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Sheath and presheath structure in the plasma–wall transition layer in an oblique magnetic field

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In this paper we present a hydrodynamic study of a current carrying transition layer that separates the plasma from electrically conducting or insulating walls. The plasma is placed in a magnetic field that could be parallel to, or intersects obliquely with the walls. The self-consistent model of the smooth presheath–sheath transition of the near-wall plasma layer developed here for a finite Debye radius to ion Larmor radius ratio (Ψ) is based on a previously developed model of the quasineutral plasma presheath [Phys. Plasmas **4**, 3461 (1997)]. The potential distribution in the presheath is found to have a positive maximum with respect to the plasma–presheath interface. In the case of a wall with a floating potential, the value of the maximum decreases with the incidence angle θ of the magnetic line force, and approaches zero when $\theta=2°$. The presheath thickness generally increases with the incidence angle, from about the electron Larmor radius up to the ion Larmor radius, and depends on the electron to ion current ratio, ion velocity at the plasma–presheath interface, and Hall parameter. Analysis of presheath-sheath transition when Ψ is in the range of 10^{-2} to 10^{-4} shows that (1) the electric field at the presheath–sheath interface is finite, (2) the critical ion velocity at the sheath edge is about (0.6–0.9) of ion sound speed, depending on the plasma parameters, and generally increases with the ion to electron current ratio. The current–voltage characteristic of the transition layer does not depend on θ, the magnetic field incidence angle, for $\theta \geqslant 5°$. © *1998 American Institute of Physics.* [S1070-664X(98)02305-2]

I. INTRODUCTION

The transition layer between a magnetized plasma and a wall is important in determining particle and energy fluxes to the wall, wall sputtering,[1] arcing,[2] and is relevant in probe theory.[3] In a case of practical interest, namely, the magnetized plasma duct in a filtered vacuum arc deposition systems,[4] a magnetic field generally parallel to the duct wall is used to limit plasma losses. However, because of the finite number of magnetic coils, the magnetic field may intersect the wall obliquely at some small angle. This characteristic of the magnetic field affects the performance efficiency of the filter. A detailed study of the plasma motion in the filter is required in order to improve its efficiency. Such a study depends, however, on detailed information about the physical properties of the wall–plasma transition region.

Generally, the plasma–wall transition layer in a plasma placed in a magnetic field consists of two regions: an electrostatic sheath and a quasineutral presheath.[5] The electrostatic sheath of the magnetized plasma, where the magnetic field obliquely intersects the wall, was studied by Daybelge and Bein[6] and Holland *et al.*[7] Chodura[8] studied the quasineutral magnetic presheath in the case of an oblique magnetic field in the collisionless limit, whereas Riemann[9] studied such a presheath in the case of a weak collision plasma. In these models, the transition layer thickness and potential distribution in the layer were calculated by assuming zero wall electrical current, entailing a floating potential on the wall. In the electrostatic sheath region, Holland *et al.*[7] calculated that a transition from a positive to negative wall potential with respect to the plasma bulk occurred for small angles of incidence (i.e., $\theta \leqslant 9°$), which only depended weakly on the electron to the ion temperature ratio. Holland *et al.* also obtained that the wall potential is negative with respect to the plasma when the electron velocity along the magnetic field lines is larger than the ion velocity across the field.[7] The thickness of the sheath decreases with the magnetic field lines incident angle.[7] In the quasineutral presheath region, the Boltzmann distribution for the electrons was used[8,9] in order to determine the electric field, and as a result the presheath potential with respect to the plasma was calculated to be negative for any incidence angle of the magnetic field lines. Moreover, under these conditions the magnetic presheath thickness is about the ion Larmor radius and depends weakly on the magnetic field lines incidence angle.

The electron density distribution may be described by the Boltzmann function when the pressure gradient is equal to the electrical force. This distribution will occur when the plasma flow is parallel to the magnetic field.[10] Keidar *et al.*[11] and Beilis *et al.*[12] showed that when the magnetic field is parallel to the wall, the electron distribution density may deviate from the Boltzmann law. Moreover, even in the case when the plasma density dependence on the electrical potential is expressed by the Boltzmann distribution, the presheath thickness deviates from that calculated using a model based

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on such a density distribution. Furthermore, it was shown that the electrical potential in the presheath, with respect to that at the plasma–presheath interface, can be either negative or positive, depending on the electron to ion current ratio and ion Hall parameter.[11,12]

The smooth presheath–sheath transition depends on the presheath plasma parameters. In earlier presheath models, the electric field approached infinity toward the sheath interface, where the ion velocity equals sound speed (Bohm criterion).[5,13–15] The presheath–sheath interface was defined by the condition that the presheath electrical field (in the presheath scale) is infinite there, while the same interface is defined by the condition that the sheath electrical field (in the sheath scale) approaches zero there. This is known as the two-scale problem for the presheath–sheath matching solution.[5,15–17] In order to match the two regions, Baksht *et al.*[15] and Riemann[5,17,18] proposed taking into account rare collisions using kinetic analysis of the presheath and sheath interface, and Godyak *et al.*[19] proposed the use of some characteristic electric field, while Beilis[20,21] proposed matching the two regions by some characteristic velocity at the presheath–sheath interface, basing the definition on a hydrodynamic model. However, it seems that no self-consistent analysis of the presheath–sheath matching problem in the presence of a magnetic field that intersects a wall has been published in the past.

In the present paper, ion velocity and the electric field at the presheath–sheath interface are considered in the presence of an oblique magnetic field. The influence of the electron to ion current ratio on the sheath and presheath parameters is determined by use of a hydrodynamic approach. In Sec. II the presheath model is developed for the case of a magnetic field obliquely intersecting the wall. The physical framework includes a general electron momentum equation, magnetic forces, and pressure gradient and drag forces. The effect of the magnetic field–wall intersection angle, in the range of small angles, on the potential distribution and presheath thickness are studied. A smooth presheath–sheath transition is considered, and the current–voltage characteristic of the transition layer is obtained (Sec. II). In Sec. IV these results are discussed.

II. PHYSICAL MODEL

In this section, the condition for the electrostatic sheath existence, a model for the electrostatic space charge sheath, a model for the quasineutral presheath, and a model for a smooth interface between these regions are formulated and presented.

