\gamma_i-1}}\right), \tag{15}$$

$$\frac{\partial u_{iy}}{\partial\xi} = \hat{Z}_i\alpha_i\hat{\delta}_i^{1/2}\left(\frac{u_{iz}\cos\theta - u_{ix}\sin\theta}{u_{ix}}\right), \tag{16}$$

$$\frac{\partial u_{iz}}{\partial\xi} = -\hat{Z}_i\alpha_i\hat{\delta}_i^{1/2}\left(\frac{u_{iy}}{u_{ix}}\right)\cos\theta, \tag{17}$$

$$\frac{\partial^2\phi}{\partial\xi^2} = \left(N_e + \sum_j\tilde{Z}_j\tilde{N}_j - \sum_i\hat{Z}_i\hat{N}_i\right), \tag{18}$$

where $N_e = \exp(\phi)$, $\hat{N}_i = \hat{\delta}_iu_{0ix}/u_{ix}$ and $\tilde{N}_j = \tilde{\delta}_j\exp(\tilde{Z}_j\phi/\tilde{\zeta}_j)$.

III. RESULTS AND DISCUSSION

Using Eqs. (15)–(18) and a fourth-order Runge-Kutta method, one can study the sheath structure of a magnetized plasma consisting of multi-charged positive and negative ion species, electrons, and neutral atoms. However, to solve these equations the boundary conditions must be specified. This leads to determine the Bohm criterion. First, we derive the generalized Bohm criterion and then we reduced it to some special cases.

The first integral of Eq. (18) leads to

$$\frac{1}{2}\left(\frac{\partial\phi}{\partial\xi}\right)^2 = \frac{1}{2}\left(\frac{\partial\phi}{\partial\xi}\right)^2_{\xi=0} - V(\phi, u_{0ix}), \tag{19}$$

where

$$V(\phi, u_{0ix}) = \int_0^\phi \left(\sum_i \hat{Z}_i\hat{N}_i - N_e - \sum_j \tilde{Z}_j\tilde{N}_j\right) d\phi. \tag{20}$$

Here, V is the Sagdeev potential satisfying the boundary conditions $V(0, u_{0ix}) = 0$ and $\partial V(0, u_{0ix})/\partial\phi = 0$.

Using the condition for maximizing V in the sheath edge, i.e., $\partial^2 V(0, u_{0ix})/\partial\phi^2 < 0$, we have

$$\left(\frac{\partial^2 V(\phi, u_{0ix})}{\partial\phi^2}\right)_{\phi=0} = \left(\sum_i \hat{Z}_i\frac{\partial\hat{N}_i}{\partial\phi} - \frac{\partial N_e}{\partial\phi} - \sum_j \tilde{Z}_j\frac{\partial\tilde{N}_j}{\partial\phi}\right)_{\phi=0} < 0. \tag{21}$$

Using definition of the normalized density of the ith positive ion ($\hat{N}_i = \hat{\delta}_i u_{0ix}/u_{ix}$) and Eq. (15), one can find

$$\left(\sum_i \hat{Z}_i\frac{\partial\hat{N}_i}{\partial\phi}\right)_{\phi=0} = \sum_i \frac{\hat{Z}_i^2\hat{\delta}_i}{u_{0ix}^2 - \gamma_i\hat{\zeta}_i}\left(1 - \frac{\alpha_i\hat{\delta}_i^{1/2} u_{0iy}\sin\theta}{\left(\frac{\partial\phi}{\partial\xi}\right)_{\phi=0}}\right). \tag{22}$$

In addition, the last two terms in the right hand side of Eq. (21) can be written as follows:

$$\left(\frac{\partial N_e}{\partial\phi}\right)_{\phi=0} = 1, \tag{23}$$

and

$$\left(\sum_j \tilde{Z}_j\frac{\partial\tilde{N}_j}{\partial\phi}\right)_{\phi=0} = \frac{\tilde{Z}_j^2\tilde{\delta}_j}{\tilde{\zeta}_j}. \tag{24}$$

Combining Eqs. (22)–(24) with Eq. (21), one can get

$$\sum_i \frac{\hat{Z}_i^2\hat{\delta}_i}{u_{0ix}^2 - \gamma_i\hat{\zeta}_i}\left(1 - \frac{\alpha_i\hat{\delta}_i^{1/2} u_{0iy}\sin\theta}{\left(\frac{\partial\phi}{\partial\xi}\right)_{\phi=0}}\right) \le 1 + \sum_j \frac{\tilde{Z}_j^2\tilde{\delta}_j}{\tilde{\zeta}_j}. \tag{25}$$

