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A magnetized plasma sheath where the ion collision frequency depends on ion flow velocity

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Abstract
Recently some correlative works have been done to investigate the effects of collisions between ion and neutral gas atoms on the characteristics of plasma sheath in an external magnetic field. In general, the momentum transferring cross section (and thus the ion collision frequency) has a power law dependence on the ion flow velocity in the depth direction. Usually in the literature, constant collision frequency and constant mean free path are treated. Here, by using a collisional fluid model, we investigate the characteristics of a magnetized plasma sheath where the ion cross section depends on the ion flow velocity. The numerical calculations show that the effects of collisions on ion characteristics are more powerful when the momentum transferring cross section is constant. However, some parameters such as the electron density distribution are independent of this dependence.

1. Introduction

The significant importance of the electrodynamic properties of the plasma sheath in plasma experiment equipment and other applications has generated interest in investigating the structure of the plasma sheath. Many researchers have recently tried to develop several works, both empirically and theoretically to study the plasma sheath [1–7]. In these works, the electrodynamic properties of the plasma sheath such as the electron and ion density distribution, ion flow velocity and electron potential have been investigated under various conditions such as in the presence of an external magnetic field or under the effect of collisions between the ion and the neutral gas atoms [7–9]. These effects are two important conditions which affect the structure of the plasma sheath. For example, Zou *et al* studied the structure of the non-collisional plasma sheath in an oblique magnetic field and showed that the magnetic field cannot be ignored compared with the electrostatic field [7]. A corresponding study for collisional magnetic plasma sheath has been done in [9], in which, by considering the ion collision frequency as a constant parameter, the effects of the magnitude of the ion collision frequency on the magnetic plasma sheath are investigated.

However, in the general case, the ion collision frequency depends on the ion velocity. As the ion flow velocity changes in the plasma sheath as a function of the depth direction, so does the ion collision frequency. Here we investigate the effect of this dependence on the characteristics of the magnetic plasma sheath.

2. Basic equations of the model

In our model, we consider a magnetic plasma sheath in contact with a planar wall. The density, velocity and mass of the ion ($n_{\rm i}$, $\boldsymbol{v}$ and $m_{\rm i}$, respectively), the external magnetic field ($\boldsymbol{B}$) and the effective ion collision frequency ($\nu_{\rm i}$) are related to each other by equations of continuity and momentum as follows:

Continuity:

$$\vec{\nabla}\cdot(n_{\rm i}\vec{v}) = 0, \tag{1}$$

Momentum:

$$(\vec{v}\cdot\vec{\nabla})\vec{v} = \frac{e}{m_{\rm i}}(-\vec{\nabla}\phi + \vec{v}\times\vec{B}) - \nu_{\rm i}\vec{v}. \tag{2}$$

We assume that the electron energy is not very high. This assumption enables us to neglect the ionization [10, 11], as we consider this fact in the equation of continuity.

The ion collision frequency can be expressed as a function of neutral gas density ($n_{\rm n}$), the momentum transferring cross section for collisions between ions and neutrals (σ) and the total drift velocity (v) as follows:

$$\nu_i = n_n \sigma v. \tag{3}$$

In many research works, two special cases are generally considered for the momentum transferring cross section σ: the case of constant cross section and the case of constant collisional drag or mobility in which the cross section has the inverse ratio to the velocity. It means if we consider a power law dependence of the cross section on velocity, the exponent is zero (constant cross section) or -1 (constant collisional mobility). However, in some examples, the exponent is not these two values. For example, for argon in the range 3–300 eV, the power law is most closely satisfied for $p = -0.25$. So, for generalization, we consider that the cross section has a power law dependence on velocity, which is used in some papers such as [3, 10]. Therefore the momentum transferring cross section can be considered as follows:

$$\sigma = \sigma_s \left(\frac{v}{c_s}\right)^p, \tag{4}$$

where $c_s = (K_B T_e/m_i)^{1/2}$ is the ion acoustic velocity, σ_s is the cross section measured at ion acoustic velocity and p is a dimensionless parameter ranging from 0 to -1. By defining a dimensionless parameter $\boldsymbol{u} = \boldsymbol{v}/c_s$, the ion collision frequency can be expressed as

$$\nu_i = n_n \sigma_s c_s u^{p+1}. \tag{5}$$

Now, considering that the sheath consists of fluid ions and using the fluid method, we investigate a collisional magnetic plasma sheath under various values of p from -1 to 0. Also, we consider that the sheath consists of isothermal electrons which are assumed to be in thermal equilibrium. So the electron density obeys the Boltzmann distribution as follows:

$$n_e = n_0 \exp(e\phi/k_B T_e), \tag{6}$$

where n_e is the electron local density, ϕ is the local potential and T_e is the electron temperature. At the sheath edge, $z = 0$, the electron density is n_0 and the electrostatic potential is taken to be zero, $\phi(z = 0) = 0$.