A. Presheath

The presheath model is based on the assumption that the quasineutral presheath thickness (L_p) is much larger than the Debye radius (r_D). Therefore, $Z_i N_i = N_e = N$, where N_i, N_e are the ion and electron densities, respectively, and Z_i is the ionicity. The presheath in an oblique magnetic field, that is not parallel to the wall, is modeled using the hydrodynamic steady-state approach. The magnetic field intersects the wall with an angle θ (Fig. 1). The wall is infinite in the y and z

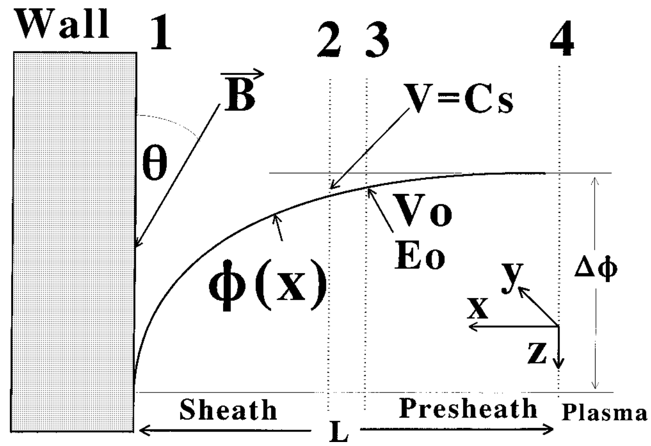

FIG. 1. The geometry of the plasma–wall transition layer.

directions. The plasma–presheath boundary ($x = 0$) is assumed to be a source of steady charged particle fluxes. In the present model, the effect of ion–ion, ion–atom collisions are approximated by an effective collision frequency of ions, which are assumed to be distributed uniformly within the presheath.[9,11,12]

The quasineutral plasma region is described by following two-fluids equation system:

$$\nabla \cdot (N_\alpha \mathbf{V}_\alpha) = 0, \tag{1}$$

$$N_i m_i (\mathbf{V}_i \cdot \nabla) \mathbf{V}_i = Z_i e N_i (\mathbf{E} + \mathbf{V}_i \times \mathbf{B}) - \nabla P_i - N_i \nu_c m_i \mathbf{V}_i + N_i \nu_{ie} m^* (\mathbf{V}_e - \mathbf{V}_i), \tag{2}$$

$$0 = -e N_e (\mathbf{E} + \mathbf{V}_e \times \mathbf{B}) - \nabla P_e - N_e \nu_{ei} m^* (\mathbf{V}_e - \mathbf{V}_i), \tag{3}$$

where $\alpha = e$, i; $\mathbf{V}_i$, $\mathbf{V}_e$ are the ion and electron fluid velocity, respectively, $\mathbf{E} = -\nabla U$ is the electric field, U is the electric potential, $\mathbf{B}$ is the magnetic field, n_α is the density of particles of type α, ν_{ei} is the electron–ion collision frequency, ν_c is the effective ion collision frequency, ϵ_0 is the permittivity of vacuum, m_e and e are electron mass and charge, respectively, m_i and Z_i are ion mass and ionicity (for simplicity we will use $Z_i = 1$), respectively, $m^* = m_i m_e/(m_i + m_e) \cong m_e$ is an effective mass, and P is the partial pressure. Electrons and ions are assumed to be ideal gases, hence $P_e = N_e k T_e$ and $P_i = N_i k T_i$, where k is the Boltzmann constant and T_e, T_i are the electron and ion temperatures, respectively, which are assumed to be constant. The density and velocity components are assumed to vary only along the direction normal to the wall. From the quasineutrality condition and Eq. (1), we have $V_{ex} = \eta V_{ix}$, where $\eta = J_e/J_i$, and where J_e and J_i are electron and ion current densities normal to the wall, respectively. It should be noted that particle fluxes in the direction transverse to the magnetic field are not constant, because of the magnetic field. However, if the presheath length is smaller than or close to the Larmor radius of a certain particle, then the flux to the wall of this particle may be considered to be steady.[7,8,9,11,12] In the present model a constant electron to ion current ratio η is used as a model parameter, as was done by Keidar *et al.*[11] and Beilis *et al.*[12]

The model is presented with the following dimensionless variables: $n_\alpha = N_\alpha / N_0$, where N_0 is the plasma region density, $\chi = x/\rho_i$, where $\rho_i = C_s/\omega_i$, $v = V_{ix}/C_s$, $u = V_{iy}/C_s$, $w = V_{iz}/C_s$, $C_s^2 = k(T_i + T_e)/m_i$, $\beta_e = \omega_e/\nu_{ei}$, $\beta_i = \omega_i/\nu_c$, $\omega_e = eB/m_e$, $\omega_i = eB/m_i$, $\phi = eU/kT_e$, $\epsilon = T_e/T_i$, and $\mu = m_e/m_i$.

In addition, the relation between β_e and β_i is used in the form of $\beta_e = \beta_i \delta/\mu$, where $\delta = \nu_c/\nu_{ei}$. Generally, the ratio δ depends strongly on the plasma ionization degree and the electron to ion temperature ratio. With these dimensionless quantities, we obtain Eqs. (1)–(3) in the final form

$$(v^2 - 1)\,\frac{v'}{v} = u - \frac{1}{1+\beta^2 \tan^2\theta} \cdot \left(\frac{\beta_i \eta \delta}{\mu} \cdot v - \beta_e \tan\theta \cdot w + u \right) - \frac{1}{\beta_i}\, v, \tag{4}$$

$$u' = \eta - 1 - \frac{u}{v\beta_i} - \frac{\beta \tan^2\theta}{1+\beta^2 \tan^2\theta} \cdot \left(\frac{\beta_i \eta \delta}{\mu} - \beta_e \tan\theta \cdot \frac{w}{v} + \frac{u}{v} \right), \tag{5}$$

$$w' = -\frac{u \tan\theta}{v} - \frac{w}{v\beta_i} - \frac{\tan\theta}{1+\beta^2 \tan^2\theta} \cdot \left(\frac{\beta_i \eta \delta}{\mu} - \beta_e \tan\theta \cdot \frac{w}{v} + \frac{u}{v} \right), \tag{6}$$

$$\phi' = -\frac{v'}{v} + \left(1 + \frac{1}{\epsilon}\right) \cdot v \cdot \frac{(\eta - 1)\mu}{\beta_i \delta} + \left(1 + \frac{1}{\epsilon}\right) \cdot \frac{1}{1+\beta^2 \tan^2\theta} \cdot \left(\frac{\beta_i \eta \delta}{\mu}\, v - \beta_e \tan\theta \cdot w + u \right), \tag{7}$$

where $'$ marks the derivative with respect to the dimensionless coordinate in the wall direction χ. Equations (4)–(6) describe the components of fluid velocity: v normal to the wall, u, and w parallel to the wall, whereas Eq. (7) describes electric potential distribution in the presheath.

