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# Properties of a warm plasma collisional sheath in an oblique magnetic field

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The properties of a warm plasma collisional sheath in an oblique magnetic field and the associated sheath criterion are investigated with a two-fluid model. In the fluid framework, a sheath criterion including effects of the magnetic field and collision is established theoretically for a wide range of ion temperature. With the sheath criterion as the plasma-sheath boundary condition, different plasma parameters including potential, electron and ion densities, and ion velocity are calculated for various ion temperatures and ion thermal motions. It is shown that the properties of the sheath depend not only on the plasma balance equations but also on the sheath boundary conditions. In addition, effects of the directions and magnitudes of the magnetic field on the plasma sheath are also discussed under different ion temperatures. © *2012 American Institute of Physics*.
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## I. INTRODUCTION

The sheath is formed when plasma is in contact with an absorbing wall. It plays a key role in determining the plasma-wall interactions. In recent years, it has received much attention due to its relevance to plasma processing technology, lab plasma, and magnetic confinement fusion plasma. For a sheath model, an appropriate boundary condition is crucial. It affects not only the sheath plasma but also the bulk plasma. For example, in a magnetic fusion device such as tokamak, investigation of the plasma edge transport process phenomena involves the sheath boundary condition.[1,2] For a weakly collisional plasma sheath in absence of magnetic field, it requires that the ions entering the sheath region satisfy Bohm criterion,[3,4]

$$V_{ix} \geq C_s = \sqrt{k_B(T_e + \gamma T_i)/m_i}, \tag{1}$$

where $C_s$ is the velocity of ion acoustic, $k_B$ is the Boltzmann constant, $T_e$ is the electron temperature, $T_i$ is the ion temperature, $m_i$ is the ion mass, $\gamma = 1$ is for isothermal approximation, and $\gamma = 3, 2, 5/3$ are for uni-, bi-, or tri-dimensional adiabatic approximation, respectively. For a weakly collisional plasma sheath in presence of an oblique magnetic field, it is shown from the numerical results that the ion velocity at the sheath edge should be higher than the ion acoustic velocity, depending on the incident angle of magnetic field $\theta$[5,6]

$$V_{ix} \geq C_s \cos\theta. \tag{2}$$

In general cases, sheath plasma is not only magnetized but collisional. In a weakly collisional sheath, the effects of the ionization and collisions on the sheath plasma can be neglected. While in a strong collisional sheath, the ionization and plasma-neutral collision can affect the sheath structure and then the Bohm criterion is modified.[7] For a collisional sheath in an oblique magnetic field, the Bohm criterion includes the effects of both magnetic field and collision.[8,9] The simulation results in Ref. 8 show that the collision can diminish the magnetic field influence on the ion velocity. However, in the Refs. 8 and 9, the modified Bohm criterion is investigated only for the cold ions sheath plasma ($T_i \sim 0$) or finite temperature ions sheath plasma ($T_i/T_e \ll 1$). In the magnetic confinement fusion plasma such as edge tokamak plasma, $T_i$ is generally comparable to $T_e$, or $T_i$ is even higher than $T_e$. For a warm plasma sheath, it is shown from Eq. (1) that ion temperature can affect the Bohm criterion. Thus, a general sheath criterion including the effects of both magnetic field and collision should be taken into account for a wide range of $T_i/T_e$.

For the interaction of the plasmas with an absorbing wall, under most conditions, a negative charge wall is formed through bombardment with the electrons before the ions can react since the time scale of electrons is far shorter than that of ions. Chodura studied this process in presence of magnetic field and divided it into three distinct regions: presheath, magnetic presheath (Chodura sheath), and Debye sheath.[5] When the effects of both collision and ionization are taken into account, two limiting regimes, namely, collisional presheath and magnetized presheath, are obtained in the presheath region.[10] Since there exists a pre-sheath region between bulk plasma and sheath plasma, the electric field in the pre-sheath ($E_p$) is formed and resultant $\vec{E} \times \vec{B}$ drift arises in a magnetized sheath. $\vec{E} \times \vec{B}$ drift can affect the ion velocity entering the sheath and sheath plasma.[1,11] In the fluid approximation, another important drift is diamagnetic $\nabla p \times B$ drift for magnetized plasma. The velocity of diamagnetic drift may be very large for warm plasma such as tokamak plasma.[12] However, the diamagnetic drift can split into a divergence-free part, which does not contribute to particle transport, and into a part due to the magnetic field gradient and curvature.[13] In this paper, only the effect of the $\vec{E} \times \vec{B}$ drift on the sheath is considered while the effect of the $\nabla p \times B$ drift is not taken into account due to the neglected magnetic field gradient and curvature.

