AIP Physics of Plasmas

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Citation: Phys. Plasmas 20, 033506 (2013); doi: 10.1063/1.4795297
View online: http://dx.doi.org/10.1063/1.4795297
View Table of Contents: http://pop.aip.org/resource/1/PHPAEN/v20/i3
Published by the American Institute of Physics.

PHYSICS OF PLASMAS **20**, 033506 (2013)

Sheath formation criterion in magnetized electronegative plasmas with thermal ions

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(Received 29 September 2012; accepted 25 February 2013; published online 18 March 2013)

Taking into account the effect of collisions and positive ion temperatures, the sheath formation criterion is investigated in a weakly magnetized electronegative plasma consisting of electrons, negative and positive ions by using the hydrodynamics equations. It is assumed that the electron and negative ion density distributions are the Boltzmann distribution with two different temperatures. Also, it is assumed that the velocity of positive ions at the sheath edge is not normal to the wall (oblique entrance). Our results show that a sheath region will be formed when the initial velocity of positive ions or the ion Mach number M lies in a specific interval with particular upper and lower limits. Also, it is shown that the presence of the magnetic field affects both of these limits. Moreover, as an practical application, the density distribution of charged particles in the sheath region is studied for an allowable value of M, and it is seen that monotonically reduction of the positive ion density distribution leading to the sheath formation occurs only when M lies between two above mentioned limits. © *2013 American Institute of Physics*.
[http://dx.doi.org/10.1063/1.4795297]

I. INTRODUCTION

Models describing electronegative discharges are of great practical importance due to the widespread use of such discharges in plasma processing. Plasmas used for materials processing often contain large amounts of negative ions. The existence of negative ions in the plasma has a large effect on the structure of the sheath as well as on the transport and spatial distribution of charged particles. For a plasma consisting of electrons and ions, the necessary condition for steady-state charged sheath existence was first obtained by Bohm and is commonly known as the Bohm criterion. This criterion determines the ion velocity at the sheath entrance, which is necessary for steady-state sheath existence in a two-component plasma.[1]

In 1950's, Thompson and Boyd studied the Bohm potential for electronegative plasmas.[2] As is well known, in collisionless electropositive plasmas with cold positive ions, the sheath begins when $v_{0i} \geq c_s = (T_e/m_i)^{1/2}$ where v_{0i} and c_s are the positive ion velocity to the wall at the sheath edge and the ion sound speed (or the Bohm speed); T_e and m_i are the electron temperatures and ion mass, respectively. For electronegative plasmas, in the cold positive ion approximation, this relation is modified as $v_{0i} \geq c_s[n_{0i}T_n/(n_{0e}T_n + n_{0n}T_e)]^{1/2}$ where T_n is the negative ion temperatures and n_{0n}, n_{0e} and n_{0i} are density of negative ions, electrons and positive ions at the sheath edge, respectively.[3]

In the past decade, many authors studied the sheath structure of electronegative plasma with positive and negative ion components as well as electropositive plasma with two and more positive ion components.[4–12] Generally, from these works, it has been concluded that sheath boundary conditions can significantly affect the transport process of plasma. For this reason, different attempts have been made to "generalize" the Bohm criterion.[13–15]

In the derivation of the Bohm criterion for sheath formation in electronegative plasmas, a multi-valued function was found by Braithwaite and Allen.[3] A multi-valued solution was found when the ratio of the electron temperatures to the negative ion temperatures becomes greater than $5 + \sqrt{24}\,(\sim 9.9)$. Furthermore, the effect of both negative ions and secondary electrons on the collisionless sheath was theoretically investigated by Deutsch *et al.*[16] Also, using a one-dimensional fluid model of ions (positive ions and negative ions), Amin *et al.* described the behavior of the unsteady sheath structure self-consistently.[17] In addition, Franklin[8] showed that in an active collisionless plasma containing more than one species of ion generated by electron impact each ion species reaches its own Bohm speed at the plasma-sheath interface when the ionization rates are constant. Using a simple cold positive ion model, Ming *et al.*[18] studied the sheath structure of an electronegative plasma. They reported on the effects of electronegativity, the temperature ratio of electrons to negative ions, and the wall potential on the density profile of each species and the sheath thickness.[18] Also, Wang *et al.*[19] studied the Bohm criterion for electronegative plasmas composed of electrons, negative and positive ions, as well as dust grains. They found that both positive ion and dust Bohm velocities increase with the growth of dust density due to the interaction between positive ions and dust grains, while both of them decrease by increasing the negative ion density owing to the sheath edge conditions modified by negative ions. Furthermore, changing one of the two Bohm velocities also may affect the other one.

