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Citation: Physics of Plasmas (1994-present) **3**, 1459 (1996); doi: 10.1063/1.871736
View online: http://dx.doi.org/10.1063/1.871736
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/3/4?ver=pdfcov
Published by the AIP Publishing

Bohm criterion for the collisional sheath

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(Received 28 November 1995; accepted 16 January 1996)

Using a two-fluid model it is shown analytically that in collisional slightly ionized plasmas, i.e., for many typical low pressure discharges, at the sheath edge the drift velocity of the plasma can be smaller than the ambipolar sound speed of the ions. This basic result was known earlier by numerical calculations but it is demonstrated here using simple analytical expressions. Under these conditions the Bohm criterion, in its usual formulation, is a sufficient condition for the existence of a positive boundary sheath, but it is not a necessary condition. © *1996 American Institute of Physics.* [S1070-664X(96)02404-8]

I. INTRODUCTION

Using a collisionless two-fluid model of the plasma Bohm[1] has derived a condition for the existence of a positive boundary sheath. At the sheath edge the drift velocity of the plasma must be greater than the ambipolar sound speed of the ions, the so-called Bohm velocity.[2,3] Harrison and Thompson[4] have derived a modified version of the so-called Bohm criterion by means of a kinetic theory of the ion gas neglecting elastic and inelastic collisions. In the limit of a very small Debye length Riemann[5] has investigated the Bohm criterion in its hydrodynamic and kinetic formulations carefully. Collisions have been taken into account by Stangeby and Allen[6] and by Andrews and Stangeby.[7] However, the critical drift velocity has not been investigated. A collisional sheath containing additionally negative ions was also treated.[8]

The sheath edge adjacent a quasineutral plasma can be defined by means of the Bohm criterion.[1–5,9,10] Thus, often the Bohm criterion is used as a boundary condition in the plasma theory.

Really, the transition between the nearly quasineutral bulk of the plasma and the sheath takes place smoothly. Numerical calculations have shown that in collisional plasmas the thickness of the sheath is substantially larger than the Debye length.[2,11–16] These calculations have also yielded that in collisional slightly ionized plasmas a positive sheath exists although the drift velocity of the ions can remain smaller than the Bohm velocity.[2,11–16]

The aim of the present paper is to show that the results of the numerical calculations can be confirmed analytically. We follow the well-known derivation of the Bohm criterion. Additionally, the generation of charged particles and collisions between ions and neutral atoms are taken into consideration.

II. MODEL AND BASIC EQUATIONS

A two-fluid model for compressible media is taken into consideration. The plasma is assumed to consist of electrons and singly charged positive ions in an immobile neutral gas. A planar geometry with insulating walls is treated. Steady-state conditions and homogeneity parallel to the walls are assumed to prevail. The electrons and ions are generated by electron collisions within the plasma. They recombine at the walls. It is well known that the walls are charged negatively with respect to the plasma. The inertia of the ions and elastic collisions between ions and atoms and the charge exchange are taken into account. The electrons are assumed to be in the thermodynamic equilibrium. The neutral gas density and the electron temperature T_e are taken as constant. The ion temperature is assumed to be zero. The space charge is taken into account.

The basic equations for the ions are the equation of continuity

$$\frac{d}{dx}\, n_i v_i = n_e \nu_{ni} \tag{1}$$

and the equation of momentum transfer

$$m_i v_i \frac{dv_i}{dx} = -e\,\frac{d\phi}{dx} - \nu_c m_i v_i\,, \tag{2}$$

with $\nu_c = \nu_i + \nu_{ni} n_e / n_i$. The electron density is given by

$$n_e(x) = n(0)\exp\{[\phi(x) - \phi(0)]/kT_e\}. \tag{3}$$

The electric potential ϕ is determined by the Poisson equation

$$\frac{d^2\phi}{dx^2} = -\frac{\epsilon_0}{e}\,(n_i - n_e). \tag{4}$$

Here, x denotes the distance from the sheath edge, n_i and n_e the number densities of the ions and the electrons, respectively, v_i the drift velocity of the ions, ν_{ni} the ionization frequency, ν_i the collision frequency between ions and atoms, m_i the mass of an ion, e the positive elementary charge, k the Boltzmann constant, and ϵ_0 the permittivity of the vacuum.

At the sheath edge we assume $n_i = n_e = n(0)$ and $\phi(0) = 0$. Within the sheath $x > 0$ holds.

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III. EXISTENCE CONDITION OF A POSITIVE BOUNDARY SHEATH

The space charge density and the potential in the vicinity of the sheath edge are calculated. Therefore, all dependent functions are expanded into Taylor series in x. The first terms are taken into account, only.

