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Citation: Phys. Plasmas **19**, 083510 (2012); doi: 10.1063/1.4747157

View online: http://dx.doi.org/10.1063/1.4747157

View Table of Contents: http://pop.aip.org/resource/1/PHPAEN/v19/i8

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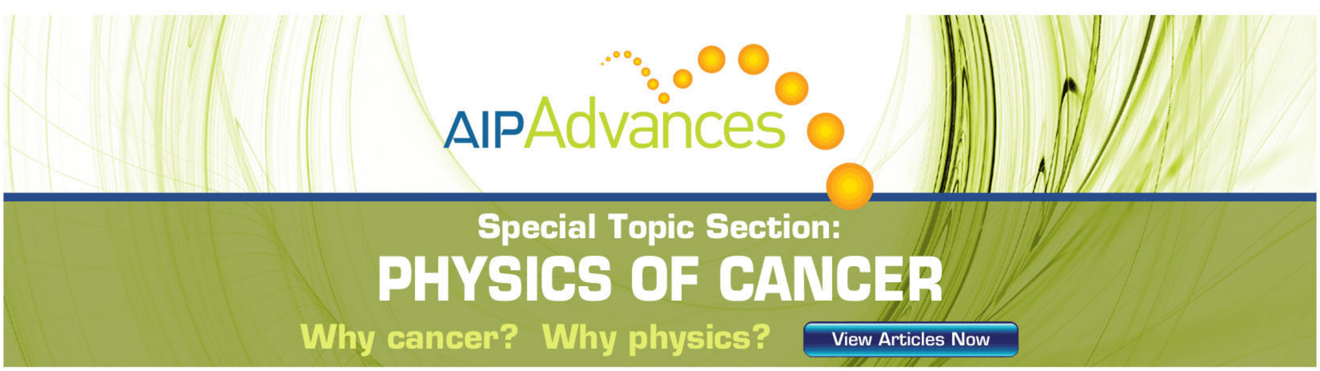

PHYSICS OF PLASMAS **19**, 083510 (2012)

Bohm's criterion in a collisional magnetized plasma with thermal ions

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(Received 28 January 2012; accepted 2 August 2012; published online 23 August 2012)

Using the hydrodynamic model and considering a planar geometry, the modified Bohm's sheath criterion is investigated in a magnetized, collisional plasma consisting of electron and positive ions with finite temperature. It is assumed that the singly charged positive ions enter into the sheath region obliquely, i.e., their velocity at the sheath edge is not normal to the wall, and the electron densities obey Boltzmann relations. It is shown that there are both upper and lower limit for the Bohm entrance velocity of ions in this case and both of these limits depend on the magnitude and direction of the applied magnetic field. To determine the accuracy of our derived generalized Bohm's criterion, it reduced to some familiar physical condition. Also, using this generalized Bohm's criterion, the behavior of the electron and positive ion density distributions are studied in the sheath region. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4747157]

I. INTRODUCTION

One of the old problems in the plasma physics is the formation of the sheath region, i.e., a thin positive space charge region with a thickness of several electron Debye length near a wall placed in the plasma. Due to the different mobilities of electrons and ions, in typical cases the wall has a negative potential with respect to the plasma. It is shielded from the neutral plasma by the sheath region.[1] Due to great importance of the characteristics of the plasma-sheath boundary in practical applications, investigation of the electrodynamic properties of the sheath region is very attractive. On the other hand, most of these characteristics are defined by the condition of the plasma-sheath boundary. Therefore, a sheath criterion is needed to provide the boundary condition to calculate the plasma parameter profiles.[1,2] For example, the choice of electric field at the sheath edge is important to accelerate ions and give them the enough velocity entering the sheath.

