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Numerical investigation of the ion temperature effect in magnetized plasma sheath with two species of positive ions

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The effect of ion temperature, magnitude of magnetic field and its orientation on a magnetized plasma sheath consisting of electrons and two species of positive ions are investigated. Using three-fluid hydrodynamic model and some dimensionless variables, the dimensionless equations are obtained and solved numerically. It is found that with the increase of the ion temperature and magnetic field strength there is a significant change in ion densities and energies in the sheath. It is also noticed that increase of magnetic field angle enhances the ion density near the sheath edge for a constant ion temperature. With increase in ion temperature and magnetic field angle, the lighter ion density near the sheath edge enhances and reverses for the heavier ion species. © *2012 American Institute of Physics*. [doi:10.1063/1.3678199]

I. INTRODUCTION

Plasma with more than one species of positive ions is widely used in material processing and microelectronics manufacturing for etching, sputtering, thin film deposition, and ion implantation.[1,2]

Due to different mobility of electrons and ions, the negative wall repels the electrons while the ions are attracted towards the wall. So a thin layer of positive space charge is formed at the wall. This non-neutral layer, with thickness of few Debye lengths, is called plasma sheath.[3–6] A number of papers can be found in the literature that study the multi-component plasma with kinetic or hydrodynamic models.[7–10] Franklin[10] has studied the plasma sheath in multi-component weakly collisional plasma with two species of positive ions and examined the stability of the sheath in the steady state. He concluded that for a significant mass difference between positive ion species, each ion species enters into the plasma sheath with its individual Bohm velocity. This leads to the excitation of instability near the sheath edge. He also considered active multi-component plasma in the collisional regime.

The importance of the plasma sheath has been investigated by many researchers[11–15] in the presence of magnetic field. Yet, the physics of the magnetized plasma sheath is still not perfectly understood. Chodura[15] has introduced a hydrodynamic model for semi-infinite plasma, which is valid for an oblique magnetic field to the wall, where ionization and collisions are not taken into account. In this model it is seen that plasma particles pass through three regions on their way towards the wall: quasineutral plasma region, the quasi-neutral magnetized pre-sheath (the Chodura layer) and a collisionless thin space charge sheath. Next, Riemann *et al.*[16] have shown that by taking into account the ionization and ion-neutral collision in plasma, the Chodura layer can be eliminated.

Khoramabadi *et al.*[17] have studied the ion temperature effect on magnetized dc plasma sheath. They have also shown that the ion temperature strongly affects the sheath properties, i.e., an increase of the ion temperature leads to a slow increase of the sheath width and to a decrease of the potential at the sheath edge. They have studied the above for an electron-ion plasma (only one species of ion). Hatami *et al.*[18] have studied the collisional effect in magnetized plasma with two species of positive ions. They have shown that, by increasing the ion-neutral collisional frequency, the amplitude of ion density fluctuation and velocity increases and have shifted towards the plasma sheath edge, and there is significant effect of heavier ion on the density distribution, velocity and kinetic energy on the lighter ions.

Unlike the earlier work of Hatami *et al.*,[18] in this paper we have reported both the effects of ion temperature and ion mass ratio inside the sheath for a plasma containing two species of positive ions. Our study is concerned with the effects of magnetic field and the ion temperature in the sheath. Nonlinear equations in a simple plasma model for both the ions (lighter and heavier) are elucidated in Sec. II. In Sec. III, dimensionless equations and variables are presented. In Sec. IV numerical results and the corresponding discussions are presented. A brief inference is presented in Sec. V.

II. BASIC EQUATIONS AND ASSUMPTIONS

Let us consider a magnetized plasma consisting of electrons and two species of positive ions in the phase space of an one dimensional spatial co-ordinate z and three dimensional velocity co-ordinate v_x, v_y, and v_z as shown in Fig. 1. Masses of the ions are different, but they have the same ionization ratio.

The magnetic induction vector B_0 is in the Z-X plane and forms an angle θ with the Z-direction. We assume that the wall is infinite along Z and Y directions. Therefore, we can write $\nabla \rightarrow \hat{z}\partial/\partial z$, while $\vec{v} = (v_x, v_y, v_z)$, where $\vec{v}$ is the ion species velocity. This means that the co-ordinate system is one-dimensional and velocity space is three-dimensional.

