Properties of plasma sheath with ion temperature in magnetic fusion devices

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(Received 10 October 2010; accepted 28 December 2010; published online 18 January 2011)

The plasma sheath properties in a strong magnetic field are investigated in this work using a steady state two-fluid model. The motion of ions is affected heavily by the strong magnetic field in fusion devices; meanwhile, the effect of ion temperature cannot be neglected for the plasma in such devices. A criterion for the plasma sheath in a strong magnetic field, which differs from the well-known Bohm criterion for low temperature plasma sheath, is established theoretically with a fluid model. The fluid model is then solved numerically to obtain detailed sheath information under different ion temperatures, plasma densities, and magnetic field strengths. © *2011 American Institute of Physics*. [doi:10.1063/1.3543757]

I. INTRODUCTION

Plasma sheath is one of the most significant and complex subjects in plasma physics. It plays a key role in many plasma applications, such as Langmuir probe, plasma surface treatment, and plasma-wall interaction in magnetic confinement fusion (MCF) devices. The sheath affects plasma energy flux, on which the impurity production and its migration depend strongly. The issues caused by the impurity include reducing the lifetime of plasma facing components, engendering instability of the core plasma, etc.

Since 1923, a number of studies have been carried out on the sheath properties both experimentally and theoretically. In the early days, people focus on the plasma free of the magnetic field;[1–3] in the later days, many works have been done on plasma in a magnetic field,[4–22] but most of them are about common laboratory plasma, i.e., in which the magnetic field is not strong enough to make an ion do a cyclotron motion in the sheath before it strikes the wall. In 1981, Chodura[4] studied the plasma boundary by dividing it into three distinct regions: presheath, magnetic presheath (Chodura sheath), and sheath. In previous studies, the third region is technically the Debye sheath. Since the Debye sheath is typically much thinner than an ion Larmor radius, the cyclotron motion of the ions is therefore ignored.[20] In 1989, Theilhaber and Birdsall[18,19] studied plasma behavior in a magnetic field parallel to the wall. In 1994, Riemann[21] systematically studied the collisional presheath in an oblique magnetic filed with a hydrodynamic model. In 1999, Ahedo[22] studied the sheath in an oblique magnetic field in the cold plasma, in which exact solutions of the Chodura layer and space-charge sheath were obtained with a multiscale analysis. In 2005, Tskhakaya[12] discussed a collision sheath with a magnetic field parallel to a wall. The study focuses on a case that electrons are magnetized and mainly analyzed the electron dynamics. However, in magnetic fusion devices, the magnetic field is extremely strong, such that the ion Larmor radius is comparable with the Debye length, as shown in Fig. 1. In general, the sheath has size of several Debye lengths. Therefore, the motion of ions in the sheath will be influenced by the magnetic field greatly. Ions behavior very differently from that in the weak magnetic field, and plasma sheath structure must be different from the classic sheath. Many researchers have examined the behavior of such a sheath in recent years. Jelić *et al.* used particle-in-cell (PIC) method to study kinetic parameters (i.e., polytropic coefficient γ) in the sheath. Kawamura and Fukuyama, using gyrokinetic Vlasov equation, studied the sheath properties with $\vec{E}\times\vec{B}$ drift velocity being considered, in which a shifted Maxwellian velocity distribution was adopted to describe ions. In the boundary of a MCF device, $\vec{E}\times\vec{B}$ drift plays an important role, as pointed out by Krasheninnikov.[23] Very recently, Krasheninnikova *et al.*[14] carried out a systematic investigation of the scaling of a one-dimensional plasma sheath with a magnetic field parallel to the wall using analytical theory and a PIC code. It was found that the scaling greatly depends on the relation between three parameters: Debye length and the Larmor radii of ions and electrons.

However, a fluid model with ions having a $\vec{E}\times\vec{B}$ drift velocity at the boundary has not been done yet. In this work, we develop a self-consistent one-dimensional collisionless fluid model with a magnetic field with an arbitrary incident angle, in which the ion $\vec{E}\times\vec{B}$ drift velocity is taken into account and the floating electric potential at the wall is derived from the assumption of the net conductive current being zero.