The electrostatic potential of these electrons is related to the ion and electron densities by the Poisson equation:

$$\nabla^2\phi = e(n_e - n_i)/\varepsilon_0. \tag{7}$$

We consider from here that the physical parameters change along the depth direction, z, i.e. $n_e = n_e(z)$, $n_i = n_i(z)$, $\phi = \phi(z)$, $\boldsymbol{v} = \boldsymbol{v}(z)$ and $\boldsymbol{\nabla} \equiv \partial/\partial z$. Also, for sake of simplicity, we define some dimensionless variables as follows:

$$\eta = -e\phi/k_B T_e, \xi = z/\lambda_D, \vec{u} = \vec{v}/c_s,$$

$$N_e = n_e/n_0, \qquad N_i = n_i/n_0, \tag{8}$$

where $\lambda_D = (\varepsilon_0 K_B T_e/n_0 e^2)^{1/2}$ is the electron Debye length.

We consider that the wall is parallel to the x–y plane and the magnetic field is embedded in the xz-plane and make θ angle in the z-direction. Substituting these dimensionless parameters into equations (1), (2), (6) and (7) and using the power law dependence of the ion collision frequency on the ion speed, equation (4), we have

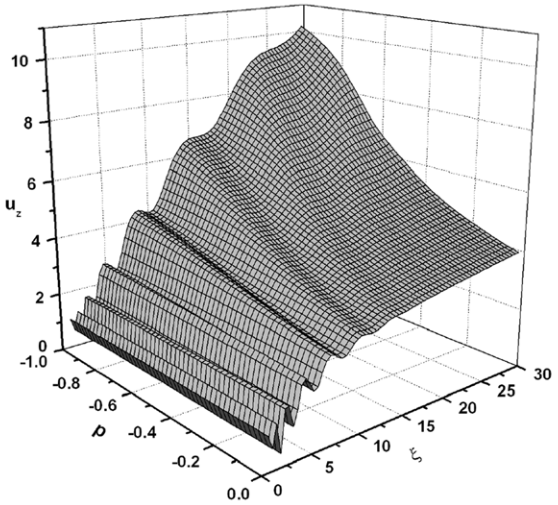

Figure 1. The ion flow velocity in the depth direction ($\alpha = 0.134$, $\theta = 40°$ and $\gamma = 5$).

$$N_e = \exp(-\eta), \tag{9}$$

$$N_i = M/u_z, \tag{10}$$

$$u_z \partial_\xi u_x = \gamma \cos\theta u_y - \alpha u^{p+1} u_x, \tag{11}$$

$$u_z \partial_\xi u_y = \gamma \sin\theta u_z - \gamma \cos\theta u_x - \alpha u^{p+1} u_y, \tag{12}$$

$$u_z \partial_\xi u_z = \partial_\xi \eta - \gamma \sin\theta u_y - \alpha u^{p+1} u_z, \tag{13}$$

$$\partial_\xi^2 \eta = M/u_z - \exp(-\eta), \tag{14}$$

where $M = u_{0z} = v_{0z}/c_s$ is the ion Mach number, $\alpha = \lambda_D n_n \sigma_s$ is a dimensionless parameter which characterizes the magnitude of the collision force independent of the ion velocity and $\gamma = \lambda_D/\rho_i$. ρ_i is the ion grain gyro radius which can be expressed as $\rho_i = c_s\tau = (T_e m_i/e^2 B_0^2)^{1/2}$ where $\tau = m_i/eB_0$ is the ion cyclotron period. Solving equations (9)–(14) the variation of the dimensionless parameters can be obtained as a function of the variation of the depth direction.

3. Results

Our goal is to solve the model presented above, i.e. equations (9)–(14), with the initial conditions: $\boldsymbol{u}|_{z=0} = (0, 0, 1)$ and $\partial_\xi \eta|_{z=0} = 0.01$. The model equations can be solved numerically. These numerical calculations enable us to investigate the effect of ion collision frequency on the sheath characteristics. As we want to calculate the variables under various p values, we consider a certain pressure and temperature. In our simulation, it means that we consider a fixed α value (for example, $\alpha = 0.134$, which corresponds to the neutral pressure of 0.1 Torr at 290 K).

