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托卡马克刮削层等离子体–器壁间过度层中的离子碰撞效应

EFFECTS OF COLLISIONALITY ON PLASMA-WALL TRANSITION IN TOKAMAK SCRAPE-OFF LAYER

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摘　要

在考虑粒子碰撞的情况下，利用二流体模型研究了等离子体与器壁之间的相互作用。首先，应用类似于分析势阱中粒子运动的方法研究了德拜 (Debye) 鞘层判据和边界条件，结果表明，原来用于推导鞘层判据 (Bohm's criterion) 的方法在有些情况下是不适用的，而且求得鞘层判据的新的表示式，其中包含碰撞的影响。此外分析了磁预鞘层 (magnetic presheath) 边界的离子流和预鞘层宽度之间的关系，发现在一定的碰撞频率下，沿磁力线方向的离子速度随预鞘层宽度的增加而减小，其规律类似于指数下降，因此可以用离子速度的e-倍衰减长度 (e-fold length) 定义预鞘层宽度，基于这一定义计算了边界离子流与碰撞之间的函数关系。

关键词：德拜鞘层　磁预鞘层　粒子碰撞
分类号：TL631.24

Effects of Collisionality on Plasma-Wall Transition in Tokamak Scrape-off Layer

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ABSTRACT

With the particle collision taken into account, the interaction of plasma with a fixed wall is studied by using a two-fluid model. Firstly the sheath criterion of plasma and boundary conditions at the Debye sheath edge are investigated by using analogy of the analysis of a particle in a potential well. It is shown that the usual method to derive the Bohm's sheath criterion is not always suitable, and a new approximated formula of sheath criterion, which includes the collision effect, is derived. Secondly the relationship between the starting value of ion flow at the presheath entrance and the presheath length is analyzed. It is found that as the presheath length increases the starting value of the ion velocity along the magnetic field lines bears analogy to exponential decrease for a fixed collisionality. The scale length of magnetic presheath can be defined by the 1/e fold length of ion velocity. Based upon this definition, the ion velocity boundary condition as a function of collisionality is evaluated.

Key words: Debye sheath, Magnetic presheath, Particle collision
Category: TL631.24

INTRODUCTION

In controlled nuclear fusion devices like tokamaks, plasma particles are confined by closed magnetic flux surfaces. Outside the last closed flux surface (LCFS), plasma is in direct contact with a solid wall in the scrape-off layer (SOL). Due to the different mobilities of electrons and ions, in typical cases the wall has a negative potential with respect to the plasma, and it is shielded from the neutral plasma by a positive space charge region (sheath) extended over several electron Debye length λ_{D}. By using a collisionless two-fluid model of the plasma Bohm[1] has derived a condition for existence of the positive boundary sheath, i. e. at the sheath edge the drift velocity of the plasma must be greater than the ambipolar sound speed of the ions, the so-called Bohm's criterion. In the limit of a very small Debye length $\lambda_{\mathrm{D}} / \rho \to 0$, where ρ is the characteristic length of plasma (ion mean free path or plasma extension), Riemann[2] investigated the Bohm criterion in its hydrodynamic and kinetic formulations carefully, and concluded that Bohm's criterion is based on the boundary condition $\mathrm{d}\chi / \mathrm{d}\varsigma \to 0$ for $\varsigma = z / \lambda_{\mathrm{D}} \to -\infty$ (where $\chi = e\phi / T_{\mathrm{e}}$).

In the tokamak discharge conditions, ion collisions in the sheath can be significant and the value $\lambda_{\mathrm{D}} / \rho$ should be finite. In this situation the transition from the nearly quasineutral bulk plasma to the sheath takes place smoothly, there is no strict boundary between the plasma and sheath. However, in order to solve the plasma and sheath problem separately, a common approach in modeling the discharge, one has to specify an appropriate boundary condition. Obviously, the sheath edge has to be chosen in a certain interval within the transition region, where a small space charge density exists. There are a few definitions[3, 4] for the sheath edge of a collisional plasma, and the boundary conditions are deduced for this edge. Unfortunately, the treatments of the problem are rather simple and leads to some mistakes[5]. Here, we will deal with this problem in some detail and in a rather different way.

As Bohm's criterion request that the ions must enter the sheath region with a high directed velocity $V_z \geq c_{\mathrm{s}}$, here c_{s} is the ion acoustic velocity, a quasi-neutral presheath, where a electric field exists, is requested to accelerate the ions to the acoustic velocity perpendicular to the surface at the sheath edge. When a magnetic field at some oblique angle to the solid wall exists in a collisionless plasma, Chodura [6] showed that a magnetic presheath region arises upstream the Debye sheath and found a criterion of the ion speed along the field lines at the "entrance" to the magnetic presheath, i. e. $V_{//} \geq c_s$. Consequently, he postulates an additional "plasma presheath", where the ions are accelerated to the ion sound speed. The function of the magnetic presheath is to turn the plasma flow toward the wall. Riemann[7] addressed the issue allowing for the additional effect of ion collisions. The inclusion of collisions provides a "velocity driver", and the ion acceleration can be accomplished within the magnetic presheath. Then the "plasma presheath" is not required.