The paper is organized as follows. In Sec. II, a steady state two-fluid model is used to investigate the sheath, through which we can get the potential at the wall and a sheath criterion. In Sec. III, with the newly obtained sheath criterion and the potential at the wall as two of boundary conditions, the fluid model is further numerically solved to discuss the effects of magnetic field and ion temperature. Finally, conclusions are presented in Sec. IV.

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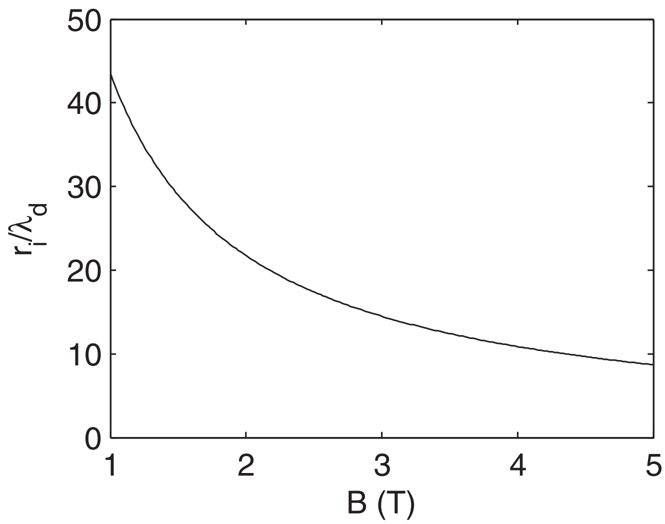

FIG. 1. Ratio of ion gyration radius (r_i) to Debye length (λ_d) varies with magnetic strength.

II. THEORETICAL MODEL AND MODIFIED BOHM CRITERION

In a MCF device, the ion temperature (T_i) cannot be neglected. Besides, a strong magnetic field present in the plasma makes the ions' behavior very different from that in low temperature plasma. The electron Larmor radius (r_e) is much shorter than the sheath length (l_s). In a typical case of high temperature plasma, $r_e/l_s \ll 1$, i.e., the electrons can complete over at least ten gyration circles during the course of traveling the sheath region. Consequently, electrons can reach thermal equilibrium. Thus, a steady magnetized plasma sheath can be described by four equations: electron Boltzmann distribution relation, continuity equation of ions, the equation of motion of ions, and Poisson equation. After reduced units (Table I) being introduced, these four equations have the following forms:

$$n_e = exp(\varphi), \tag{1}$$

$$u_{ix0} = n_i u_{ix}, \tag{2}$$

$$\vec{u}_i \cdot \nabla \vec{u}_i = -\nabla \varphi - \beta \nabla \ \ln n_i + \alpha \vec{u}_i \times \hat{b}, \tag{3}$$

TABLE I. Units for normalizing.

Parameter	Units
Electric potential	$\varphi = \frac{\phi}{kT_e}$
Length	$\xi = \frac{x}{\lambda_d}, (\lambda_d = \sqrt{(kT_e\epsilon_0)/(n_0e^2)})$
Velocity	$u = \frac{v}{c_s}, (c_s = \sqrt{kT_e/m_i})$
Particle number density	$n = \frac{n}{n_0}$
Ion mass	$m_i = \frac{m_i}{m_e}$
Charge	e

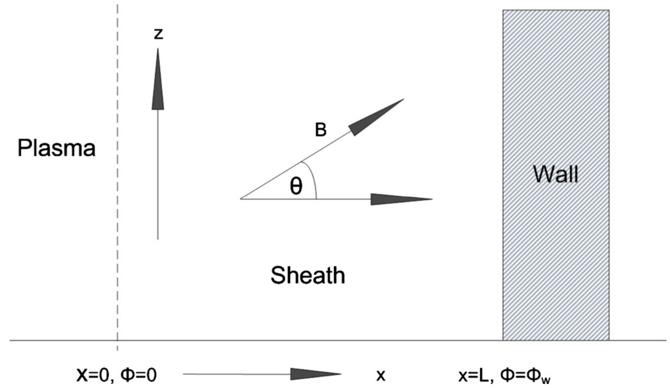

FIG. 2. (Color online) Geometry of the magnetized sheath.

$$\frac{\partial^2 \varphi}{\partial \xi^2} = -(n_i - n_e). \tag{4}$$

Here, $\alpha = \omega_c/\omega_p$, $\omega_c = eB/m_i$, $\omega_p = \sqrt{n_0 e^2/(\epsilon_0 m_i)}$, $\beta = T_i/T_e$, and $\hat{b} = \vec{B}/B_0$. Subscripts e and i denote quantities for electrons and ions, respectively, and $x0$ denotes the quantities at $\xi = 0$.