In [9] the effect of collisions on the plasma sheath has been investigated for a constant ion mean free path or a constant cross section; $p = 0$. Here we investigate these effects for various values of p from -1 to zero.

Figure 1 shows the ion flow velocity in the depth direction versus the variation of the p value, where we take $\alpha = 0.134$, $\theta = 40°$ and $\gamma = 5$. The figure shows that the variation of

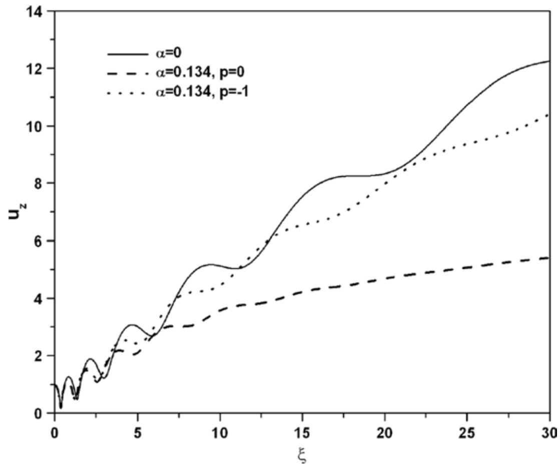

Figure 2. The ion flow velocity in the depth direction for three cases: $\alpha = 0$ (collisionless plasma sheath) and $\alpha = 0.134$ ($p = 0$; constant mean free path and $p = -1$; constant collisional frequency).

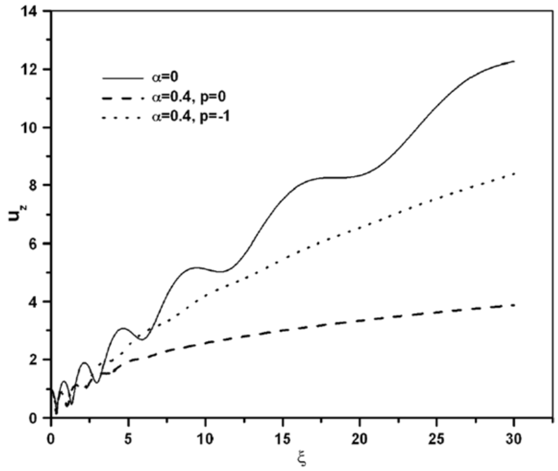

Figure 4. The ion flow velocity in the depth direction for three cases: $\alpha = 0$ (collisionless plasma sheath) and $\alpha = 0.4$ ($p = 0$; constant mean free path and $p = -1$: constant collisional frequency).

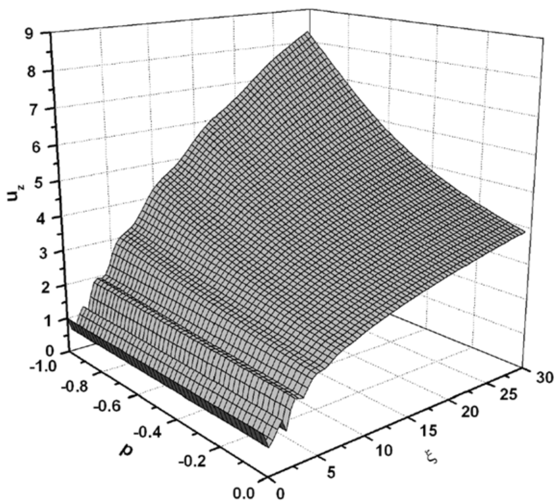

Figure 3. The ion flow velocity in the depth direction ($\alpha = 0.4$, $\theta = 40^{\circ}$ and $\gamma = 5$).

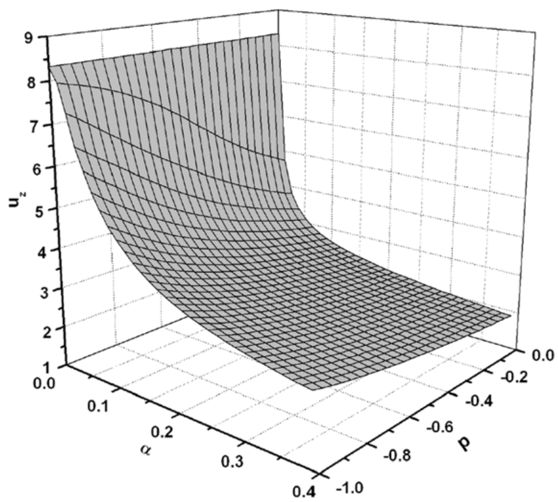

Figure 5. The ion flow velocity in depth direction at a specific distance from the sheath edge; $z = 20\lambda_D$ ($\theta = 40^{\circ}$ and $\gamma = 5$).