B. Sheath

The electrostatic sheath is collisionless ($r_D/\lambda_c < 1$, where λ_c is the mean-free path) and unmagnetized ($r_D/\rho_e \ll 1$, where ρ_e is the electron Larmor radius) in the considered range of plasma parameters. Taking this into account, the system of equations for the sheath parameters has the following dimension form:

$$\nabla \cdot (N_i \mathbf{V}_i) = 0, \tag{8}$$

$$m_i (\mathbf{V}_i \cdot \nabla) \mathbf{V}_i = e\mathbf{E}, \tag{9}$$

$$\nabla^2 U = e/\epsilon_0 (N_e - N_i), \tag{10}$$

$$N_e/N_0 = e^{-eU/kT_e}. \tag{11}$$

Assuming that all plasma parameters in the sheath vary only in the x direction (Fig. 1), it follows from Eqs. (8)–(9) that the ion density in the sheath has the following expression in the dimensionless form:

$$n_i = \frac{1}{\sqrt{1 - 2(\phi - \phi_0)\left(\frac{C_s}{V_0}\right)^2 \frac{1}{1+1/\epsilon}}}, \tag{12}$$

where V_0/C_s and ϕ_0 are ion velocity and potential at the sheath–presheath interface. It can be seen that ion density n_i in the sheath depends on the electron to ion temperature ratio ϵ, and on the ion initial velocity at the sheath–presheath interface, V_0.

We rewrite Eq. (11) as

$$n_e = \exp(-\phi). \tag{13}$$

Using Eqs. (12) and (13), the equation for the electrical potential [Eq. (10)] can be expressed in dimensionless form as

$$\Psi^2 \frac{d^2\phi}{d\chi^2} = \left(e^{-\phi} - \frac{1}{\sqrt{1 - 2(\phi - \phi_0)\left(\frac{C_s}{V_0}\right)^2 \frac{1}{1+1/\epsilon}}} \right) = \frac{\Delta n}{n_e}, \tag{14}$$

where $\Psi = r_D/\rho_i$ and $\Delta n = n_e - n_i$. In Eq. (14) the spatial coordinates are also normalized by the ion Larmor length, as was done in the presheath equations. Equation (14) describes the potential distribution in an unmagnetized sheath, and because of the normalization, there is a dependence on the magnetic field (parameter Ψ). Generally, the parameter Ψ is proportional to the sheath and presheath thicknesses ratio.

The solution of Eq. (14) requires the specification of the boundary conditions for the potential, electric field, and ion velocity V_0 at the presheath–sheath interface (boundary 3, Fig. 1). These boundary conditions at the sheath–presheath interface are considered in the next section.

C. Sheath and presheath: Smooth transition model

As was mentioned above, it is necessary to know the sheath edge location and matching conditions at the presheath–sheath interface in order to solve the presheath–sheath transition problem. The necessary condition for the existence of a continuous solution for the sheath problem, when the electric field at the *sheath edge* is assumed to be equal to zero, is known as the Bohm criterion, which states that the ion velocity at the sheath edge is equal to $(kT_e/m_i)^{1/2}$ (Bohm velocity). When V_0 equals Bohm velocity, the *presheath* electric field at the sheath–presheath interface approaches infinity (a singular point). In this case, a smooth matching of the presheath and sheath solutions is impossible. However, if V_0 differs slightly from the Bohm velocity, the electric field becomes a continuous function, increasing from a relatively small value (but not zero) at the plasma–sheath interface up to maximal value at the

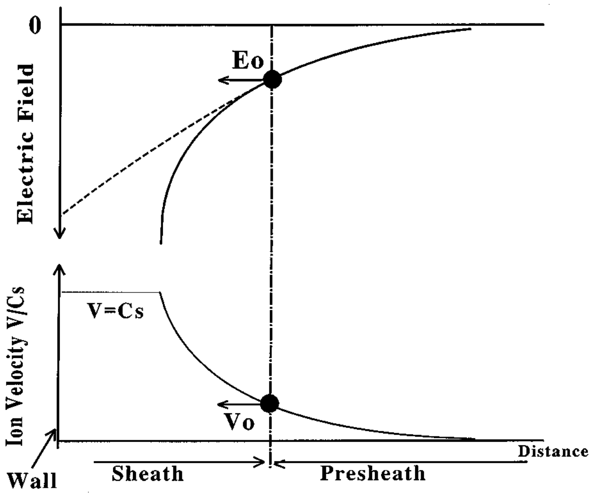

FIG. 2. Ion velocity and electric field distribution in the neighborhood of the presheath–sheath interface. The broken line shows the electric field in the sheath represent electric field distribution after smooth matching.

wall.[5,15,19] Maintaining a finite electric field at the sheath edge, a continuous solution is obtained for the potential distribution in the sheath, provided the ion velocity at the sheath–presheath interface is smaller than the Bohm velocity.[15,17,21]

We apply this approach in the present work, and the model presented here consists of finding the minimal V_0 at the presheath–sheath interface that results in a continuous solution for potential distribution in the sheath. The general connection between velocity V_0 and the electric field, and the behavior of the electric field in the neighborhood of the presheath–sheath interface (solid line) and sheath (broken line) are plotted schematically in Fig. 2.

For a smooth matching of the sheath and the presheath, we use the condition that both the electric field and its divergence should be continuous functions.[15] This entails that quasineutrality should be fulfilled in the presheath up to the presheath–sheath interface. (It was shown in Ref. 15 that this is attained if $\Psi \ll 1$.) Furthermore, it was also shown in Ref. 15 that a large electric field enables the matching of the presheath and the sheath, and collisions between plasma particles can be neglected in the sheath analysis. Thus, the presheath–sheath interface location is derived assuming a collisionless sheath. As was stated above, quasineutrality in the presheath is assumed.

The presheath and sheath regions are matched for a finite value of r_D/ρ_e under the following conditions: (i) $U_{sh}(3) = U_{pr}(3) = U_0$, and (ii) $E_{sh}(3) = E_{pr}(3) = E_0$, where subscripts ''sh'' and ''pr'' correspond to values of the potential and the electric field at the sheath and presheath sides of boundary 3 (Fig. 1), respectively. The value of V_0 (boundary 3, Fig. 1) is the minimal velocity for which [taking into account condition (ii)] the potential, obtained from Eq. (13), is continuous in the sheath. It follows that in the present approach the presheath–sheath interface shifts from boundary 2 (where $V_0 = C_s$ and electric field is singular) to boundary 3 (where $V_0 < C_s$ and electric field is finite) in Fig. 1.