Relation (25) is the Bohm criterion in magnetized multi-component plasmas. In this expression, the normalized charge densities are related by the quasineutrality condition

$$\sum_j \tilde{Z}_j\tilde{\delta}_j + 1 = \sum_i \hat{Z}_i\hat{\delta}_i. \tag{26}$$

Considering $\vec{E}\times\vec{B}$ drift velocity at the sheath edge, we have $u_{0iy} = -E_0\sin\theta/\alpha_i$ where $E_0 = -(\partial\phi/\partial\xi)_{\phi=0}$. Substituting u_{0iy} into inequality (25), it is easily found that in weak magnetic fields the ion Bohm velocity is independent of B and depends only on the direction of the applied magnetic field.

Now we are going to investigate the validity of our modified Bohm criterion by reducing it to some special cases studied previously by authors:

(a) Unmagnetized plasma including singly charged cold positive ions ($i = 1, j = 0$):
In this case, $\hat{Z}_1 = 1$, $\hat{\zeta}_1 = 0$, $\tilde{\delta}_1 = 0$, $\alpha_1 = 0$, and $\hat{\delta}_1 = 1$ (or $n_{0e} = \hat{n}_{01}$) in relations (25) and (26). Therefore, the Bohm criterion is

$$u_{01x} \ge 1, \tag{27}$$

which is the most familiar shape of Bohm criterion derived by Bohm for isothermal ion flow ($\gamma_1 = 1$).[2] This inequality says that if the ion velocity becomes equal or greater than the ion sound velocity, the ion can enter into the sheath region.

(b) Unmagnetized plasma consisting of singly charged warm positive ions ($i = 1, j = 0$):
By considering $\hat{Z}_1 = 1$, $\tilde{\delta}_1 = 0$, $\alpha_1 = 0$, $\hat{\delta}_1 = 1$ and $\hat{\zeta}_1 \neq 0$ in inequality (25), we have

$$u_{01x} \ge (1 + \gamma_1\hat{\zeta}_1)^{1/2}, \tag{28}$$

which is the same as Ref. 29.

(c) Unmagnetized plasma including of singly charged negative and cold positive ions ($i = 1, j = 1$):
In this case, $\hat{Z}_1 = 1$, $\tilde{Z}_1 = 1$, $\alpha_1 = 0$, $\hat{\zeta}_1 = 0$, and $\hat{\delta}_1 = 1 + \tilde{\delta}_1$ (or $n_{0e} + \tilde{n}_{01} = \hat{n}_{01}$). Therefore,

$$u_{01x} \ge \left(\frac{\tilde{\zeta}_1(1 + \tilde{\delta}_1)}{\tilde{\zeta}_1 + \tilde{\delta}_1}\right)^{1/2}, \tag{29}$$

which is the well-known Bohm criterion derived by Franklin and Snell for isothermal ion flow.[27]

(d) Unmagnetized plasma consisting of singly charged negative and warm positive ions:
Assuming $\hat{Z}_1 = 1$, $\tilde{Z}_1 = 1$, $\alpha_1 = 0$, and $\hat{\delta}_1 = 1 + \tilde{\delta}_1$, the Bohm criterion can be written as follows:

$$u_{01x} \ge \left(\frac{\tilde{\zeta}_1(1 + \tilde{\delta}_1)}{\tilde{\zeta}_1 + \tilde{\delta}_1} + \gamma_1\hat{\zeta}_1\right)^{1/2}, \tag{30}$$

which is the same results of Braithwaite and Allen.[15]

(e) Unmagnetized multiple-ion-species plasma consisting of multi-charged warm positive ions:
In this case, $\tilde{\delta}_i = 0$ and $\alpha_i = 0$. Therefore, the Bohm criterion drives as follows:

$$\sum_i \frac{\hat{Z}_i^2\hat{\delta}_i}{u_{0ix}^2 - \gamma_i\hat{\zeta}_i} \le 1, \tag{31}$$

which is the generalized Bohm criterion reported by Riemann for multi-component electropositive plasmas.[14] In this case, the quasineutrality condition takes the following form:

$$\sum_i \hat{\delta}_i = 1. \tag{32}$$

(f) Unmagnetized multiple-ion-species plasma consisting of multi-charged negative and warm positive ions:

Assuming $\alpha_i = 0$, the Bohm criterion and the quasineutrality condition can be written as follows:

$$\sum_i \frac{\hat{Z}_i^2 \hat{\delta}_i}{u_{0ix}^2 - \gamma_i \hat{\zeta}_i} \leq 1 + \sum_j \frac{\tilde{Z}_j^2 \tilde{\delta}_j}{\tilde{\zeta}_j}, \tag{33}$$

and

$$\sum_j \tilde{Z}_j \tilde{\delta}_j + 1 = \sum_i \hat{Z}_i \hat{\delta}_i, \tag{34}$$

which is the same as the Bohm criterion derived in Ref. 23.