Recently, Palop *et al.* presented a general model to analyze the general aspects of the interface between an electronegative plasma and a metallic plane surface and introduced

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a simple criterion for the values of the density ratio of negative ions to electrons in the plasma (n_{0n}/n_{0e}) from which the sheath begins to be governed by negative ions.[20] Also, Crespo *et al*. studied the sheath in front of a plane probe immersed in an electropositive/electronegative plasma in the low ionization regime. In the case of low-pressure electronegative plasmas, in contrary to the commonly accepted view, they showed that the Bohm criterion unambiguously determines the potential at the sheath edge in electronegative plasmas even when the electronegativity of the plasma corresponds to values for which the electric potential has an oscillatory behavior.[21] Moreover, in the absence of an external magnetic field, the sheath formation criterion in a collisional electronegative plasma is examined in Ref. 7. They showed that in a collisional sheath with finite positive and negative ion temperatures, there will be upper and lower limits for the sheath velocity criterion and the characteristics of the negative ions such as temperature and initial density only affect the lower limit.

Boundary and the sheath region properties of a magnetized plasma have been treated basically in Refs. 22–26. In this case, Chodura[22] was introduced a hydrodynamic model for a semiinfinite plasma emerged in an oblique magnetic field where ionization and collisions were not taken into account. It was shown that plasma particles pass through three regions on their way toward the wall: quasi-neutral plasma region, the quasi-neutral magnetized presheath region (the Chodura layer), and a collisionless thin space charge sheath. Riemann[23] showed that the Chodura layer is eliminated by accounting for ion collisions with gas atoms. He confirmed that collisions provide an adequate transport mechanism to accelerate a quiescent plasma to sonic/supersonic conditions. Also, using a macroscopic model of the plasma and the presence of an oblique uniform magnetic field to the wall, Ahedo[24] presented a different plasma-wall structures, depending on the relative magnitudes of the three important scale lengths: the Debye length, the ion gyroradius, and the collision mean free path.

To modeling the plasma boundary, it is common to use two-scale theory.[25] According to the two-scale theory, the sheath region is scaled by the electron Debye length, while the presheath is scaled by the size of the plasma. Therefore, plasma boundary problem can be split into a presheath and a sheath solution, which are found separately. On the sheath scale, the presheath is infinitely far from the wall where the electric field is zero while on the presheath scale, the sheath is infinitesimally thin and the electric field there is infinite. It means that there is an ambiguous behavior at the sheath boundary (the sheath-presheath interface) which has to be removed when solving the matching problem.[7,25] In our previous work,[26] we investigated the plasma boundary layer on the sheath scale while in Ref. 23 the effect of collisions on the boundary layer of a magnetized plasma investigated on the presheath scale.

The object of the present paper is to derive the sheath formation criterion for a collisional and magnetized electronegative plasma on the sheath scale by analyzing the Sagdeev potential throughout the entire sheath. Taking into account, an oblique constant magnetic field applied to the sheath region and the finite positive and negative ion temperatures, a multi-component fluid model of the plasma is used to investigate the sheath criterion and the behavior of the plasma components in the sheath region. In Sec. II, we present the basic equations describing the electronegative plasma in dimensional and dimensionless forms. Derivation of the modified sheath criterion and its examination in some interesting physical conditions are done in Sec. III, and a brief summary is presented in Sec. IV.

II. BASIC EQUATIONS

In this section, the basic equations are used to investigate the planar bounded electronegative plasma-wall problem. Using a multi-component fluid model, we consider a low pressure collisional magnetized plasma consisting of electrons and positive and negative ions with finite temperature. We assume that an external constant magnetic field is exerted on the sheath region making angle θ with the x coordinate (Fig. 1). Therefore, the unit vector along the external magnetic field can be written as follows:

