Stopping power of ions in a strongly magnetized plasma

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(Received 19 September 1995; accepted 30 November 1995)

The energy loss of heavy ions in a strongly magnetized plasma is studied by means of Vlasov and Particle-in-Cell (PIC) simulations. The topic is of importance for many practical applications, as for example in the electron cooling of heavy ion beams. Measurements of the longitudinal cooling force show strong deviations from the well known linear theories. Moreover, the possibility of heavy ions to reach high charge states Z_p increases the interest in understanding the plasma regime with coupling parameter $Z_p/N_D > 1$ (N_D is the number of electrons in a Debye sphere), when nonlinear phenomena are important. In this regime a linearization of the drift kinetic Vlasov–Poisson system is not possible, so that a fully nonlinear numerical approach is unavoidable. Comparisons between the two numerical schemes are made for the stopping power and the potential created in the plasma by a moving ion, together with the dielectric theory of ion stopping. Our results show strong deviations from the dielectric theory, and our investigations show the importance of the role played by the electrons trapped in the potential troughs excited by the ion in the stopping process. © *1996 American Institute of Physics.* [S1070-664X(96)02003-9]

I. INTRODUCTION

The stopping power of ions in a magnetized plasma is an important quantity for the electron cooling of ion beams,[1,2] a technique used in accelerator physics to reduce the phase space volume of ion beams. The cooling process is based on the energy loss due to Coulomb interaction of the ions in a superimposed cold electron beam moving with the same average velocity and guided by a longitudinal magnetic field B. Typical parameters for an electron cooling device are $T \simeq 10^{-1}-10^{-4}$ eV and $n \simeq 10^6-10^8$ cm^{-3}, which means that $N_D \simeq 10^{-2}-10^4$, i.e., the plasma regime covers a wide region of parameters, running from strongly coupled ($N_D < 1$) to ideal plasmas. Furthermore, the longitudinal magnetic field is quite strong ($B \approx 1$ kG). The process is characterized by a cooling time and a final ion-beam temperature, and these quantities may be obtained when friction (drag) and diffusion in ion-velocity space are determined. Recent experimental results[3] of the longitudinal cooling force for fully stripped medium and heavy ions show deviations from the well known linearized theories, the dielectric and binary collision theories. The reason is addressed to nonlinear plasma phenomena, the interaction being quite strong ($Z_p/N_D > 1$). Interest arises in understanding these phenomena. Another point of interest is the question of whether the stopping power reaches a stationary state during the cooling process. The interaction time between the incoming ion and the electron beam in the cooling section is of the order of a few plasma periods. This means that the time evolution is of great importance in determining the cooling process of the ion beam. In this paper we study the interaction of a heavy ion moving in a strongly magnetized plasma, with particular interest for the nonlinear regime of interaction in a classical and collisionless plasma ($N_D > 1$). We shall assume $r_L \ll \lambda$ in the following, where r_L is the electron Larmor radius and $\lambda = v_0/\omega_P$ the dynamical screening length, with $\omega_P^2 = 4\pi n_0 e^2/m_e$ the plasma frequency, in order to apply the approximation of an infinite magnetic field. The electron dynamics in the transverse direction is effectively frozen in the strong magnetic field, and the motion reduces to one dimension. In this case the drift kinetic Vlasov equation coupled with the Poisson equation can be used to study the collective electron response.[1] Within the framework of the linearized drift kinetic Vlasov–Poisson system a similar problem has been already addressed in Refs. 4 and 5, where the stopping power and wake field are calculated for superthermal electrons in a magnetized plasma. Following O'Neil,[6] it is claimed that the usual dielectric theory is not valid for the problem considered in the present work. The argumentation is based on O'Neil's one-dimensional solution for the steady-state amplitude of an initial periodic perturbation. He showed that the usual Landau damping is correct only for short times, so that the steady-state solution cannot be described by the usual dielectric theory. The electrons trapped in the wave are essential to determine the asymptotic ($t \to \infty$) wake field. It must be noted that this argumentation is valid only if the velocity phase space is one-dimensional. In more dimensions wave-electron energy transfer is possible via small angle deflections and the usual dielectric approach can be used to describe also the asymptotic behavior in time. The nonlinear stopping power of a negative ion in a one-dimensional plasma has been treated in Ref. 7, where a nonlinear analytic solution is obtained for the stopping power in the limit of small velocities, $v_0 \ll v_{th}$. This solution is different from the dielectric result. Also the problem of the existence of a steady-state stopping power in a collisionless plasma has been considered there. For the determination of the longitudinal cooling force, and also for a general understanding of nonlinear plasma effects connected with ion stop-