(g) Magnetized plasma consisting of singly charged negative and cold positive ions:

By considering $\hat{Z}_1 = 1, \tilde{Z}_1 = 1, \hat{\zeta}_1 = 0$, and $\alpha_1 \neq 0$ in inequality (25), we have

$$u_{01x} \geq \left(\frac{\hat{\delta}_1 \left(1 - \hat{\delta}_1^{1/2} \sin^2\theta\right)}{1 + \dfrac{\hat{\delta}_1 - 1}{\tilde{\zeta}_1}} \right)^{1/2}, \tag{35}$$

which is the Bohm criterion derived by Zou *et al*. for $\gamma_1 = 1$.[30]

(h) Magnetized plasma consisting of singly charged warm positive ions:

In this case, to drive the Bohm criterion it is sufficient to take $\hat{Z}_1 = 1, \alpha_1 \neq 0, \hat{\delta}_1 = 1$, and $\tilde{\delta}_1 = 0$

$$u_{01x} \geq (\hat{\zeta}_1 + \cos^2\theta)^{1/2}, \tag{36}$$

which is the Bohm criterion reported in Ref. 11 for isothermal ion flow. Relation (36) says that the entrance velocity of ion into the sheath can be lower than the ion acoustic velocity, depending on the incident angle of magnetic field and the ratio of the ion temperature to electron temperature.

In the following, we are going to investigate in detail how the presence of negative ion species affects on the velocity of positive ions at the sheath edge of a magnetized plasma sheath. To prevent more complexity, we assume that there is one species of positive ions in the plasma ($i = 1$). Therefore, the modified Bohm criterion takes the following form:

$$u_{01x}^2 > \gamma_1 \hat{\zeta}_1 + \frac{\hat{Z}_1^2 \hat{\delta}_1 \left(1 - \hat{\delta}_1^{1/2} \sin^2\theta\right)}{1 + \sum_j \dfrac{\tilde{Z}_j^2 \tilde{\delta}_j}{\tilde{\zeta}_j}}, \tag{37}$$

where $\hat{Z}_1 \hat{\delta}_1 = 1 + \sum_j \tilde{Z}_j \tilde{\delta}_j$.

In Fig. 2, we illustrate the variation of the entrance velocity of positive ions into the sheath versus the incident angle of magnetic field for different values of negative ion parameters. As it is seen, Figure 2 consists of 6 curves labeled as "A"–"F". The curve "A" corresponds to a magnetized electropositive plasma consisting of electrons and singly charged positive ions with $\gamma_1 = 1$ and $\hat{T}_1 = 0.02\,T_e$, while the curve "B" corresponds to an electronegative plasma consists of electrons, positive ions with $\hat{Z}_1 = 1$, $\gamma_1 = 1$, $\hat{T}_1 = 0.02\,T_e$, and singly charged negative ions with $\tilde{\delta}_1 = 0.03$ and $\tilde{T}_1 = 0.01\,T_e$. Comparing the curves "A" and "B" show that the presence of negative ions causes to decrease the entrance velocity of positive ions into the sheath. Also, contrary to what has been reported in Ref. 11 for magnetized electropositive plasmas, our results show that in the presence of negative ions the entrance velocity of positive ions into the sheath is lower than the ion acoustic velocity, regardless on the incident angle of magnetic field and the ratio of the positive ion temperature to electron temperature.