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Recently, the properties of a plasma collisional sheath in an oblique magnetic field have been widely examined with a two-fluid model.[14–16] It is shown that the plasma sheath characteristic is a sensitive function of the magnetic field and ion-neutral collision. Among these works, for various magnetic field and ion-neural collision frequency, the fixed boundary condition for the ion velocity entering the sheath is used. In the strongly magnetized case, results obtained from the case with fixed boundary condition will somewhat overestimate the absolute value of the wall potential.[16] Due to the sheath criterion associated with the magnetic field and collision, when the effects of the magnetic field and collision on the sheath structure are investigated, the boundary condition of ion velocity entering the sheath should be changed synchronously. Based on the sheath structure affected by the sheath boundary conditions, with a variable boundary conditions which depend on the $T_i/T_e$, the plasma parameters are examined and it is found that the sheath width decreases with the increasing ion temperature.[17] The sheath structures are investigated further with modified Bohm criterion including the effects of both incident angle of magnetic field and $T_i/T_e$.[18] However, in Ref. 18, only the case of $T_i/T_e \ll 1$ is considered. Under this assumption, it is not important that the ions are assumed to be isothermal or adiabatic. For the large $T_i/T_e$, ion thermal motion affects the sheath properties obviously.[19] Furthermore, the interplay between the ion pressure and magnetic field on the sheath should be explored under different ion temperatures. For these purpose, in Sec. II, a self-consistent one-dimensional fluid model for a warm plasma collisional sheath in an oblique magnetic field is implemented, in which the ion $\vec{E} \times \vec{B}$ drift velocity is also taken into account. With this model, a modified sheath criterion including effects of magnetic field and collision is established theoretically in Sec. III. Then, in Sec. IV, properties of the sheath are studied for various ion temperatures and ion thermal motions. Effect of the magnetic field on the sheath is also shown under the different ion temperatures. Finally, concluding remarks are provided in Sec. V.

## II. SHEATH MODEL

In our fluid model, two-component plasmas consisting of electrons and singly charged ions are considered. In the electron temperature range of $T_e = 1-100\,\mathrm{eV}$, the electron-neutral collision rate is $\langle \sigma_s v \rangle < 10^{-13}\,\mathrm{m^3 s^{-1}}$ for hydrogen gas,[20] and with a neutral density $n_n < 10^{20}\,\mathrm{m^{-3}}$, the resulting electron-neutral collision frequency is $\nu_e = n_n \langle \sigma_s v \rangle < 10^7\,\mathrm{s^{-1}}$. On the other hand, the electron plasma frequency for an electron density $n_0 = 10^{19}\,\mathrm{m^{-3}}$ (typical plasma density at present edge tokamak) is $\omega_{pe} = \sqrt{n_0 e^2/(\varepsilon_0 m_e)} \approx 10^{11}\,\mathrm{s^{-1}}$, where $e$ is the electron charge, $m_e$ is electron mass, and $\varepsilon_0$ is permittivity of free space. Since the electron-neutral collision frequency is smaller than the electron plasma frequency, and the electron flow velocity in the sheath is much smaller than its thermal velocity, apparently, the pressure is much larger than the friction force. Therefore, for the distribution of electron density $n_e$, the Boltzmann relations is satisfied well even in the presence of an oblique magnetic field

$$n_e = n_0 \exp\left(\frac{e\phi}{k_B T_e}\right), \tag{3}$$

where $\phi$ is the electric potential in the sheath. For the ions, the equation of continuity and motion in the steady states are described as

$$\nabla.(n_i \vec{V}_i) = \gamma_I n_e, \tag{4}$$

$$\vec{V}_i.\nabla\vec{V}_i = -\frac{e}{m_i}\nabla\phi + \frac{e}{m_i}\vec{V}_i \times \vec{B} - \frac{1}{m_i n_i}\nabla P_i - \nu_i \vec{V}_i, \tag{5}$$

where $m_i$, $n_i$, and $\vec{V}_i$ denote the ion mass, density, and velocity, respectively. $\gamma_I$ is ionization frequency and $\nu_i$ is the total collision frequency. The ion partial pressure is $P_i = k_B T_i \frac{n_i^\gamma}{n_0^{\gamma-1}}$.

A schematic description of the sheath configuration is shown in Fig. 1. The plasma-sheath boundary is located at x = 0 with the plasma filling the half space x < 0. The magnetic field is in x-z plane and $\vec{B}/B = (\cos\theta, 0, \sin\theta)$, which means that the y direction corresponds to the direction of $\vec{E} \times \vec{B}$ drift. It is assumed that the sheath is homogeneous in y and z directions and the physical parameters in the sheath change only along the x direction. Then, the Eqs. (4) and (5) can be written as one-dimensional problem

$$\frac{\partial n_i V_{ix}}{\partial x} = \gamma_I n_e, \tag{6}$$

$$V_{ix}\frac{\partial V_{ix}}{\partial x} = -\frac{e}{m_i}\frac{\partial\phi}{\partial x} + \frac{e}{m_i}BV_{iy}\sin\theta - \frac{1}{m_i n_i}\frac{\partial P_i}{\partial x} - \nu_i V_{ix}, \tag{7}$$

$$V_{ix}\frac{\partial V_{iy}}{\partial x} = \frac{e}{m_i}B(V_{iz}\cos\theta - V_{ix}\sin\theta) - \nu_i V_{iy}, \tag{8}$$

$$V_{ix}\frac{\partial V_{iz}}{\partial x} = -\frac{e}{m_i}BV_{iy}\cos\theta - \nu_i V_{iz}. \tag{9}$$