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ping, the drift kinetic Vlasov–Poisson system must be solved numerically. The results of the numerical solution will be presented, together with the results obtained by Particle-in-Cell (PIC) simulations. With the help of the full time dependent solution, analytical models can be checked. In Sec. II the drift kinetic Vlasov–Poisson system is introduced and a short review of some standard analytical approaches (Sec. III), together with some global considerations and scaling laws (Sec. IV), is presented. To describe the process of ion stopping in a classical thermal plasma it is useful to introduce the following dimensionless variables: $r \to r/\lambda_D$, $v = v_e/v_{th}$, $t \to \omega_P t$, $f \to v_{th}^3 f/n_0$, $\phi \to e\phi/kT$, $n = n_e/n_0$, $Z = Z_p/N_D$, where n_0 is the initial electron density, $\lambda_D^2 = kT/4\pi n_0 e^2$ the Debye length, $N_D = n_0 \lambda_D^3$ the number of particles in the Debye sphere, $v_{th}^2 = kT/m_e$ the electron thermal velocity, T the electron plasma temperature, k the Boltzmann constant, e and m_e the electron charge and mass respectively, Z_p the charge state of the ion, $f = f(\vec{r}, v_z, t)$ is the electron distribution function, and $\phi = \phi(\vec{r}, t)$ the electrostatic potential. Finally we want to stress that in this paper we neglect all atomic physics processes as ionization and recombination, being not directly of concern with the subject of this work.

II. DRIFT KINETIC VLASOV–POISSON SYSTEM

A classical and collisionless electron plasma with a homogeneous ion background and immersed in a strong longitudinal magnetic field ($\vec{B} = B\vec{e}_z$) can be treated in the framework of the Vlasov–Poisson equations. If $r_L \ll \lambda$ holds, one can think of electron guiding centers moving only along the magnetic field lines, and the motion of the electrons being one-dimensional. The drift kinetic Vlasov equation for the distribution function $f(\vec{r}, v_z, t)$,[8] coupled with the Poisson equation for the electrostatic potential $\phi(\vec{r}, t)$ thus becomes (see also Ref. 1):

$$\frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} + \frac{\partial \phi(\vec{r}, t)}{\partial z} \frac{\partial f}{\partial v_z} = 0, \tag{1}$$

$$-\frac{\partial^2 \phi}{\partial \vec{r}^2} = 1 - n(\vec{r}, t) + Z\delta(\vec{r} - v_0 \vec{e}_z t), \tag{2}$$

where

$$n(\vec{r}, t) = \int dv_z f(\vec{r}, v_z, t) \tag{3}$$

is the electron density. In Eq. (1) the δ-function represents the point like projectile. Once the solution for the potential is known, the stopping power is defined as

$$-\frac{dE}{dz} = Z N_D \frac{\partial \phi_{\text{ind}}(\vec{r} = \vec{v}_0 t)}{\partial z}. \tag{4}$$

The magnetic field cannot be regarded as infinite for those electron guiding centers passing the ion with impact parameters $b \lesssim r_L \ll \lambda$. The exact inclusion of effects due to the finite size of the Larmor radius is quite complicated, nevertheless the stopping power due to these electrons can be approximated by neglecting the magnetic field and using the standard binary collision theory (BCT) for a bare Coulomb potential with an upper cut-off $b_{\max} = r_L$ (see Ref. 9 for a detailed description of the BCT):

$$-\frac{dE}{dz}\bigg|_{\text{BCT}} = \frac{Z^2 N_D}{4\pi} \int_{-\infty}^{\infty} \int_0^{\infty} dv_z dv_r \times f(v_r, v_z) \frac{\lambda(u)}{u^3} (v_0 - v_z) \tag{5}$$

with

$$\lambda(u) = \frac{1}{2} \ln\left(\frac{b_{\max}^2}{b_\perp^2} + 1 \right). \tag{6}$$

Here $b_\perp = Z/(4\pi u^2)$ is the impact parameter for 90° deflection in the Coulomb potential, $u^2 = (v_0 - v_z)^2 + v_r^2$ the relative velocity and $f(v_r, v_z)$ the Maxwell distribution of the plasma electrons. The stopping power can be calculated as the sum of both contributions, Eq. (4) for $b \geqslant r_L$ and Eq. (5) for $b \leqslant r_L$. The separation in these two contributions is possible as long as $r_L \ll \lambda$ holds.

III. REVIEW OF SOME ANALYTICAL APPROACHES

Before starting to present our numerical results, we want to review the well known theories and point out some peculiarities of the ion stopping in a strongly magnetized plasma. As far as we know, these peculiarities do not appear to have been previously recognized in literature.

A. Binary collision theory

An alternative approach to Eq. (4) for the calculation of the stopping power is to look at the energy transfer ΔE along the characteristics of a test electron. Since velocity space is one-dimensional, energy can be transferred only if an electron is reflected by the potential. This holds as long as the ion is moving along the B-field. The energy transferred by a reflected electron with impact parameter b and initial velocity v_z is:

$$\Delta E(b) = 2v_0(v_0 - v_z), (v_z - v_0)^2 < -2\phi_{\min}(b). \tag{7}$$

The stopping power is

$$-\frac{dE}{dz} = 2\pi N_D \int_{-\infty}^{\infty} dv_z f(v_z) \frac{u}{v_0} \int_0^{\infty} b\,db\,\Delta E(b), \tag{8}$$

where $u = |v_0 - v_z|$ is the relative velocity and $f(v_z)$ is a Maxwell distribution. We notice that Eq. (8) is exact, as far as the exact expression for the potential $\phi_{\min}(b)$ is known. In conclusion, contrary to the three-dimensional case, stopping is due only to collective effects. Since potential energy humps ($V_{\max} = -\phi_{\min}$) can be induced only by clumps of free plasma electrons, it is $\phi_{\min}(b) = \phi_{\text{min,ind}}(b)$.