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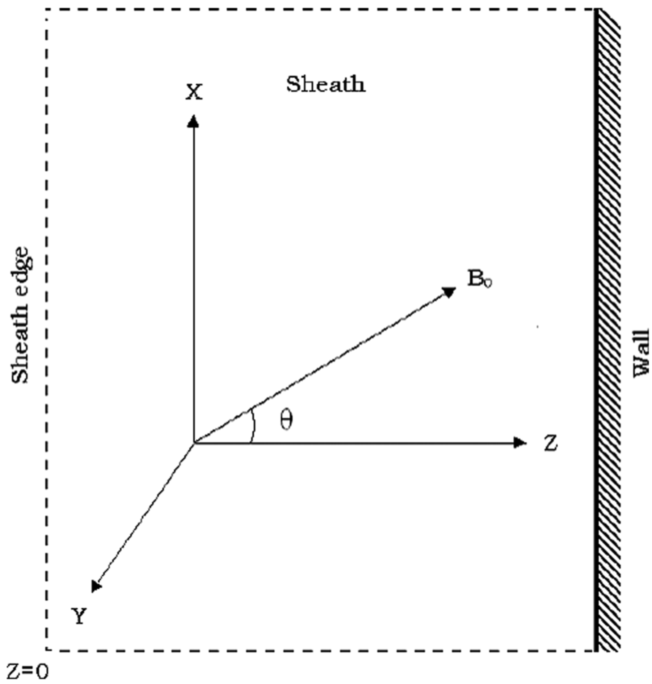

FIG. 1. Schematic configuration of the magnetized sheath in the assumed model. The magnetic field vector is in the z-x plane and it makes the θ angle with z-axis.

We chose the Z-axis normal to the target surface (wall), and the position of the plasma sheath interface at z = 0. In z < 0, there is a quasineutral plasma with $\phi \sim 0$. The ions are generated in the bulk of the plasma (z < 0) are then accelerated in a pre-sheath by a weak electric field towards the sheath, and enter into the sheath with a non-zero velocity (Bohm velocity).

The electrons are in thermal equilibrium and obey the Boltzmann density distribution as

$$n_e = n_{e0} exp\left(\frac{e\phi}{T_e}\right), \tag{1}$$

where n_{e0}, T_e, and ϕ are the electron density at the sheath edge, electron temperature, and electric potential, respectively.

The quasineutrality condition in plasma is

$$n_{i0} = n_{10} + n_{20} = n_{e0}, \tag{2}$$

where n_{10} and n_{20} are the lighter and heavier ion species density in plasma, respectively.

In the steady state ($\partial/\partial t = 0$), the continuity equations for ions are as follows:

$$\nabla.(n_1\vec{v}_1) = 0, \tag{3}$$

$$\nabla.(n_2\vec{v}_2) = 0, \tag{4}$$

where n_1, n_2 are the ion species densities and $\vec{v}_1$, $\vec{v}_2$ are the ion species velocity vectors in the sheath.

Considering the collisions between the ion species and neutrals, the momentum transport equations for ions are as follows:

$$m_1 n_1 \vec{v}_1.\nabla\vec{v}_1 = -en_1\nabla\phi + en_1\vec{v}_1 \times \vec{B} - \nabla p_1 - n_1\vec{F}_{c1}, \tag{5}$$

$$m_2 n_2 \vec{v}_2.\nabla\vec{v}_2 = -en_2\nabla\phi + en_2\vec{v}_2 \times \vec{B} - \nabla p_2 - n_2\vec{F}_{c2}, \tag{6}$$

where m_1 and m_2 are the mass of the ion species and $\vec{F}_{c1}$ and $\vec{F}_{c2}$ are the ion drag forces that each ion species experiences during travel through the sheath. These forces can be written as

$$\vec{F}_{cj} = m_j \upsilon_{cj}\vec{v}_j, \tag{7}$$

where υ_c is the effective collision frequency of ion species with neutrals, and j = 1, 2 (1 for lighter ions and 2 for heavier ions). It is well known that ion collision frequency is the function of ion velocity in the direction of the wall, i.e., a function ofv_z. We assume that there is a linear relationship between υ_c and v_z as $\upsilon_c = \alpha v_z/\lambda_{De}$, where $\lambda_{De} = (T_e/n_{e0}e^2)^{1/2}$ is the electron Debye length and α is the dimensionless parameter that characterizes the magnitude of the collision force or collision frequency. In our model there are two different types of ion having different cross-sections. So we use two different $\alpha(\alpha_1, \alpha_2)$ for two different ion species. The positive ion partial pressure is $P_j = k_B T_j \nabla n_j$, where j = 1, 2; k_B is Boltzmann constant, and T_j is the ion temperature. Here we assume that the ions are isothermal, so that $T_1 = T_2 = T_i$.

Finally, we use Poisson's equation which relates the electron and ions number densities to the electric potential,

$$\nabla^2\phi = -\frac{e}{\varepsilon_0}(n_1 + n_2 - n_e). \tag{8}$$

According to the general criterion for the stationary sheath formation in the fluid approximations,[19–21] the isothermal ions have to enter the sheath region with the normal to the wall initial velocity defined by

$$v_{jz0} = \sqrt{\frac{T_e}{m_j}}\left(\sqrt{1 + \frac{T_j}{T_e}}\right) \geq C_{sj}, \tag{9}$$

where j = 1, 2. Conversely, referring to the geometry sketched in Fig. 1 and assuming that $B = B_0(\hat{z}\text{Cos}\theta + \hat{x}\text{Sin}\theta)$, we can obtain a series of equations from Eqs. (5) and (6) as follows:

$$m_1 v_{1z}\frac{dv_{1x}}{dz} = eB_0\text{Cos}\theta v_{1y} - m_1\upsilon_{c1}v_{1x}, \tag{10}$$

$$m_1 v_{1z}\frac{dv_{1y}}{dz} = eB_0[\text{Sin}\theta v_{1z} - \text{Cos}\theta v_{1x}] - m_1\upsilon_{c1}v_{1y}, \tag{11}$$

$$m_1 v_{1z}\frac{dv_{1z}}{dz} = -e\frac{d\phi}{dz} - eB_0\text{Sin}\theta v_{1y} - \frac{T_i}{n_1}\frac{dn_1}{dz} - m_1\upsilon_{c1}v_{1z}, \tag{12}$$

$$m_2 v_{2z}\frac{dv_{2x}}{dz} = eB_0\text{Cos}\theta v_{2y} - m_2\upsilon_{c2}v_{2x}, \tag{13}$$

$$m_2 v_{2z}\frac{dv_{2y}}{dz} = eB_0[\text{Sin}\theta v_{2z} - \text{Cos}\theta v_{2x}] - m_2\upsilon_{c2}v_{2y}, \tag{14}$$

$$m_2 v_{2z}\frac{dv_{2z}}{dz} = -e\frac{d\phi}{dz} - eB_0 \mathrm{Sin}\theta v_{2y} - \frac{T_i}{n_2}\frac{dn_2}{dz} - m_2 \upsilon_{c2} v_{2z}. \tag{15}$$

III. DIMENSIONLESS EQUATIONS AND VARIABLES

For simplification, the ion species velocities are scaled by the lighter ion species sound velocity as $\vec{u}_1 = \vec{v}_1/C_{s1}$ and $\vec{u}_2 = \vec{v}_2/C_{s1}$, where $C_{s1} = (T_e/m_1)$ is the lighter ion species sound velocity. The density of ion species and electrons are scaled by the electron density in the plasma as $N_1 = n_1/n_{e0}$, $N_2 = n_2/n_{e0}$, and $N_e = n_e/n_{e0}$, respectively. We scale electric potential and ion temperature by electron temperature as $\psi = -e\phi/T_e$, $T = T_i/T_e$, respectively. The distance z is scaled by the electron Debye length ($\lambda_{De} = \sqrt{\varepsilon_0 T_e/n_{e0}e^2}$) as $\xi = z/\lambda_{De}$. Use of dimensionless variables reduces equations (1), (3), (4), (8), (10)–(15), into following forms, respectively:

$$N_e = exp(-\psi), \tag{16}$$

$$N_1 u_{1z} = \frac{M_1}{1+\delta}, \tag{17}$$

$$N_2 u_{2z} = \frac{\delta M_2}{1+\delta}, \tag{18}$$

$$u_{1z}\frac{du_{1x}}{d\xi} = \beta \mathrm{Cos}\theta u_{1y} - \alpha_1 u_{1x} u_{1z}, \tag{19}$$

$$u_{1z}\frac{du_{1y}}{d\xi} = \beta[\mathrm{Sin}\theta u_{1z} - \mathrm{Cos}\theta u_{1x}] - \alpha_1 u_{1y} u_{1z}, \tag{20}$$

$$\left[u_{1z} - \frac{T}{u_{1z}}\right]\frac{du_{1z}}{d\xi} = \frac{d\psi}{d\xi} - \beta \mathrm{Sin}\theta u_{1y} - \alpha_1 u_{1z}^2, \tag{21}$$

$$u_{2z}\frac{du_{2x}}{d\xi} = \mu\beta \mathrm{Cos}\theta u_{2y} - \alpha_2 u_{2x} u_{2z}, \tag{22}$$

$$u_{2z}\frac{du_{2y}}{d\xi} = \mu\beta[\mathrm{Sin}\theta u_{2z} - \mathrm{Cos}\theta u_{2x}] - \alpha_2 u_{2y} u_{2z}, \tag{23}$$

$$\left[u_{2z} - \frac{T}{u_{2z}}\mu\right]\frac{du_{2z}}{d\xi} = \mu\left[\frac{d\psi}{d\xi} - \beta \mathrm{Sin}\theta u_{2y}\right] - \alpha_2 u_{2z}^2, \tag{24}$$

$$\frac{d^2\psi}{d\xi^2} = [N_1 + N_2 - N_e], \tag{25}$$

In the above equations $M_{1,2} = v_{z1,2}(z=0)/C_{s1}$ is the ion Mach number and $\delta = n_{20}/n_{10}$, $\mu = m_1/m_2$, and $\beta = \lambda_{De}/\rho_i$, where $\rho_i = (m_1 T_e/e^2 B_0^2)^{1/2}$ is the lighter ion species gyroradius.