the p value has no effects on the fluctuation of the ion flow velocity near the sheath edge. However, by increasing the distance from the sheath edge, the effect of collisions is more obvious for the constant ion mean free path. To compare the effects of collision with the collisionless magnetic sheath, in figure 2 we plot the ion flow velocity in the depth direction for collisionless and collisional magnetic plasma sheaths (with constant mean free path, $p = 0$, and constant collisional frequency, $p = -1$). As can be seen, for $\alpha = 0.134$, the collisional magnetic plasma sheath with constant collisional frequency behaves approximately like a collisionless magnetic plasma sheath even far from the sheath edge. However, for a constant mean free path the difference is more obvious corresponding to the increase in the ξ value.

In figures 3 and 4 the same calculations are made for $\alpha = 0.4$. As the α value corresponds to the amplitude of the collision force, the effects of collision which diminish the intensity of the fluctuations are more obvious compared with figures 1 and 2.Again, the effects of collision with a constant mean free path are much more than the constant collisional frequency (increasing the p value is corresponding to the increase in the effect of collision).

Figure 5 shows the ion flow velocity in the depth direction at a specific distance from the sheath edge ($z = 20\lambda_D$) versus α and p values. It is obvious that the collision between the ion and neutral gas decreases the ion velocity; however, this decreasing depends on the p value. This dependence decreases when the α value increases. For any specific α value the lowest value of ion flow velocity corresponds to the constant mean free path. However by increasing the α value, the dependence of ion flow velocity on the p value decreases. Thus the effect of p on the plasma sheath characteristics is more important for a lower value of α. These facts can be seen near the sheath edge too. Figure 6 shows u_z versus α and p values in $z = 2\lambda_D$. As can be seen, near $\alpha = 0.1$ the ion flow velocity increases by increasing the α value. This is due to the fact that for a small α value, the fluctuation of the ion velocity exists near the sheath edge and increasing the collision force only makes some changes in fluctuations.

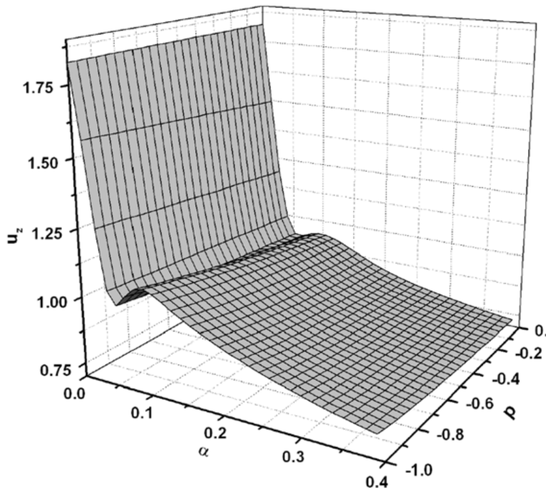

Figure 6. The ion flow velocity in depth direction at a specific distance from the sheath edge; $z = 2\lambda_D$ ($\theta = 40^\circ$ and $\gamma = 5$).

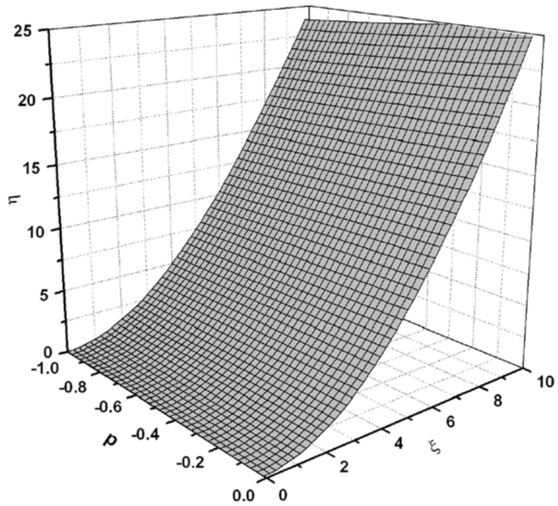

Figure 8. The electrostatic field ($\alpha = 0.134$, $\theta = 40^\circ$ and $\gamma = 5$).

