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Bohm Criterion in a Magnetized Plasma Sheath*

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Abstract Bohm criterion in a magnetized plasma sheath is investigated with a fluid model. The upper and lower limits for the sheath criterion in different states of applied magnetic field are studied. The results from numerical simulation reveal that the magnetic field affects significantly the ion Mach number. The variation of the ion Mach number with the incident angle of the ions is also presented.

Keywords: plasma sheath, magnetic field

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1 Introduction

Sheath plays an important role in many plasma processing techniques, such as the modification of the chemical and physical properties of a surface, plasma etching and film deposition. The sheath is a transition layer from the plasma to the solid surface. Ions accelerated by the electric field in sheath are incident upon the solid surface. Elastic and inelastic collisions occur between ions and solid atoms. In most cases, the profiles of plasma parameter in the sheath are determined by the conditions at the plasma-sheath boundary, Bohm criterion. So the study in the Bohm criterion has received a considerable amount of interests [1~22].

RIEMANN and FRANKLIN et al. [1~13] carried out lots of work on the Bohm criterion for collisional and collisionless sheath without an applied magnetic field. Most previous studies [1~9] showed that the electric field at the plasma-sheath interface is defined based on the Bohm criterion. These studies only provide a lower limit of the ion-entering velocity, i.e., the ion velocity entering the sheath must exceed the ion acoustic speed. In 2003, LIU [10] studied the sheath criterion in a collisional sheath with a fluid model, and introduced an upper limit of the Bohm criterion in addition to the lower limit.

So far, the sheath criterion with an applied magnetic field has scarcely been reported [14~18]. In 1982, CHODURA [14] studied the effects of an applied magnetic field on the transition layer between plasma and an absorbing wall based on a kinetic treatment. He concluded that the velocity parallel to the magnetic field must be equal to or greater than the ion acoustic speed. In 1994, RIEMANN [15] worked on the theory of the collisional pre-sheath in an oblique magnetic field (nearly parallel to the wall) with a fluid model. In 1995, STANGEBY [16] developed a new theoretical model to resolve the contradiction arising in the flow condition of the ions at the entrance of the magnetic pre-sheath. He came to the similar conclusions on the sheath criterion. In 2001 VALSAQUE [17] conducted a systematic numerical study for several sheath models, one of which was a magnetized sheath neglecting collision and ionization. Also in 2001, SZIKORA [18] characterized a DC planar magnetron discharge measured by Langmuir probe. Furthermore the Bohm's theory was proved to be valid.

In this paper, a collisional sheath model in an applied oblique magnetic field is established. Based on the theoretical analysis, a sheath criterion is obtained, then both upper and lower limits for the sheath criterion are discussed. Furthermore, the effects of the magnetic field and ion incident angles on the ion Mach number are numerically studied.

2 Sheath model and basic equations

Consider a collisional sheath model, shown in Fig. 1, with one dimension in configuration space and three dimensions in velocity space. The magnetic field, which lies in the (x, z) plane and makes an angle θ with the x-axis direction, is spatially uniform and constant in time. At the edge of the sheath, $x = 0$, the electrostatic potential is set to be zero, $\phi = 0$.

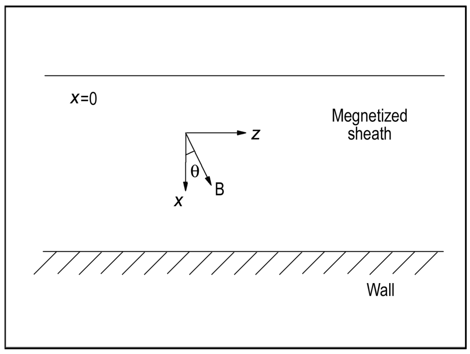

Fig.1 Geometry for the magnetized plasma sheath model

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The electrons are assumed to be in a thermal equilibrium state, thus the density n_e takes the form of the Boltzmann distribution [15~17]

$$n_e = n_{e0}\exp(\frac{e\phi}{T_e}), \quad (1)$$

where T_e (eV) is the electron temperature.