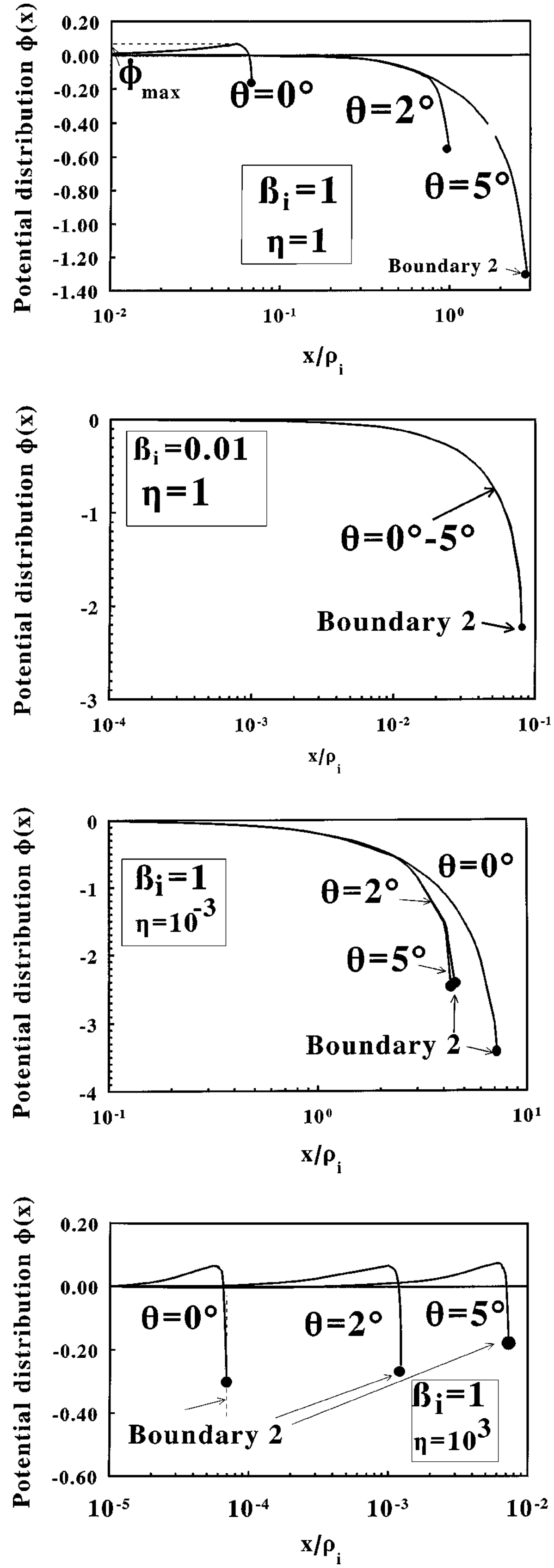

FIG. 3. The potential distribution in the presheath with the intersection angle θ as the parameter for a Cu plasma, with $\delta = 10^{-3}$, $\epsilon = 10$. (a) $\beta_i = 1$, $\eta = 1$; (b) $\beta_i = 0.01$, $\eta = 1$; (c) $\beta_i = 1$, $\eta = 10^{-3}$; (d) $\beta_i = 1$, $\eta = 10^3$.

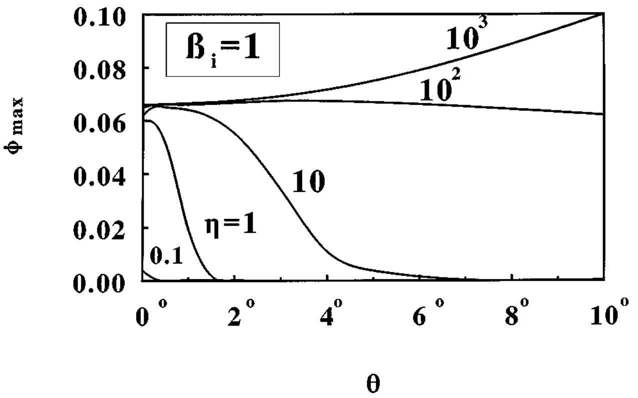

FIG. 4. The dependence of the maximum presheath potential ϕ_{max} on the intersection angle θ, with the electron to ion current ratio η as a parameter for Cu plasma with $\beta_i = 1$, $\delta = 10^{-3}$, $\epsilon = 10$.

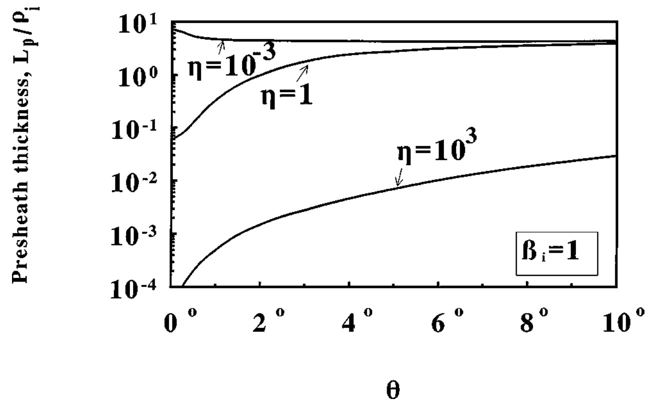

FIG. 5. The magnetic presheath thickness (in units of the ion gyroradius) as a function of the intersection angle θ, with the electron to ion current ratio η as a parameter, for a Cu plasma and $\beta_i = 1$.

III. RESULTS

A. Presheath parameter distribution in an oblique magnetic field

The presheath parameters were calculated numerically [Eqs. (4)–(7)] for the ion to electron temperature ratio in the range of $\epsilon = 1 - 100$, and the ratio of ion collision frequency to ion–electron collision frequency (δ) equals 10^{-3}. As was shown in Ref. 12, the value of δ affects only weakly the presheath thickness, and electrical potential and ion velocity in the presheath.[12] Ion velocity at the plasma-presheath boundary V_x should also be specified to allow the solution of Eqs. (4)–(7). This velocity is unknown and depends on the plasma parameters, such as ion temperature, plasma flow etc. The effect of V_x on the presheath thickness is included in the present model by considering it as a free parameter.