B. Dielectric theory

The stopping power can be calculated via the induced electric field at the position of the ion, $\partial\phi_{\text{ind}}/\partial z$, as expressed in Eq. (4). This approach is the starting point for any mean-field theory, including the dielectric theory that will be briefly discussed here. In the dielectric theory the Vlasov–Poisson system (1),(2) is linearized by assuming $Z \ll 1$, and solved by using the Fourier and Laplace transform techniques in space and time, respectively. The potential is

$$\phi(\vec{r}) = \frac{Z}{(2\pi)^3}\int d^3k \frac{\exp(i\vec{k}\vec{r})}{k^2\epsilon(\vec{k},\vec{k}\vec{v_0})}, \tag{9}$$

where $\epsilon(\vec{k},\omega)$ is the dielectric function for a strongly magnetized plasma:

$$\epsilon(\vec{k},v_0k_z) = 1 + \frac{1}{k^2}\left(X\left(\frac{v_0k_z}{|k_z|}\right) + iY\left(\frac{v_0k_z}{|k_z|}\right)\right), \tag{10}$$

$$X(s) = 1 - s\ \exp\left(\frac{-s^2}{2}\right)\int_0^s dy\ \exp\left(\frac{y^2}{2}\right), \tag{11}$$

$$Y(s) = \sqrt{\frac{\pi}{2}}s\ \exp\left(-\frac{s^2}{2}\right). \tag{12}$$

In passing we remember that the solutions of the equation $\epsilon(\vec{k},\omega) = 0$ are plasma modes. Plasma waves are excited for $k_z < 1$ and for $v_0 \gg 1$. The evaluation of Eq. (9) along the z-axis $(\rho = 0)$ leads to

$$\phi(z) = \frac{Z}{2\pi^2 z}\int_0^\infty dk \times k\frac{(k^2 + X(v_0))\sin(kz) + Y(v_0)(1-\cos(kz))}{(k^2+X(v_0))^2 + Y^2(v_0)}. \tag{13}$$

The stopping power is

$$-\frac{dE}{dz} = \frac{Z^2N_D}{(2\pi)^2}\int_0^{k_{\max}} dkk\int_{-1}^1 duu\ \mathrm{Im}\left(\frac{1}{\epsilon(\mathbf{k},kuv_0)}\right), \tag{14}$$

where a cut-off $k_{\max}$ has been introduced to avoid the divergence of the k-integral due to close collisions. This expression can be evaluated exactly in the one-dimensional case:

$$-\frac{dE}{dz} = \frac{Z^2N_D}{4\pi^2}G(v_0,k_{\max}), \tag{15}$$

where

$$G(w,k_{\max}) = Y(w)\left(\ln\ (k_{\max}) + \frac{1}{4}\ln\frac{(1+X(w)/k_{\max}^2)^2 + Y(w)^2/k_{\max}^4}{X^2(w)+Y^2(w)}\right) - \frac{X(w)}{2Y(w)}\left(\arctan\frac{k_{\max}^2 + X(w)}{Y(w)} - \arctan\frac{X(w)}{Y(w)}\right), \tag{16}$$

Since the stopping power in the strong magnetic field approximation is due only to the excitation of collective modes, $k_{\max} \approx 1$ can be chosen for a positive ion. Electron plasma modes with $k > 1$ are strongly damped and can be neglected in the dielectric approach. This is the opposite to the ion stopping without an external magnetic field, where the large angle deflections in the Coulomb potential produce density perturbations on scales smaller than λ_D. Notice that the dielectric theory predicts a scaling of the stopping power as Z^2N_D, for a given ion velocity v_0. In the limit of $v_0 \gg 1$ Eq. (14) can be calculated without the need of any cutoff. We obtain:

$$-\frac{dE}{dz} = \frac{Z^2N_D}{8\pi v_0^2}. \tag{17}$$

For the energy loss of a negative ion in a strong magnetic field $k_{\max} = 1/b_\perp$ must be set in order to treat the reflected electrons.