The ion's flow is treated as a cold fluid governed by the continuity equation and momentum equation

$$\nabla\cdot(n_i\mathbf{v}_i) = 0, \quad (2)$$

$$m_i(\mathbf{v}_i\cdot\nabla)\mathbf{v}_i = -e\nabla\phi + e\frac{\mathbf{v}_i\times\mathbf{B}}{c} - m_i(n_n\sigma v_i)\mathbf{v}_i, \quad (3)$$

where n_i, v_i and m_i are the ion density, ion velocity, and ion mass, respectively. n_n is the neutral gas density. $\sigma = \sigma_s(v_i/c_{is}^{\beta})$ is the cross section of momentum-transfer for ion-neutral collision. σ_s is the cross section, and $c_{is} = (T_e/m_i)^{1/2}$ is the ion acoustic speed. β is an index and set as that $\beta = 0$ for constant ion mean-free path and $\beta = -1$ for constant ion mobility.

Finally, the system is completed with the Poisson equation:

$$\frac{\partial^2\phi}{\partial x^2} = -4\pi e(n_i - n_e). \quad (4)$$

For simplicity, a normalization is taken as $\Phi = -e\phi/T_e$, $\xi = x/\lambda_D$, $\mathbf{u}_i = \mathbf{v}_i/c_{is}$, $N_e = n_e/n_{e0}$, $N_i = n_i/n_{i0}$ and $\alpha = \lambda_D n_n\sigma_s$. $\omega_{ic} = eB/m_ic$ is the ion gyro frequency and $\lambda_D = (T_e/4\pi n_{e0}e^2)^{1/2}$ is the electron Debye length. At the sheath edge $x = 0$, from quasi-neutral condition, we have $n_{e0} = n_{i0}$. All the functions are assumed to vary only in the direction normal to the wall, $\nabla \to (\partial/\partial x)\hat{x}$

Taking $\hat{B}_0 = \hat{x}\cos\theta + \hat{z}\sin\theta$, then Eqs. (1)~(4) can be written as

$$N_e = \exp(-\Phi), \quad (5)$$

$$N_i = \frac{M_i}{u_{ix}}, \quad (6)$$

$$u_{ix}\frac{\partial u_{ix}}{\partial\xi} = \frac{\partial\Phi}{\partial\xi} + \gamma_i u_{iy}\sin\theta - \alpha u_{ix}^{2+\beta}, \quad (7)$$

$$u_{ix}\frac{\partial u_{iy}}{\partial\xi} = \gamma_i(u_{iz}\cos\theta - u_{ix}\sin\theta) - \alpha u_{iy}^{2+\beta}, \quad (8)$$

$$u_{ix}\frac{\partial u_{iz}}{\partial\xi} = \gamma_i(-u_{iy}\cos\theta) - \alpha u_{iz}^{2+\beta}, \quad (9)$$

$$\frac{d^2\Phi}{d\xi^2} = N_i - N_e. \quad (10)$$

In Eq. (6) $M_i = v_{ix0}/c_{is}$ is the ion Mach number, and in Eq. (7) $\gamma_i = \omega_{ic}/\omega_{pi}$ is the ratio of ion gyro frequency to ion plasma frequency, $\omega_{pi} = (4\pi n_{i0}e^2/m_i)^{1/2}$.

At the edge of the sheath $x = 0$, $\Phi \to 0$, $N_i \to 1$, and $d\Phi/d\xi = E_0 \neq 0$. From Eqs. (5)~(7) and (10) one obtains

$$M_i^2 \geq 1 + \frac{\gamma_i u_{iy0}\sin\theta - \alpha M_i^{2+\beta}}{E_0}. \quad (11)$$

Since there is a neutral drag to the ions, and the boundary condition at the sheath edge $x = 0$: $\partial u_{ix}/\partial\xi \geq 0$, $d\Phi/d\xi = E_0 > 0$, from Eqs. (7) and (11), one can find

$$(\frac{E_0 + \gamma_i u_{iy0}\sin\theta}{\alpha + E_0})^{1/2} \leq M_i \leq (\frac{E_0 + \gamma_i u_{iy0}\sin\theta}{\alpha})^{1/2}$$

$$(\beta = 0),$$

$$[(1 + \frac{\alpha^2}{4E_0^2} + \frac{\gamma_i u_{iy0}\sin\theta}{E_0})^{1/2} - \frac{\alpha}{2E_0}]$$

$$\leq M_i \leq \frac{E_0 + \gamma_i u_{iy0}\sin\theta}{\alpha} \quad (\beta = -1). \quad (12)$$

It is clearly shown that γ_i, θ and u_{iy0} are three crucial factors to determine the ion Mach number in the collisional sheath under the applied magnetic field. γ_i, is determined by $\mathbf{B}$, indicates the strength of the magnetic field. These relations are the Bohm criterion of a collisional sheath in an applied magnetic field.