The electric potential distribution in the presheath, with respect to the potential at the plasma–presheath interface, is plotted in Figs. 3(a)–3(d) with θ as a parameter. The condition $\eta = 1$ is fulfilled when the wall is floating, i.e., when the electron wall current is equal to the ion wall current. When the magnetic field is perfectly parallel to the wall ($\theta = 0°$), the potential with respect to the plasma–presheath boundary is positive initially and increases toward the wall up to a maximal value ϕ_{max}. As may be seen in Figs. 3(a)–3(d), the gradient of the electrical potential increases strongly at the presheath–sheath interface, relative to its value in the presheath. It follows from the data in Figs. 3(a) and 3(d) that, when $\beta_i = 1$ and $\eta \geqslant 1$, the presheath thickness (defined by the condition that $v = 1$ at the presheath–sheath interface) increases as a function of θ, for θ in the region $(0\backslash 2°)$. However, for smaller values of β_i and η [Fig. 3(b), Fig. 3(c)] the presheath thickness is practically constant. The effect of η (the electron to the ion current ratio) on the dependence of ϕ_{max} on θ is presented in Fig. 4. In the $\eta = 1$ case the value of ϕ_{max} decreases with θ, and approaches zero for $\theta \geqslant 1.5°$, while in the $\eta = 10$ case the ϕ_{max} approaches zero for $\theta \geqslant 4°$. In the case of relatively large electron currents, $\eta = 10^2$, ϕ_{max} is not affected by the magnetic field incidence angle, and when $\eta = 10^3$ the value of ϕ_{max} increases with θ.

The dependence of the presheath thickness on the incidence angle is shown in Fig. 5, with η as a parameter. When $\eta = 1$ the presheath thickness increases up to the ion Larmor radius ρ_i and saturates for incidence angles $\theta \geqslant 4°$. When $\eta = 10^{-3}$ (i.e., when there is a positive space charge in the sheath) the presheath thickness slightly decreases with θ (by a factor of 2) and saturates for $\theta \geqslant 2°$. In the opposite case of large η (e.g., $\eta = 10^3$, resulting in a negative space charge in the sheath) the presheath thickness increases with θ for values of θ up to 10°.

The presheath thickness dependence on the Hall parameter β_i for various values of η and two values of θ ($\theta = 0$ and $\theta = 2°$) is shown in Fig. 6. When $\eta = 1$ and $\theta = 0°$ the normalized presheath thickness as a function of β_i increases up to $\beta_i = 0.03$, where it reaches a maximum value of 0.1, and then decreases for larger β_i. When $\theta = 2°$ the presheath thickness increases with β_i and saturates for $\beta_i > 10$. When $\theta = 0°$ and $\eta = 1000$ (i.e., relatively large electron currents), the presheath thickness decreases with β_i, while when $\theta = 2°$ $\eta = 1000$ the presheath thickness has a minimum at $\beta_i \sim 0.1$, and increases up to twice the ion Larmor radius for $\beta_i > 0.1$. In the range of a relatively large Hall parameter ($\beta_i > 10$) and $\theta = 2°$, the presheath thickness shown in Fig. 6 is approximately equal to the ion Larmor radius. This is

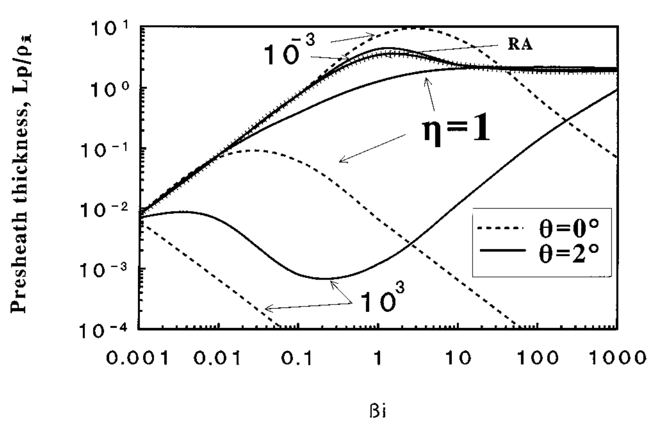

FIG. 6. The magnetic presheath thickness (in units of the ion gyroradius) as a function of the Hall parameter β_i with the electron to ion current ratio η as parameter, for Cu plasma. The broken curve (RA) is from a model[9] with a Boltzmann density distribution.

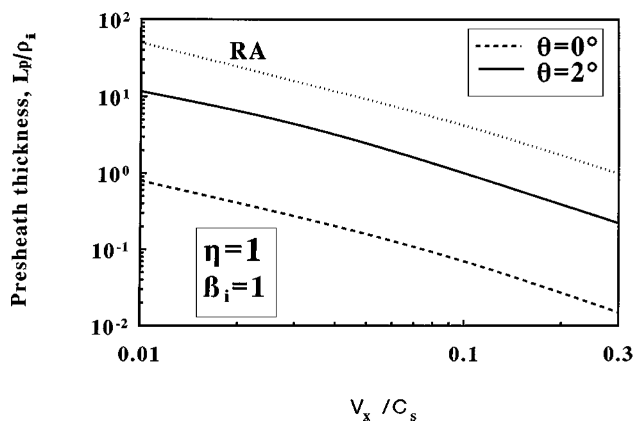

FIG. 7. The magnetic presheath thickness (in units of the ion gyroradius) as a function of the ion velocity V_x. The broken curve corresponds to the model[9] with a Boltzmann density distribution.

quite different than the $\theta=0$ case, where the presheath thickness strongly decreases when $\beta_i>0.1$.

The dependence of the presheath thickness on the Hall parameter β_i, as calculated by the model of Riemann,[9] is also plotted in Fig. 6 for $\eta=1$, $\theta=0°$ (the dashed line marked RA). Riemann used a Boltzmann density distribution for the electrons. It can be seen that, in this case, the potential distribution varies slightly, depending on the incidence angle θ (the curves are practically coincided), in contrast to our model, which predicts significant dependence on θ with transition from the perfect parallel magnetic field to the small (2°) incidence angle.

The effect of the normalized ion velocity at the plasma-presheath interface (V_x/C_s, at boundary 4 in Fig. 1) on the presheath thickness is shown in Fig. 7, for the case $\beta_l=1$, $\eta=1$, and $\theta=0$, and $\theta=2°$. The presheath thickness at $\theta=2°$ is significantly larger than that for $\theta=0°$, for all considered values of the initial ion velocity V_x. In contrast, using the Boltzmann distribution function for the electrons,[9] a weak dependence on the incidence angle of the presheath thickness (curves RA are coincided in Fig. 7) is obtained.