Integrating Eqs. (1) and (2) by using this approximation yields

$$n_i v_i = n_0 (v_0 + \nu_{ni} x) \tag{5}$$

and

$$v_i = v_0 \left(1 - \frac{2e\phi}{m_i v_0^2} - \frac{2\nu_c}{v_0} x \right)^{1/2}, \tag{6}$$

where $n(0)=n_0$ and $v_i(0)=v_0$. We introduce

$$\begin{gathered} v_B = (kT_e/m_i)^{1/2}, \quad \omega_{pi} = (e^2 n_0/\epsilon_0 m_i)^{1/2}, \\ l_{ni} = \frac{v_B}{\nu_{ni}}, \quad l_i = \frac{v_B}{\nu_i}, \quad l_D = \frac{v_B}{\omega_{pi}}, \end{gathered} \tag{7}$$

and, furthermore,

$$\eta = -e\phi/kT_e, \quad s = x/l_D. \tag{8}$$

Then, from Eqs. (3), (5), and (6) we obtain

$$n_e/n_0 = 1 - \eta \tag{9}$$

and

$$\frac{n_i}{n_0} = 1 - \left(\frac{v_B}{v_0} \right)^2 \eta + \left(\frac{1}{l_i} + \frac{2}{l_{ni}} \right) l_D s. \tag{10}$$

Inserting Eqs. (7)–(10) into Eq. (4) gives

$$\frac{d^2\eta}{ds^2} = \alpha^2 \eta + \beta s, \tag{11}$$

where

$$\alpha^2 = 1 - v_B^2/v_0^2, \quad \beta = (\nu_i + 2\nu_{ni})/\omega_{pi}. \tag{12}$$

The necessary condition for the existence of a positive sheath reads

$$\frac{d^2\eta}{ds^2} > 0 \quad \text{if } s > 0, \tag{13}$$

i.e., at the beginning of the sheath edge, the curvature of η has to be positive. It is well known that this condition is satisfied if $\alpha^2 > 0$, i.e., $v_0 > v_B$.[1] However, it can be seen that the condition (13) can also be fulfilled if α^2 is slightly negative since $\beta > 0$.

The differential equation (11) has the solution

$$\eta(s) = \sin \gamma s + (\beta/6) s^3, \tag{14}$$

where $\gamma = i\alpha$. Equation (11) and, thus, also Eq. (14) are approximations, valid for small s. Restricting to $|\gamma| s \ll 1$ by using (13) and (14) leads to $|\gamma^3| < \beta$ and, further on, to

$$v_0 > v_c, \tag{15}$$

where

$$v_c = v_B/(1+\beta^{2/3})^{1/2}. \tag{16}$$

Collisions affect the critical velocity v_c substantially if $\beta \geqslant 1$, i.e., $l_D \geqslant l_i$ or $l_D \geqslant l_{ni}$. Such conditions can exist in discharges.

It is useful to characterize plasmas in low-pressure discharges by means of the reciprocal K of the modified Knudsen number and the space charge number a_0 where

$$K = \frac{l_{ni}}{l_i} = \frac{\nu_i}{\nu_{ni}}, \quad a_0 = \left(\frac{l_D}{l_{ni}} \right)^2. \tag{17}$$

One obtains $l_D/l_i = a_0^{1/2} K$ and

$$\beta = a_0^{1/2}(2+K). \tag{18}$$

It follows $\beta = l_D/l_i$ if $K \gg 1$. For instance, under diffusion conditions in the positive column $K=10^3$ and $a_0 \geqslant 10^{-6}$ can occur.

Slightly ionized plasmas used in technics and experiments possess a degree of ionization in the range 10^{-8}–10^{-4}. We consider a positive column in argon. Assuming $kT_e = 1.5$ eV, a tube radius $R=1$ cm and $n_e/n_n = 10^{-7}$ where n_n is the neutral gas density leads to $K=10^3$, $n_n = 2\times10^{17}$ cm^{-3}, $a_0 = 10^{-5}$, $l_i = 5\times10^{-3}$ cm and $l_D = 6\times10^{-3}$ cm, approximately. The results $\beta \geqslant 1$ and $v_c/v_B \leqslant 1/\sqrt{2}$ confirm the results of numerical calculations under corresponding conditions. The numerical investigations have used the complete set of the nonlinear differential equations (1)–(4).[2,11–16] They have yielded that $v_c < v_B$ can occur for values of K and a_0 that are a little smaller than the results of the linear theory given here.

Note, that the condition (13) is a little stronger than necessary. In the model used here, $d\eta/ds > 0$ is sufficient to yield a positive sheath in front of a negatively charged wall. This leads to $\beta > (2/\pi^2)|\gamma^3|$.

IV. SUMMARY

Bohm's sheath criterion gives a condition for the existence of a positive boundary sheath if no collisions occur within the sheath. A two-fluid model is used to investigate the influence of elastic collisions, the charge exchange, and the plasma generation on the sheath criterion analytically. In collisional slightly ionized plasmas the thickness of the sheath is substantially greater than the Debye length. Furthermore, the Debye length can have the same order of magnitude as the mean-free path of the ions. At the sheath edge the critical drift velocity of the plasma is a little smaller than the so-called Bohm velocity. Under these conditions, in its usual formulation, the Bohm criterion is a sufficient condition, but, it is not a necessary condition. Conditions of this kind can occur in typical low-temperature plasmas in technics and experiments.

Foregoing numerical results given in several papers are confirmed by the analytical results.

ACKNOWLEDGMENTS

This work was supported by the Bundesministerium für Forschung und Bildung under Contract No. 13-N-6198.

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