$$\hat{B}_0 = \hat{x}\cos(\theta) + \hat{z}\sin(\theta), \tag{1}$$

where $\hat{x}$ and $\hat{z}$ are the unit vector along the x and z coordinates. Here, similar to our previous work,[6] we discuss only the case when the external magnetic field is weak and the low-pressure plasma is not highly electronegative as well. Therefore, negative ion density has the Boltzmann distribution.[6] In addition, the wall potential is specified by a given value φ_w which can be the floating potential or a more negative value for which Boltzmann distribution holds, i.e., $|\varphi_w| > 3kT_e/e$.[6] Therefore, the electron and negative ion density distributions are written as follows:

$$n_e = n_{0e}\exp\left(\frac{e\varphi}{T_e}\right), \tag{2}$$

$$n_n = n_{0n}\exp\left(\frac{e\varphi}{T_n}\right), \tag{3}$$

where n_{0e}, n_{0n} are the electron and negative ion densities at the sheath edge; T_e, T_n are the electron and negative ion temperatures and φ the electrostatic potential.

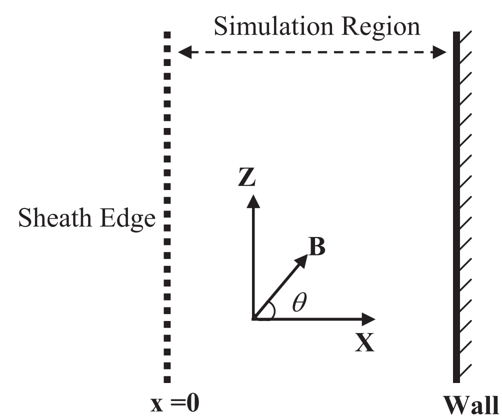

FIG. 1. Magnetized plasma sheath configuration.

In the sheath, the ionization can be neglected due to the low density of the electrons. Therefore, by ignoring the effect of the ionization and recombination, the steady state continuity equations for positive ions in the sheath region can be written as follows:[6]

$$div(n_i\vec{v}_i) = 0, \tag{4}$$

where n_i and $\vec{v}_i$ are the positive ion densities and velocities, respectively.

Assuming that the plasma $x < 0$ contains sources which maintain stationary particle fluxes across $x = 0$ and taking into account the collisions between positive ions and neutrals and the finite positive ion temperatures, the motion equation of positive ions is as follows:

$$m_i(\vec{v}_i.grad)\vec{v}_i = \vec{F}_c + e(\vec{E} + \vec{v}_i \times \vec{B}_0) - \frac{1}{n_i}grad(P_i), \tag{5}$$

where m_i is the mass of the ions, $P_i = n_i k_B T_i$, k_B and T_i are the ion pressures, Boltzmann constant, and ion temperatures, respectively; $\vec{F}_c = -m_i\nu_{i0}\vec{v}_i$ is the drag forces that ions experience during travel through the sheath, $\nu_{i0} = n_0\sigma v_i$ is the effective collision frequency of positive ions with neutrals, n_0 is the neutral gas density, and σ is the momentum transfer cross section for collision of positive ions and neutral particles. Generally, we can consider a power law dependence between σ and v_i as follows:

$$\sigma = \sigma_s\left(\frac{v_i}{c_s}\right)^{\gamma}, \tag{6}$$

where c_s is the ion sound velocity, σ_s is the cross section measured at that velocity, and γ is a dimensionless parameter ranging from -1 to 0. Therefore, the ion collision frequency can be expressed as

$$\nu_{i0} = n_0\sigma_s\left(\frac{v_i}{c_s}\right)^{\gamma} v_i. \tag{7}$$

Finally, the electrostatic potential and the electric field are related to each other by Poisson's equation

$$div\,\vec{E} = e(n_i - n_n - n_e)/\epsilon_0, \tag{8}$$

and

$$\vec{E} = -grad(\varphi), \tag{9}$$

where ϵ_0 is the electric permittivity of free space. Considering the quasineutrality condition at the plasma-sheath edge, the following relation can be written as:

$$n_{0i} = n_{0e} + n_{0n}. \tag{10}$$

Similar to our previous work,[6] it is assumed that the cathode is significantly larger in the y and z directions. Therefore, the quantities change only in the $\hat{x}$ direction, i.e., $\nabla \to \hat{x}\partial/\partial x$. However, the velocity of positive ions has three components, i.e., $\vec{v}_i = (v_{ix}, v_{iy}, v_{iz})$. This means that we have a one-dimensional spatial coordinate system and a three-dimensional velocity coordinate system.