C. An oscillator model for the stopping power

When $v_0 \gg 1$, one can think that stopping power is due to the excitation of undamped linear waves. The electrons, moving in a strong magnetic field, react as one-dimensional oscillators driven by the moving ion. The eigenfrequency of the oscillators is the plasma frequency, $\omega = 1$. If $\xi_\parallel$ is the displacement along the B-field and $F_\parallel$ the projection of the force (see also Ref. 10), the equation of motion is

$$\ddot{\xi}_\parallel + \xi_\parallel = F_\parallel \tag{18}$$

where

$$F_\parallel(t,b) = \frac{Z}{4\pi}\frac{v_0t + \xi_\parallel}{((v_0t+\xi_\parallel)^2 + b^2)^{3/2}}.$$

The solution is

$$\xi_\parallel(t,b) = \int_{-\infty}^t dt' F_\parallel(t',b)\sin(t-t'). \tag{19}$$

The total energy gained by one single oscillator is

$$\Delta E(b) = \frac{1}{2}\ (\dot{\xi}_\parallel^2 + \xi_\parallel^2)|_{t\to\infty} = \frac{1}{2}\left|\int_{-\infty}^\infty dtF_\parallel(t,b)\exp(it)\right|^2. \tag{20}$$

Equation (20) can be calculated for small displacements $\xi_\parallel \ll 1$:

$$\Delta E(b) = \frac{Z^2}{4\pi^2v_0^4}K_0^2\left(\frac{b}{v_0}\right). \tag{21}$$

For $v_0 \gg 1$ the internal motion of the plasma electrons can be neglected and Eq. (8) can be simplified in order to obtain the stopping power:

$$-\frac{dE}{dz} = 2\pi N_D\int_0^\infty bdb\Delta E(b) = \frac{Z^2N_D}{8\pi v_0^2}. \tag{22}$$

Equation (22) coincides with Eq. (17), and gives the collective energy loss. For a negative ion the energy loss due to the reflected electrons must be added.

IV. SCALING LAWS

Since no analytical solution can be given for the stopping power when $Z \geq 1$, in this section we want to make some simple estimates to obtain approximate scaling laws for the stopping power in the form of $-dE/dz \sim Z^\alpha$, in the two limiting cases of $v_0 \gg 1$ and $v_0 \ll 1$. As discussed in Sec. III B, plasma waves are excited by fast ions. In this case the dielectric theory could be used, which predicts a Z^2 scaling law for the stopping power. But, as a matter of fact this scaling can be modified by nonlinear wave excitation. Since the stationary state of the excited wave is determined only by the trapped electrons,[6] strong modifications of the dielectric theory results are expected even for $Z \ll 1$, contrary to the three-dimensional case. A linearized theory cannot account for trapped electron effects. In order to obtain the scaling law for low velocities $v_0 \ll 1$ we write Eq. (4) in terms of the solution of the Poisson equation in the rest frame of the projectile ion:

$$-\frac{dE}{dz} = ZN_D \int d^3r' \, \frac{\vec{r'}\vec{v_0}}{v_0} \, \frac{q(r')}{(r')^3}, \tag{23}$$

where $q(r) = 1 - n(r)$ is the total charge density of the plasma. For small velocities the electron screening cloud around the ion is ellipsoidal, shortened in the direction of ion motion by $\lambda = v_0 \ll 1$.[11,12] This polarization effect causes the stopping force on the slowly moving ion. On the other hand, trapped electrons cause no polarization effect if the ion is moving. If we assume for the plasma free electrons a Maxwellian distribution, $f(r, v_z) = (1/\sqrt{2\pi})\exp(-v_z^2/2 + \phi)$ for $v_z^2 \geqslant 2\phi$, we get for the electron density

$$n(\phi) = \exp(\phi)(1 - \operatorname{erf}(\sqrt{\phi})). \tag{24}$$

Near the core $(r \ll 1)$ the potential can be approximated by the pure Coulomb potential, $\phi \approx Z/(4\pi r) \gg 1$, and $n(r) \approx 1/\sqrt{\pi\phi}$ holds. That is, in the stationary case the velocity of the free electrons increases near the core (energy conservation) and the density decreases. This means that the plasma free electrons antishield the projectile (a discussion of screening in a strongly magnetized plasma can be found in Ref. 13). Finally, in Eq. (23) $q \sim 1$, and we get simply $-dE/dz \sim Z$, the stopping power scales linearly with the perturbation for $v_0 \ll 1$.

V. NUMERICAL RESULTS

In this section we present results obtained by solving numerically the coupled drift kinetic Vlasov–Poisson system (1),(2) using a splitting scheme (Appendix A), and compare them with PIC simulations (Appendix B). It is worth noticing at this point that the results that will be presented in the following concerning the PIC simulations refer to the very strong nonlinear regime $(Z > 10)$, since for lower Z the level of numerical noise increases and becomes comparable with the intensity of the perturbation. The reason is addressed to the poor statistics in the phase space, which causes the loss of some microscopic details, but nevertheless for $Z > 10$ the obtained stopping power shows good agreement with the Vlasov simulations. This will be pointed out below. Vlasov simulations, on the other hand, show very high resolution for every Z, the price to pay being a very high memory allocation and computer time. Since no particle transport exists between different B-field lines, the problem is ideal for parallel computing. Initially a Maxwell distribution is chosen for the electrons. The projectile ion velocity v_0 is held constant during the interaction. Figures 1 and 2 show the stopping power versus time for $v_0 = 4$ respectively from the Vlasov $(Z = 5)$ and from PIC $(Z = 30)$ simulations. In both cases the stopping power reaches an oscillating steady-state after