Initially, by using a collisionless two-fluid model of the plasma, Bohm has derived a condition for the existence of the positive boundary sheath. At the sheath edge the drift velocity of the plasma must be greater than the ambipolar sound speed of the ions, the so called Bohm's criterion.[3]

Many authors investigate the sheath structure and ion-entering velocity, i.e., Bohm's criterion in their works.[4–13] For example, Harrison and Thompson derived a modified version of the so-called Bohm's criterion by means of a kinetic theory of the ion gas neglecting elastic and inelastic collisions.[4] Using the fluid model approximation as well as the kinetic approach, Riemann investigated the Bohm's criterion in the limit of the very small Debye length.[5] Considering a collisional slightly ionized plasma and using a two-fluid model, it was shown analytically that at the sheath edge the drift velocity of the plasma is little smaller than the so-called Bohm velocity meaning that under these conditions, the Bohm's criterion is a sufficient condition but not a necessary condition.[7] Franklin and Kaganovich focused on how to patch plasma and sheath in their work.[14–17] Collisions have been taken into account by Liu *et al.*[18] to investigate the sheath criterion in a collisional plasma sheath by a two-fluid model. They have shown that there are upper and lower limits for the sheath criterion when collisions between ions and neutrals are taken into account. Furthermore, Sternberg and Poggie presented a model of the plasma-wall problem in the presence of an applied magnetic field. They showed that the plasma boundary can be specified as the surface on which the component of the ion velocity normal to the wall reaches the ion sound speed (Bohm's criterion).[8] In addition, Das *et al.* studied the plasma sheath formation and Bohm's criterion in a collisionless thermal plasma.[10] In Ref. 19, using two-fluid model for a collisional plasma sheath, the effect of the finite ion temperature on the Bohm's criterion and ion velocity in the sheathpresheath boundary was investigated, and it was concluded that there is a velocity interval for the entrance of the ion into the sheath. However, the transition condition for collisional thermal plasma sheath in the presence of an applied magnetic field has not been studied yet.

In the present paper, we extend the results of the previous work[8,10,19] to include an oblique external magnetic field and find analytically the Bohm's criterion in a magnetized plasma sheath under different physical conditions such as collisionless or collisional plasma, cold or warm plasma, and normal or oblique entrance of positive ion into the sheath region. Using a two-fluid model and hydrodynamic equations, the generalized Bohm's criterion is calculated. In this case, it is shown that there are upper and lower limits for the Bohm's criterion and both of these limits depend on the magnitude and direction of the applied magnetic field. Also, using this modified Bohm's criterion, we confirm the results of previous work and determine the ion Mach number of some interesting physical situations. Finally, to show the correction of our result to determine the ion transition condition for sheath formation, we investigate the behavior of the

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density distribution of electrons and positive ions in a collisional magnetized plasma with thermal ions.

This work is organized into four sections including the introduction as the first section. In Sec. II, we explain our model and basic equations. In Sec. III, we calculate the modified Bohm's criterion analytically and examine it in some interesting physical conditions. Finally a brief conclusion is presented in Sec. IV.

II. MODEL AND BASIC EQUATIONS

In this section the model and the basic equations are used to investigate the planar bounded plasma-wall problem, and the existence condition of the electrostatic sheath is presented. Using a two-fluid model, we consider a collisional magnetized plasma system composed of electrons and singly charged positive ion with finite temperature. We assume that the electrons are in thermal equilibrium due to their mobility, so the electron density obeys the Boltzmann distribution[11–13]

$$n_e = n_0 \exp\left(\frac{e\varphi}{T_e}\right), \tag{1}$$

where T_e, φ, and n_0 are the electron temperature, the electrostatic potential, the electron and ion density at the sheath edge where $x=0$ and $\varphi=0$, respectively. Also, we assume that the electron energy is not very high so that the ionization can be ignored.[9] For such a plasma we consider a one-dimensional spatial coordinate x and three-dimensional velocity coordinates $\vec{v}=(v_x, v_y, v_z)$. Without loss of generality, we can choose the coordinate axes such that the external magnetic filed $\vec{B}_0$ is in x–z plane, i.e., $\vec{B}_0=(B_{0x},0,B_{0z})$, and form an angle θ with the x coordinate. As it is seen in Fig. 1, the x axis is directed normal to the wall, and the boundary between plasma ($x<0$) and sheath ($x>0$) is the plane of $x=0$.