Equations (2)–(5), (8), and (9), the basic equations describing the electronegative plasma, have to be dimensionless. Here, we normalize each charged particle density distribution to its density distribution at the sheath edge, i.e.,

$$N_i = \frac{n_i}{n_{0i}}, \quad N_e = \frac{n_e}{n_{0e}}, \quad N_n = \frac{n_n}{n_{0n}}.$$

Also, the speed of positive ion, the distance from the sheath edge, and the electrostatic potential are normalized as follows:

$$\vec{u} = \frac{\vec{v}_i}{c_s}, \quad \xi = \frac{x}{\lambda_{De}}, \quad \phi = \frac{-e\varphi}{T_e},$$

where $\lambda_{De} = (\epsilon_0 T_e/n_0 e^2)^{1/2}$ is the electron Debye length.

Using these variables, the equations of ion continuity, ion motion, and Poisson's equation (Eqs. (4), (5), and (8)) take the following form:

$$u_x\frac{\partial u_x}{\partial \xi} = \frac{\frac{\partial \phi}{\partial \xi} + \rho u_y \sin\theta - \alpha u^{\gamma+1}u_x}{\left(1 - \frac{T}{u_x^2}\right)}, \tag{11}$$

$$u_x\frac{\partial u_y}{\partial \xi} = \rho[-u_x\sin\theta + u_z\cos\theta] - \alpha u^{\gamma+1}u_y, \tag{12}$$

$$u_x\frac{\partial u_z}{\partial \xi} = -\rho u_y\cos\theta - \alpha u^{\gamma+1}u_z, \tag{13}$$

$$\frac{\partial^2\phi}{\partial \xi^2} = \frac{\frac{M}{u_x} - \delta\exp(-\varepsilon\phi) - (1-\delta)\exp(-\phi)}{1-\delta}, \tag{14}$$

where $M = u_{0x} = v_{ix}(x=0)/c_s$ is the ion Mach number; $\alpha = n_0\sigma_s\lambda_{De}$ is a dimensionless parameter characterizing the degree of collisionality in the sheath, $\delta = n_{0n}/n_{0i}$, $\varepsilon = T_e/T_n$, $T = T_i/T_e$ and $\rho = \lambda_{De}/r$ where $r = c_s/\omega_c$ is the positive ion gyroradius and ω_c is the ion cyclotron frequency. It is clearly that ρ is proportional to the magnitude of the external magnetic field. Therefore, similar to previous works,[7,10,26] to transition among various limits (collisionless/collisional, unmagnetized/magnetized, cold/warm plasma) and study the effects of external magnetic field, ion-neutral collision and positive ion temperatures on the characteristics of the plasma sheath, it is sufficient to solve Eqs. (11)–(14) under various values of ρ, α, and T, respectively.

III. RESULTS AND DISCUSSION

In this section, the sheath formation criterion for a magnetized and collisional electronegative plasma with warm positive and negative ions is derived. Hereafter, for simplicity, the prime symbol denotes the differentiation with respect to ξ. Derivation of the ion transition condition into the sheath region can be done by integrating Eq. (14) under the condition $\phi_0 = 0$ and $\partial\phi_0/\partial\xi \neq 0$ at the plasma-sheath edge ($\xi = 0$) which yields,

$$\frac{1}{2}\phi'^2 = \frac{1}{2}\phi_0'^2 - Y(\phi, M), \tag{15}$$

and

$$Y(\phi, M) = 1 - \exp(-\phi) + \frac{\delta}{\varepsilon(1-\delta)}(1 - \exp(-\varepsilon\phi)) - \frac{M}{1-\delta}\int_0^{\phi}\frac{d\phi}{u_x}, \tag{16}$$

where ϕ' and Y are the dimensionless electric field at the sheath edge and Sagdeev potential,[5,7] respectively, which satisfies the boundary conditions $Y(0, M) = 0$ and $Y'(0, M) = 0$.