IV. NUMERICAL RESULTS AND DISCUSSION

Here we consider a collisional plasma of helium ($m_1 = 4$) and a few percent of argon ($m_2 = 40$). Hence, we have $\mu = 1/10$ and $\delta = 0 - 0.1$. If $\delta = 0$, the collisional plasma consists of only one species of positive ion, i.e., the lighter ion species. To solve the equation of ion species motion and Poisson's equation, we use, $u_{1z}(\xi=0) = 1$, $u_{1y}(\xi=0) = u_{1x}(\xi=0) = 0$, similarly $u_{2z}(\xi=0) = 0.3$, $u_{2y}(\xi=0) = u_{2x}(\xi=0) = 0$ at the sheath edge. Here for the isothermal ions, we assume that $M_1 = 1$ and $M_2 = M_1\sqrt{m_1/m_2} \approx 0.3$. The Mach number depends upon ion-neutral collisional frequency[18] and the ratio of lighter to heavier ion-neutral collisional frequency is $\sim$0.3 (the ion-neutral cross-section is taken to be $1.6 \times 10^{-15}\mathrm{cm}^2$ for helium and $5 \times 10^{-15}\mathrm{cm}^2$ for argon). It is noted that for the above values of M_1, M_2, and δ the generalized Bohm criteria[22] for the multi-component plasmas hold. It is explained that at the sheath edge ($\xi = 0$) the electrostatic potential ψ and electric field $\partial\psi/d\xi$ is zero.[1] To prevent the divergence of the numerical results, we assume that $d\psi(\xi=0)/d\xi$ takes an infinitesimal value. Therefore, we assume that at the sheath boundary the following conditions hold: $\psi(\xi=0) = 0$ and $d\psi(\xi=0)/d\xi = 0.01$. To examine the sheath structure, we have to investigate the profile of the particle density, electric potential and ion velocity. For this purpose, we solve Eqs. (16)–(25) by Runge-Kutta fourth-order method.

A. Ion temperature affect on particle density

Density of lighter ion species as a function of distance (from the sheath edge to the wall) is plotted for different ion temperatures is shown in Fig. 2. Fig. 2(a) shows that in the absence of magnetic field β, the aggregation of lighter ion

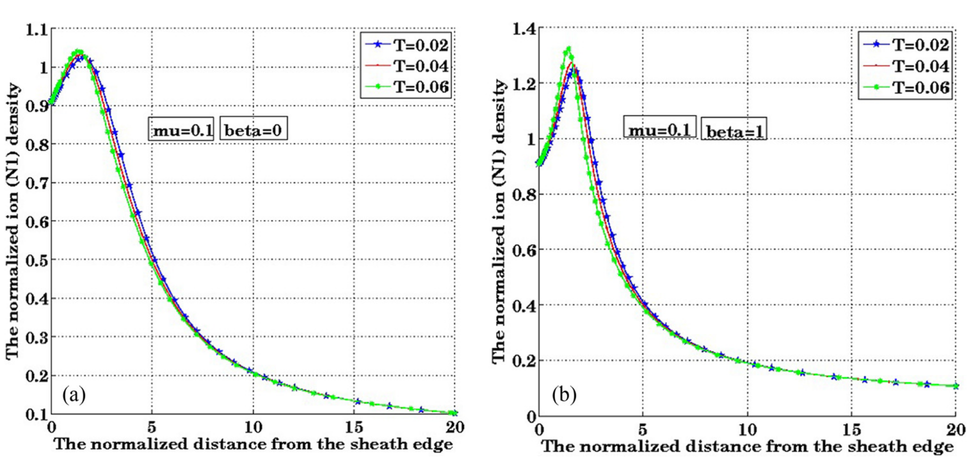

FIG. 2. (Color online) (a) The normalized lighter ion species density distributions for $\theta = 30^0$, $T_e = 1$ eV, $\beta = 0$ and different ion temperature as a function of normalized distance from the sheath edge. (b) The normalized lighter ion species density distributions for $\theta = 30^0$, $T_e = 1$ eV, $\beta = 1$, and different ion temperature as a function of normalized distance from the sheath edge.

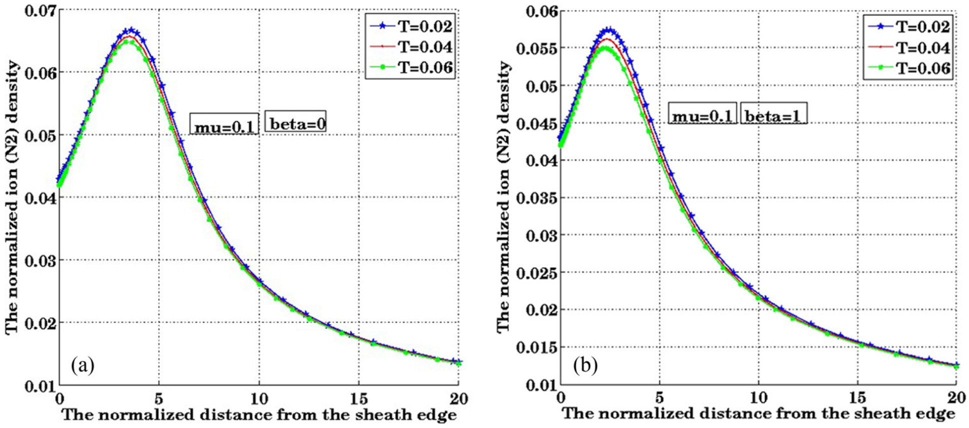

FIG. 3. (Color online) (a) The normalized heavier ion species density distributions for $\theta = 30^0$, $T_e = 1\,\text{eV}$, $\beta = 0$ and different ion temperature as a function of normalized distance from the sheath edge. (b) The normalized heavier ion species density distributions for $\theta = 30^0$, $T_e = 1\,\text{eV}$, $\beta = 1$ and different ion temperature as a function of normalized distance from the sheath edge.