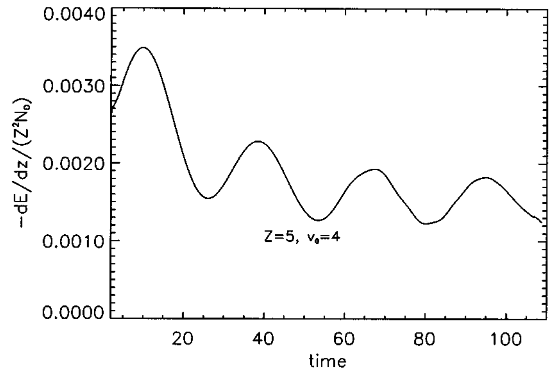

FIG. 1. The stopping power as a function of time for $Z = 5$, $v_0 = 4$ from Vlasov simulations.

an initial transient which depends in general on both Z and v_0. The oscillating structure of the stopping power is found to be a global characteristic for the problem under consideration, due to electrons trapped in the potential troughs excited by the projectile. The PIC simulations cannot resolve these details (Fig. 2). The frequency of these oscillations is directly related to the bounce frequency of the electrons trapped in the potential energy minima behind the ion. This is demonstrated in Fig. 3, where the contour plot of the electron distribution function is shown for $r = 0.2$ at different time steps. Here $Z = 5$ and $v_0 = 4$. The time step is equal to half of the oscillation period of the stopping power seen in Fig. 1. The closed loops at $z \approx -40$ and $z \approx -70$ around $v_0 = v_z = 4$ represent electrons trapped in the excited wave field. (We remind that since the distribution function is not stationary in time, the contour lines cannot be directly related to the electron trajectories in the $z - v_z$ phase space.) Notice that the rotation frequency of the centers of the closed loops about each other is the same as half the oscillation frequency of the stopping power shown in Fig. 1. At $t \approx 90$ the centers of the closed loops are nearly on a vertical line, while at $t \approx 105$ they are on a horizontal line. This leads to oscillations of the density in the frame of the ion and so of the stopping power. The PIC code is unable to resolve these

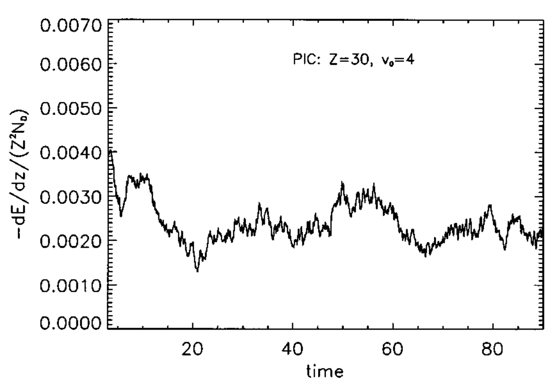

FIG. 2. The stopping power as a function of time for $Z = 30$, $v_0 = 4$ from PIC simulation.

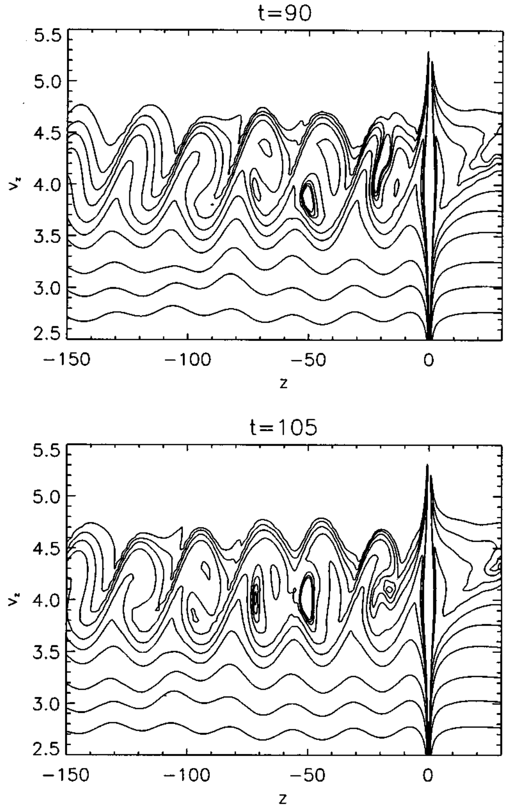

FIG. 3. Contour plots of the electron distribution function $f(r=0.2,z,v_z)$ for $Z=5$ and $v_0=4$ at different times.

details due the poor statistics in the phase-space regions where particle trapping occurs. In these regions, which are crucial for the problem considered in this work, PIC simulations lack an adequate number of simulation particles to display the distribution function (see, for example, Ref. 14, where, however, no explicit comparison between the two numerical approaches have been made). Figure 4 shows results from our simulations for the normalized stopping power $(dE/dx)/(Z^2 N_D)$ versus v_0, compared with both the dielec-