The formulation is completed with the Poisson equation for electric potential

$$\frac{\partial^2\phi}{\partial x^2} = -\frac{e}{\varepsilon_0}(n_i - n_e). \tag{10}$$

For convenience, the following dimensionless quantities are defined:

$$\xi = x/\lambda_{De}, \quad \varphi = e\phi/k_B T_e, \quad N_e = n_e/n_0, \quad N_i = n_i/n_0,$$
$$u_i = V_i/V_{te}, \quad \alpha = \omega_c/\omega_p, \quad \Delta = \gamma_I/\nu_i,$$

where $\lambda_{De} = \sqrt{\varepsilon_0 k_B T_e/n_0 e^2}$ is Debye length, $V_{te} = \sqrt{k_B T_e/m_i}$ is the thermal velocity, $\omega_c = eB/m_i$ is the ion gyrofrequency, and $\omega_p = \sqrt{n_0 e^2/(\varepsilon_0 m_i)}$ is the ion plasma frequency.

The following normalized set of Eqs. (3), (6)–(10) can be rewritten as

$$N_e = \exp(\varphi), \tag{11}$$

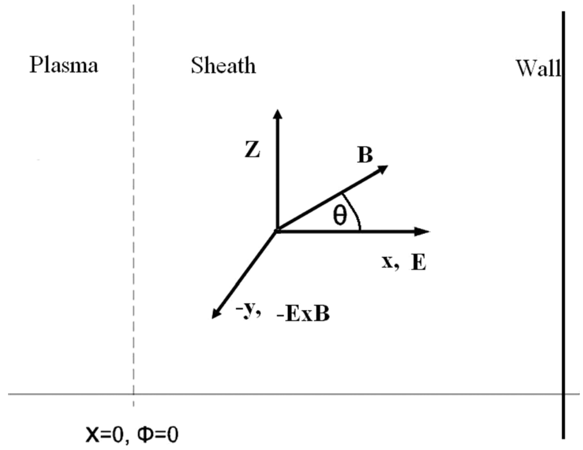


FIG. 1. Schematic geometry of the sheath model.

$$\frac{\partial N_i}{\partial \xi}=\frac{N_i}{u_{ix}^2-\gamma N_i^{\gamma-1}T_i/T_e}\left[\frac{\partial\varphi}{\partial\xi}-u_{iy}\alpha\sin\theta+\frac{u_{ix}\nu_i}{\omega_p}\left(1+\Delta\frac{N_e}{N_i}\right)\right], \tag{12}$$

$$\frac{\partial u_{ix}}{\partial \xi}=-\frac{u_{ix}}{u_{ix}^2-\gamma N_i^{\gamma-1}T_i/T_e}\left[\frac{\partial\varphi}{\partial\xi}-u_{iy}\alpha\sin\theta +\frac{u_{ix}\nu_i}{\omega_p}\left(1+\Delta N_e\gamma N_i^{\gamma-2}\frac{T_i}{T_e}\frac{1}{u_{ix}^2}\right)\right], \tag{13}$$

$$\frac{\partial u_{iy}}{\partial \xi}=-\alpha\sin\theta+\frac{u_{iz}}{u_{ix}}\alpha\cos\theta-\frac{\nu_i}{\omega_p}\frac{u_{iy}}{u_{ix}}, \tag{14}$$

$$\frac{\partial u_{iz}}{\partial \xi}=-\frac{u_{iy}}{u_{ix}}\alpha\cos\theta-\frac{\nu_i}{\omega_p}\frac{u_{iz}}{u_{ix}}, \tag{15}$$

$$\frac{\partial^2\varphi}{\partial\xi^2}=N_e-N_i. \tag{16}$$

The boundary conditions for these equations can be expressed as follows. At the sheath edge ($\xi=0$), the electric potential is zero ($\varphi_0=0$) and the charge neutrality condition is satisfied ($N_{i0}=N_{e0}=1$).