In the two-dimensional domain as shown in Fig. 2, the coordinate origin is defined at the boundary of sheath with electric potential equal to zero. The magnetic field is in the $x-z$ plane, making an angle of θ with the x axis. Thus, the magnetic field can be expressed as $\vec{B}_0 = B_0(\cos\theta, 0, \sin\theta)$. The floating electric potential at the wall (ξ_w) is denoted as ϕ_w. We can separate Eq. (3) into three components

$$u_{ix}\frac{\partial u_{ix}}{\partial \xi} = -\frac{\partial \varphi}{\partial \xi} - \beta \frac{\partial \ln n_i}{\partial \xi} + \alpha u_{iy} \sin\theta, \tag{5}$$

$$u_{ix}\frac{\partial u_{iy}}{\partial \xi} = -\alpha u_{ix}\sin\theta + \alpha u_{iz}\cos\theta, \tag{6}$$

$$u_{ix}\frac{\partial u_{iz}}{\partial \xi} = -\alpha u_{iy}\cos\theta. \tag{7}$$

Here, subscripts ix, iy, and iz denote, respectively, the components of ion velocity in the x, y, and z directions.

As mentioned above, the boundary conditions for these equations can be expressed as follows. At the sheath edge ($\xi = 0$), the electric potential is zero ($\varphi_0 = 0$) and the charge neutrality condition is satisfied, i.e.,

$$n_{i0} = n_{e0} = n_0. \tag{8}$$

At the wall ($\xi = \xi_w$), the floating electric potential is φ_w and, under the steady condition, the current is zero, i.e.,

$$j_{ix}(\xi_w) + j_{ex}(\xi_w) = 0. \tag{9}$$

Here, j is current density.

A. Particle flux and electric potential at wall

As aforementioned, the T_i should not be neglected in investigation of the plasma sheath in a MCF device. We assume that the velocity distribution of ions has a form

$$f(\vec{u}) = n_i\left(\frac{1}{2\pi\beta}\right)^{3/2}\exp\left[-\frac{1}{2\beta}(\vec{u}-\vec{u}_i)^2\right]. \tag{10}$$

We can get the ion current at $\xi=0$ by integrating the Eq. (10) over the x and y velocity components

$$j_{ixw} = j_{ix0} = \sqrt{\frac{\beta T_e}{2\pi m_i}}\exp\left(-\frac{1}{2\beta}u_{ix0}^2\right) + \frac{1}{2}u_{ix0}c_s\left[1+erf\left(\sqrt{\frac{1}{2\beta}}u_{ix0}\right)\right] \tag{11}$$

and the electron current density at the wall

$$j_{exw} = -\exp(\varphi_w)\sqrt{\frac{T_e}{2m_e}}. \tag{12}$$

Combining Eqs. (11) and (12) with Eq. (9), we can arrive at the electric potential at wall

$$\varphi_w = \ln\left\{\left(\frac{\beta}{m_i}\right)^{1/2}\exp\left(-\frac{1}{2\beta}u_{ix0}^2\right) + \sqrt{\frac{\pi}{2m_i}}u_{ix0}\left[1+erf\left(\sqrt{\frac{1}{2\beta}}u_{ix0}\right)\right]\right\}, \tag{13}$$

which is a complex function of β, m_i, and u_{ix0}.