density near the sheath edge is increased with respect to ion temperature. Fig. 2(b) shows that in the presence of magnetic field, the aggregation of lighter ion density near the sheath edge is also increased with respect to temperature. As the ions enter to the sheath, they are decelerated by Lorentz force in the z-direction and therefore the ion density increases. The increase in ion temperature T also intensifies the effect and due to this, a shifting of lighter ions occurs towards the sheath edge.[17] In Fig. 2(b), it is seen that in the presence of magnetic field, the peaks of lighter ion density at the sheath edge is more than the density peaks for lighter ions for absence of magnetic field (Fig. 2(a)). According to the equation of continuity, increase of the ion density tends to decrease its velocity. It is to be noted from Fig. 2, ion density decreases towards the wall, which indicates the increase in ion velocity.

Figure 3 is plotted for the heavier ion density against the sheath distance for the absence and presence of magnetic field for different ion temperatures. It is seen that increase in ion temperature, the heavier ion density decreases, which has been explained in the next sub-section and also shows no shifting towards the sheath edge.

B. Affect of ion mass ratio on ion density

We have studied the temperature affect on different mass ratio (μ). Fig. 4 shows that in presence of magnetic field the density of lighter ions are more than the density of heavier one for a constant ion temperature ($T = 0.02$) and a constant mass ratio ($\mu = 0.1$) (which have been used throughout in our problem). The heavier ion densities are much smaller than the lighter ions due to a small δ.

As we increase the value of μ from 0.1 to 0.35 there is no shifting of heavier ions towards the sheath edge and decreases with increase in ion temperature. Further increase of mass ratio to 0.38, it can be seen that [Fig. 5(a)] increase in temperature decreases the heavier ion density, but with a shifting towards the sheath edge. An increase of μ to 0.4, Fig. 5(b) shows that the heavier ion density peaks are same magnitude irrespective of ion temperatures. Further increase of μ, 0.42 to 1 shows that with increase of the temperature the heavier ion density increases [Fig. 5(c)]. This could be explained as follows. The Bohm velocity of lighter ions is more than the heavier ions, because the Bohm velocity is inversely proportional to the mass of the ion. So the lighter ions enter inside the sheath first and create a positive space charge near the wall, and the slower heavier ions react to that positive space charge, hence the density of heavier ions decrease. When the mass difference between the two ion species is very small, they enter inside the sheath almost at the same velocity, so the rise in ion temperature have simultaneous effect on both the species.

Figure 6 shows the lighter ion density profiles for various ion mass ratios. It is seen that for increase in temperature, the lighter ion density peaks in the sheath increases for a constant mass ratio. The trends are same in further increase in mass ratio.

C. Affect of temperature on ion velocity

The normalized ion velocity in magnetized plasma is shown in Figs. 7(a) and 7(b) for lighter and heavier ions, respectively. The ion velocity is a function of normalized distance from the sheath edge to the wall. For both the cases, increase in ion temperature increases the z-component of ion

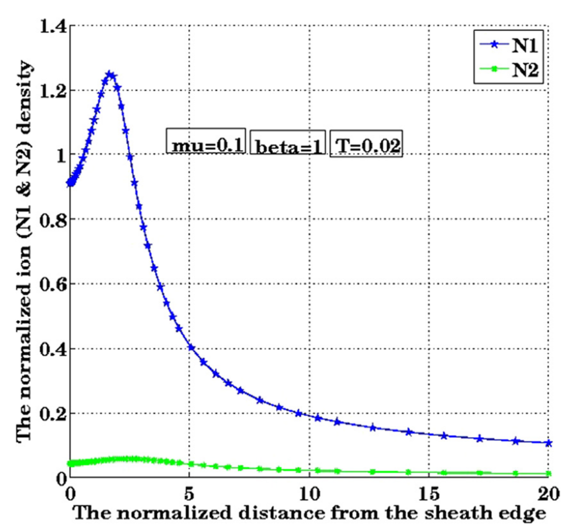

FIG. 4. (Color online) The normalized heavier and lighter ion species density distributions for $\theta = 30^0$, $T_e = 1$ eV, T = 0.02 eV, $\beta = 1$, as a function of normalized distance from the sheath edge.