The ion velocity at the entrance to the magnetic presheath is an important boundary condition in the models and codes are used for both one and two-demensional SOL analysis[8~10]. In this paper we will discuss this problem in a collisional magnetic presheath in some details.

1 MODEL AND BASIC EQUATIONS

The geometry of the model for plasma-wall transition is shown in Fig.1. The plasma state is assumed to depend on the coordinate perpendicular to the wall only. The electric field $\boldsymbol{E} = -\nabla\phi$ is parallel to the z direction. The magnetic field $\boldsymbol{B}$, which is in the x-z plane, is assumed to be static and homogeneus. The z=0 plane seperates the presheath ($z < 0$) from the sheath ($z > 0$) that extends several Debye length λ_D to the particle absorbing wall. z=−L (which is not well defined) presents the entrance to the magnetic presheath. The motion of ions in the self-consistent electric field and prescribed magnetic field is governed by a fluid model,

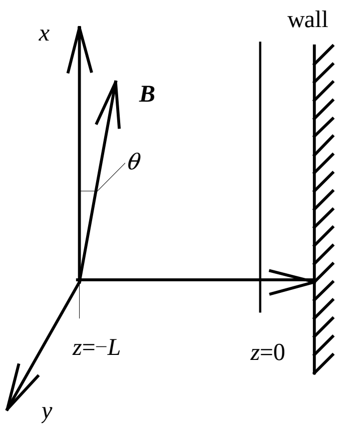

Fig. 1 Geometry for the plasma-wall transition model

The magnetic field vector is in the x-z plane and z=−L indicates the entrance to the magnetic presheath, z=0 the sheath edge.

$$\nabla \bullet (n\boldsymbol{V}) = 0 \tag{1}$$

$$m_{\rm i}\boldsymbol{V}\bullet\nabla\boldsymbol{V}=e(\boldsymbol{E}+\boldsymbol{V}\times\boldsymbol{B})-\frac{1}{n_{\rm i}}\nabla p_{\rm i}-\nu_{\rm c}m_{\rm i}\boldsymbol{V} \tag{2}$$

$$\nabla p_{\rm i}=\gamma T_{\rm i}\nabla n_{\rm i} \tag{3}$$

where $m_{\rm i}$, $T_{\rm i}$ and $n_{\rm i}$ are the ion mass, temperature and density respectively, $\boldsymbol{V}$ is the average ion flow velocity, $\nabla p_{\rm i}=T_{\rm i}\nabla n_{\rm i}$ is the ion pressure gradient, and $\nu_{\rm c}$ is the effective ion collision frequency used to show the plasma collision effects.

Poisson's equation relates the electron and ion densities to the self-consistent electric field:

$$\varepsilon_0\frac{{\rm d}^2\phi}{{\rm d}z^2}=-e(n_{\rm i}-n_{\rm e}) \tag{4}$$

where ε_0 is the vacuum permitivity. The electrons are assumed to be in the thermodynamic equilibrium and their density obeys the Bolzmann relation,

$$n_{\rm e}=n_0\exp(e\phi/T_{\rm e}) \tag{5}$$

where ϕ is the local potential, $T_{\rm e}$ is the electron temperature. n_0 is the electron density at the point where ϕ=0.

2 SHEATH BOUNDARY CONDITIONS

In the narrow sheath region ($\lambda_{\rm D}/L<<1$) the electric field $\boldsymbol{E}$ is dominated, and the magnetic force can be neglected. We consider a plasma consisting of electrons and singly charged positive ions in an immobile neutral gas. It is assumed that the sheath is source-free, and ions enter the sheath as a cold beam with a velocity V_z and experience a collisional drag inside the sheath. The cold ions obey the source free, steady-state equation of continuity,

$$\frac{\rm d}{{\rm d}z}(n_{\rm i}V_z)=0 \tag{6}$$

and motion,

$$m_{\rm i}V_z\frac{{\rm d}V_z}{{\rm d}z}=-e\frac{{\rm d}\phi}{{\rm d}z}-F_{\rm c} \tag{7}$$

here a drag force as the ion fluid travels through the sheath, $F_{\rm c}$ is used to indicate the collision effect, which describes that an ion, on average, losses all its momentum in a collision with a neutral particle. It is given by

$$F_{\mathrm{c}} = m_{\mathrm{i}}(n_{\mathrm{n}}\sigma V_z)V_z \tag{8}$$

where n_{n} is the neutral gas density and σ is the momentum transfer cross section for collisions between ions and neutrals. Ion charge exchange is the main collision mechanism in the sheath, and its cross section is almost constant over the energy range of interest (1～100 eV).