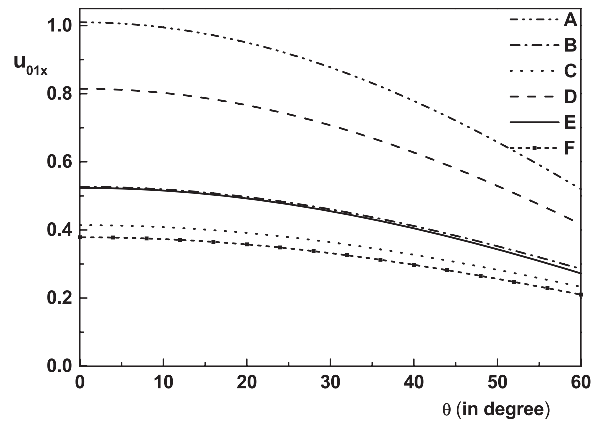

FIG. 2. Variation of the normalized velocity of positive ions at the sheath edge versus θ for $\gamma_1 = 1$, $\hat{Z}_1 = 1$, $\hat{T}_1 = 0.02\,T_e$ (curve "A"), $\tilde{Z}_1 = 1$, $\tilde{\delta}_1 = 0.03$, $\tilde{T}_1 = 0.01\,T_e$ (curve "B"), $\tilde{Z}_1 = 1$, $\tilde{\delta}_1 = 0.06$, $\tilde{T}_1 = 0.01\,T_e$ (curve "C"), $\tilde{Z}_1 = 1$, $\tilde{\delta}_1 = 0.03$, $\tilde{T}_1 = 0.05\,T_e$ (curve "D"), $\tilde{Z}_1 = 1$, $\tilde{\delta}_1 = 0.05$, $\tilde{T}_1 = 0.03\,T_e$, $\tilde{Z}_2 = 1$, $\tilde{\delta}_2 = 0.05$, $\tilde{T}_2 = 0.03\,T_e$ (curve "E"), and $\tilde{Z}_1 = 1$, $\tilde{\delta}_1 = 0.05$, $\tilde{T}_1 = 0.03\,T_e$, $\tilde{Z}_2 = 2$, $\tilde{\delta}_2 = 0.05$, $\tilde{T}_2 = 0.03\,T_e$ (curve "F"). The other parameters in curves "B"–"F" are the same with curve "A."

In the curve "C," we show the effect of electronegativity ($\tilde{\delta}_1 = \tilde{n}_{01}/n_{0e}$) on the Bohm velocity of positive ions. The parameters of this curve are the same with the curve "B" but we assume $\tilde{\delta}_1 = 0.06$ in curve "C." Comparing the curves "B" and "C" indicate that an increase in the electronegativity causes to a decrease in u_{01x}.

The effect of temperature of negative ions on u_{01x} is depicted in the curve "D." In this curve, we assume that $\tilde{T}_1 = 0.05\,T_e$. The other parameters of this curve are the same with the curve "B." Comparing the curves "D" and "B" shows that the velocity of positive ions at the sheath edge increases by increasing the temperature of negative ions. Therefore, depending on the value of $\tilde{T}_1$, u_{01x} may become greater than unity.

To investigate the effect of presence of different negative ion species on the entrance velocity of positive ions into the sheath, we plot the curve "E" for $\tilde{\delta}_1 = \tilde{\delta}_2 = 0.05$, $\tilde{T}_1 = \tilde{T}_2 = 0.03\,T_e$, and $\tilde{Z}_1 = \tilde{Z}_2 = 1$. In spite of the fact that electronegativity and temperature of the negative ion species have been increased in the curve "E," but there is not any considerable difference between the curves "E" and "B." On the other hand, our results show that if the negative ion species have different charge numbers, the presence of the second species of negative ions will considerably affect u_{01x} (see, for example, the curve "F" which has been plotted for $\tilde{Z}_1 = 1$, $\tilde{Z}_2 = 2$ and the other parameters of the curve "E"). Therefore, it can be concluded that the presence of negative ion species with different charge numbers is more effective in decreasing the Bohm velocity of the positive ions (u_{01x}) in comparison with the case in which negative ion species are singly charged.

In addition, as can be seen in Fig. 2, the positive ion velocity entering the sheath decreases by increasing the incident angle of the applied magnetic field, regardless of positive and negative ion species parameters, and becomes less than the ion-acoustic speed for $\theta > 10°$ in both electropositive and electronegative plasmas.