B. Sheath criterion with magnetic field

The well-known Bohm sheath criterion says that ions must enter the sheath region with a velocity greater than the acoustic velocity c_s. In a similar way,[24] we can obtain the criterion with considering $\vec{E}\times\vec{B}$. Multiplying Eq. (4) by $\partial\varphi/\partial\xi$ and then integrating it with the boundary condition, we can arrive at

$$\frac{1}{2}\left(\frac{\partial\varphi}{\partial\xi}\right)^2 = \frac{1}{2}\left(\left.\frac{\partial\varphi}{\partial\xi}\right|_{\xi=0}\right)^2 - V(\varphi), \tag{14}$$

where $V(\varphi)=\int_0^{\varphi}(n_i-n_e)d\varphi$ is a quasipotential called the Sagdeev potential. It is apparent that if Eq. (14) has a real number solution, the right hand side (RHS) of Eq. (14) should be positive. With $V(0)=0$ and $\partial V/\partial\varphi|_{\varphi=0}=0$, the RHS of Eq. (14) must satisfy the following relation:

$$\left.\frac{d^2V}{\partial\varphi^2}\right|_{\varphi=0} = \left.\left(\frac{\partial n_i}{\partial\varphi}-\frac{\partial n_e}{\partial\varphi}\right)\right|_{\varphi=0} < 0. \tag{15}$$

With the relation of Eq. (1), the second term in the RHS of Eq. (15) equals to

$$\left.\frac{\partial n_e}{\partial\varphi}\right|_{\varphi=0} = 1. \tag{16}$$

Thus, making use of Eqs. (2) and (5), we can write $\partial n_i/\partial\varphi$ as

$$\left.\frac{\partial n_i}{\partial\varphi}\right|_{\varphi=0} = \frac{1-\alpha u_{iy0}\sin\theta/(\partial\varphi/\partial\xi)|_{\xi=0}}{u_{ix0}^2-\beta}. \tag{17}$$

Substituting Eqs. (16) and (17) into Eq. (15), we can have a criterion,

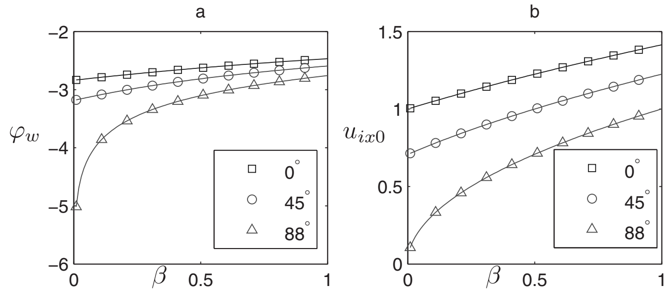

FIG. 3. Electric potential at wall (φ_w) and x component of ion velocity at the sheath boundary (u_{ix0}) profiles with $\theta=0°$, $45°$, $88°$.

$$u_{ix0}^2 > 1+\beta+\frac{\alpha\sin\theta u_{iy0}}{-\left.\frac{\partial\varphi}{\partial\xi}\right|_{\xi=0}}. \tag{18}$$

Clearly, the criterion is a complex function, unlike Bohm criterion $u_{ix0}^2>1$. Since there exists a presheath region between core plasma and Debye sheath, the electric field in the presheath (E_0) cannot be neglected. The $\vec{E}_0$ is in the x direction and the $\vec{B}$ is on the $x-z$ plane, so there is a $\vec{E}\times\vec{B}$ drift in the y direction

$$u_{iy0} = -\frac{E_0\sin\theta}{\alpha}. \tag{19}$$

Thus, Eq. (18) becomes

$$u_{ix0}^2 > \beta+\cos^2\theta. \tag{20}$$

Equation (20) is a modified Bohm criterion, which considers both the effects of the ion temperature and the $\vec{E}\times\vec{B}$ drift. Combining Eqs. (13) and (20), we can get the electric potential drop cross the sheath. Figure 3 shows the φ_w and u_{ix0} as functions of β. The amplitude of φ_w decreases with either β or θ, while the u_{ix0} behaves oppositely. In the model, the ions' collisions are ignored. As the parameter β decreases, the ions have less ability countering the accumulation of electrons at the wall. Consequently, the wall potential is more negative. When the magnetic field is nearly parallel to the wall and β is small, the x component of ion velocity is very small, namely, the x component of ion current drops to very low levels. Since few ions arrive at the wall, it needs a highly negative φ_w to reduce the electron current to make $j_{ix}+j_{ex}=0$. In contrast with the Bohm criterion, the modified criterion says that the ion velocity entering the sheath u_{ix0} can be lower than the ion acoustic velocity, depending on the incident angle of magnetic field and the ratio of the ion temperature to electron temperature.