The governing equations can be made dimensionless by using the following notations,

$$\chi = e\phi / T_{\mathrm{e}}, \quad \varsigma = z / \lambda_{\mathrm{D}}, \quad \dot{z} = V_z / c_{\mathrm{s}}, \quad \alpha = \lambda_{\mathrm{D}} / \lambda_{\mathrm{i}} \tag{9}$$

where $\lambda_{\mathrm{D}} = (\varepsilon_0 T_{\mathrm{e}} / n_0 e^2)^{1/2}$, $c_{\mathrm{s}} = (T_{\mathrm{e}} / m_{\mathrm{i}})^{1/2}$ is the ion acoustic velocity (for the cold ion case), and $\lambda_{\mathrm{i}} = 1 / n_{\mathrm{n}}\sigma$ is the mean free path of ions.

After the dimensionless variables are substituted into Eq. (7) and the Poisson's equation, we obtain

$$uu' = -\chi' - \alpha\dot{z}^2 \tag{10a}$$

$$\chi'' = \exp(\chi) - \frac{n_{\mathrm{i0}}}{n_0}\frac{M}{\dot{z}} \tag{10b}$$

where the prime indicates derivative with respect to ζ, n_{i0} is the ion density at the sheath edge, $M = \dot{z}_0 = V_{z0} / c_{\mathrm{s}}$ is the ion Mach number at the sheath edge. To solve these equations boundary conditions must be specified. It is the choice of the boundary conditions based on the definition of the sheath edge that leads to the Bohm criterion and other modified forms to the sheath.

To clarify the approach we start with a collisionless case. In the collisionless limit α=0, Eq. (10a) gives

$$\frac{1}{2}(\dot{z}^2 - M^2) = -(\chi - \chi_0) \tag{11}$$

where χ_0 is the value of χ at the sheath edge. It is known that only the potential difference of two points has physical significance, we can always set $\chi_0 = 0$.

For quasineutral presheath there is no space charge in this region, then $n_{\mathrm{i0}} = n_{\mathrm{e0}} = n_0$. The transition from presheath to sheath should be smooth, so $n'_{\mathrm{i0}} = n'_{\mathrm{e0}}$. Combining Eqs. (1) and (5) yields $\dot{z}'_0 / M = -n'_0 / n_0 = -\chi'$. Substituting it into Eq. (10a) we get

$$(1 - M^2)\chi'_0 = 0 \tag{12}$$

So for a collisionless sheath with a quasineutral presheath, the boundary condition is M=1 or $\chi_0' = 0$.

The Poisson's equation becomes

$$\chi'' = \exp(\chi) - (1 - \frac{2\chi}{M^2})^{-1/2} \tag{13}$$

with the boundary conditions $\chi_0 = 0$, $\chi_0' = 0$ (or M=1). Integrating once, one obtains

$$\frac{1}{2}\chi'^2 + U(\eta, M) = \frac{1}{2}\chi_0'^2 \tag{14}$$

with the Sagdeev potential

$$U(\chi, M) = 1 - \mathrm{e}^{\chi} + M^2[1 - (1 - 2\chi / M^2)^{1/2}] \tag{15}$$

Although Eq. (14) can, in principle, be integrated again, the analysis of the Sagdeev potential $U(\chi, M)$ will give all the necessary information we need [11, 12]. Eq. (14) is analogous to the energy conservation law for a classical particle in the potential well $U(\chi, M)$, with $(\chi_0')^2/2$ being the analogous total energy.

It is well known that for the existence of a positive sheath, $\chi' \le 0$ is needed all over the sheath, which means that χ should monotonically decrease from χ=0 at ζ=0. By analogy of the analysis of a particle in a potential well, it needs a monotonically decreasing function

$$U(\chi, M) \le \chi_0'^2 / 2 \tag{16}$$

Fig. 2 presents the Sagdeev potential $U(\chi, M)$ on the (χ, M) plane. The extreme value of $U(\chi, M)$ is reached at the point

$$\frac{\partial}{\partial\chi}U(\chi, M) = \exp(\chi) - (1 - \frac{2\chi}{M^2})^{-1/2} = 0 \tag{17}$$