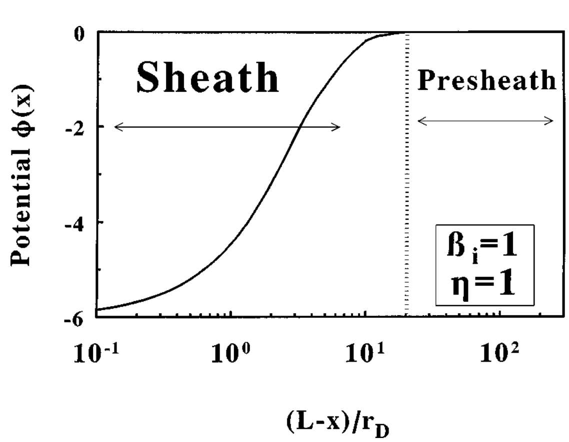

FIG. 8. The potential distribution in the plasma–wall transition layer. Here $\theta=0$; $\Psi=3\times10^{-4}$.

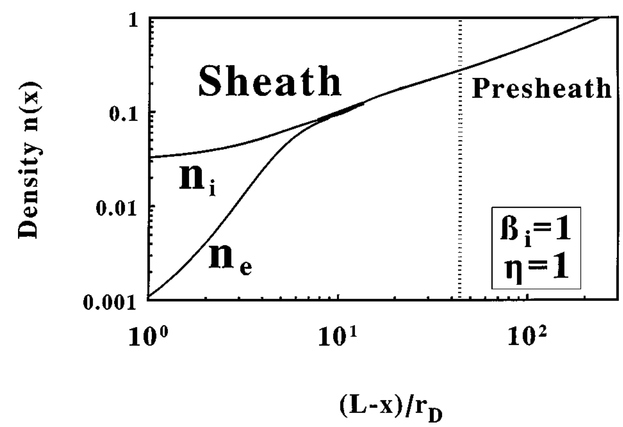

FIG. 9. The density distribution in the plasma–wall transition layer. Here $\theta=0$; $\Psi=3\times10^{-4}$.

B. Current–voltage characteristic of the transition layer

1. Electrical potential and density distribution

The potential distribution in the presheath–sheath region is shown in Fig. 8 for the case $\beta_l=1$, and $\eta=1$. It can be seen that the potential changes mainly in the sheath ($\sim 6kT_e/e$), while in the presheath the potential is less than $1kT_e/e$. The density distribution in the transition layers is shown in Fig. 9 for the case $\beta_l=1$, and $\eta=1$. It can be seen that both electron density n_e and ion density n_i decrease toward to wall, with n_i falling more slowly than n_e, and thus the region adjacent to the wall has in this case a positive space charge.

2. Conditions for a regular sheath–presheath transition

A regular (smooth) transition (that is no singularity in the electrical potential at the interface) was obtained using the boundary conditions for electrical field, potential, and initial ion velocity at the sheath–presheath interface to solve the sheath problem (see above in Sec. II). Hence, the solutions depend on the parameter $\Psi=r_D/\rho_i$.

The critical ion velocity V_0 we calculated for the case $\beta_l=1$, and $\theta=0°$, and found to be smaller than the ion

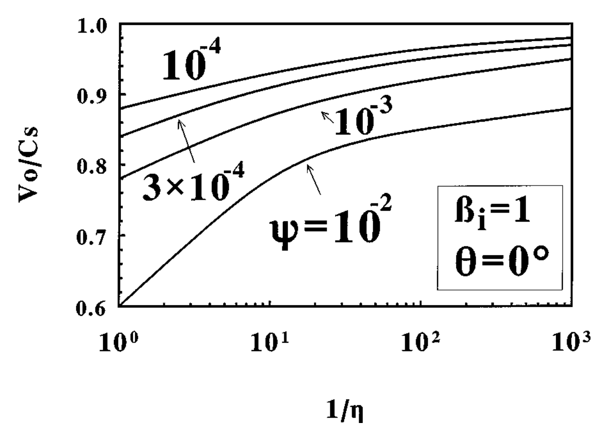

FIG. 10. The critical ion velocity V_0 at the presheath–sheath interface as a function of the electron to ion current ratio η with Ψ as a parameter for the Cu plasma.

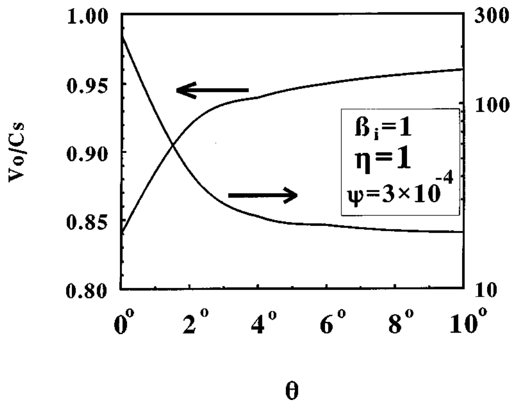

FIG. 11. The critical ion velocity V_0 and electric field E_0 at the presheath–sheath interface as a function of the intersection angle θ. Here $\eta=1$, $\Psi=3\times10^{-4}$, $\beta_i=1$, Cu plasma.

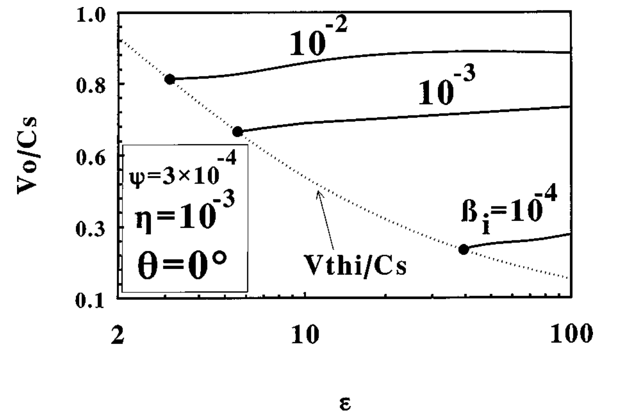

FIG. 13. The critical ion velocity V_0 at the presheath–sheath interface as a function of the electron to ion temperature ratio ϵ, with β_i as a parameter. Here $\eta=1$, $\Psi=3\times10^{-4}$, Cu plasma.

sound velocity C_s, is shown in Fig. 10, where $\Psi=r_{\mathrm{D}}/\rho_i$ is a finite parameter. The velocity V_0 is about $(0.6–0.9)C_s$ for $\Psi=10^{-2}–10^{-4}$, and decreases with η. When $\Psi>10^{-3}$, V_0 significantly decreases. It should be noted that the case $\Psi\sim10^{-2}$ is the limiting case for the present approach if $\rho_e\sim r_{\mathrm{D}}$.