Now, we are going to show the importance of our derived modified Bohm criterion in numerical investigation of the sheath properties. In order to do this, we consider a plasma consisting of electrons and singly charged positive and negative ion species with following parameters $T_e = 1.5\,\text{eV}$, $\hat{T}_1 = 0.03\,\text{eV}$, $\tilde{T}_1 = 0.03\,\text{eV}$, $\phi_0 = \phi(\xi = 0) = 0$, $E_0 = 0.01$, $B_0 = 0.01\,\text{T}$, and $\gamma_1 = 5/3$.[31] It should be noted that the sheath structure for $E_0 = 0$ is very similar to the case when E_0 is a small value.[32] Therefore, to prevent the divergence of the numerical results, we assumed that E_0 takes an infinitesimal value. Using these parameters in inequality (25) and assuming $\theta = 7°$ and $\tilde{\delta}_1 = 0.02$, it is easily found that the sheath formation criterion takes the form $u_{01x} > 0.8$, i.e., the lowest allowable velocity of positive ion to enter into the sheath region is $u_{01x} = 0.8$.

Figure 3 shows the behavior of the normalized density distribution of the charged particles versus the normalized distance from the sheath edge ($\xi = 0$) for $u_{01x} = 0.9$ and 0.7, respectively. It is seen that when the entrance velocity of the positive ions into the sheath region satisfies the sheath formation criterion ($u_{01x} > 0.8$), the sheath is formed ($\hat{N}_1 > N_e + \tilde{N}_1$), and the curve of the normalized density distribution of positive ions $\hat{N}_1$ decreases monotonically through the sheath region; while for $u_{01x} = 0.7$ the sheath formation criterion is violated and consequently the sheath region is not formed. In this case, similar to the result of Ref. 23, the curves of the charged particle density distributions have an fluctuating behavior and do not decrease monotonically.

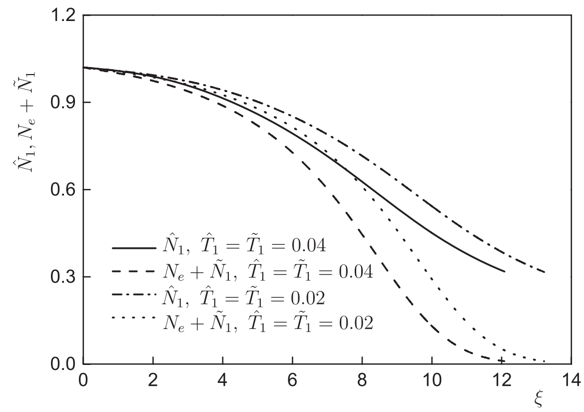

FIG. 4. Normalized density distribution of the charged particles versus the normalized distance for $u_{01x} = 1$ and different values of positive and negative ion temperatures. The other parameters are the same with Fig. 3.

At the end of this section and as a practical application of the obtained Bohm criterion, we investigate the effect of the magnetic field and positive and negative ion temperatures and concentration on the sheath thickness of a magnetized plasma with the above mentioned parameters. Here, we determine the wall position from the location of zero electron density.

Figure 4 shows the effect of positive and negative ion temperatures ($\hat{T}_1$ and $\tilde{T}_1$) on the normalized density distribution of the charged particles ($\hat{N}_1$, $N_e + \tilde{N}_1$) for $T_e = 1.5\,\text{eV}$, $E_0 = 0.01$, $B_0 = 0.01\,\text{T}$, $\gamma_1 = 5/3$, $\theta = 7°$, $\tilde{\delta}_1 = 0.02$, and $\hat{Z}_1 = \tilde{Z}_1 = 1$. Also, we consider $u_{01x} = 1$ which is acceptable for the values of $\hat{T}_1$ and $\tilde{T}_1$ in Fig. 4. From this figure, it is seen that normalized density distribution of the charged particles reduces by increasing $\hat{T}_1$ and $\tilde{T}_1$. In addition, it is seen that an increase in $\hat{T}_1$ and $\tilde{T}_1$ leads to a decrease in the sheath width. These results are in agreement with the results of Ref. 10.

In Fig. 5, normalized density distribution of the charged particles is displayed for $\hat{T}_1 = 0.03\,\text{eV}$, $\tilde{T}_1 = 0.03\,\text{eV}$, $u_{01x} = 1$, and different values of θ. The other parameters are similar to Fig. 4. As mentioned above, it is assumed that the

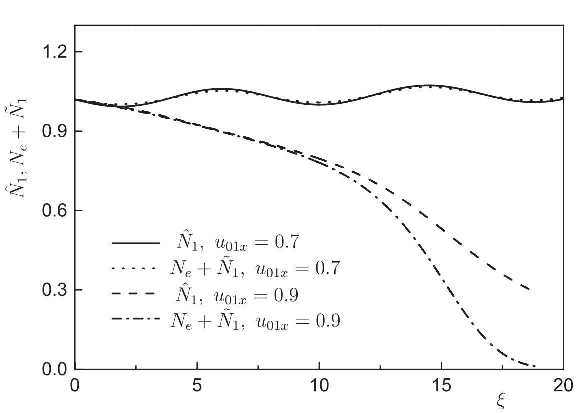

FIG. 3. Normalized density distribution of the charged particles versus the normalized distance for $T_e = 1.5\,\text{eV}$, $\hat{T}_1 = 0.03\,\text{eV}$, $\tilde{T}_1 = 0.03\,\text{eV}$, $E_0 = 0.01$, $B_0 = 0.01\,T$, $\gamma_1 = 5/3$, $\theta = 7°$ and different values of u_{01x}.