## III. SHEATH CRITERION

To obtain the sheath criterion for the warm ion plasma including the effects of the magnetic field and collision, we use the similar method as in Refs. 18 and 21. Introducing the Sagdeev potential $V_S(\varphi)=\int_0^\varphi (N_i-N_e)d\varphi$, then the integrated form of the Eq. (16) with the boundary condition can be written as

$$\frac{1}{2}\left(\frac{\partial\varphi}{\partial\xi}\right)^2=\frac{1}{2}\left(\frac{\partial\varphi}{\partial\xi}\bigg|_{\xi=0}\right)^2-V_S(\varphi). \tag{17}$$

At the plasma-sheath edge, Sagdeev potential satisfies $V_S(0)=0$ and $\partial V_S/\partial\varphi|_{\varphi=0}=0$. Due to the monotonic potential drop across the sheath, Eq. (17) at the right hand side must satisfy the condition

$$\frac{d^2V_S}{d\varphi^2}\bigg|_{\varphi=0}=\left(\frac{\partial N_i}{\partial\varphi}-\frac{\partial N_e}{\partial\varphi}\right)\bigg|_{\varphi=0}<0. \tag{18}$$

From Eq. (12), $\partial N_i/\partial\varphi|_{\varphi=0}$ can be written as

$$\frac{\partial N_i}{\partial\varphi}\bigg|_{\varphi=0}=\frac{1}{u_{ix0}^2-\gamma T_i/T_e}\left[1.0+\left(-u_{iy0}\alpha\sin\theta+\frac{u_{ix0}\nu_i}{\omega_p}(1+\Delta)\right)\Big/(\partial\varphi/\partial\xi)|_{\varphi=0}\right], \tag{19}$$

and from Eq. (11),

$$\partial N_e/\partial\varphi|_{\varphi=0}=1. \tag{20}$$

Combining Eqs. (19) and (20) with Eq. (18), two inequalities are obtained:

$$u_{ix0}^2>\frac{T_e+\gamma T_i}{T_e}+\left[-u_{iy0}\alpha\sin\theta+\frac{u_{ix0}\nu_i}{\omega_p}(1+\Delta)\right]\Big/(\partial\varphi/\partial\xi)|_{\varphi=0},\quad u_{ix0}^2>\frac{\gamma T_i}{T_e}. \tag{21}$$

Eq. (21) is the modified Bohm criterion including effects of the magnetic field and collision. However, Eq. (21) is a complex function depending on several free parameters, and it should be reduced in order to be compared with the simple expressions Eqs. (1) and (2).

In y direction, only the $\vec{E}\times\vec{B}$ drift is considered and $u_{iy0}$ can be written as

$$u_{iy0}=-\frac{E_0\sin\theta}{\alpha}, \tag{22}$$

where $E_0=-\frac{\partial\varphi}{\partial\xi}|_{\varphi=0}$ is the normalized electric field at the plasma-sheath edge.

Substituting Eq. (22) into Eq. (21), then we can obtain

$$u_{ix0}^2>\frac{\gamma T_i}{T_e}+\cos^2\theta-\frac{(1+\Delta)\alpha u_{ix0}}{E_0\beta},\quad u_{ix0}^2>\frac{\gamma T_i}{T_e}, \tag{23}$$

or

$$u_{ix0}>\left[\frac{\gamma T_i}{T_e}+\cos^2\theta+\left(\frac{(1+\Delta)\alpha}{4E_0\beta}\right)^2\right]^{1/2}-\frac{(1+\Delta)\alpha}{2E_0\beta},\quad u_{ix0}>\sqrt{\frac{\gamma T_i}{T_e}}. \tag{24}$$

Here, $\beta=\frac{eB}{m_i\nu_i}$ is defined as a factor set to characterize the ratio of ion gyrofrequency to plasma collision frequency. Usually, $\beta>10$ is for the typical parameters of tokamak edge with magnetic field 1-10 T, neutral density $n_n<10^{20}\,\mathrm{m^{-3}}$, and hydrogen ion-neutral collision rate $\langle\sigma_s v\rangle<10^{-13}\,\mathrm{m^3s^{-1}}$.

A modified Bohm criterion (Eq. (23) or (24)), which differs from the well-known Bohm criterion for low temperature plasma sheath, is established theoretically for the wide

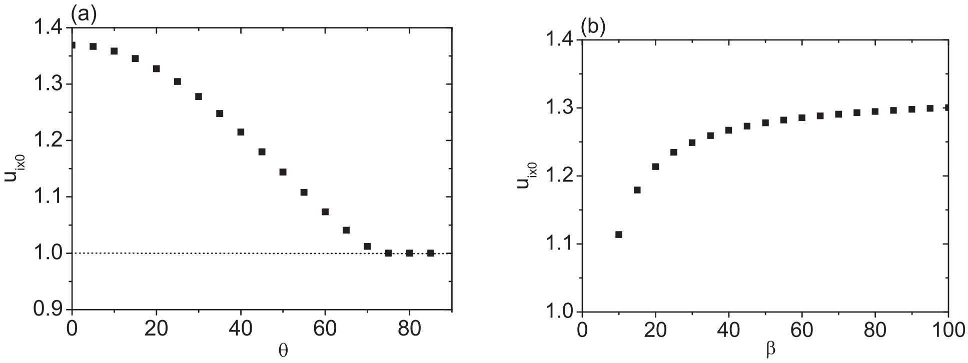


FIG. 2. Profiles of x component of hydrogen ions velocity at the sheath boundary $u_{ix0}$ for the parameters $T_i = T_e$, $\gamma = 1$, $\Delta = 1$, $n_0 = 10^{19}\,\mathrm{m}^{-3}$, $B = 1\,\mathrm{T}$, $E_0 = 0.01$, and (a) $\beta = 50$ with the varied $\theta$; (b) $\theta = 30°$ with the varied $\beta$.