Since it is assumed that only elastic collisions occur between positive ions and neutral atoms, ionization will not happen in the sheath region, so the continuity equation for positive ions is written as follows:

$$\nabla.(n_i\vec{v}_i)=0, \tag{2}$$

where n_i, v_i are the density and velocity of ions. Taking into account the collisions between ions and neutrals and the finite ion temperatures, the motion equation of ions is as follows:

$$m_i\vec{v}_i.\nabla\vec{v}_i = -e\nabla\varphi + e\vec{v}_i\times\vec{B}_0 - \frac{1}{n_i}\nabla P_i - \vec{F}_c, \tag{3}$$

where m_i is the mass of the ions, $P_i=n_ik_BT_i$, k_B, and T_i are the ion pressures, Boltzmann constant, and ion temperatures, respectively, and $\vec{F}_c$ is the drag forces that ions experience during travel through the sheath. This force can be written as follows:

$$\vec{F}_c = m_i\nu_{i0}\vec{v}_i, \tag{4}$$

where ν_{i0} is the effective collision frequency of ions with neutrals.

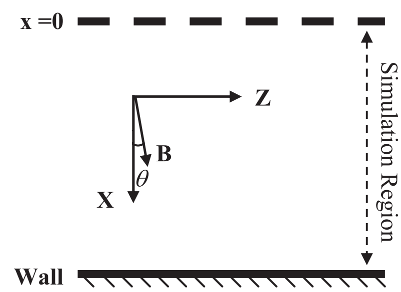

FIG. 1. Geometry of the magnetized plasma sheath.

The ion collision frequency of positive ions and neutral particles ν_{i0} is expressed as follows:[2]

$$\nu_{i0} = n_n\sigma v_i, \tag{5}$$

where n_n and σ are the neutral gas density and the momentum transfer cross section for collision of positive ions and neutral particles, respectively. In general case, we can consider a power law dependence between σ and v_i as follows:

$$\sigma = \sigma_s\left(\frac{v}{c_s}\right)^g, \tag{6}$$

where $c_s=(T_e/m_i)^{1/2}$ is the ion acoustic velocity, σ_s is the cross section measured at that velocity, and g is a dimensionless parameter ranging from -1 to 0. Therefore, the ion collision frequency can be expressed as

$$\nu_{i0} = n_n\sigma_s\left(\frac{v}{c_s}\right)^g v_i. \tag{7}$$

Equation (7) contains two well-known special cases. The first special case is known as constant cross section corresponding to $g=0$ in Eq. (7), and the second special case is known as constant collisional mobility which corresponds to $g=-1$ in Eq. (7).

The Poisson's equation for such an electropositive discharge consisting of singly charged positive ions and electrons is written as follows:

$$\nabla^2\varphi = e(n_i-n_e)/\epsilon_0, \tag{8}$$

where ϵ_0 is the electric permittivity of free space.

Here, we assume that the physical quantities like density of positive ions and electrons change only along the x direction (normal to the wall) and so ∇ is replaced with $\partial/\partial x$ in the above equations.

To normalize the model equations, i.e., Eqs. (3) and (8), we introduce the following variables:

$$\vec{u}=\vec{v}_i/c_s, \quad T=T_i/T_e, \quad \xi=\frac{x}{\lambda_{De}}, \quad \phi=-e\varphi/T_e,$$

where $\lambda_{De}=(\epsilon_0T_e/n_0e^2)^{1/2}$ is the electron Debye length.

With these dimensionless quantities we obtain Eqs. (3) and (8) in the final form

$$\left(1-\frac{T}{u_x^2}\right)u_x u_x{}' = \phi' + \rho u_y \sin\theta - \alpha u^{g+1}u_x, \quad (9)$$

$$u_x u_y{}' = \rho[-u_x \sin\theta + u_z \cos\theta] - \alpha u^{g+1} u_y, \quad (10)$$

$$u_x u_z{}' = -\rho u_y \cos\theta - \alpha u^{g+1} u_z, \quad (11)$$

$$\phi'' = \frac{M}{u_x} - \exp(-\phi), \quad (12)$$

where the prime symbol denotes the differentiation with respect to ξ and $M = u_{0x} = v_x(x=0)/c_s$. $\alpha = n_n \sigma_s \lambda_{De}$ is a dimensionless parameter characterizing the degree of collisionality in the sheath and $\rho = \lambda_{De}/r$, where ρ is proportional to the magnitude of the external magnetic field and $r = (m_i T_e/e^2 B_0^2)^{1/2}$ is the positive ion gyroradius.

III. RESULTS AND DISCUSSION

Investigation of a collisional and magnetized plasma sheath consisting of positive ions with finite temperature can be carried out by solving Eqs. (9)–(12). However, to solve these equations the boundary conditions must be specified. This leads to determine the Bohm's criterion. First, we derive the generalized Bohm's criterion, and then we reduced it to some special cases.

