Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Vacuum 73 (2004) 681–685
VACUUM
SURFACE ENGINEERING, SURFACE INSTRUMENTATION & VACUUM TECHNOLOGY
www.elsevier.com/locate/vacuum

Plasma sheath in a magnetic field

Xiu Zou*, Jin-Yuan Liu, Ye Gong, Zheng-Xiong Wang, Yue Liu, Xiao-Gang Wang

State Key Laboratory of Materials Modification by Laser, Ion and Electron Beams, Dalian University of Technology, Dalian 116024, China

Abstract

The structure of the plasma sheath in an oblique magnetic field with the fluid method was investigated. In various magnitude and directions of the magnetic field, the electron and ion density distribution, ion flow velocity, electron potential and Bohm's criterion have been calculated. It is shown that magnetic field has obvious effect on the plasma sheath. Under the action of electrostatic and Lorentz forces, the ion flow makes a helical movement, and the ion density distribution fluctuates.

Keywords: Magnetic sheath; Plasma; Magnetic field

1. Introduction

The study of plasma sheath is one of the important problems in the experiment equipments of plasma. So the correlative works both of experiment and theory are developed widely during the past several years [1–8]. In some sense, the effect of the magnetic field cannot be ignored, but the introduction of the magnetic field does make the problem more complex. Many authors [3–8] have investigated the structure of the plasma sheath in an oblique magnetic field using the dynamic theory, and their work is mainly about the presheath. In this paper, using fluid method we studied the characteristics of the plasma sheath in an oblique magnetic field which cannot be ignored compared to the electrostatic field. It is shown that the structure of the magnetic sheath is really different from the plasma sheath without magnetic field.

2. Mathematical formulation and basic equations

We consider a plasma sheath, which has one dimension coordinate space and three dimensions speed space. The external magnetic field is embedded in the (x, z) plane, and θ is the angle between magnetic field and x direction (see Fig. 1). At the edge of the sheath, $x = 0$, the electrostatic potential is taken to be zero, $\phi = 0$.

The sheath is consisting of isothermal electrons and fluid ions. The electrons are assumed to be in thermal equilibrium with the density given by

$$n_e = n_0 \exp(e\phi/T_e). \tag{1}$$

The ion fluid equations are

$$\nabla \cdot (n_i\vec{v}) = 0, \tag{2}$$

*Corresponding author.
E-mail address: zouxiu@student.dlut.edu.cn (X. Zou).

0042-207X/$ - see front matter
doi:10.1016/j.vacuum.2003.12.099

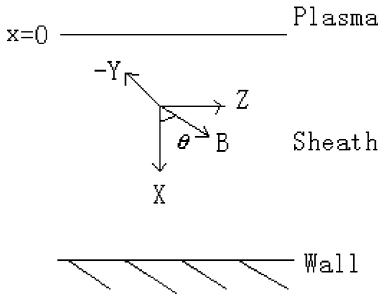

Fig. 1. Geometry of the magnetic sheath model.

$$m_i\vec{v}\cdot\nabla\vec{v} = -e\nabla\phi + e\vec{v}\times\vec{B}_0 - m_i v_i\vec{v}, \tag{3}$$

where $v_i = \pi v_x/2\lambda_i$ is the effective ion collision frequency, and λ_i is free path.

The system is completed with the Poisson equation

$$\partial^2\phi/\partial x^2 = -e(n_i - n_e)/\varepsilon_0. \tag{4}$$

We consider only the physical parameters changing along x direction, $\nabla\to\hat{x}\partial/\partial x$, and we define $c_s = (T_e/m_i)^{1/2}$ as the ion sound speed, $\omega_c = eB_0/m_i$ is the ion cyclotron frequency, and $\tau = \omega_c^{-1}$ is the ion cyclotron period. For simplicity, we introduce dimensionless quantities: $\eta = -e\phi/T_e$, $\xi = x/(c_s\tau)$, $\vec{u} = \vec{v}/c_s, v = v_i/\omega_c$, $N_e = n_e/n_0$ and $N_i = n_i/n_0$.