Considering the condition $\partial^2 Y(0, M)/\partial\phi^2 < 0$, i.e., maximizing the Sagdeev potential at the sheath edge, we have

$$\left[-\exp(-\phi) - \left(\frac{\varepsilon\delta}{1-\delta}\right)\exp(-\varepsilon\phi) + \left(\frac{M}{1-\delta}\right)\frac{u'_x}{\phi' u_x^2}\right]_{\phi=0} < 0, \tag{17}$$

leading to

$$M\phi'_0(1 + \delta(\varepsilon - 1)) > u'_{0x}. \tag{18}$$

Moreover, at the sheath edge ($\xi = 0$) from Eq. (11) we obtain

$$Mu'_{0x} = \frac{\phi'_0 + \rho u_{0y}\sin\theta - \alpha u_0^{\gamma+1}M}{\left(1 - \frac{T}{M^2}\right)}, \tag{19}$$

where $u_0 = (M^2 + u_{0y}^2 + u_{0z}^2)^{1/2}$.

It is evident that $u'_{0x} \geq 0$ due to neutral drag to the positive ions in the plasma. Therefore, the necessary condition of entrance the positive ions into the sheath region is $\phi'_0 > 0$ which means that there must exist an accelerating force to overcome the drag force. This condition is not specified for the sheath formation. It holds in the quasi-neutral plasma bulk and the presheath as well.

Considering Eq. (19) and the above mentioned conditions, we get

$$\frac{\phi'_0 + \rho u_{0y}\sin\theta - \alpha M u_0^{\gamma+1}}{\left(1 - \frac{T}{M^2}\right)} \geq 0. \tag{20}$$

Similar to Ref. 7, using inequality (18) and Eq. (19), it can easily be found that when $T < [(\phi'_0 + \rho u_{0y}\sin\theta)/\alpha]^{2/(2+\gamma)}$, the following inequality have to be evaluated to determine the modified lower sheath formation criterion:

$$(M^2 - T)\left(1 + \frac{\varepsilon\delta}{1-\delta}\right) - \frac{1}{\phi'_0(1-\delta)} \times [\phi'_0 + \rho u_{0y}\sin\theta - \alpha M(M^2 + u_{0y}^2 + u_{0z}^2)^{(\gamma+1)/2}] \geq 0. \tag{21}$$

Moreover, the upper limit of the sheath criterion, which is adopted to avoid the accumulation of ions in collisional sheath, is determined as follows:[11]

$$M(M^2 + u_{0y}^2 + u_{0z}^2)^{(\gamma+1)/2} \leq \left(\frac{\phi'_0 + \rho u_{0y}\sin\theta}{\alpha}\right). \tag{22}$$

Therefore, when $M < M_{up}$ for an ion, the driving force exceeds the drag force and so the ion velocity increases in plasma sheath.

On the other hand, when $T > [(\phi'_0 + \rho u_{0y}\sin\theta)/\alpha]^{2/(2+\gamma)}$, only the upper limit of sheath criterion (inequality (22)) changes and, regardless of the magnitude of γ, takes the simple form $M \leq \sqrt{T}$. Therefore, there are both upper and lower limits for the ion Mach number (M_{up}, M_{low}).

Using inequalities (21) and (22) and assuming $T < [(\phi'_0 + \rho u_{0y}\sin\theta)/\alpha]^{2/(2+\gamma)}$, the sheath criterion for the collisional magnetized electronegative plasma with warm positive ions is

$$\left[\left(\frac{\frac{\alpha}{2\phi'_0}}{1+\delta(\varepsilon-1)}\right)^2 + \frac{\left(1 + \frac{\rho}{\phi'_0}u_{0y}\sin\theta\right) + T(1+\delta(\varepsilon-1))}{1+\delta(\varepsilon-1)}\right]^{1/2} - \left(\frac{\frac{\alpha}{2\phi'_0}}{1+\delta(\varepsilon-1)}\right) \leq M \leq \left(\frac{\phi'_0 + \frac{\rho}{\phi'_0}u_{0y}\sin\theta}{\alpha}\right), \tag{23}$$

for $\gamma = -1$. To show the validity of our derived generalized sheath formation criterion (inequality (23)), we reduce our general sheath formation criterion to some special cases as follows:

(a) collisionless and unmagnetized electronegative plasma with cold positive ions: It is sufficient to take $T = 0$ (cold), $\alpha = 0$ (collisionless) and $\rho = 0$ (unmagnetized) in inequality (21). Therefore, the Bohm criterion is

$$M \geq \left(\frac{1}{1+\delta(\varepsilon-1)}\right)^{1/2}, \tag{24}$$

which is the well-known Bohm criterion derived by Franklin *et al.*[28] for an electronegative plasma. Besides, in opposition to a cold collisionless electropositive plasma, the positive ions can enter into the sheath region of a cold collisionless electronegative plasma with a velocity smaller than the ion sound velocity.