The first integral of Eq. (12) leads to

$$\frac{1}{2}(\phi'^2 - \phi_0'^2) = -V(\phi, M), \quad (13)$$

where

$$V(\phi, M) = 1 - \exp(-\phi) - \int_0^\phi \frac{M}{u_x} d\phi. \quad (14)$$

Here ϕ_0' and V are the dimensionless electric field at the sheath edge and Sagdeev potential satisfying the boundary conditions $V(0, M) = 0$ and $\partial V(0, M)/\partial\phi = 0$.

From Eq. (14), we have

$$\frac{\partial^2 V(\phi, M)}{\partial \phi^2} = -\exp(-\phi) + \frac{M u_x{}'}{\phi' u_x^2}. \quad (15)$$

From Eq. (15), by considering the condition for maximizing the Sagdeev potential at the sheath edge, i.e., $\partial^2 V(0, M)/\partial\phi^2 < 0$, we have

$$M\phi_0' > u_{0x}'. \quad (16)$$

In addition, at the sheath edge ($\xi = 0$), Eq. (9) takes the following form:

$$M u_{0x}{}' = \frac{\phi_0' + \rho u_{0y}\sin\theta - \alpha u_0^{g+1} M}{\left(1 - \frac{T}{M^2}\right)}, \quad (17)$$

where $u_0 = (M^2 + u_{0y}^2 + u_{0z}^2)^{1/2}$.

It is evident that $u_{0x}{}' \geq 0$ due to neutral drag to the positive ions in the plasma. Therefore, the necessary condition of entrance of the positive ions into the sheath region is $\phi_0' > 0$ which means that there must exist an accelerating force to overcome collision drag.

Considering Eq. (17) and the above mentioned conditions, we get

$$\frac{\phi_0' + \rho u_{0y}\sin\theta - \alpha M u_0^{g+1}}{\left(1 - \frac{T}{M^2}\right)} \geq 0. \quad (18)$$

From inequalities (16) and (17), it is seen that when $T < [(\phi_0' + \rho u_{0y}\sin\theta)/\alpha]^{2/(2+g)}$, the new Bohm's criterion will be obtained

$$M_1 \leq M \leq M_1', \quad (g = 0), \quad (19)$$

where M_1 and M_1' satisfy the following inequalities, respectively:

$$-M^2 - \frac{\alpha}{\phi_0'} M (M^2 + u_{0y}^2 + u_{0z}^2)^{1/2} + \left(1 + T + \frac{\rho u_{0y}\sin\theta}{\phi_0'}\right) \leq 0 \quad (20)$$

and

$$\phi_0' + \rho u_{0y}\sin\theta \geq \alpha M (M^2 + u_{0y}^2 + u_{0z}^2)^{1/2}. \quad (21)$$

Also, for $g = -1$ we drive

$$M_2 \leq M \leq \frac{\phi_0' + \rho u_{0y}\sin\theta}{\alpha} \quad (g = -1), \quad (22)$$

where M_2 satisfies the following inequality:

$$-M^2 - \frac{\alpha}{\phi_0'} M + \left(1 + T + \frac{\rho u_{0y}\sin\theta}{\phi_0'}\right) \leq 0. \quad (23)$$

On the other hand, when $T > [(\phi_0' + \rho u_{0y}\sin\theta)/\alpha]^{2/(2+g)}$, the new Bohm's criterion will be as follows:

$$M_1 \leq M \leq \sqrt{T} \quad (g = 0) \quad (24)$$

and

$$M_2 \leq M \leq \sqrt{T} \quad (g = -1), \quad (25)$$

where M_1 and M_2 satisfy again inequalities (21) and (23).

Therefore, there are both upper and lower limits for the ion Mach number. It is worthwhile to mention that the upper limit shows the balance between the driving (the first two terms in Eq. (9)) and drag (the last term in Eq. (9)) forces.[18]

Now to investigate the validity of the derived generalized Bohm's criterion, we would like to reduce our general Bohm's criterion to some special cases studied previously by authors in the limit of $T < [(\phi_0' + \rho u_{0y}\sin\theta)/\alpha]^{2/(2+g)}$.