range of ion temperature plasma collisional sheath in an oblique magnetic field. Obviously, ion temperature influences the ion velocity entering the sheath $u_{ix0}$. $u_{ix0}$ becomes large with the increase in $T_i/T_e$. Besides the effect of $T_i/T_e$, the modified criterion expresses that $u_{ix0}$ also depends on the incident angle of magnetic field and the collision (including ionization). From the Fig. 2(a), we can see that when the angle of magnetic field incidence with surface varies from near perpendicular to grazing, $u_{ix0}$ becomes small. In Fig. 2(b), with the increase in collision ($\beta$ decreases), qualitatively similar to the results in Ref. 7, it tends to reduce the ion velocity entering the sheath. If the collision in the sheath is weak, the effect of the collision on the modified sheath criterion, i.e., the third term in the first inequality in the Eq. (23), can be neglected, and then the modified Bohm criterion is consistent with results in Ref. 18. The second inequality in Eq. (23) shows that $u_{ix0}$ has a lower limit that is not zero with the variation of the magnetic field angle and collision. The lower limit is determined by the ion thermal motion and the ratio of the ion temperature to electron temperature.

## IV. SIMULATION RESULTS

With the plasma-sheath boundary conditions Eqs. (22), (24), $u_{iz0} = u_{ix0}\tan\theta$ and $\varphi_0|_{\xi=0} = 0$, $E_0 = 0.01$, $N_{i0} = N_{e0} = 1$, the set of Eqs. (11)–(16) can be solved numerically to investigate the sheath structure using a fourth-order Runge-Kutta method. Started from $\xi = 0$, until the wall position located at zero electron density, the width of the plasma sheath is determined in this process instead of self-consistent calculation of a given constant electric potential. Although the results may be applicable to a wide variety of gas discharge, tokamak plasma is considered in this paper. For the simulation, the hydrogen gas is only discharge gas and parameters are chosen from the typical parameters at present edge tokamak as $T_i/T_e = 0.2 - 2.0$, $B = 1 - 5\,\mathrm{T}$, $T_e = 25\,\mathrm{eV}$, $n_0 = 10^{19}\,\mathrm{m}^{-3}$, and $\Delta = 1$.

### A. Effect of the ion pressure on the sheath

The ion pressure influence appears in the ions momentum of the Eq. (5). For the cold ion plasma ($T_i/T_e \ll 1$), the ion pressure term can be neglected due to the low ion temperature. When $T_i$ is comparable to $T_e$, the ion pressure becomes important. To demonstrate the effect of the ion temperature on the sheath, Figs. 3(a)–3(d) show the normalized potential, electron and ion densities, and ion velocity in x direction as a function of distance from the plasma-sheath edge for different $T_i/T_e$. According to the Eq. (23), the ions velocity entering the sheath is greater for the larger $T_i/T_e$.