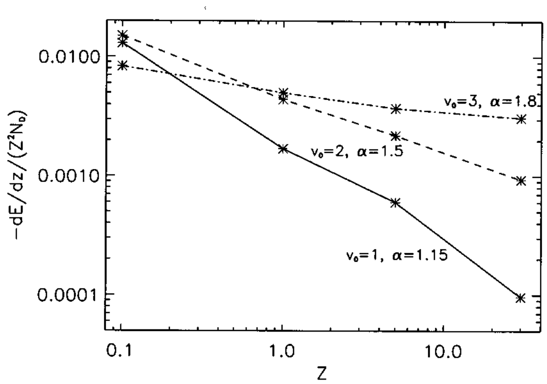

FIG. 5. The scaling of the normalized stopping power $-dE/dz \sim Z^{\alpha}$ for different v_0.

tric theory and the oscillator model. Results for $Z=0.1$, 1, 5 are obtained with the Vlasov simulations, while $Z=30$ are PIC results. Here we have defined the stopping power as the mean value in time. Within the dielectric theory the normalized stopping force is independent of Z for a given velocity v_0, as shown in Sec. III B. The simulations show that this is not the case. For $Z=0.1$ and $v_0<4$ the stopping power can be approximated by Eq. (15) if we choose $k_{\max}=2$. For increasing Z and for $v_0 \leqslant 4$ the normalized stopping power is considerably lower than the result predicted by the dielectric theory. In this regime it is not possible to fit the stopping power by using a Z dependent $k_{\max}$ in the dielectric theory. On the other hand, for $v_0>4$ the normalized stopping force lies slightly higher than predicted by the dielectric theory or oscillator model. This is due to the excitation of nonlinear plasma waves. Figure 5 shows the resulting scaling of the stopping power in Z. Notice that for $v_0=1$ we obtain the linear Z scaling, as described in detail in the previous section, even for $Z<1$. Increasing the velocity we recover the Z^2 scaling. To improve our understanding on the nonlinear stopping power it is worth performing a detailed study of the nonlinear electrostatic potential excited by the projectile. In Fig. 6 the wake field (divided by Z) obtained from Vlasov

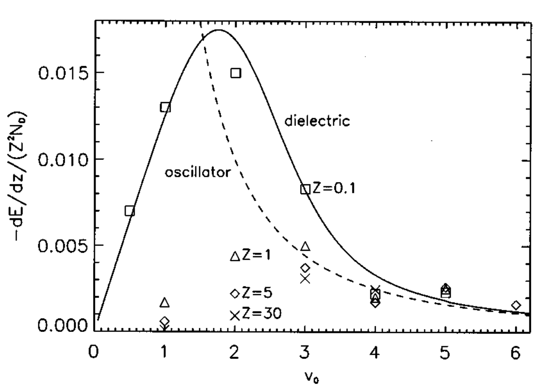

FIG. 4. Comparison of the normalized stopping power (i.e. divided by $Z^2 N_D$) for $Z=0.1$, 1, 5, 30 with the oscillator model and the dielectric theory [$k_{\max}=2$ is set in (15)].

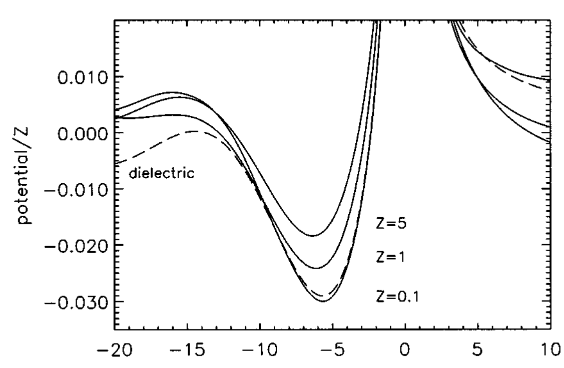

FIG. 6. The total potential divided by Z along the z-axis as calculated from the Vlasov simulation for $v_0=3$ and $Z=0.1$, 1, 5 in comparison with the dielectric theory.

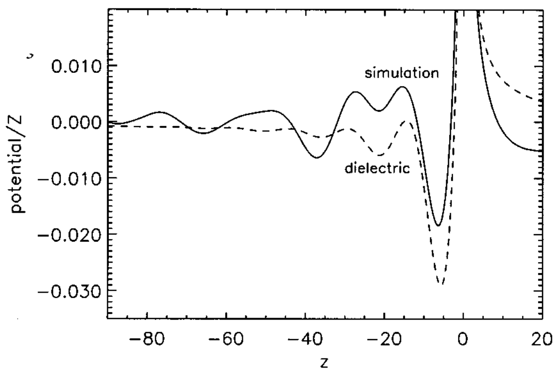

FIG. 7. The total potential divided by Z along the z-axis as calculated from the Vlasov simulation for $Z=5$ and $v_0=3$ in comparison with the dielectric theory.