(a) Collisionless and unmagnetized plasma with cold positive ions:

In this case $T = 0$, $\alpha = 0$, and $\rho = 0$ in inequalities (19) or (22). Therefore, the Bohm's criterion is

$$M \geq 1, \tag{26}$$

which is the most familiar shape of Bohm's criterion derived by Bohm[3] and say that if the ion velocity becomes equal or greater than the ion sound velocity, the ion can enter into the sheath region.

(b) Collisionless and unmagnetized plasma with warm positive ions:

By considering $T = 0$, $\alpha = 0$, and $\rho \neq 0$ in inequality (22), we have

$$M \geq \sqrt{T+1}. \tag{27}$$

Inequality (16) says that in a collisionless plasma with finite ion temperature, the ions can enter into the sheath region even when there is not a initial electric field as predicted in Ref. 19.

(c) Collisional and unmagnetized plasma with cold positive ions:

In this case $T = 0$, $\rho = 0$, but $\alpha \neq 0$. Therefore,

$$\sqrt{\frac{\phi_0'}{\phi_0' + \alpha}} \leq M \leq \sqrt{\frac{\phi_0'}{\alpha}}, \tag{28}$$

for $g = 0$ and

$$\left(\sqrt{\left(\frac{\alpha}{2\phi_0'}\right)^2 + 1} - \frac{\alpha}{2\phi_0'}\right) \leq M \leq \frac{\phi_0'}{\alpha}, \tag{29}$$

for $g = -1$, which are the same results of Liu *et al.*[18] As it is mentioned by Liu *et al.*,[18] in a collisional plasma with cold positive ion the Mach number is between upper and lower limits. The lower limit shows the reduction of the ion entering speed u_{0x} due to the existence of the neutral drag, and the upper limit shows the balance between the driving and drag forces.

(d) Collisional and unmagnetized plasma with warm positive ions:

Assuming $\rho = 0$ and $\alpha, T \neq 0$, the Bohm's criterion can be written as follows:

$$\sqrt{\frac{1+T}{1+\alpha/\phi_0'}} \leq M \leq \sqrt{\frac{\phi_0'}{\alpha}}, \tag{30}$$

for $g = 0$ and

$$\left(\sqrt{\frac{\alpha^2}{4\phi_0'^2} + (1+T)} - \frac{\alpha}{2\phi_0'}\right) \leq M \leq \frac{\phi_0'}{\alpha}, \tag{31}$$

for $g = -1$ which confirms the results of Ref. 19.

(e) Collisionless and magnetized plasma with cold positive ions:

In this case, $T = 0$, $\alpha = 0$, but $\rho \neq 0$. Therefore, the Bohm's criterion drives as follows:

$$M \geq \sqrt{1 + \frac{\rho}{\phi_0'} u_{0y} \sin\theta}. \tag{32}$$

(f) Collisionless and magnetized plasma with warm positive ions: Here, to drive the Bohm's criterion it is sufficient to take $\alpha = 0$ but ρ and $T \neq 0$.

$$M \geq \sqrt{1 + T + \frac{\rho}{\phi_0'} u_{0y} \sin\theta}. \tag{33}$$

It should be noted that the Bohm's criterion for warm positive ions entering into the sheath region of a collisional and magnetized plasma "normally to the wall" is the same as the Bohm's criterion derived above in the case (d). On the other hand, if positive ions enter the sheath region obliquely, using inequalities (22) and (23), the generalized Bohm's criterion or $g = -1$ can be written as follows:

$$\left(\sqrt{\frac{\alpha^2}{4\phi_0'^2} + \left(1 + T + \frac{\rho}{\phi_0'} u_{0y} \sin\theta\right)} - \frac{\alpha}{2\phi_0'}\right) \leq M \leq \frac{\phi_0' + \rho u_{0y} \sin\theta}{\alpha}. \tag{34}$$