Taking $\hat{B}_0 = \hat{x}\cos\theta + \hat{z}\sin\theta$, then we have

$$N_e = \exp(-\eta), \tag{5}$$

$$N_i u_x = M, \tag{6}$$

$$u_x\partial u_x/\partial\xi = \partial\eta/\partial\xi + u_y\sin\theta - vu_x, \tag{7}$$

$$u_x\partial u_y/\partial\xi = -u_x\sin\theta + u_z\cos\theta - vu_y, \tag{8}$$

$$u_x\partial u_z/\partial\xi = -u_y\cos\theta - vu_z, \tag{9}$$

$$\mathrm{d}^2\eta/\mathrm{d}\xi^2 = [N_i - \exp(-\eta)]/\gamma^2, \tag{10}$$

where, in (8) $M = v_{x0}/c_s$ is the ion Mach number, in (10) $\gamma = \lambda_e/\rho_i$, and $\lambda_e = (\varepsilon_0 T_e/n_0 e^2)^{1/2}$ is electron Debye length, $\rho_i = (T_e m_i/e^2 B_0^2)^{1/2}$ is the ion grain gyro radius. From Eqs. (5)–(10), we can obtain the electron and ion density distribution, ion flow velocity and the electrostatic potential.

2.1. *Bohm's criterion*

Introducing Eq. (6) in (7), we obtain

$$u_y\sin\theta = -A(\eta)\,\mathrm{d}\eta/\mathrm{d}\xi, \tag{11}$$

where $A(\eta) = 1 + M^2/N_i^3\,\mathrm{d}N_i/\mathrm{d}\eta$. At the edge of sheath, $\eta\to 0$, $u_{y0} = 0$, $N_i\to 1, \mathrm{d}\eta/\mathrm{d}\xi\neq 0$, and from the relation of $[\mathrm{d}N_i/\mathrm{d}\eta + \exp(-\eta)]_{\eta=0}\geqslant 0$, we obtain the Bohm's criterion

$$M^2\geqslant 1. \tag{12}$$

The Bohm's criterion is the same as without magnetic field. That is to say, only when the ion velocity is higher than, or at least equal to, the ion sound speed, can it enter the magnetic sheath.

2.2. *Energy conservation law*

Multiplying Eqs. (7)–(9) by u_x, u_y, u_z respectively and adding, we obtain

$$u_x^2\partial u_x/\partial\xi + u_x u_y\partial u_y/\partial\xi + u_x u_z\partial u_z/\partial\xi = u_x\partial\eta/\partial\xi. \tag{13}$$

Both sides of Eq. (13) being divided by u_x, and then integrating, we get the energy conservation

$$u_x^2 + u_y^2 + u_z^2 = 2\eta + M^2. \tag{14}$$

Here, using the boundary conditions that $\xi\to 0$, $u_{x0} = M$, $u_{y0} = 0$, $u_{z0} = 0$, $\eta\to 0$.

3. Numerical results and discussion without collision

We consider the effect of magnetic field to the plasma sheath without collision. In the following numerical results (Figs. 2–8), we adopt the parameters as $v = 0$, $M = 1.0$ and the original electric field $\partial\eta/\partial\xi\big|_{\xi=0} = 0.01$.

3.1. *Movement of the ion flow in the magnetic sheath*

Fig. 2 shows the ion flow velocity in three directions while $\gamma = 1$, in another word the ion grain gyro radius $\rho_i = \lambda_e$. Since the magnetic field is weak, the ion flow in the sheath cannot finish

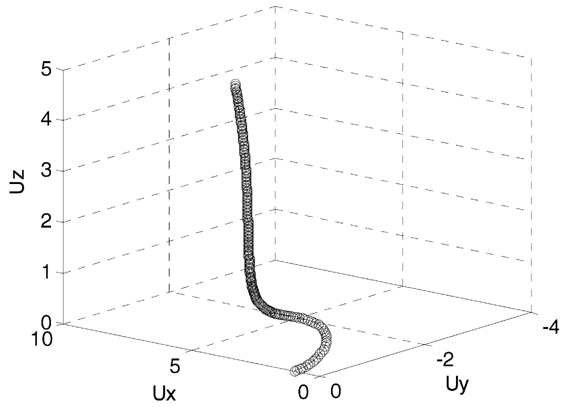

Fig. 2. The ion flow velocity ($\gamma = 1$).

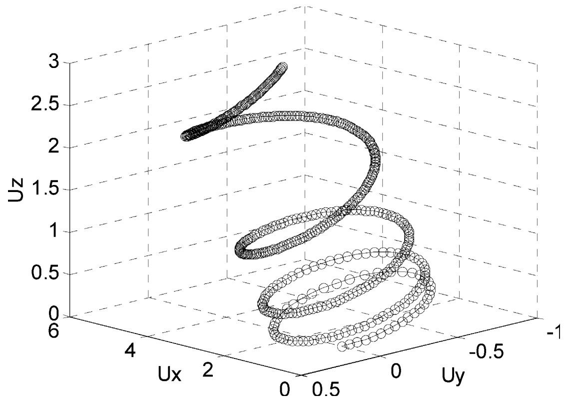

Fig. 3. The ion flow velocity ($\gamma = 5$).