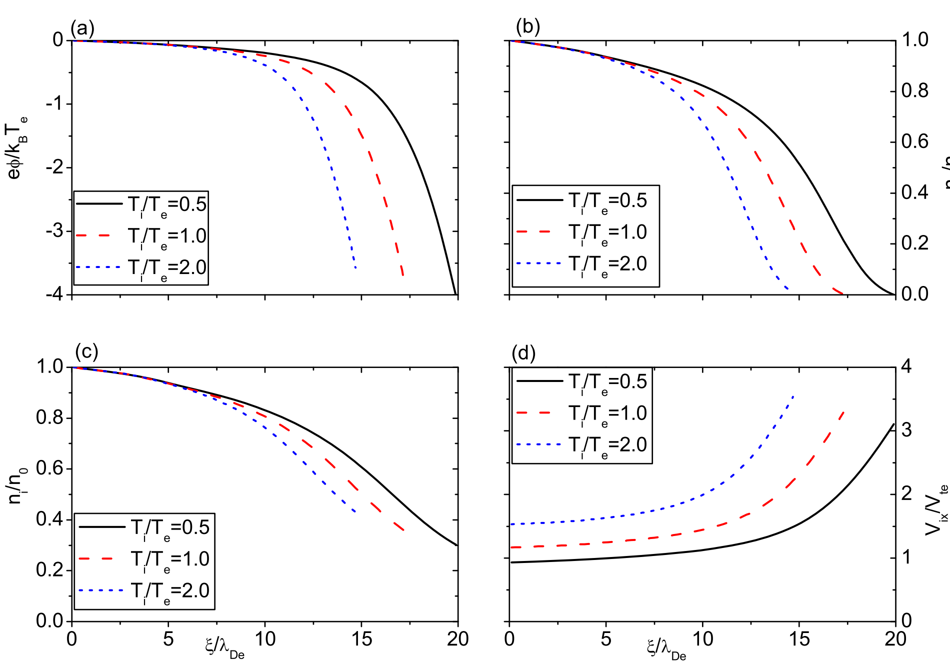


FIG. 3. Variation of normalized (a) sheath potential, (b) electron density, (c) ion density, and (d) ion velocity for different $T_i/T_e$ with $\gamma = 1$, $B = 1\,\mathrm{T}$, $\theta = 50°$, and $\beta = 100$.

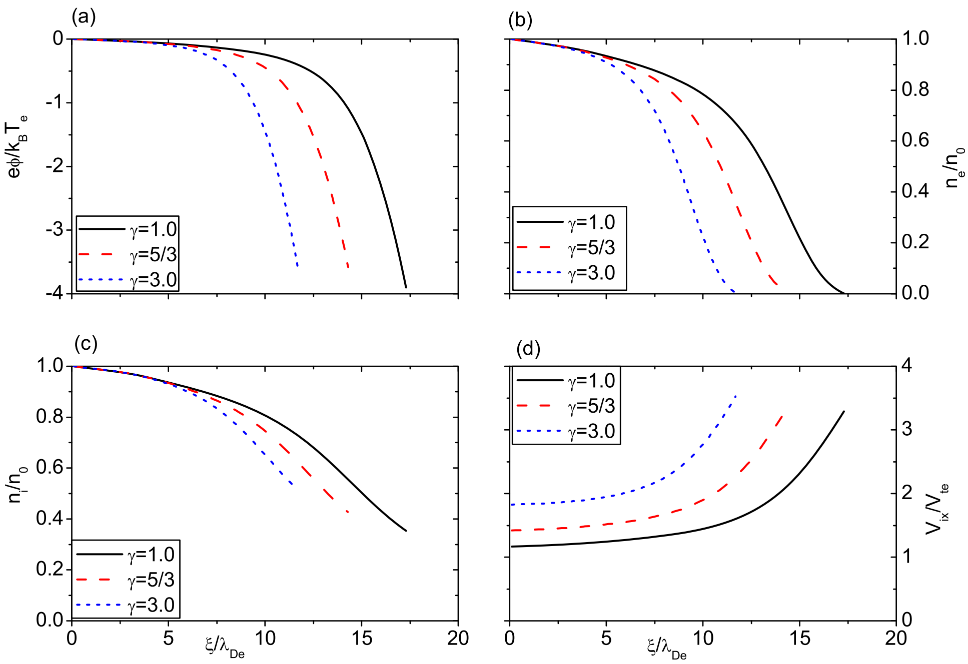


FIG. 4. Variation of normalized (a) sheath potential, (b) electron density, (c) ion density, and (d) ion velocity for different $\gamma$ with $T_i/T_e = 1$, $B = 1\,\mathrm{T}$, $\theta = 50^\circ$, and $\beta = 100$.

As long as the ions enter the sheath, they are scattered responding to the sheath electric field and cause the variation of the ion density. For the higher ion temperature, the gradient of the ion pressure is steeper. As a result, ions hit the wall more quickly due to the higher velocity entering sheath and the larger ion pressure. This effect, of course, causes significant decrease in the sheath width by the increase in $T_i/T_e$ (Figs. 3(a) and 3(b)). Near the plasma-sheath edge region, with the variation of the $T_i/T_e$, a strong change in the profiles of the ion velocity is determined by the boundary condition at the plasma-sheath edge. In contrast, the corresponding ion density is insensitive to the variation of the $T_i/T_e$.

Ion thermal motion in the sheath includes isothermal and adiabatic approximations. As shown by Riemann, a fluid approximation must become one-dimensionally adiabatic at the sheath edge.[22] For $T_i/T_e \ll 1$, it is not important for the ion thermal motion to be considered as isothermal or adiabatic. For $T_i/T_e \sim 1$, ion thermal motion can affect both the ion velocity entering the sheath region and ion momentum in the sheath. For the larger $\gamma$, the ion velocity entering the sheath is larger and effect of ion pressure on the ion momentum is stronger. In Figs. 4(a)–4(d), the sheath behavior is investigated with the parameters $T_i/T_e = 1$, $B = 1\,\mathrm{T}$, $\theta = 50^\circ$, and $\beta = 100$ for different ion thermal motions. The sheath width is shorter for adiabatic motion than that for isothermal motion. In the three cases of the ion thermal motion, the sheath width is the shortest for uni-dimensional adiabatic motion due to the largest $\gamma$. Based on boundary condition of the ion velocity dependent on the $\gamma$, near the plasma-sheath edge region, the variation of the ion velocity is more