It can be deduced from this equation that χ=0 is such a point for all M's. At χ=0, $U(\chi, M)$ reaches its maximum [$U(\chi, M)$=0] for M>1, and minimum for M<1. For M=1, this point is a turning point. It is shown from Fig. 2 that the point where $U(\chi, M)$ reaches its maximum is not always very near to the sheath edge. Thus the usual method to derive the sheath criterion, expanding all relevant functions into Taylor series near the sheath edge, is not suitable. Fig. 3 shows the maximum of $U(\chi, M)$, $U_{\mathrm{m}}\,(\times 10^4)$ in the region $\chi \le 0$ as a function of M. It is obvious that the inequality (16) for $\chi_0' = 0$ requires $M \geq 1$. Thus a proper boundary condition for collisionless sheath with quasineutral presheath is M=1.

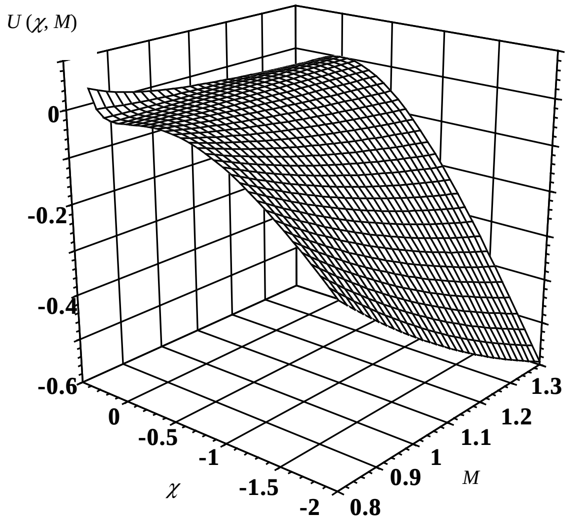

Fig. 2 The Sagdeev potential $U(\chi, M)$ for collisionless sheath

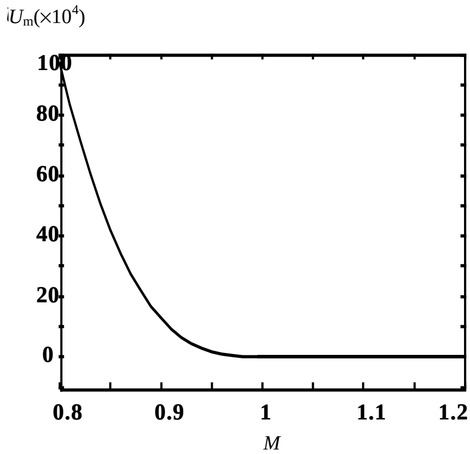

Fig. 3 The maximum (U_{m}) of the Sagdeev potentia l $U(\chi, M)$ for collisionless sheath in the region of $\chi \leq 0$ as a function of M

In the tokamak discharge conditions, the collisionality of ions in the sheath can not be ignored, then Eq. (12) becomes

$$\chi_0' = -\frac{\alpha M^2}{1 - M^2} \tag{18}$$

The Persson's criterion M<1 [3, 13], and the electric field at the sheath edge $\chi_0' \neq 0$ could be obtained immediately from this equation.

The boundary condition $\chi_0 = 0$ is still tenable. Unfortunately, the velocity of ions $\dot{z}$ can not be expressed analytically with sheath potential χ in this situation, and we can not get a explicit Sagdeev potential $U(\chi, M)$. Nevertheless, the function $\dot{z}(\chi, M)$ could be always obtained from the numerical analysis of Eq. (10), and then the Sagdeev potential $U(\chi, M)$ could be calculated. For simplicity, we use a polynomial to simulate the function $\dot{z}(\chi, M)$ based on the least square method in practice. It is found that a second order polynomial would be sufficient to complete this job (because we only need the results not far from the sheath edge). Fig. 4 shows the results, where M=0.5, α=0.1.

The Sagdeev potential $U(\chi, M)$ will also reach its maximum U_{m}>0 , which, however, does not satisfy the inequality (16) in some conditions. Fig. 5 is an example, where the parameters are the same as in Fig. 4. It means that there still exists a lower limit for *M* in this situation. We present the curve of *M*, which satisfies the equality $U_{\mathrm{m}} = \chi_0'^2/2$, as a function of α in Fig. 6 where we also show the

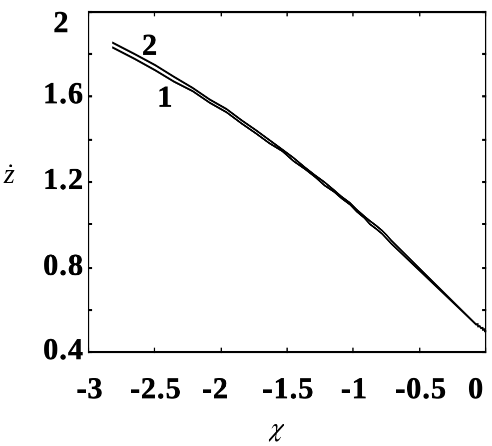

Fig. 4 Ion velocity $\dot{z}$ as a function of potential χ for a collision sheath at M=0.5, α=0.1

Curve 1 is the numerical calculation result, and curve 2 gives the simulation result.