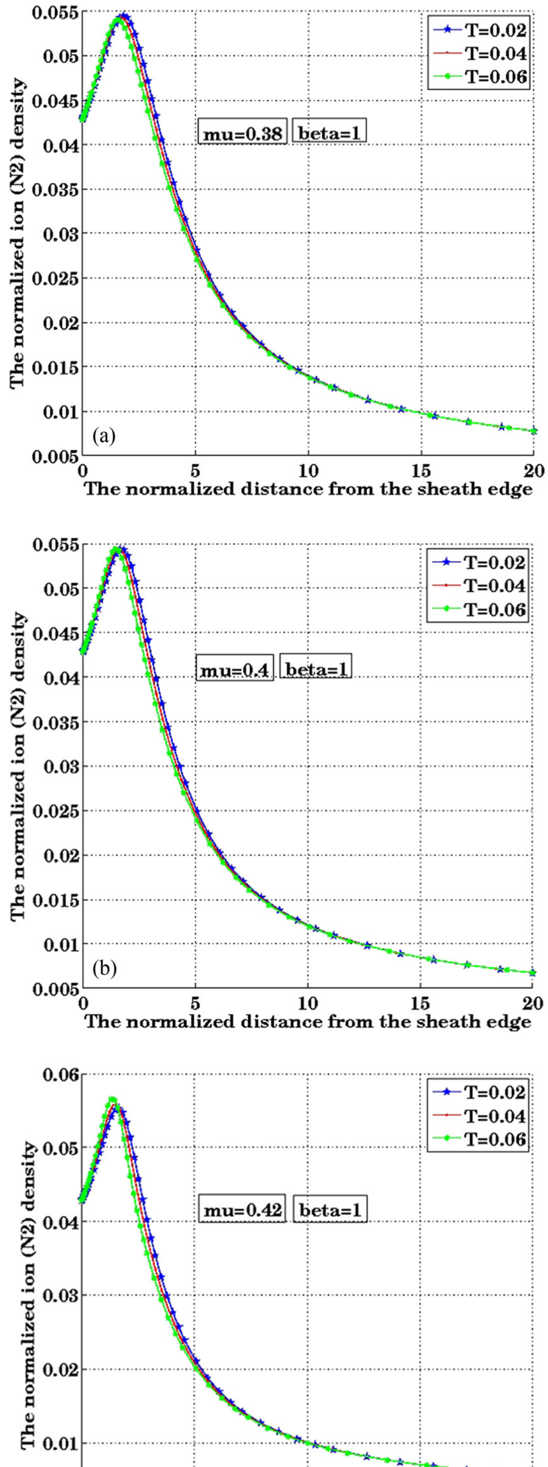

FIG. 5. (Color online) (a) The normalize heavier ion species densities distributions for$\theta = 30^0$, $T_e = 1$ eV, $\mu = 0.38$, $\beta = 1$ and different ion temperature as a function of normalized distance from the sheath edge. (b) The normalize heavier ion species densities distributions for $\theta = 30^0$, $T_e = 1$ eV, $\mu = 0.4$, $\beta = 1$ and different ion temperature as a function of normalized distance from the sheath edge. (c) The normalize heavier ion species densities distributions for $\theta = 30^0$, $T_e = 1$ eV, $\mu = 0.42$, $\beta = 1$ and different ion temperature as a function of normalized distance from the sheath edge.