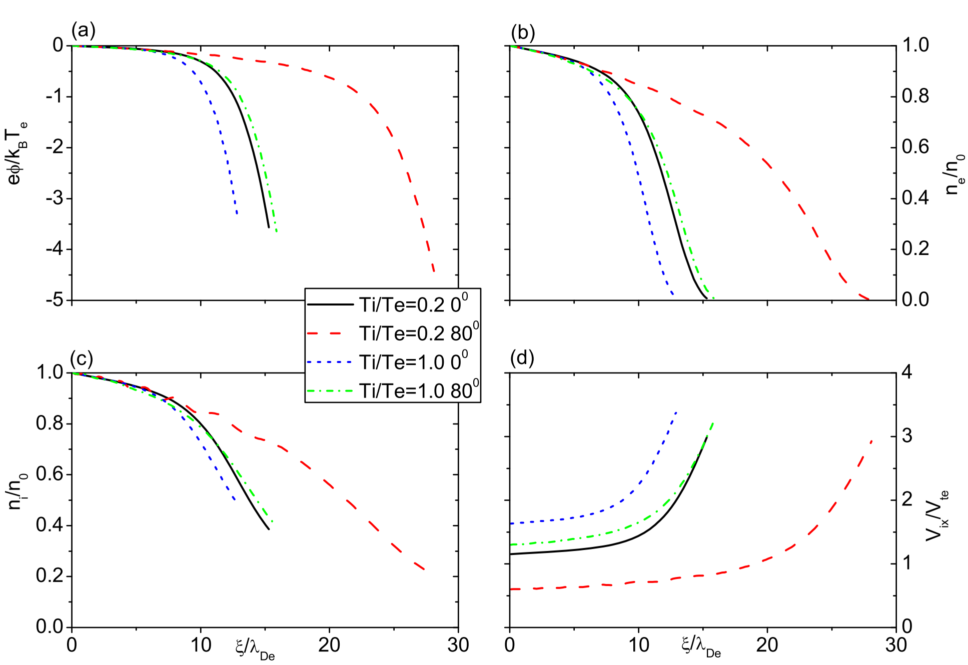


FIG. 5. Comparison of normalized (a) sheath potential, (b) electron density, (c) ion density and (d) ion velocity for different $\theta$ with $\gamma = 5/3$, $B = 1\,\mathrm{T}$ and $\beta = 1000$ in the case $T_i/T_e = 0.2$ and $T_i/T_e = 1$.

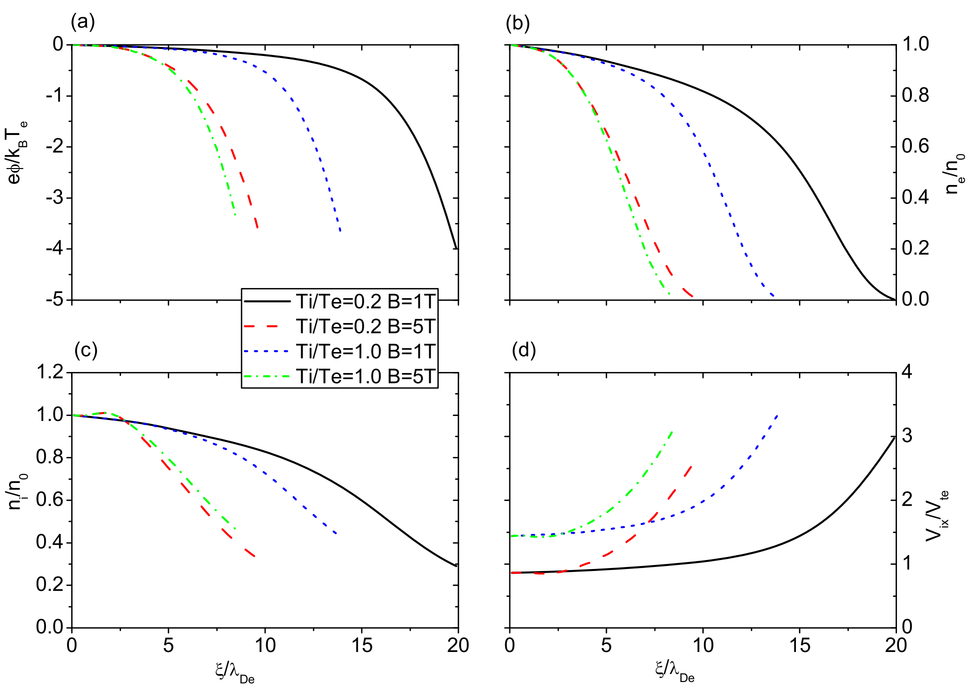


FIG. 6. Comparison of normalized (a) sheath potential, (b) electron density, (c) ion density and (d) ion velocity for different $B$ with $\gamma = 5/3$, $\theta = 50°$, and $\beta = 1000B$ in the case $T_i/T_e = 0.2$ and $T_i/T_e = 1$.

pronounced than that of the ion density (Figs. 4(c) and 4(d)). The tendency is similar to the results caused by the variation of $T_i/T_e$ in Figs. 3(c) and 3(d).