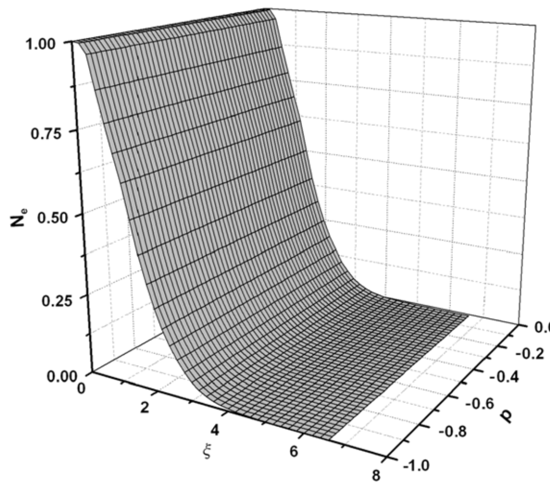

Figure 7. The electron density distribution ($\alpha = 0.134$, $\theta = 40^\circ$ and $\gamma = 5$).

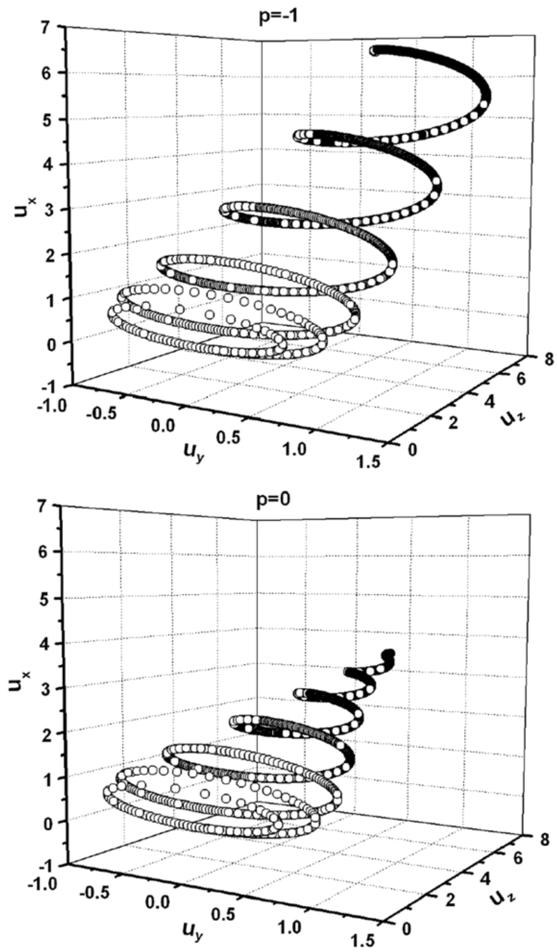

Figure 9. The ion flow velocity in three directions in two cases: $p = 0$ and $p = -1$.

Figures 7 and 8 show the electron density distribution and the electrostatic field ($\alpha = 0.134$, $\theta = 40^\circ$ and $\gamma = 5$). Although the electrostatic field and thus the electron density distribution depend on the ion flow velocity in the depth direction, these figures show that these characteristics of the plasma sheath are independent of the p value. Many numerical simulations with different values and directions of the magnetic field and different collision forces show this independence of the p value.

In figure 9, we plot the ion flow velocity in three directions in two cases: $p = 0$ and $p = -1$. This enables us to investigate the effect of ion collision frequency on the ion gyration movement. As can be seen, in the plasma sheath with a constant mean free path the ion flow velocity increases slowly and the ion gyro radius decreases faster compared with the constant collision frequency.

4. Conclusions

We have solved numerically a magnetic collision sheath model. A power law dependence on the ion flow velocity is considered for ion collision frequency. The effects of collisions between ion and neutral gas atoms on the plasma sheath characteristics (ion flow velocity in the depth direction, the electron density distribution and the electrostatic potential and the ion gyration movement) are investigated under all ranges of the power law from a constant collision frequency to a constant mean free path. The results show that near the sheath edge, the plasma sheath characteristics are independent of the power law dependence. However, far from the sheath edge, by increasing the power (p value in the model) the collision force increases. So by increasing the p value, the ion flow velocity decreases more rapidly and the ion gyro radius decreases faster. Therefore, most effects of the collisions correspond to the constant mean free path. However, the simulations show that the electron density distribution and the electrostatic potential are independent of the power law dependence of the ion collision frequency.

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