III. DISCUSSION

In this section, the authors feel it is necessary to discuss the effects of magnetic field and ion temperature with the modified Bohm criterion and the potential at the wall as two of the boundary conditions. With boundary conditions Eqs. (13), (19), and (20), $\varphi_0=0$, and $E_0=\delta_E\ll 1$, the set of Eqs. (4)–(7) and the relation between the electric potential

TABLE II. The ranges of parameters used in the calculation.

Number density n (m^{-3})	10^{19}–10^{20}
Magnetic strength B (T)	1–5
Electron temperature T_e (eV)	10–100
Ion temperature T_i (eV)	10–100

and electric field can be solved numerically. Here, the atomic hydrogen ions are assumed to be the unique ionic species. (The detailed parameters refer to Table II.)

A. Magnetic field effects

As shown in Fig. 4, both the sheath width and electric potential drop increase with θ. The ion Larmor radius increases with position. In the ion energy density profiles, the ion energy density (E_i) is not a monotonous function of position.

A special thing is worth pointing out for the velocity u_{iy}. As we can see from Fig. 5, u_{iy} is in a good linear with position, especially when $\theta \to \pi/2$. From Eq. (6), we can get $\partial u_{iy}/\partial\xi = -\alpha u_{ix}\sin\theta + \alpha u_{iz}\cos\theta / u_{ix} = -\alpha = -0.0132$ when $\theta = \pi/2$. This analytical result agrees well with the numerical evaluation.

From Fig. 6, it is clearly seen that when the magnetic field strength increases, the floating potential ϕ_w does not change, but the sheath width reduces. The ion energy density behaves similarly at different magnetic field strengths. The sheath width is in inverse proportion to the magnetic field strength. The transition position is inversely proportional to the magnetic field strength; as can be seen from Fig. 6(d): $\xi_{t1}:\xi_{t2}:\xi_{t3} \simeq 62:41:24 \simeq 1/B_1:1/B_2:1/B_3$.

B. Ion temperature effects

In the fusion plasmas, the effects of ion temperature on the sheath properties cannot be ignored. The electric potential of sheath, ion density, perpendicular component of ion momentum, and ion energy density are presented as functions of the length at different ratios of ion temperature to electron temperature. As can be seen from Fig. 7, when the ratio β increases, the sheath length increases, but the floating potential reduces. Higher ion temperature leads to a higher ion current. In others words, for the same electric potential drop, less ion velocity is needed to balance the electron current when the value of β increases.

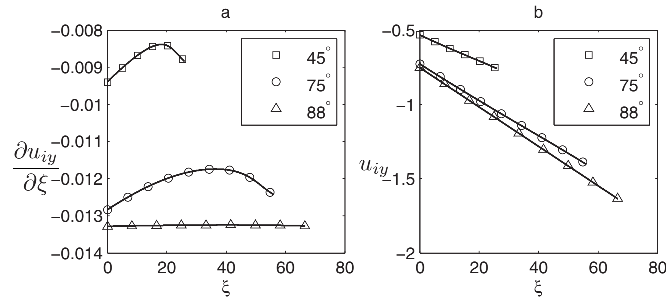

FIG. 5. $\partial u_{iy}/\partial\xi$ and u_{iy} profiles at parameters $n_0 = 3\times10^{19}\ \mathrm{m}^{-3}$, $\beta = 0.1$, $B = 1$ T, and $\theta = 45°$, 75°, 88°.

IV. CONCLUSIONS

In this paper, a sheath criterion for ion magnetized plasmas is obtained by considering the effects of ion temperature. Under the influence of magnetic field, the sheath criterion is a function involving the electric field, magnetic field, and the ion velocity. Theoretical analysis shows that ion flux incident to the wall and the electric potential on the wall are both affected by the ion temperature. With the assumption of atomic hydrogen ions being the unique ionic species, a numerical analysis is carried out to see some details of sheath structure. The results further show that strong magnetic field makes the ion momentum nonlinear with the position.