(b) collisional and unmagnetized electronegative plasma with warm positive ions: Assuming $\rho = 0$ and $T < [(\phi'_0 + \rho u_{0y}\sin\theta)/\alpha]^{2/(2+\gamma)}$, the sheath formation criterion will be written as

$$\left(\left[\left(\frac{\alpha}{2\phi'_0}\right)\left(\frac{1}{1+\delta(\varepsilon-1)}\right)\right]^2 + \frac{1 + T(1+\delta(\varepsilon-1))}{1+\delta(\varepsilon-1)}\right)^{1/2} - \left(\frac{\alpha}{2\phi'_0}\right)\left(\frac{1}{1+\delta(\varepsilon-1)}\right) \leq M \leq \frac{\phi'_0}{\alpha}, \tag{25}$$

for $\gamma = -1$ which is in complete agreement with the result of Ref. 7.

From inequality (23), it is seen that the direction and magnitude of the external magnetic field applied to the sheath region affect both upper and lower limits of the ion Mach number. To show this fact, we draw the upper and lower limits of the ion Mach number as a function of the normalized electric field ϕ_0' for $\gamma = -1$, $\theta = 15°$, $\alpha = 0.1$, $\delta = 0.01$, $\varepsilon = 5$, $T_i = 0.3$ eV, $n_{0e} = 10^8$ cm^{-3}, $T_e = 1$ eV and different values of magnetic field ($B_0 = 0$, $B_0 = 0.05$ T) in Fig. 2.

According to Ref. 27, the electric field in the sheath of the electronegative plasma is $E_0 \approx 1 - 10$ V/cm where $E_0 = T_e\phi_0'/e\lambda_{De}$. Therefore, there is a good agreement between the initial electric field we use here and the experimental results.

Comparing the curves in Fig. 2 shows that the presence of the external magnetic field changes both upper and lower interval limits of the ion Mach number and both of these limits increase in comparison with the case of unmagnetized plasma sheath ($B_0 = 0$). Also, from this figure, one can see that for $\phi_0' < 0.05$ the upper limit of ion Mach number becomes smaller than the lower limit. It stems from this fact that the inequality (23) is satisfied only for $T < [(\phi_0' + \rho u_{0y}\sin\theta)/\alpha]^2$ and, as it is mentioned above, in the opposite case, the upper limit of the ion Mach number (the right hand side of inequality (23)) changes to $\sqrt{T}$. Considering the assumed values for γ, α, δ, θ and other parameters in Fig. 2, it is easily found that for $\phi_0' < 0.05$ the inequality $T < [(\phi_0' + \rho u_{0y}\sin\theta)/\alpha]^2$ does not hold and the left hand side of inequality (23) becomes greater than $\sqrt{T}$. Therefore, for $T < [(\phi_0' + \rho u_{0y}\sin\theta)/\alpha]^2$ in a weakly electronegative plasma sheath with warm and collisional positive ion, the permissible values of ion Mach number correspond to $\phi_0' > 0.05$.

Before ending this section, using inequality (23), we investigate the characteristics of charged particle density distributions in the sheath region of an electronegative plasma to verify the validity of our derived modified sheath formation criterion. To do this, we choose the following parameters in the following figures: $\gamma = -1$, $\alpha = 0.1$, $T_i = 0.3$ eV, $n_{0e} = 10^8$ cm^{-3}, $T_e = 1$ eV, $T_n = 0.2$ eV, $\delta = 0.01$, $\phi_0' = 0.1$ ($E_0 \approx 1.4$ V/cm), $\theta = 15°$. Using these parameters in