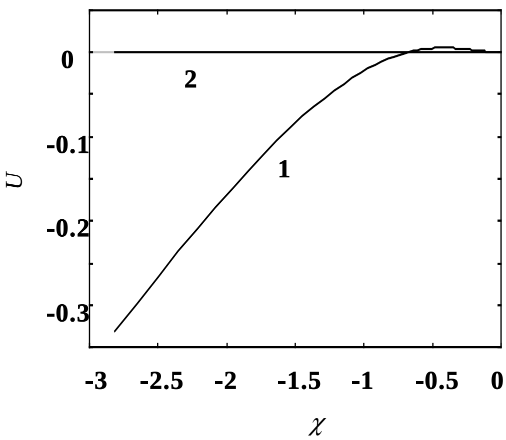

Fig. 5 The Sagdeev potential $U(\chi)$ for a collision sheath at M=0.5, α=0.1 (curve 1)

The analogous total energy $\chi_0'^2/2$ is also shown (curve 2).

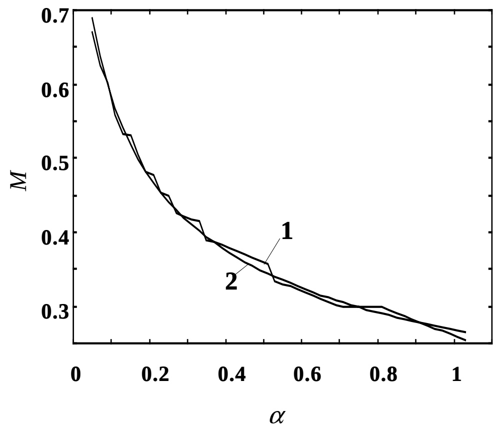

Fig. 6 Mach number M as a function of α that satisfies the equality $U_{\mathrm{m}} = \chi_0'^2/2$ (curve 1) for a collisional sheath.

The simulation results are also shown (curve 2) simulation results with the usual adopted form of modified Bohm sheath criterion[3, 4] $M=(1+c\alpha^{b})^{-1/2}$ based on the least square method. The simulated result is c=12.9, b=0.8.

3 ION FLOW AT THE PRESHEATH ENTRANCE

In the magnetic presheath, it is assumed that the ion flow is isothermal, and the plasma is in the condition of quasi-neutrality (i.e. the plasma on the presheath scae L>>λ_D). For the sake of convenience, we use the plasma frequency ω_{pi} in the undisturbed bulk plasma to define natural measure for time. We introduce the following notations:

$$\delta=\frac{\omega_z}{\omega_{pi}\sqrt{1+\gamma\xi}},\qquad \sigma=\frac{\omega_x}{\omega_{pi}\sqrt{1+\gamma\xi}},\qquad \nu=\frac{\nu_c}{\omega_{pi}\sqrt{1+\gamma\xi}} \tag{19}$$

$$\dot{x}=V_x/c_s,\qquad \dot{y}=V_y/c_s,\qquad \dot{z}=V_z/c_s \tag{20}$$

where $\boldsymbol{\omega}=e\boldsymbol{B}/m_i$, accounting for the ion temperature the ion accoustic velocity $c_s=\sqrt{(T_e+\gamma T_i)/m_i}$, and $\xi=T_i/T_e$ (for cold ions $\xi=0$). With these notations used the dimensionless governing equations are obtained,

$$\dot{z}\dot{x}'=\delta\dot{y}-\nu\dot{x} \tag{21a}$$

$$\dot{z}\dot{y}'=\sigma\dot{z}-\delta\dot{x}-\nu\dot{y} \tag{21b}$$

$$(\dot{z}-\frac{1}{\dot{z}})\dot{z}'=-\sigma\dot{y}-\nu\dot{z} \tag{21c}$$

The dimesionless potential is related to the flow velocity by

$$\chi=\ln(1/\dot{z}) \tag{22}$$

In the tokamak SOL, the toroidal magnetic field is very strong ($\theta<<1$) and $\nu<<1$. Chodura's claim of a sonic plasma flow along the field lines seems reasonable and is usually used as a boundary condition in the SOL analysis[8~10]. But when collision is neglected, we obtain the energy conservation law from Eq. (21),

$$\dot{x}^2+\dot{y}^2+\dot{z}^2=2\ln\frac{\dot{z}}{\dot{z}_L}+u \tag{23}$$

and

$$\dot{x}=(u+\frac{1}{u})\Big/\cos\theta-\frac{\delta}{\sigma}(\dot{z}+\frac{1}{\dot{z}}) \tag{24}$$