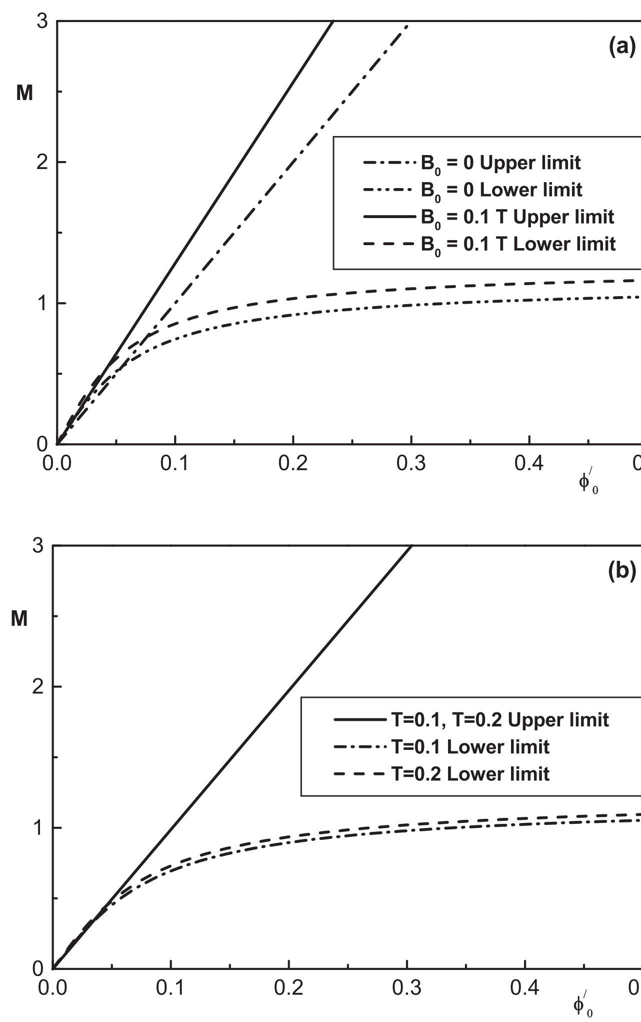

FIG. 2. The lower and upper limits of Bohm's sheath criterion for the $g = -1$ as function of the normalized electric field ϕ_0' for $n_{0e} = 10^8\,\mathrm{cm}^{-3}$, $\theta = 30°$ and (a) $\alpha = 0.1$, $T = 0.3$ and different values of the magnetic field, and (b) $\alpha = 0.13$, $B_0 = 0.1\ T$ and different values of the ion temperatures.

At the end of this section, we examine our derived modified Bohm's criterion for the case of $g = -1$ represented in inequality (34). We consider an quasineutral argon plasma with following parameters $n_{0e} = 10^8\ \mathrm{cm}^{-3}$, $T_e = 1$ eV, and $\phi_0' = 0.1$ at the sheath edge.[18–20] Therefore, it is found that $u_{0y} = 0$ for $B_0 = 0$ (unmagnetized plasma sheath) and $u_{0y} = 0.4$ for $B_0 = 0.1\ T$.

In Fig. 2(a), the upper and lower limits of the Bohm's criterion (Mach number limits) versus the normalized electric field at the sheath edge are seen for $\alpha = 0.1$, $T = 0.3$, $u_{0y} = 0$ (for $B_0 = 0$) and $u_{0y} = 0.4$ (for $B_0 = 0.1\ T$), respectively. Also, in Fig. 2(b), these limits are sketched for $\alpha = 0.13$, $B_0 = 0.1\ T$, $u_{0y} = 0.4$ and different values of the ion temperature (T). As it is noted, the permissable values of the Bohm's criterion (permissable values of Mach number) lie between these two limits. From these figures, it is seen the external magnetic field affects both upper and lower limits of the allowable Mach number while the ion temperature affects only the lower limit of the allowable Mach number which is in agreement with the result of Ref. 19.

As it is mentioned above, inequality (34) is satisfied for $T < [(\phi_0' + \rho u_{0y} \sin\theta)/\alpha]^2$. However, this inequality does not