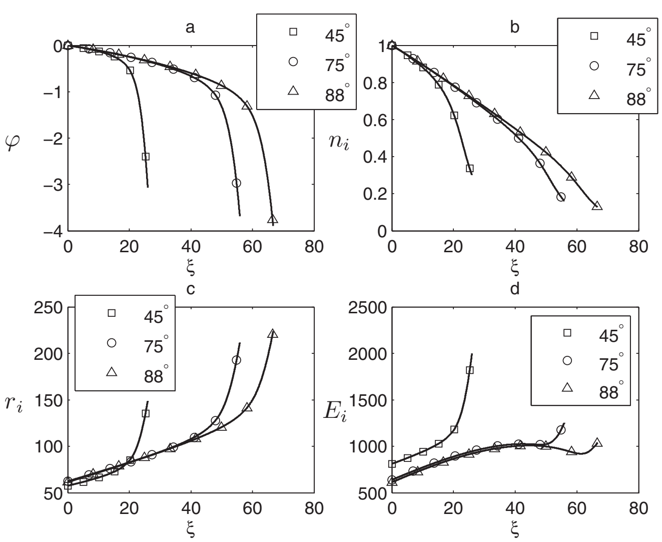

FIG. 4. Electric potential (φ), ion density (n_i), ion gyration radius (r_i), and ion energy density (E_i) profiles at parameters $n_0 = 3\times10^{19}\ \mathrm{m}^{-3}$, $\beta = 0.1$, $B = 1$ T, and $\theta = 45°$, 75°, 88°.

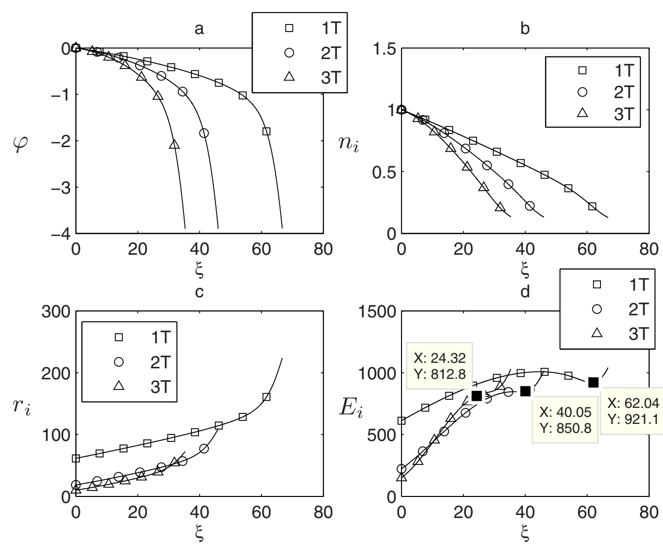

FIG. 6. (Color online) Electric potential (φ), ion density (n_i), ion gyration radius (r_i), and ion energy density (E_i) profiles at parameters $n_0 = 3\times10^{19}\ \mathrm{m}^{-3}$, $\beta = 0.1$, $\theta = 88°$, and $B = 1$, 2, 3 T; black squares denote the transition positions.

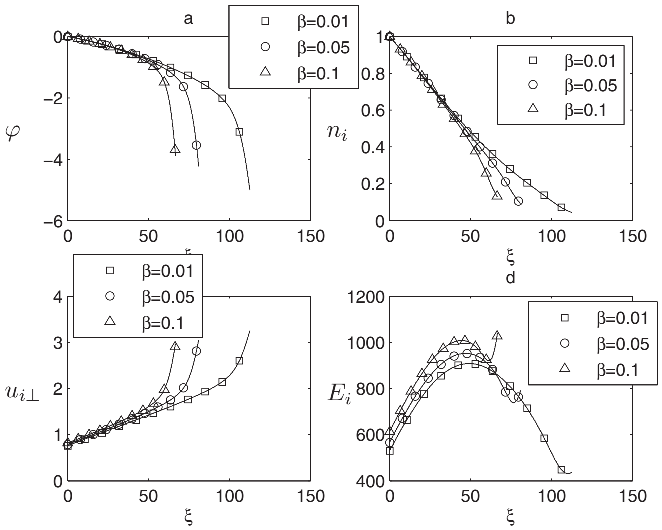

FIG. 7. Potential (φ), ion density (n_i), ion velocity in perpendicular to magnetic field ($u_{i\perp}$), and ion energy (E_i) at parameters $n_0=3\times 10^{19}$ m^{-3}, $\theta=88°$, $B=1$ T, and $\beta=0.01$, 0.05, 0.1.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant No. 10875024). The authors wish to thank Professor Xiaogang Wang for helpful discussions.

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