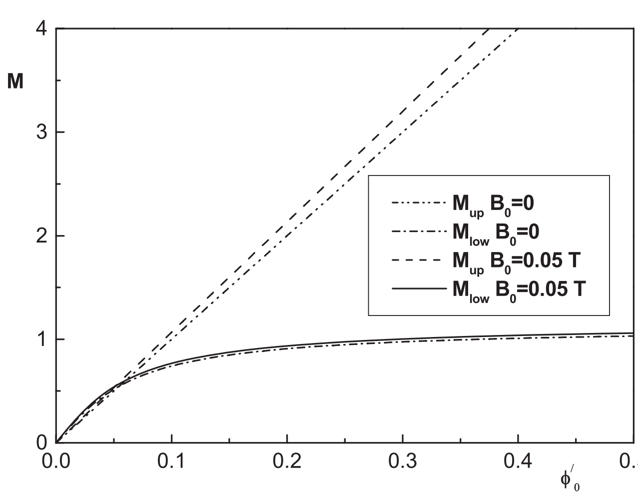

FIG. 2. The lower and upper limits of ion entrance velocities at the sheath region (ion Much number) versus the normalized electric field ϕ_0' for $\gamma = -1$, $n_0 = 10^8$ cm^{-3}, $\theta = 15°$, $\alpha = 0.1$, $\delta = 0.01$, $\varepsilon = 5$, $T = 0.3$ and different values of the magnetic field.

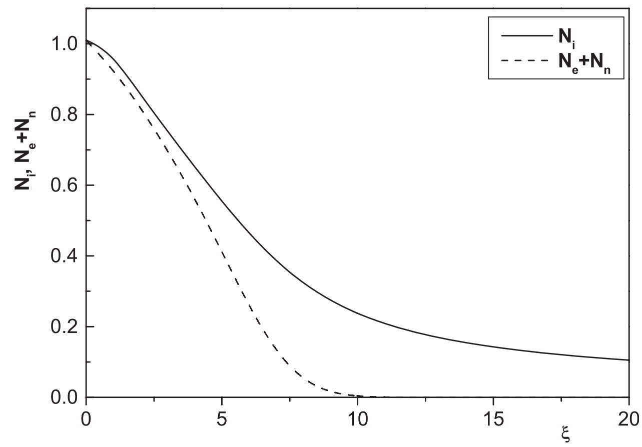

FIG. 3. The normalized density distribution of positive ions and negative charged particles (electrons and negative ions) versus the normalized distance from the sheath edge for $\gamma = -1$, $M = 0.9$ and $B_0 = 0.5$ T. The other parameters are the same with Fig. 2.

inequality (23), we have $M_{low} = 0.74$ and $M_{up} = 1$ in unmagnetized case ($B_0 = 0$, $u_{0y=0}$), and $M_{low} = 0.76$ and $M_{up} = 1.06$ for the case of $B_0 = 0.05$ T ($u_{0y} = 0.45$).

Figure 3 shows the behavior of the charged particle normalized density distributions versus the normalized distance from the sheath edge ($\xi = 0$) for $M = 0.9$ and $B_0 = 0.05$ T.

In this case, the entrance velocity of the positive ions in to the sheath region satisfies the sheath formation criterion (inequality (23)), i.e., $M_{low} < u_{0x}(= M) < M_{up}$. Therefore, the sheath is formed and the curve of the normalized density distribution of positive ions N_i and, consequently, the curve of the positive ion velocities decreases monotonically through the sheath region ($\xi > 0$).

On the other hand, when the positive ion entrance velocity lies outside the permissible region of the ion Mach number ($M < M_{low}$ or $M > M_{up}$), the sheath formation criterion is violated and consequently the sheath region is not formed. In this case, the curve of N_i does not decrease monotonically. This fact was shown in Refs. 7 and 11 for electropositive and electronegative plasmas, respectively. The reason of non-monotonically reduction of N_i can be explained as follows: when the ion entrance velocity into the sheath is small ($M < M_{low}$), the accelerating forces on the ions exceed the neutral collision force which plays the role of the decelerating force on ions and then ions would be accelerated (see Eq. (5)). Therefore, the number density of ions is decreased rapidly, making the number density of ions less than that of electrons at the sheath edge. The sheath formation criterion (inequality (23)) is not satisfied in this case. On the other hand, when $M > M_{up}$, the neutral collision force on the ions exceeds the accelerating force on the ions and thus ions are decelerated, resulting in the accumulation of ions. Here, the sheath formation criterion is not satisfied too.