3 Discussion

For a collisionless sheath without an applied magnetic field ($\gamma_i = 0$, $\sin\theta = 0$, $\alpha = 0$), from Eq. (11), one gets the well-known Bohm criterion

$$M_i^2 \geq 1. \quad (13)$$

It means the ion flow velocity entering the sheath must be equal to or greater than the ion acoustic speed. It also indicates that the electric field at the boundary should give an appropriate ion velocity entering the sheath.

For a collisional sheath without an applied magnetic field ($\gamma_i = 0$, $\sin\theta = 0$, $\alpha \neq 0$), from Eq. (12), one obtains

$$(\frac{E_0}{\alpha + E_0})^{1/2} \leq M_i \leq (\frac{E_0}{\alpha})^{1/2} \ (\beta = 0),$$

$$\left[(1 + \frac{\alpha^2}{4E_0^2})^{1/2} - \frac{\alpha}{2E_0}\right] \leq M_i \leq \frac{E_0}{\alpha} \ (\beta = -1). \quad (14)$$

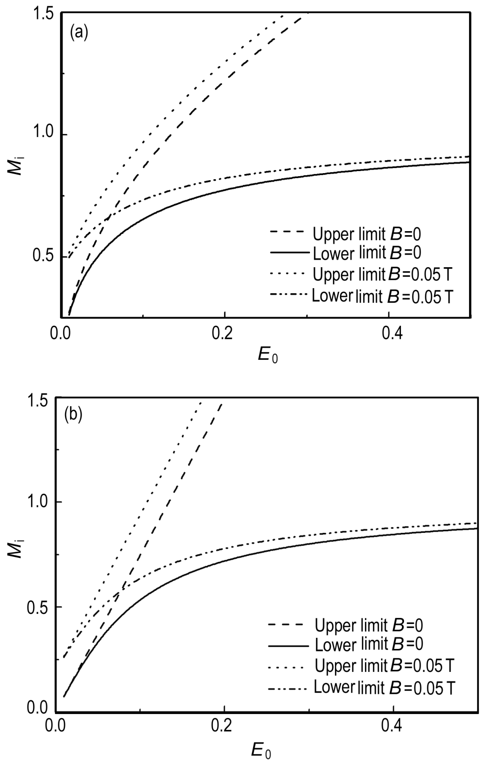
These relations were obtained in LIU's work [10], in which the upper and lower limits for the sheath criterion were found. It is revealed that the electric field at the plasma-sheath boundary determines the sheath criterion.

Here, for a collisional sheath with the applied magnetic field ($\gamma_i \neq 0, \sin\theta \neq 0$, $\alpha \neq 0$), two cases of ion incidence are considered. One is that as ions enter the sheath only along the direction normal to the electrode ($u_{iy0} = 0$), we still obtain Eq. (14). The Bohm criterion for this sheath model is the same as that without an applied magnetic field. Another is that as ions enter the sheath with an oblique incident angle ($u_{iy0} \neq 0$), the range of the ion Mach number is determined by Eq. (12).

In the following numerical simulation, we adopt $\alpha = 0.134$, which corresponds to a neutral pressure of

$p = 13.33$ Pa at a temperature of 290 K. The density of argon plasma is 5.0×10^7 cm^{-3}.