Here $u(=V_{//L}/c_s)$ is the ion flow velocity along magnetic field line at the presheath entrance (z=–L), and according to the Chodura's claim[6], there exist the following boundary conditions:

$$\dot{z} \to \dot{z}_L = u\sin\theta \qquad \dot{y}_L = 0 \qquad \dot{x}_L = u\cos\theta \tag{25}$$

Substituting $\dot{x}$ and $\dot{y}$ from Eqs. (24) and (21c) into Eq. (23) yields

$$\dot{z}' = \sigma \frac{\dot{z}}{(1-\dot{z}^2)}\sqrt{f(\dot{z})} \tag{26}$$

with

$$f(\dot{z}) = u^2 + 2\ln(\dot{z}/\dot{z}_L) - \dot{z}^2 - \left[\frac{u+1/u}{\cos\theta} - (\dot{z}+1/\dot{z})\tan\theta\right]^2 \tag{27}$$

The expression shows that the electric field goes to infinitive as approaching to the sheath edge where the sheath boundary condition requires M=1 (Eq.(12)). By integrating Eq. (26) we obtain the solution of Chodura's model,

$$\varsigma = -\frac{1}{\sigma}\int_{\dot{z}}^{1} \frac{1-w^2}{w}\left[f(w)\right]^{-1/2} \mathrm{d}w \tag{28}$$

rsulting in the asymptotic solution: $\dot{z} \to \dot{z}_L$ for $\varsigma \to -\infty$.

As indicated by the analysis above, the neglect of collisions moves Chodura's sonic point to $z \to -\infty$ and with a finite ν the sonic point $V_{\|} = c_s$ is gradually shifted toward the wall, which would lead to difficulties on defining the ion velocity at the boundary in the SOL models.

The presheath edge and ion velocity at the edge could not be well defined for a collisional magnetic presheath. If the presheath boundary moves towards the sheath edge, what results can we get? The sheath criterion, as dicussed in last section, should be satisfied certainly. The other two components of ion velocity have still to be obtained from the presheath analysis. Thus the analysis for the boundary conditions at the entrance to the magnetic presheath are necessary for the SOL models. In the tokamak SOL, generally $\nu << \theta << 1$. The ion flow at the entrance to the magnetic presheath can be considered as being composed of a motion along the magnetic field lines and an $\boldsymbol{E}\times\boldsymbol{B}$ drift as discussed by Riemann[7]:

$$\dot{z}_L = \frac{\delta}{\sigma}\dot{x}_L = u\sin\theta\,, \qquad \dot{y}_L = \frac{\nu}{\delta}\frac{u\cos\theta}{1-[1+(\delta/\sigma)^2]u^2\cos^2\theta} \tag{29}$$

It has been known that the function of the presheath is to accelerate the ion velocity perpendicular to the wall from $\dot{z}_L = u\sin\theta$ at the presheath edge to satisfy the criterion at the sheath edge. There must be a relationship between $\dot{z}_L$ and the presheath length *L* in which the acceleration is accomplished. The relationship can be achieved by numerical evaluation of Eqs. (21) with the initial conditions (Eq. (29)). When $\theta = 6°$ ($B_\mathrm{p} / B_\mathrm{r} \approx 0.1$, where B_p and B_r are the poloidal and toroidal magnetic field, respectively), which is a typical tokamak parameter, curve 1 (for $\nu = 2\times10^{-3}$) and curve 2 (for $\nu = 5\times10^{-3}$) in Fig. 7 are obtained showing the functional relation of the requested ion velocity along the magnetic field at the presheath edge *u* with respect to the presheath length *L*. It is found that as the presheath length increasing, the starting value of the ion velocity along the magnetic field lines bears analogy to an exponential decrease for a fixed collisionality. Thus the scale length of presheath can be determined by the 1/e fold length of *u*, e.g. $L \approx 30\rho_\mathrm{i}$ (where ρ_i is the ion Larmor radius) for $\nu = 5\times10^{-3}$. When the collisionality decreases, the scale length increases. If we define $L \approx 30\rho_\mathrm{i}$, the ion velocity at the presheath edge can be expressed as a function of collisionality as shown in Fig. 8 (curve 1).