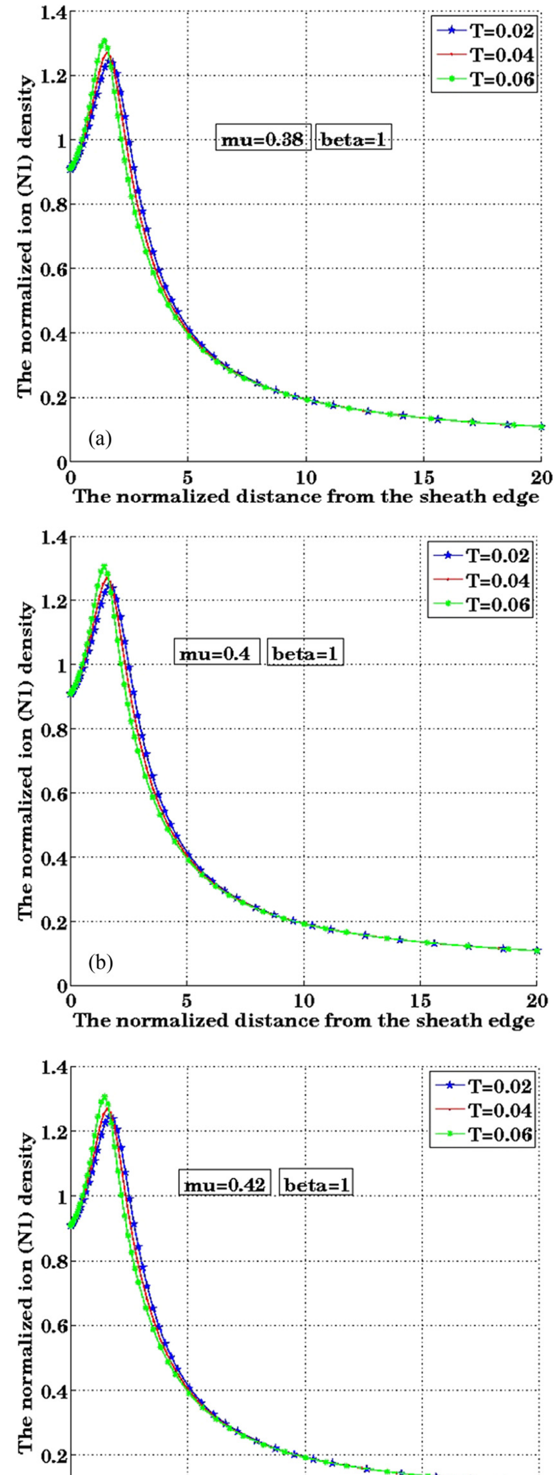

FIG. 6. (Color online) (a) The normalize lighter ion species densities distributions for $\theta = 30^0$, $T_e = 1$ eV, $\mu = 0.38$, $\beta = 1$ and different ion temperature as a function of normalized distance from the sheath edge. (b) The normalize lighter ion species densities distributions for $\theta = 30^0$, $T_e = 1$ eV, $\mu = 0.4$, $\beta = 1$ and different ion temperature as a function of normalized distance from the sheath edge. (c) The normalize lighter ion species densities distributions for $\theta = 30^0$, $T_e = 1$ eV, $\mu = 0.42$, $\beta = 1$ and different ion temperature as a function of normalized distance from the sheath edge.

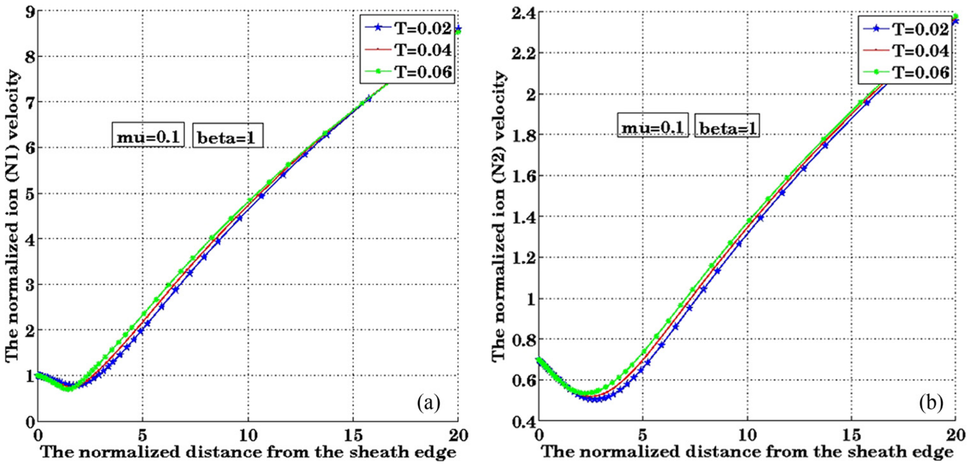

FIG. 7. (Color online) (a) The normalized lighter ion species flow velocity in the z-direction for $\theta = 30^0$, $T_e = 1$ eV, $\beta = 1$ and different ion temperature as a function of normalized distance from the sheath edge. (b) The normalized heavier ion species flow velocity in the z-direction for $\theta = 30^0$, $T_e = 1$ eV, $\beta = 1$ and different ion temperature as a function of normalized distance from the sheath edge.

velocity. In the presence of magnetic field, the transverse component (parallel to the wall) of magnetic field gyrates the ions and causes the ion density and velocity to fluctuate. The Figures show that the z-component of ion velocity in presence of magnetic field initially decreases near the sheath edge, but increases with the ion temperature as one moves from the sheath edge towards the wall.

D. Affect of temperature on electric potential

From Fig. 2, it can be seen that the lighter ion density decreases towards the wall (from sheath edge to the wall). From the ion continuity equations (3) and (4), the z-component of ion velocity is inversely proportional to the ion density. So the ion z-component velocity increases towards the wall as density decreases. On the basis of energy conservation law, the electric potential energy of the ions in the sheath is equal to their kinetic energy, so the electric potential and ion velocity distribution across the sheath are proportional to each other. Figs. 8(a) and 8(b) show the normalized electric potential for various temperatures for absence and presence of magnetic field, respectively. It is seen that for the, absence and presence of magnetic field, the normalized electric potential increases with temperature.

E. Variation of ion density with magnetic field angle

Fig. 9(a) shows that with the increase of the magnetic field angle θ the lighter ion density increases for a constant magnetic field and temperature. When magnetic field angle increases, the longitudinal velocity component decreases and transverse velocity component increases, which is responsible to enhance the ion density. As we have seen, sheath thickness decreases with the increase of ion density; hence increasing the magnetic field angle reduces the sheath thickness. The reverse is the case with heavier ion [see Fig. 9(b)]; the reason has been explained in sub-section B above.