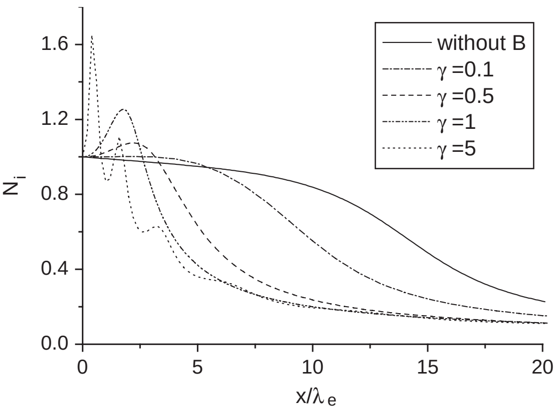

Fig. 4. The ion density distribution ($\theta = 30°$).

one circle of gyration. It only has negative velocity in y direction. Fig. 3 shows the ion flow velocity in three directions while $\gamma = 5$, that is to say the ion

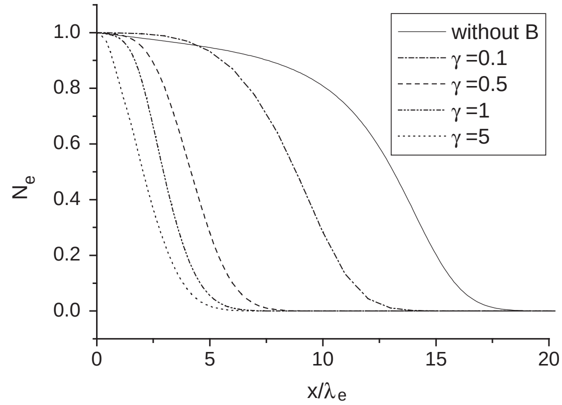

Fig. 5. The electron density distribution ($\theta = 30°$).

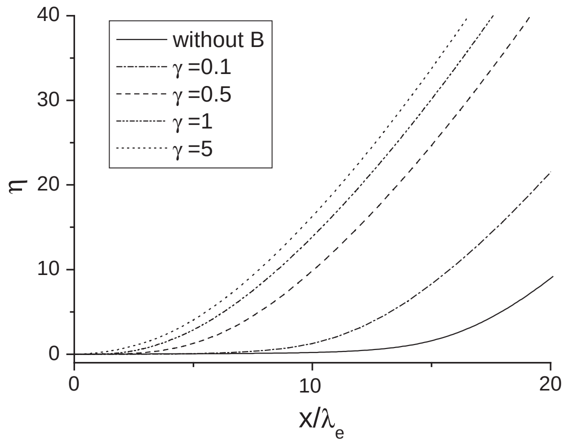

Fig. 6. The electron potential ($\theta = 30°$).

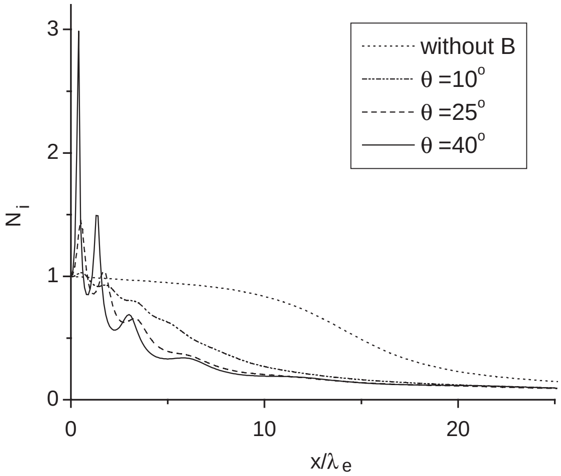

Fig. 7. The ion density distribution ($\gamma = 5$).