When the neutral density near the solid surface is extremely high, e.g. nearby

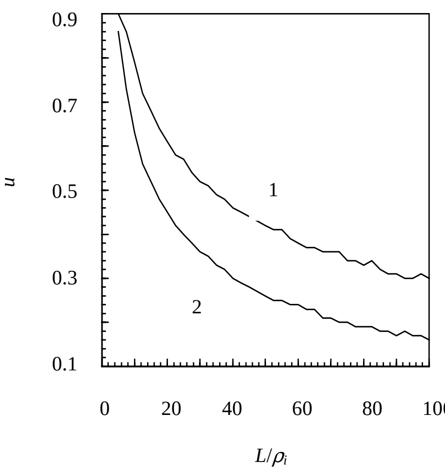

Fig. 7 Ion velocity along the magnetic field at the presheath entrance vs the presheath length for $\nu = 2\times10^{-3}$ (curve 1), and $\nu = 5\times10^{-3}$ (curve 2). $\theta = 6°(B_\mathrm{p} / B_\mathrm{r} \approx 0.1)$.

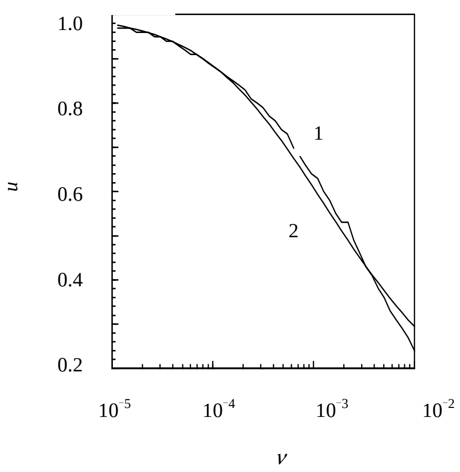

Fig. 8 Ion velocity along the magnetic field at the presheath eentrance as a function of collisionality (curve 1) in the condition of $\theta = 6°$, and $L = 30\rho_i$. Curve 2 is the simulated result.

the target plate of detached divertor, or when the grazing angle of the magnetic field is very small, the above consideration of the ion flow at the entrance (Eq. (29)) could not be hold. In these situations, $\nu \approx \theta << 1$, and the ion flow diverges a little from the magnetic field. It can be assumed that the ion flow in the y direction is just a $\boldsymbol{E} \times \boldsymbol{B}$ drift caused by the local electric field of the presheath, i. e. $\dot{y}_L = \frac{1}{(1+\gamma\xi)\sigma} \frac{\dot{z}'_L}{\dot{z}_L}$. By using Eq. (21c), we obtain the starting values of velocity in the y direction as a function of $\dot{z}_L$:

$$\dot{y}_L = -\frac{\nu}{\sigma} \frac{\dot{z}_L}{(1+\gamma\xi)\dot{z}_L{}^2 - \gamma\xi} \tag{30}$$

Referring to the geometry sketched in Fig. 1, we have

$$u = \dot{z}_L \sin\theta + \dot{x}_L \cos\theta \tag{31}$$

For the case of cold ions, $\xi = 0$, we can express the starting value of the ion velocity in both x and y directions as the functions of $\dot{z}_L$,

$$\dot{x}_L = \frac{\sigma}{\delta} \dot{z}_L \left[1 - \left(\frac{\delta}{\sigma} \right)^2 \frac{1}{\dot{z}_L{}^2} \right], \qquad \dot{y}_L = -\frac{\nu}{\sigma} \frac{1}{\dot{z}_L} \tag{32}$$

The similar procedures as achieving the results shown in Figs. 7 and 8 can be

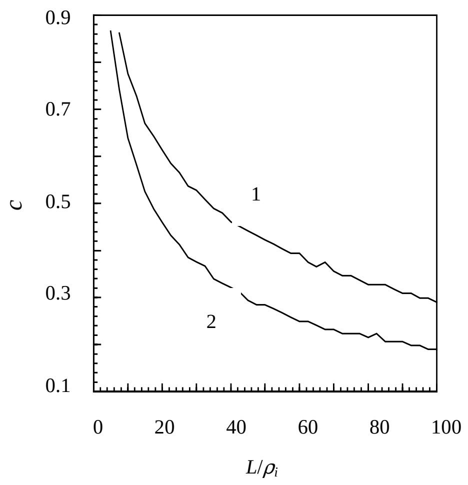

Fig. 9 Ion velocity along the magnetic field at the presheath entrance vs the presheath length for $\nu = 2\times10^{-3}$ (curve 1), and $\nu = 5\times10^{-3}$ (curve 2). $\theta = 6°$