V. SUMMARY

In this paper, we have presented a general description of a magnetized plasma sheath composed of two types of ion species and Boltzmann distributed inertia-less electrons. The affect of ion temperature has investigated for ion densities, ion velocities and electric potential across a magnetized plasma sheath. The summary of the computer simulation works are given bellow:

(a) On increasing of ion temperature, the lighter ion density increases and the heavier ion density decreases near the sheath edge.

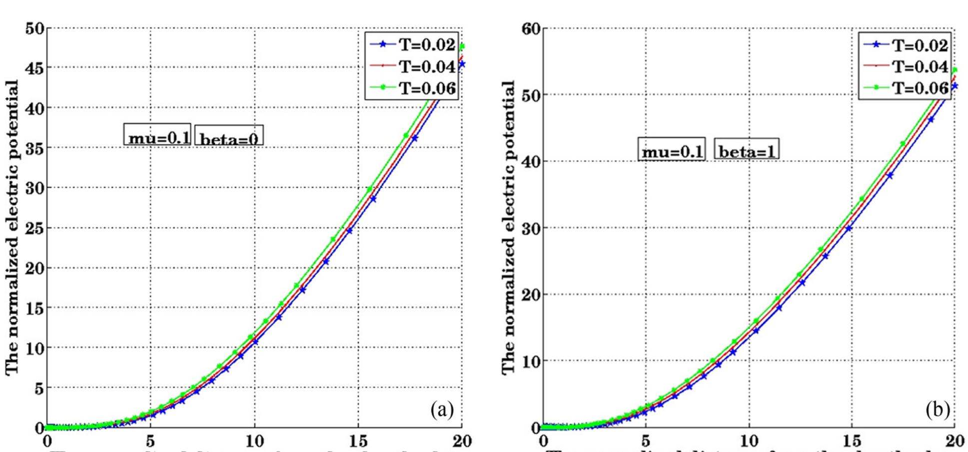

FIG. 8. (Color online) (a) The normalized potential for $\theta = 30^0$, $T_e = 1$ eV, $\beta = 0$ and different ion temperature as a function of normalized distance from the sheath edge. (b) The normalized potential for $\theta = 30^0$, $T_e = 1$ eV, $\beta = 1$ and different ion temperature as a function of normalized distance from the sheath edge.

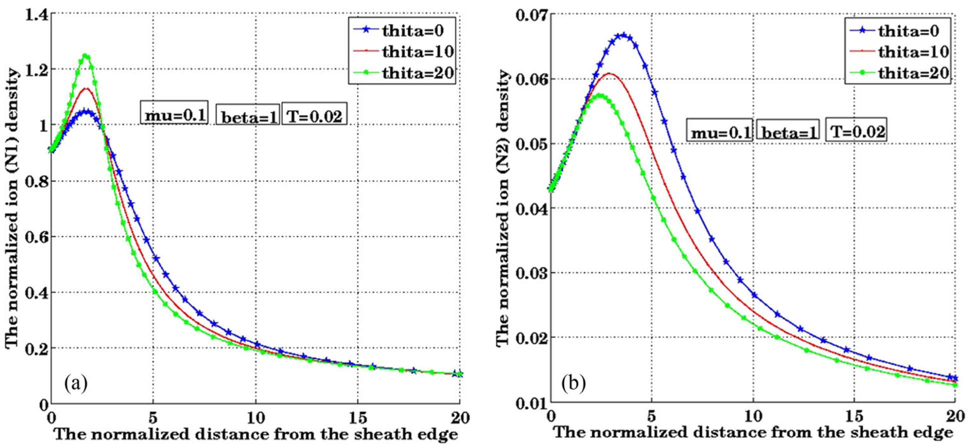

FIG. 9. (Color online) (a) The normalized lighter ion species density distributions for $\beta = 1$, $T_e = 1$ eV, $T = 0.02$ and different magnetic field angle as a function of normalized distance from the sheath edge. (b) The normalized heavier ion species density distributions for $\beta = 1$, $T_e = 1$ eV, $T = 0.02$ and different magnetic field angle as a function of normalized distance from the sheath edge.

(b) On increasing of ion temperature, both the lighter and heavier ion velocities decreases near the sheath edge and increases towards the wall. We have seen that in the absence of magnetic field, there is no effect on the sheath parameters. But the transverse component of magnetic field is responsible to aggregation of ion density near the sheath edge and the ion temperature further enhances this effect. Moreover the magnetic field increases the transverse drifting motion of the ions and the ion temperature enhances their longitudinal drifting motion.

(c) On increasing of ion temperature, electric potential increases in the sheath.

(d) For higher mass ratio (lighter to heavier), both the lighter and heavier ion densities increases in the sheath.

(e) We have also studied the affect of magnetic field angle on ion density in the sheath. With the increase of the magnetic field angle, the lighter ion density increases and the heavier ion density decreases for a constant ion temperature.

(f) Our model can be applied to study the sheath in various plasma-processing phenomena, such as, secondary electron emission guns (SEEGs).

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