## B. Effect of magnetic field on the sheath

Effect of the magnetic field on the sheath has been investigated widely for the case $T_i/T_e \ll 1$.[14,16] The plasma-sheath width is a sensitive function of the magnitudes and directions of the magnetic field. The magnetic field term can decrease the influence of the electric field together with the collision term on the ion acceleration. Due to the effect of Lorentz force, the magnetic field causes the ion flow to gyrate and the ion density experiences fluctuations.[14] In the tokamak, effect of the magnetic field on the sheath cannot be ignored because the magnetic field is strong (1–5 T), and the range of the variation of the angle of the magnetic field incident with the wall surface is wide. On the other hand, $T_i$ and $T_e$ is comparable, and the resultant effect of the ion pressure on the sheath becomes important. To show the interplay between the ion pressure and magnetic field on the sheath, Figure 5 compares the effect of angles of magnetic field incidence on the sheath in the two cases $T_i/T_e = 0.2$ and $T_i/T_e = 1$. It is shown that sheath width decreases when the angle between the magnetic field and the wall increases ($\theta$ decreases). For the $T_i/T_e = 0.2$, the corresponding ion pressure is small, and the variation of the sheath width, as well as ion velocity, is very pronounced with the variation of the angle. For the $\theta = 80°$, the magnetic field becomes almost perpendicular to the acoustic wave propagation direction. The ion density experiences fluctuations because the Lorentz force accelerates and decelerates the ions. For the $T_i/T_e = 1$, the effect of the ion pressure on the sheath becomes noticeable and the ion pressure diminishes the effect of the magnetic field. As a result, with variation of the angle, the sheath width variation is small, although the boundary condition of ion velocity changes with the angle.

The magnetic field intensity does not affect the boundary condition, but it is a variable in the ion momentum equation. From the Figs. 6(a)–6(d), we can see that the sheath width decrease with the increase in the intensity of the magnetic field in the both cases of $T_i/T_e = 0.2$ and $T_i/T_e = 1$. Based on the interplay between ion pressure and magnetic field on the sheath, similar to the effect of angle on the sheath structure in Fig. 5, the sheath parameters show a more pronounced change in the case $T_i/T_e = 0.2$ than those in the case $T_i/T_e = 1$ for different intensity of the magnetic fields.

## V. CONCLUSIONS

A two-fluid model for a plasma collisional sheath in an oblique magnetic field is implemented. With this model, a sheath criterion including the effects of the magnetic field and collision is obtained theoretically for a wide range of ion temperature. The modified criterion expresses that the ion velocity entering the sheath is associated with the incident angle of magnetic field and the collision. When the magnetic field makes a variety of angles with surface, from near perpendicular to grazing, or the collision increases, the ion velocity entering the sheath becomes small but has a lower limit instead of tendency to zero. The lower limit depends on the ion thermal motion in the sheath and the ratio of the ion temperature to electron temperature.

With the modified Bohm criterion as sheath boundary condition, the plasma parameters including the electron and ion densities, ion flow velocity, and electric potential are calculated. For various ratios of ion temperature to electron temperature, or the different ion thermal motions in the sheath, the structure of the sheath changes under the influence of the variation of the ions velocity entering the sheath and contribution of ion pressure to the ion momentum together. The sheath width decreases with the increase in the ratios of ion temperature to electron temperature, or the ions adiabatic motion instead of isothermal motion. The sheath structure is also a sensitive function of the magnetic field. Sheath width decreases when either the angle between the magnetic field and the wall or the intensity of the magnetic field increases. Under the different ion temperatures, the angle of magnetic field incidence has an obvious effect on the plasma sheath. Effects of the directions and magnitudes of the magnetic field on the plasma sheath are more pronounced for lower ion temperature than those for higher ion temperature due to the interplay between the ion pressure and magnetic field.

Finally, it must be pointed out that the model for the sheath is based on several assumptions. For example, it should be advisable to solve electron equations instead of the applicability of the Boltzmann relation in the model. To further investigate the sheath structure and the associated sheath criterion in a real situation of magnetic confinement fusion device such as the divertor region of a tokamak, the variation of the magnetic field gradient and curvature, and the resulting diamagnetic drift must be taken into account. Under some conditions in the divertor region of a tokamak, the electric field is strong at the plasma-sheath edge and also has a component tangential to the wall leading to $\vec{E} \times \vec{B}$ drift towards the wall.[1] Actually, the sheath becomes a two-dimensional problem with surface-parallel dependence of all parameters and a two-dimensional sheath model should be considered.[23]

## ACKNOWLEDGMENTS

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