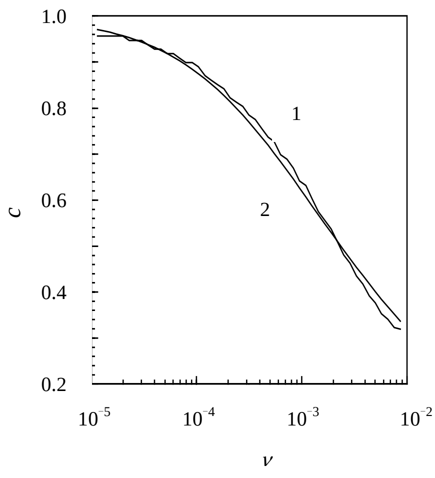

Fig. 10 Ion velocity along the magnetic field at the presheath entrance as a function of collisionality (curve 1) in the case of $\theta = 6°$, and L=30ρ_i; Curve 2 is the simulated result

carried on with the new form of boundary conditions, and the corresponding results are obtained as shown in Figs. 9 and 10. Here the grazing angle is also $\theta = 6°$. It is shown that when the new boundary conditions are used, in order to satisfy the boundary condition at the sheath edge the requested ion velocity along the magnetic field at the presheath edge is nearly unchanged in the case of low collisionality. Nevertheless, it is higher for large collisions. To show the ion collision effect explicitly for the two different boundary conditions, the simulated results (the curve 2 in Fig. 8 and Fig. 10) with the formula $u = (1 + a\nu^{b})^{-1/2}$ by using the least square method are used, which describe the functional relationship of u with respect to the collisionality ν. For the first boundary condition (Eq. (29)), $a \approx 396.5$, and $b \approx 0.79$; but when the second boundary condition is used in the case of $\nu \approx \theta << 1$, then $a \approx$ 251.7, and $b \approx 0.73$.

4 SUMMARY

The effects of collisionality on the plasma-wall transition have been investigated by using a two-fluid model. Bohm's sheath criterion is based on the collisionless sheath and a quasi-neutral presheath. When the ion collision in the sheath is taken into account, a sheath criterion is obtained by using analogy of the analysis of a particle in a potential. It is shown that the usual method to derive the sheath criterion, expanding all relevant functions into Taylor series near the sheath edge, is not always suitable. Under the collisionless conditions, the Bohm's sheath criterion $M \geq 1$ is correct. Although there is no strict boundary between the plasma and sheath for the collisional sheath, a sheath criterion exists for the appearing of a positive boundary sheath based on the definition of the sheath edge. An approximated formula of sheath criterion $M \geq (1 + 12.9\alpha^{0.8})^{-1/2}$ can be used for the collisional sheath situations, where α is used to indicate the collisional effect. It is noted that for collisional sheath the Persson's criterion $M<1$ should also be fulfilled.

When a magnetic field at some oblique angle to the solid surface presents in a collisionless plasma, a magnetic presheath region arises upstream the Debye sheath. The inclusion of collisions provides a "velocity driver", and the ion acceleration can be accomplished within the magnetic presheath. Generally in the tokamak SOL, the ion flow at the entrance to the magnetic presheath can be considered as being composed of a motion along the magnetic field line and an $\boldsymbol{E} \times \boldsymbol{B}$ drift as discussed by Riemann. The relationship between the starting value of the ion flow at the presheath entrance and the presheath length is analysed. It is shown that there is no restriction to the ion flow velocity at the entrance of magnetic presheath if the presheath length has no limit. As the presheath length L increasing, the starting value of ion flow along the magnetic field lines bears analogy to decreasing exponentially with respect to *L*. If we define the 1/e fold length as the scale length of presheath, then the scale length of presheath is a decreasing function against the collisionality.

When the neutral density near the solid surface is extremely high, e.g. nearby the target plate of detached divertor, or when the grazing angle of the magnetic field is very small, Riemann's consideration of ion flow at the presheath edge could not be hold. In these situations, the ion flow diverges a little from the magnetic field. We can assume that the ion flow in the direction perpendicular to the magnetic field is just a $\boldsymbol{E}\times\boldsymbol{B}$ drift caused by the local electric field of the presheath. By using the new form of boundary conditions, the computation results show that the requested ion velocity along the magnetic field at the entrance of presheath is nearly unchanged in the case of low collisionality, but it is higher for large collisions. The simulated results with the formula $u=(1+a\nu^{b})^{-1/2}$ by using the least square method are used, which describe the functional relationship of *u* with respect to the collisionality ν. For the first boundary condition, $a\approx396.5$, and $b\approx0.79$; but when the second boundary condition is used in the case of $\nu\approx\theta<<1$, $a\approx251.7$, and $b\approx0.73$. The analysis demonstrate that the collisionality does affect characteristics of the magnetic preshaeth in the tokamak scrape-off layer.

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