The dependence of V_0 and the normalized electric field $E_0=eE\rho_i/(kT_e)$ on θ is shown in Fig. 11 for the case $\beta_i=1$, $\eta=1$, and $\Psi=3\times10^{-4}$. It can be seen that V_0 increases with the incidence angle from 0.84 up to 0.96, the latter value obtained when $\theta=10°$, while at the same time the normalized electric field decreases from 230 down to 20. The influence of the ion Hall parameter β_i on velocity V_0 and electric field E_0 for several values of θ for the $\eta=1$ and $\Psi=3\times10^{-4}$ is shown in Fig. 12. When $\theta=0$, the velocity V_0 has a maximum while the electric field E_0 has a minimum at $\beta_i=0.1$. When $\theta=2°$, V_0 increases and E_0 decreases with β_I. The above calculations were done, assuming that the initial ion velocity $V_x/C_s=0.1$.

The influence of ϵ, the electron to ion temperature ratio, on V_0 is shown in Fig. 13, assuming that the initial ion velocity equals to the thermal velocity $V_{\mathrm{th}}=\sqrt{8kT_i/\pi m_i}$, as was done in Refs. 20 and 21. The curve of $V_{\mathrm{th}i}(\epsilon)/C_s$ is shown as a boundary for possible solutions, provided that in this case V_0 cannot be smaller than the thermal velocity.

The fulfillment of the quasi-neutrality condition $(\Delta n/n_e<1)$ at the presheath–sheath interface can be calculated from Eq. (14) using a calculated value of $\Psi^2\partial^2\phi/\partial\chi^2$ corresponding to the presheath solution. The behavior of $\Delta n/n_e$ is shown in Fig. 14 as a function of Hall parameter β_i. It can be seen that quasineutrality is fulfilled in the range of $\beta_i=0.003–1$ when $\theta=0$ while when $\theta=2°$ this condition is fulfilled when $\beta_i>0.003$.

3. Voltage drop across the transition layer

The current–voltage characteristic of the plasma–wall transition layers with the magnetic field incidence angle θ as a parameter is shown in Fig. 15(a), where $\Delta\phi$ is the total voltage drop across the transition layer (in the sheath and presheath). The voltage drop decreases with η, or increases with ion to electron current ratio $1/\eta$, depending on η that varies in the range of small angles $(\theta\leqslant5°)$, and does not depend on η when $\theta>5°$. The change of $\Delta\phi$ with θ is mainly due to the change in the voltage across the presheath, as presented in Figs. 3(a)–3(d). The dependence of $\Delta\phi$ on

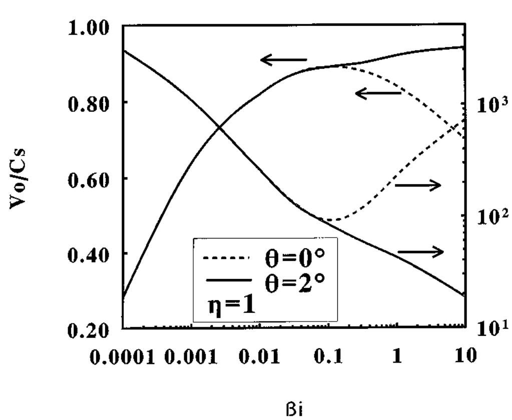

FIG. 12. The critical ion velocity V_0 and electric field E_0 at the presheath–sheath interface as a function of ion Hall parameter β_i with the intersection angle θ as a parameter. Here $\eta=1$, $\Psi=3\times10^{-4}$, Cu plasma.

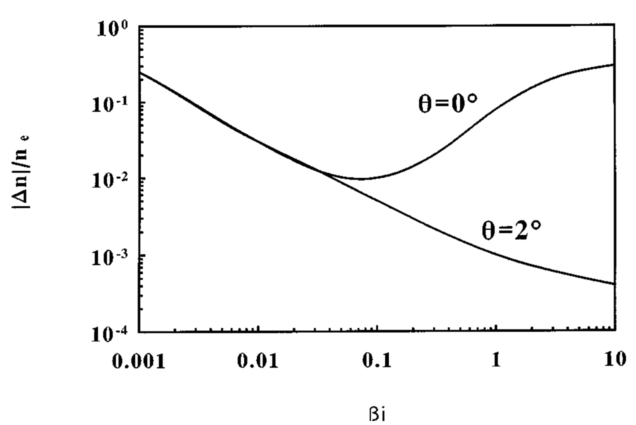

FIG. 14. The dependence of the difference between electron and ion densities $\Delta n/n_e$ at the presheath–sheath interface as a function of Hall parameter β_i with incidence angle as a parameter. Here $\eta=1$, $\Psi=3\times10^{-4}$, Cu plasma.

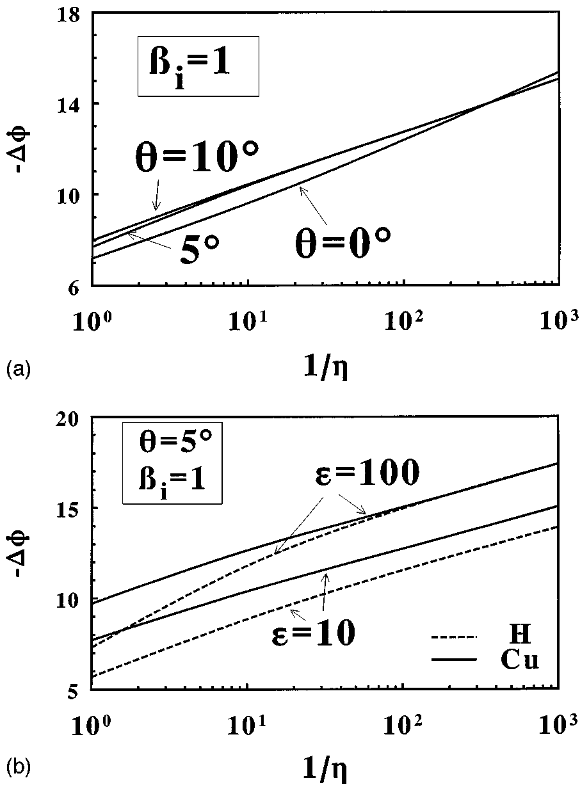

FIG. 15. The current–voltage characteristic of the transition layer (ion branch) as a function of the electron to ion current ratio η. (a) Intersection angle θ as a parameter, Cu plasma. (b) ϵ as a parameter for Cu and H plasmas.

the electron to ion temperature ratio, ϵ as a parameter, is shown in Fig. 15(b) for Cu and H plasmas. It can be seen there that $\Delta\phi$ generally increases with ϵ. The maximum potential drop $\sim 17kT_e/e$ [Fig. 15(a)] occurs for $\eta = 10^{-3}$.