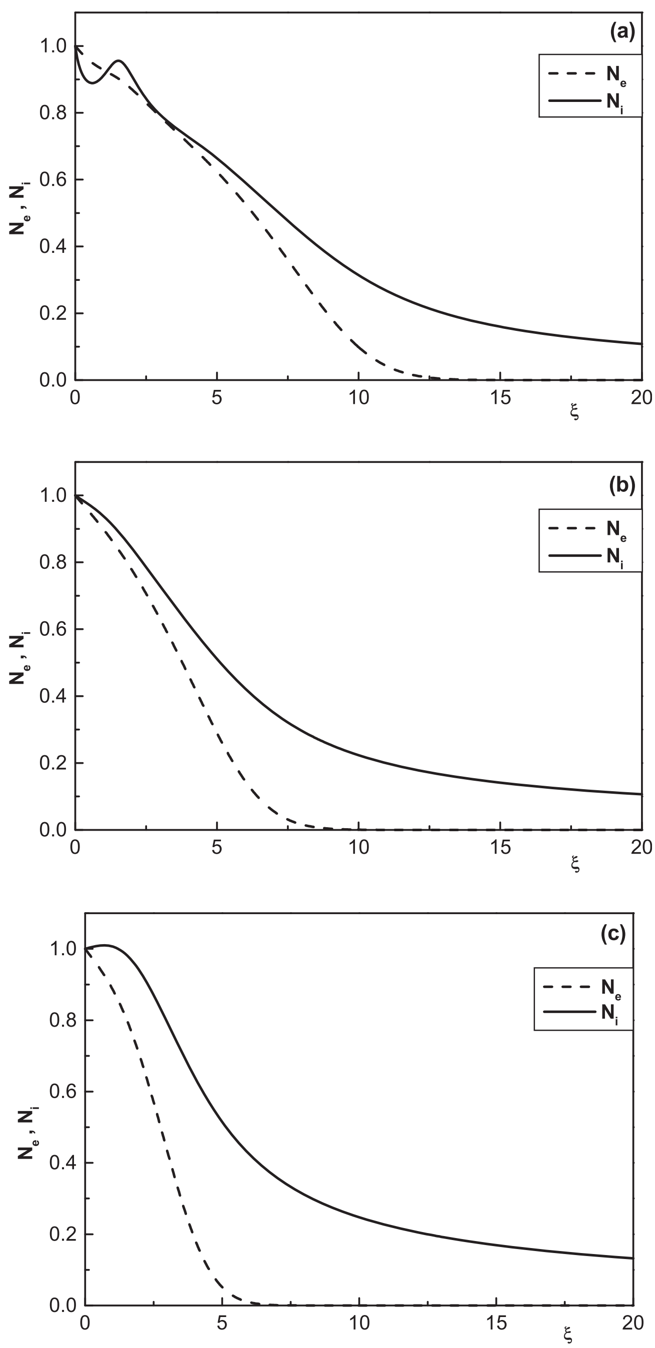

FIG. 3. The normalized density distribution of electrons and positive ions at the sheath region for $B_0 = 0$, $g = -1$, and (a) $M = 0.6$, (b) $M = 0.8$, and (c) $M = 1.3$. The other parameters are the same with Fig. 2(a).