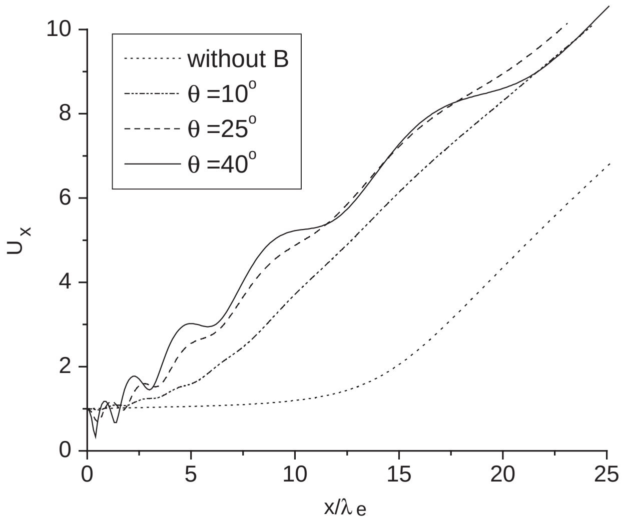

Fig. 8. The ion flow velocity in the x direction ($\gamma = 5$).

grain gyro radius $\rho_i = 1/5\lambda_e$. At this time, the magnetic field is strong enough to make the ion flow gyrate. Adding the action of the electrostatic force, the ion flow makes a helical movement. And the axis of the helical movement is almost the direction of the magnetic field, here the angle of the magnetic field $\theta = 30°$.

3.2. Effect of the magnitude of the magnetic field to sheath

Fig. 4 indicates the ion density distribution in a magnetic field of various magnitudes while the angle of the magnetic field $\theta = 30°$, and makes a comparison with it without magnetic field. It is shown that the magnetic field makes the ion density distribution rise at first and then drop. The reason for it is that the Lorentz force slows down the ion flow velocity in the x direction at first. The decrease of the ion flow velocity brings the increase of the ion density distribution. The stronger the magnetic field, the more obvious the ion density distribution increases. We assume that the magnetic field cannot affect the electron density directly, and the electron density is a function of electron potential (Eq. (1)). From the Poisson equation, the increase of the ion density distribution would lead to the increase of the electron potential η (see Fig. 6), in fact ϕ drops, and the increase of the electrostatic field. The electron density distribution drops with the decrease of the electron potential ϕ (see Fig. 5). With the increase of the electrostatic field, the electrostatic force is increased, which accelerates the ion flow velocity. The increase of the ion flow velocity in the x direction brings the drop of the ion density distribution.

We also can conclude from Fig. 4 that the ion flow cannot gyrate one circle, while $\gamma \leqslant 1$, so the ion density distribution rises for a short time and then drops. When $\gamma = 5$, the ion flow makes a helical movement, and Lorentz force speeds up and slows down the ion flow respectively in each period. So the ion density distribution has periodically fluctuation.

3.3. Effect of the direction of the magnetic field to sheath

Fig. 7 indicates the ion density distribution in a magnetic field of various angles, while $\gamma = 5$. It is shown that the bigger the angle, in other words the stronger the magnetic field in the z direction, the more obvious the density distribution fluctuation. Fig. 8 shows the ion flow velocity in the x direction in a magnetic field of various angles, while $\gamma = 5$. We can see that when the magnetic field in the z direction reaches a certain value, the ion flow velocity in the x direction fluctuates. If the velocity of the ion flow decreases to a considerable negative value at the first period, the ion could escape the sheath.

4. Conclusion

From the numerical conclusions we know that the Bohm's criterion of the magnetic sheath is the same as without magnetic field. The magnetic field makes the electron density distribution drop and the electrostatic potential η rise (in fact ϕ drop). The ion density distribution rises at first and then drops comparing with it without magnetic field. If the magnetic field in the z direction reaches a certain value, it will make the ion flow do a helical movement and fluctuate the ion density distribution.

Acknowledgements

This work has been supported by the National Natural Science Foundation of China (No. 10175013,19875007) and International Collaboration Funds (No.10010760807, 1016420799).

References

[1] Ma TC. J Dalian Univ Technol 1997;37:125–8 [in Chinese].
[2] Liu DY, Wang DZ, Liu JY. Acta Phys Sin 2000;49:1094–9 [in Chinese].
[3] Liu JY, Wang DZ, Ma TC. Vacuum 2000;59:126–34.
[4] Chodura R. Phys Fluid 1982;25:1628–838.
[5] Riemann KU. Phys Plasmas 1994;1:552–8.
[6] Kim GH, Hershkowitz N, Diebold DA, Cho MH. Phys Plasmas 1995;2:3222–33.
[7] Ahedo E. Phys Plasmas 1997;4:4419–30.
[8] Bornali S, Sarma A, Joyanti C. Phys Plasmas 2002;9: 683–90.