IV. DISCUSSION

It is known that the existence of a positive space charge in a sheath when no magnetic field is applied on the plasma requires the ion drift velocity to be equal or to exceed the ion sound velocity at the sheath entrance (the Bohm condition). This sheath existence criterion was obtained by considering the electric field at the sheath edge toward to the plasma. The necessary ion acceleration takes place in the presheath region. This constitutes a quasineutral plasma region, in which an accelerating electric field and electric currents may exist. Using this electric field as boundary condition for the sheath problem, it is possible to obtain a continuous solution of Poisson's equation, even though the ion velocity at the sheath entrance might be smaller than the sound velocity. The electric field together with the ion velocity determine the existence of the positive space charge sheath. This result also remains true when a magnetic field is applied to the plasma, provided that the sheath itself remains unmagnetized. By an unmagnetized sheath is meant a sheath with a width smaller than the ion Larmor radius. The magnetic field affects only the presheath, whose width is equal to or larger than the ion Larmor radius. The transition from the presheath to the sheath takes place in an interface where the variation in the electrical field is very large (near a singularity point). The presheath is characterized by a gradual increase in the degree of quasineutrality violation, being the smallest near the plasma–presheath interface and largest at the presheath–sheath interface.[15] The proposed model here makes it possible to locate the presheath–sheath interface with a high accuracy due to the strong electric field change at the interface.

The presheath thickness depends in an involved way on the magnetic field (i.e., on the Hall parameter β_i). In the case of a small Hall parameter and wall-parallel magnetic field, the presheath is collision dominated and, therefore, its thickness is determined by the mean-free path length λ_c. Taking into account that $\beta_i \sim \lambda_c/\rho_i$, it is expected that the normalized presheath thickness maintains $L_p \sim \beta_i$, as shown in Fig. 6. When the Hall parameter is sufficiently large, the magnetic force becomes dominant in the presheath. The scale length of the region where the plasma is influenced by the magnetic field is about an electron Larmor radius in the parallel magnetic field case. In the oblique magnetic field case, the electrons move toward the wall along the magnetic field lines (i.e., the electron flux to the wall is not influenced by the magnetic field strength), while the ions move across the magnetic field. Therefore, the magnetic field acts mainly on the ions, and the presheath thickness is about the ion Larmor radius. In the case of a floating potential ($\eta = 1$) and a perfectly oriented wall-parallel magnetic field, the potential in the plasma–presheath interface is positive. A transition to an oblique magnetic field enables the electrons to move unimpeded toward the wall along magnetic field lines, and, therefore, the electrical potential decreases and changes sign. In order to provide large electron currents ($\eta \sim 1000$), the potential in the presheath entrance must be positive with respect to the plasma. The ions flux toward the wall slightly increases with incidence angle θ due to ion flow along magnetic field lines and the potential in the presheath remains positive, slightly increasing with the incidence angle. The gradient of the potential strongly increases at the presheath–sheath edge. This behavior of the potential is connected with the ion velocity approaching a singular point (sound speed), where the gradient of plasma density strongly increases and, hence, the pressure gradient becomes the major term in Eq. (7). Other terms in this equation do not depend as much on the density gradient. This effect causes the potential to decrease from $\phi_{\max}$, and to be a negative at the presheath edge (toward the wall).

The presheath thickness decreases significantly when the ion velocity V_x at the plasma–presheath interface increases. As a result, all gradients at the presheath–sheath interface increase, leading to a breakdown of quasineutrality.

The present model maintains that the electrical potential in the presheath may be negative or positive for different values of the electron to the ion current ratio η, Hall parameter β_i, and intersection angle θ. However, if the Boltzmann density distribution is assumed to hold for the electrons in the presheath, the potential at every point in the presheath is always negative with respect to the plasma–presheath interface. Along the magnetic field the electrons have a Boltzmann distribution and, therefore, there is a good agreement between the present model and Riemann's results. An agreement with Riemann's results was also obtained in the case $\beta_i < 1$, $\eta \ll 1$, where again the Boltzmann density distribution may be used for the electrons. However, even in those cases where the plasma density dependence on the local potential in the presheath approximates the Boltzmann distribution, the presheath thickness obtained in the present model differs markedly from that obtained by a model based on the Boltzmann distribution. Thus, the Boltzmann distribution for the electrons cannot be used for obtaining the presheath thickness, although in some cases it may be used for calculating the plasma density distribution in the magnetic presheath.

The present calculations show that in the case of Debye length to the ion Larmor radius ratio $\Psi > 10^{-2}$ (i.e., the sheath becomes magnetized), the critical velocity V_0 decreases significantly, implying that the Bohm criterion no longer holds under such conditions. This result is confirmed by the analyses of the high-magnetized sheath.[7] It was shown in Ref. 7 that the ion velocity at the sheath entrance is no longer constrained if the magnetic field incidence angle is smaller than some critical value.[7] However, for larger incidence angles the Bohm condition must be satisfied.[7]

V. CONCLUSIONS

The physical characteristics of the sheath and presheath in the plasma–wall transition layers in a plasma situated in a magnetic field whose force lines incident obliquely on the wall were calculated by use of a hydrodynamic model. The following conclusion holds.

(1) A finite electric field and a critical ion velocity for the existence of a positive space charge steady-state electrical sheath at the sheath edge were determined. The velocity is smaller than the ion sound speed and depends on the magnetized presheath plasma parameters.

(2) The dependencies of the presheath thickness and the potential distribution in the presheath on the magnetic field incidence angle are determined by the values of the combination of the ion to electron current ratio, ion Hall parameter, and ratio of Debye length to the Larmor radius. The potential distribution in the presheath has a maximum. This maximum decreases as function of the incidence angle and can disappear in an oblique magnetic field when the ion to electron ratio is smaller than 10^{-2}.

(3) The critical velocity increases strongly with the Hall parameter β_i for $\beta_i < 0.1$, has a maximal value as function of β_i for an oblique magnetic field, while it increases monotonously for a magnetic field parallel to a wall.

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