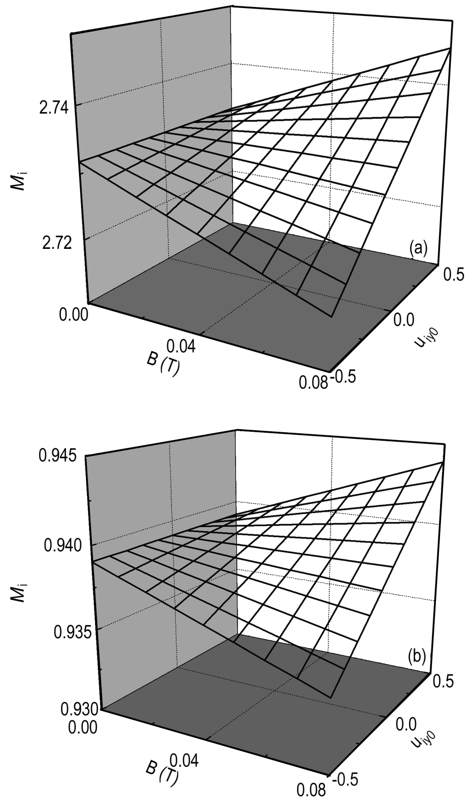
Fig. 2 shows the dependence of the ion Mach number of the collisional sheath varying upon the electric field at the sheath edge for $\beta = 0$ and $\beta = -1$. A comparison for the collisional sheath with and without an applied magnetic field indicates that the applied magnetic field affects significantly the range of the ion Mach number. It is shown that both upper and lower limits of ion Mach number vary with the magnetic field. The electric field at the sheath edge can accelerate ions sufficiently. That is to say, the electric field at the plasma-sheath boundary should give an appropriate ion velocity entering the sheath.



(a) $\beta = 0$, (b) $\beta = -1$

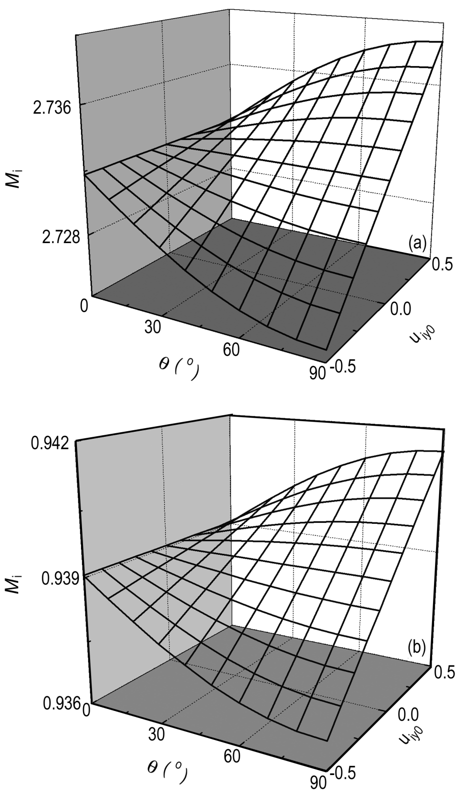
Fig.2 Dependence of ion Mach number upon the normalized electric field at sheath edge at $\theta = 30^{\circ}$ and $u_{iy0} = 0.5$ for different magnetic field applied

The dependences of both upper and lower limits of the ion Mach number upon both magnitude and angle of the magnetic field, as well as the initial velocity of ion flow in the y-axis direction at $\beta = 0$ and $E_0 = 0.2$ are shown in Figs. 3 and 4. As $u_{iy0} > 0$, both the upper and the lower limits of the ion Mach number increase with the increase in magnitude and angle of the magnetic field, which causes the range of the ion Mach number to move above. As mentioned previously [21,22], the reason is that the Lorentz force affects the ion flow velocity in the x-axis direction. As $u_{iy0} < 0$, the result is on the contrary. Under the same conditions, Lorentz force can not only speed up but also slow down the ion flow in the x-direction related to the directions of the ion velocity. As a result, the range of the ion Mach number can move down also.



(a) Upper limit, (b) Lower limit

Fig.3 Limit of ion Mach number as a function of B and u_{iy0} at $\theta = 30^{\circ}$



(a) Upper limit, (b) Lower limit

Fig.4 Limit of ion Mach number as a function of θ and u_{iy0} at $B = 0.02$ T

4 Conclusions

In conclusion, by using a fluid model for the collisional sheath, the sheath criterion with an applied oblique magnetic field is studied. It is shown that the electric field at the plasma-sheath boundary determines the sheath criterion. There exists upper and lower limits in the Bohm criterion, and both limits depend upon the magnetic field conditions. Also due to the effects of Lorentz force on the ion velocity, the ion Mach number varies with the incident angle of the ions. Thus the model, though with some limitations, can contribute to the understanding of the sheath criterion in a magnetized plasma.

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