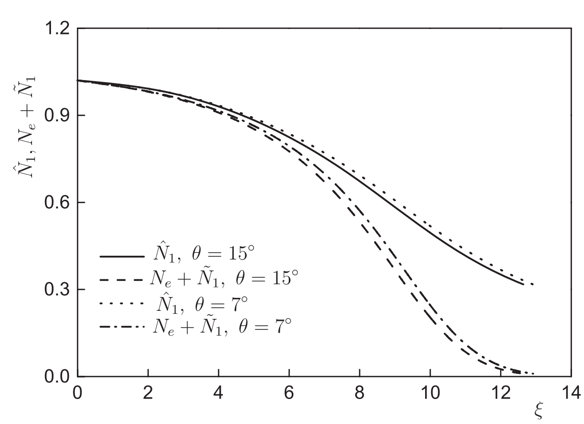

FIG. 5. Normalized density distribution of the charged particles versus the normalized distance for $u_{01x} = 1$ and different values of angles of incidence of the magnetic field with the same parameters of Fig. 3.

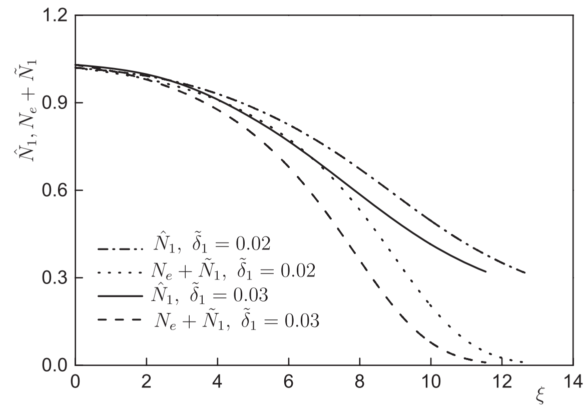

FIG. 6. Normalized density distribution of the charged particles versus the normalized distance for $u_{01x} = 1$ and different ratios of initial negative ion density to electron density. The other parameters are the same with Fig. 3.

external magnetic field is weak ($B_0 = 0.01\,\mathrm{T}$) and the angle θ at which the field lines intersect the wall is small. Similar to the result of Ref. 11, one can see that by increasing θ the normalized density distribution of the charged particles and the sheath thickness decrease.

Finally, Figure 6 shows the normalized density distribution of the charged particles for the mentioned parameters in Fig. 4 and different ratios of initial negative ion density to electron density ($\tilde{\delta}_1 = \tilde{n}_{01}/n_{0e}$). From this figure, it is seen that an increase in $\tilde{\delta}_1$ leads to a decrease in normalized density distribution of the charged particles. In addition, the sheath width decreases with increasing $\tilde{\delta}_1$.

IV. CONCLUSION

A many-fluid model was used to investigate the Bohm criterion in multi-component plasmas consisting of electrons and multi-charged positive and negative ion species. It was assumed that an external magnetic field is applied obliquely to the sheath region and the velocity of the positive ions is not normal to the wall at the sheath edge. Using these assumptions, a modified Bohm criterion was derived which limits the minimum allowable sheath entrance velocity of the positive ion at the sheath edge. It was shown that this limit and then the Mach number depend strongly on the orientation of the applied magnetic field, the charge number and temperature of positive and negative ion species and the ionic concentration. Also, to show the accuracy of our modified Bohm criterion, we confirmed the results of previous works and determined the ion Mach number of some interesting physical situations. Finally, using the obtained sheath formation criterion, the density distribution of charged particles in the sheath region of a plasma consisting of electrons and singly charged positive and negative ion species was investigated and it was found that by increasing the temperature of positive and negative ion and the incident angle of the applied magnetic field to the wall and also increasing the negative ion concentration the density distribution of charged particles in the sheath region and also the sheath width decrease.

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