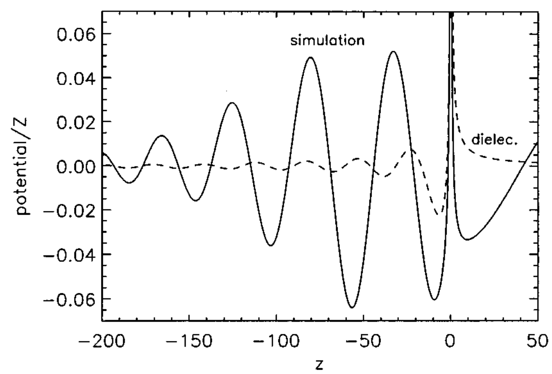

FIG. 9. The total potential divided by Z along the z-axis as calculated from the Vlasov simulation for $Z=5$ and $v_0=5$ in comparison with the dielectric theory.

simulations for $v_0=3$ is shown along the z-axis for different values of Z. We see that the higher the perturbation, the lower the scaling of the induced potential in Z in comparison with the dielectric theory. Only for $Z=0.1$ the first wave hump agrees with the dielectric calculation. Figure 7 shows the wake field for $Z=5$, $v_0=3$ compared with the dielectric theory Eq. (13), always from Vlasov simulations. The ion excites behind itself aperiodic modes in this velocity region. This is due to the absence of a linear damping mechanism. The waves are 'damped' because of the electrons trapped in the excited waves, leading to a steady-state Bernstein–Green–Kruskal (BGK) like wake field.[15] Increasing the ion velocity ($v_0 \geqslant 4$), the wake field amplitudes become much higher than predicted by the dielectric theory. Figures 8 and 9 show the ion wake field for $v_0=4$ and $v_0=5$, where we notice that the absence of an effective damping mechanism due to trapped electrons, which decrease for increasing velocity, leads to a slightly higher stopping power as compared to the linear theory. The potential obtained by the Vlasov simulations shows also a potential energy hump in front of the ion. This is due to the reflection of electrons by the first wave maximum. The wave field acts like a small piston moving through the plasma. The PIC code is not able to reproduce these features (Fig. 10), but the lower damping of the wake field can be clearly seen. It is worth reminding that all these results are not stationary solutions, since a strongly magnetized plasma cannot reach an asymptotic stationary configuration because of the one-dimensional phase space.

VI. SUMMARY AND CONCLUSIONS

The energy loss of heavy ions moving in a classical, collisionless and strongly magnetized plasma has been studied numerically both within the framework of the drift kinetic Vlasov–Poisson system and Particle-in-Cell simulation. We find that for velocities $v_0 \leqslant 4$ the dependence of the stopping power with Z is always weaker than predicted by the usual dielectric theory, even for $Z<1$. Our results show a scaling as $\approx Z^{1.15}$ for $v_0=1$ instead of the Z^2 scaling of the dielectric theory. The lower scaling in Z is due to nonlinear plasma phenomena and can be explained using simple arguments. For $Z \ll 1$ the stopping power can be fitted by the dielectric theory if a Z-dependent $k_{\max}$ is used. In the velocity region $v_0>4$, on the other hand, the wake field and the stopping power are slightly higher than in the dielectric

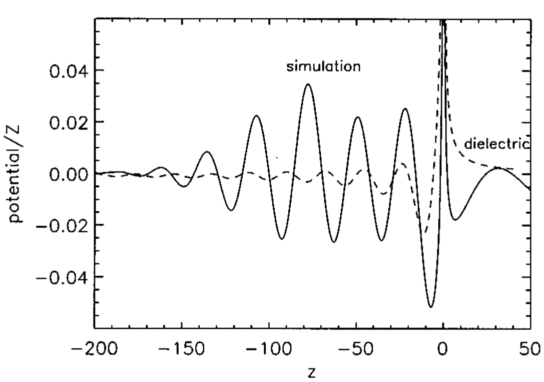

FIG. 8. The total potential divided by Z along the z-axis as calculated from the Vlasov simulation for $Z=5$ and $v_0=4$ in comparison with the dielectric theory.

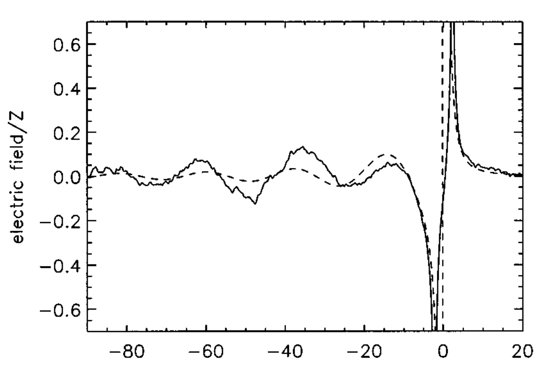

FIG. 10. The electric field divided by Z along the z-axis as calculated from the PIC simulation for $Z=30$ and $v_0=4$ in comparison with the dielectric theory (dashed line).

theory or the oscillator model, but the scaling with Z^2 is recovered. For all Z we find that the stopping power reaches a steady-state due to the electrons trapped in the excited wake field. This steady-state is oscillating in time with a period which is the bouncing period of the trapped electrons, and is found to increase with the ion velocity. For ion velocities which correspond to an efficient trapping process $(v_0 \approx 3)$, BGK like modes are excited by the ion, while for higher velocities $(v_0 > 4)$, when trapping of electrons is less efficient, ions excite large amplitude plasma waves. The strongly enhanced wake field as compared to the dielectric theory is important for the determination of the ion-ion correlations in an electron cooler. We expect that ions with $v_0 > 4$ can interact strongly since the amplitudes of the excited waves are large.