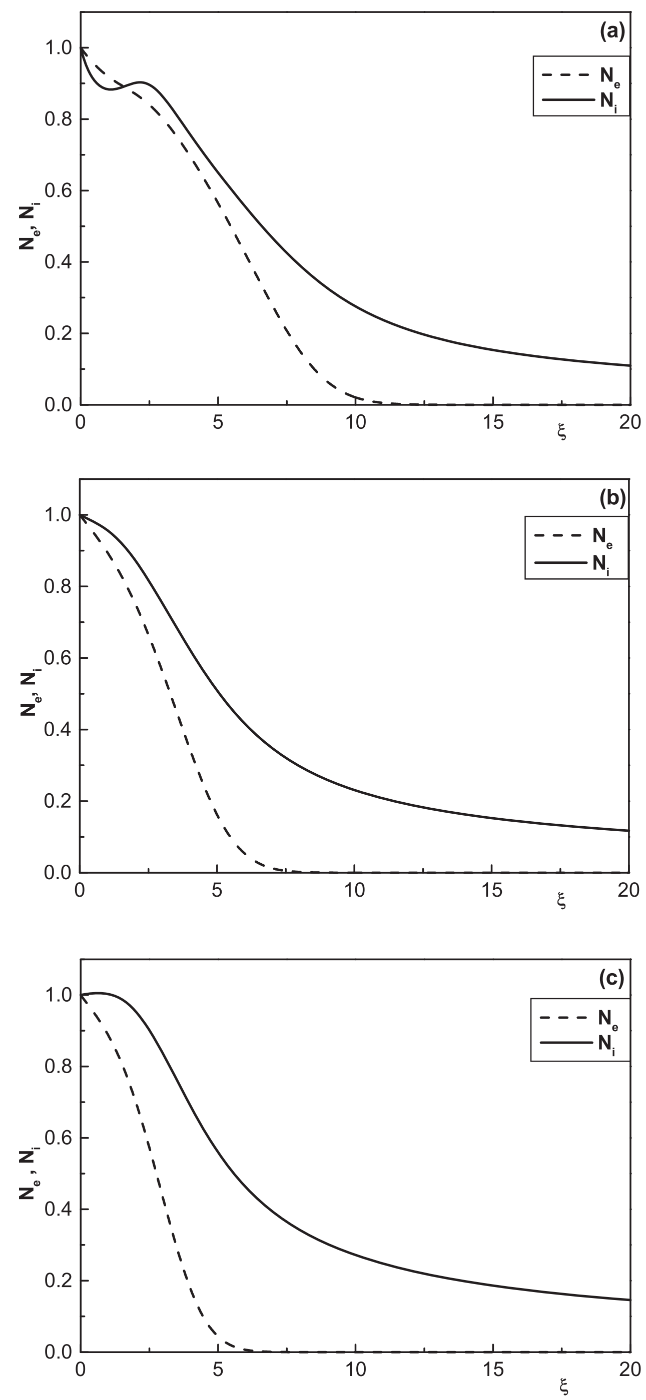

FIG. 4. The normalized density distribution of electrons and positive ions at the sheath region for $B_0 = 0.1\ T$, $g = -1$ and (a) $M = 0.7$, (b) $M = 1$, and (c) $M = 1.5$ with the same parameters of Fig. 2(a).

hold when $\phi_0' < 0.05$ and $B_0 = 0$ in Fig. 2(a). As a result, any value of M corresponding to these values of ϕ_0' is not satisfied in inequality (34).

Using inequality (34) and assuming $\alpha = 0.1$, $T = 0.3$, $\phi_0' = 0.1$, and $\theta = 30°$ (Refs. 2, 11, 12, and 18–20), it is concluded that $0.745 < M < 1.0$ for $B_0 = 0$ and $0.85 < M < 1.28$ for $B_0 = 0.1\ T$, respectively. In Figs. 3 and 4 the electron and positive ion density distributions are sketched versus the distance from the sheath edge for $B_0 = 0$ and $B_0 = 0.1\ T$, respectively. As it is seen from Figs. 3(a), 3(c), 4(a), and 4(c) if the value of the entrance velocity of positive ion u_{0x} lies out of the allowable range, e.g., $u_{0x} = M = 0.6 < M_{min} = 0.756$, the positive ion density distributions suffer some fluctuations at sheath edge while for those values of M satisfying the Bohm's sheath criterion, e.g., $0.745 < M = 0.8 < 1$, these fluctuations disappear and the positive ion density distributions decrease smoothly in the sheath region (see Figs. 3(b) and 4(b)). The reason of these fluctuations can be explained as follows: when the ion entrance velocity into the sheath is small $(M < u_{0min})$, the accelerating forces on the ions, i.e., the first three terms in the right hand side of Eq. (3), exceed the neutral collision force which plays the role of the decelerating force on ions and then ions would be accelerated. Therefore, the number density of ions is decreased rapidly, making the number density of ions less than that of electrons at the sheath edge. The Bohm's sheath criterion (inequality (34)) is not satisfied in this case. On the other hand, when $M > u_{0max}$, the neutral collision force on the ions exceeds the accelerating force on the ions and thus ions are decelerated, resulting in the accumulation of ions. Here the Bohm's sheath criterion is not satisfied too.

IV. CONCLUSION

A two-fluid model was used to investigate the Bohm's criterion in a collisional electropositive plasma consisting of electrons and positive ions with finite temperature. It is assumed that an external magnetic field is applied obliquely to the sheath region and the velocity of the positive ions is not normal to the wall at the sheath edge. Using these assumptions, a modified Bohm's criterion was derived which limits both maximum and minimum allowable sheath entrance velocity of the positive ion at the sheath edge. It was shown that both of these limits and then the Mach number depend on the magnitude and direction of the applied external magnetic field. To show the accuracy of our modified Bohm's criterion, it was shown that our modified Bohm's criterion confirms the previous derived Bohm's criterion for some especial cases. Finally, the density distribution of charged particles in the sheath region of such electropositive plasma was investigated.

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