IV. CONCLUSION

A fluid approach was used to investigate the sheath formation criterion for a quiescent and weakly magnetized electronegative plasma sheath consisting of electrons and singly charged positive and negative ions. It was assumed that the plasma is collisional and the effects of ionization and recombination were ignored. Also, in the present work, it was assumed that the positive ions have finite temperature and the plasma is low-pressure and not highly electronegative and consequently the electron and negative ion density distributions are assumed to be Maxwellian. Taking into account these assumptions, the modified sheath formation criterion was derived, and it was shown that there are upper and lower limits for positive ions velocity to enter to the sheath region. Also, it was shown that the presence of the external magnetic field affects both of these limits and by going from unmagnetized to magnetized plasma both of them increases. Finally, the validity of the modified criterion was proved by confirming the results of previous works and studying the behavior of charged particle density distributions in the sheath region. From this study, it was concluded that a positively charged space region, i.e., a sheath region, is formed only if the ion Mach number satisfies the modified criterion derived in the present work.

[1]D. Bohm, *The Characteristics of Electrical Discharges in Magnetic Fields*, edited by A. Guthrie and R. Wakerling (McGraw-Hill, New York, 1949).
[2]R. L. F. Boyd and J. B. Thompson, Proc. R. Soc. London, Ser. A **252**, 102 (1959).
[3]N. St. J. Braithwaite and J. E. Allen, J. Phys. D: Appl. Phys. **21**, 1733 (1988).
[4]T. E. Sheridan, J. Phys. D **32**, 1761 (1999).
[5]B. Alterkop, J. Appl. Phys. **104**, 123301 (2008).
[6]M. M. Hatami, B. Shokri, and A. R. Niknam, Phys. Plasmas **15**, 123501 (2008).
[7]H. Ghomi, M. Khoramabadi, P. K. Shukla, and M. Ghorannevis, J. Appl. Phys. **108**, 063302 (2010).
[8]R. N. Franklin, J. Phys. D: Appl. Phys. **33**, 3186 (2000).
[9]D. Lee, L. Oksuz, and N. Hershkowitz, Phys. Rev. Lett. **99**, 155004 (2007).
[10]M. M. Hatami, A. R. Niknam, B. Shokri, and H. Ghomi, Phys. Plasmas **15**, 053508 (2008).
[11]J. Y. Liu, Z. X. Wang, and X. Wang, Phys. Plasmas **10**, 3032 (2003).
[12]A. E. Dubinov and L. A. Senilov, Tech. Phys. **57**, 1090 (2012).
[13]A. V. Godyak and N. Sternberg, IEEE Trans. Plasma Sci. **18**, 159 (1990).
[14]H. B. Valentini, Phys. Plasmas **3**, 1459 (1996).
[15]k. U. Riemann, IEEE Trans. Plasma Sci. **23**, 709 (1995).
[16]R. Deutsch and E. Rauchle E, Phys. Rev. A **46**, 3442 (1992).
[17]A. Amin, D. Aossey, B. T. Nguyen, H. Kim, J. L. Cooney, and K. E. Lonngren, Phys. Fluids B: Plasma Phys. **5**, 3813 (1993).
[18]M. Li M, M. A. Vyvoda, S. K. Dew, and M. J. Brett, IEEE Trans. Plasma Sci. **28**(1), 248 (2000).
[19]Z.-X. Wang, Y. Liu, L.-W. Ren, J.-Y. Liu, and X. Wang, Thin Solid Films **506–507**, 637–641 (2006).
[20]J. I. Fernandez Palop, J. Ballesteros, M. A. Hernandez, and R. Morales Crespo, Plasma Sources Sci. Technol. **16**, S76 (2007).
[21]R. Morales Crespo, J. I. Fernandez Palop, J. Ballesteros, M. A. Hernandez, M. V. Lucena-Polonio, and J. M. Daz-Cabrera, Plasma Sources Sci. Technol. **19**, 025012 (2010).
[22]R. Chodura, Phys. Fluids **25**, 1628 (1982).
[23]K. U. Riemann, Phys. Plasmas **1**, 552 (1994).
[24]E. Ahedo, Phys. Plasmas **4**, 4419 (1997).
[25]T. M. G. Zimmermann, M. Coppins, and J. E. Allen, Phys. Plasmas **15**, 072301 (2008).
[26]M. M. Hatami, B. Shokri, and A. R. Niknam, J. Phys. D: Appl. Phys. **42**, 025204 (2009).
[27]K. Takizawa, A. Kono, and K. Sasaki, Appl. Phys. Lett. **90**, 011503 (2007).
[28]R. N. Franklin and J. Snell, J. Phys. D: Appl. Phys. **31**, 2532 (1998).