ACKNOWLEDGMENTS

The authors would like to thank Professor P. Mulser from TH Darmstadt, Dr. M. Lontano from IFP-CNR Milano and Dr. I. Hofmann from GSI Darmstadt for valuable discussions. The authors would also like to thank the GSI Computing Center and the Fraunhofer Institut für Graphische Datenverarbeitung in Darmstadt for computer time allocation.

O. B. F. is supported by Bundesministerium für Bildung und Forschung (BMBF), Bonn, Germany. J. D. is supported by ENEA, Frascati, Italy, and has performed part of this work during his stay in GSI Darmstadt.

APPENDIX A: VLASOV SIMULATIONS

The time dependent Vlasov–Poisson system is solved numerically for an ion moving with the constant velocity v_0 through a cylindrical box with a total length L and radius R. We use for the projectile the smoothed potential $\phi_0(r) = Z/(4\pi r)\,\mathrm{erf}(r/b_c)$ with $b_c \ll 1$. The use of such a smoothed potential takes into account the finite grid size. The smoothing factor b_c is sufficiently low if the total stopping power as a function of b_c becomes stationary. This has been extensively tested. A sufficient low b_c also implies that the shape of the total potential stays constant for lower b_c and that for $r \rightarrow b_c$ the Coulomb potential is recovered. The spatial grid is chosen such that $dr, dz \leqslant b_c$. Typical values $(Z=1)$ are $b_c = 0.2$, $dr, dz = 0.15$. In order to resolve the excited wake field $L \gg \lambda$ and $R \gg 1$ (λ_D) must hold. In the simulation $L = 600\lambda_D$ and $R = 18\lambda_D$ is chosen. Concerning the parameters in velocity space, there are two main requirements. The first is to resolve the Maxwell distribution and the oscillations of electrons in the wake field. The second requirement is to resolve the maximum velocity of the electrons for $r \approx b_c$. In the simulation $dv_z, dv_r = 0.125$, $v_{\max} = 6$ is set. Initially a Maxwellian distribution is chosen for the electrons. The Vlasov–Poisson system is advanced using the well known method of fractional steps first described in Ref. 16. This method has been used successfully for the simulation of nonlinear plasma phenomena in two and three phase-space dimensions (see for example Refs. 14, 17, and 18) and can easily be implemented on parallel computers. Let us define $t_n = n\Delta t$ and $t_{n+1/2} = (n + \frac{1}{2})\Delta t$. At each time step the calculation of the distribution function $f(r, z, v_z, t)$ is performed through two main steps:

Step 1:

$$f_1(r, z, v_z, t_{n+1/2}) = f\left(r, z - v_z \frac{\Delta t}{2}, v_z, t_{n+1/2}\right);$$

Step 2:

$$f_2(r, z, v_z, t_{n+1/2}) = f_1(r, z, v_z + E_z(t_{n+1/2})\Delta t, t_{n+1/2}).$$

Performing Step 1 a second time leads to the complete time step. The interpolation is done using an off-center three point interpolation.[19] The Poisson equation is solved at time $t_{n+1/2}$ using a Fast Fourier Transform (FFT) in the z-direction, solving the resulting tridiagonal systems and then performing the inverse FFT. Periodic boundary conditions are used at $z = 0, L$. At $r = 0, R$ reflecting boundary conditions are used for the Vlasov equation and for the Poisson equation $\phi(R) = 0$ is set. The total number of grid points used is $N_r \times N_z \times N_{v_z} = 120 \times 4000 \times 100$. The code is implemented on workstation clusters. This is done by using a master-slave scheme. The phase space is divided into equal partitions along the r-axis. Every slave process handles only one such partition. Before Step 2, the charge density is calculated locally and sent to the master process which solves the Poisson equation. The master sends the calculated potential back to each slave. The public domain software PVM (Parallel Virtual Machine) is used as a message passing library.

APPENDIX B: PIC SIMULATIONS

Our PIC simulation is based on the standard technique of solving the classical equations of motion within a leapfrog method in an $r - z$ geometry (see for example Ref. 20), coupled with a Poisson solver.[21] The 'area weighting' technique is used to deposit the charge on the four nearest grid points in order to obtain the spatial distribution of the simulation particles on the $r - z$ mesh. In the PIC simulation we choose $L \approx 200\lambda_D$ and $R = 18\lambda_D$, and $dz, dr = 0.15$ as in the Vlasov simulations.

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