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The dynamics of ions entering the magnetized plasma sheath obliquely – collisional and collisionless situations

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Abstract. The characteristics of ions that enter the plasma sheath with an oblique incident angle have been investigated in the presence of an external magnetic field. The ion dynamics in a collisional and collisionless magnetized plasma sheath have been numerically calculated by using a fluid model. Several values for the ion velocity at the sheath edge, orientation and strength of the magnetic field and the ion-neutral collision frequency have been considered. The results show that in a collisionless magnetized plasma sheath, the behaviour of ions that obliquely enter the sheath with some specific velocities at the sheath edge and at some specific orientations and strengths of magnetic field, is more complicated than that of ions with normal entrance angles. For the oblique entrance of ions, the weak magnetic fields cause some fluctuations in ion velocity around its boundary value, i.e. the ion velocity does not accelerate. However, the numerical calculations show that the ion dynamics in a collisional magnetized plasma sheath are the same for both normal and inclined entrance of ions into the sheath.

1 Introduction

Recently, there has been considerable interest in studying the electrodynamic properties of a plasma sheath, which is a space charged region that separates plasma from a metallic boundary [1–6]. The effects of a magnetic field and ion-neutral collisions on the dynamics of ions in a plasma sheath have been investigated in some correlative works in recent years. Most of these works have studied the dynamics of ions that perpendicularly enter the sheath through its boundary [7–22]. It is shown that the structure of a plasma sheath in an external magnetic field is different from that of a non-magnetized plasma sheath. This characteristic truly depends on the orientation and strength of the magnetic field [6,7]. The effect of collisions between ions and neutral particles are also investigated in magnetised and non-magnetised plasma sheath [10–13].

Zou et al. have shown in their new article that the effects of an external magnetic field are more complicated when the ions enter the collisionless sheath with an oblique incident angle [6]. Here, we study the dynamics of ions using a fluid model for different ion velocities at the plasma sheath edge. We also investigate the effect of ion-neutral collisions on the magnetised plasma sheath in this case.

The layout of the paper is as follows: in Section 2, we introduce the basic equations of the fluid model for ion movement in a plasma sheath by considering the electromagnetic force and ion-neutral collisions. We will also normalize the equations by using some dimensionless parameters. The normalized equations will be solved numerically in Section 3 and some characteristics of the ion will be investigated for strong and weak magnetic fields by using the numerical results.

2 Sheath model and basic equations

Following some correlative works [6,7,9,10], we consider a plasma sheath in contact with a planar wall in the presence of an external magnetic field. We also consider the z-direction as the depth direction from the sheath boundary ($z = 0$) to the planar wall. The geometry of the magnetic sheath is illustrated in Figure 1. As presented in the figure, it is assumed that the magnetic field is embedded in the xz plane with an angle θ in the z-direction. In addition, it is assumed that the sheath characteristics change only with depth, i.e., the plasma sheath has a one-dimensional coordinate space. However, as the magnetic field changes the direction of the ion velocity, it can be considered as a plasma sheath with three-dimensional speed space.

The sheath consists of isothermal electrons, which are assumed to be in thermal equilibrium and obey the Boltzmann relation as follows:

$$n_e = n_0 \exp(e\phi/k_B T_e), \tag{1}$$

where n_e is the local electron density, ϕ is the local potential and T_e is the electron temperature. At the sheath

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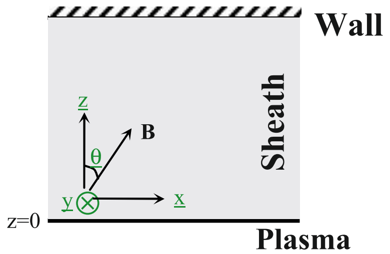

Fig. 1. Geometry of the magnetic sheath.

edge, $z = 0$, the electron density is n_0 and the electrostatic potential is taken to be zero, $\phi(z = 0) = 0$.

The ions obey the equations of continuity and momentum as follows:

$$n_i(z)v_z(z) = n_{i0}v_{z0}, \tag{2}$$

$$v_z \frac{d\vec{v}}{dz} = -\frac{e}{m_i}\frac{d\phi}{dz}\vec{k} + \vec{v} \times \omega_c - \nu_i\vec{v}, \tag{3}$$

where m_i, n_i and $\vec{v}$ are mass, local density and velocity of the ion respectively, n_{i0} and v_{z0} are the density and the ion velocity in the depth direction at the sheath edge respectively, $\omega_c = eB/m_i$ is the ion cyclotron angular velocity and $\vec{k}$ is a unit vector in the depth direction. The two first terms in equation (3) denote the electromagnetic force and the third term corresponds to ion-neutral collisions where ν_i is the effective ion collision frequency. In general, ν_i is considered to have a power law dependency on the ion velocity as follow [21]:

$$\nu_i = \frac{\alpha}{\lambda_{\mathrm{D}}} c_s \left(\frac{v}{c_s}\right)^{p+1}, \tag{4}$$

where $\lambda_{\mathrm{D}} = (\varepsilon_0 K_B T_e/n_0 e^2)^{1/2}$ is the electron Debye length, $c_s = (K_B T_e/m_i)^{1/2}$ is the ion acoustic velocity and p and α are dimensionless parameters. p ranges from 0 to -1 and $\alpha = \lambda_{\mathrm{D}} n_n \sigma_s$ where n_n is the neutral gas density and σ_s is the collision cross section measured at ion acoustic velocity.

The model is completed by Poisson equation as follows:

$$\frac{d^2\phi}{dz^2} = e(n_e - n_i)/\varepsilon_0. \tag{5}$$

Here we introduce some dimensionless variables, which are used for the numerical solution of equations (1)–(5):

$$\eta = -e\phi/k_B T_e,\ \xi = z/\lambda_{\mathrm{D}},\ \vec{u} = \vec{v}/c_s,$$
$$N_e = n_e/n_0 \text{ and } N_i = n_i/n_0. \tag{6}$$

Substituting these dimensionless parameters into equations (1) and (2), we have

$$N_e = \exp(-\eta), \tag{7}$$
$$N_i = M/u_z, \tag{8}$$

where M is the ion Mach number, which is the boundary value (at the sheath edge) of the dimensionless velocity in the depth direction.

Equation (3), which is three equations in x, y and z directions, can be expressed by dimensionless parameters as follows:

$$u_z\partial_\xi u_x = \gamma\cos\theta\, u_y - \alpha\, u^{p+1}u_x, \tag{9}$$

$$u_z\partial_\xi u_y = \gamma\sin\theta\, u_z - \gamma\cos\theta\, u_x - \alpha\, u^{p+1}u_y, \tag{10}$$

$$u_z\partial_\xi u_z = \partial_\xi\eta - \gamma\sin\theta\, u_y - \alpha\, u^{p+1}u_z, \tag{11}$$

where ∂_ξ denotes $d/d\xi$, $\gamma = \lambda_{\mathrm{D}}/\rho_i$, and $\rho_i = c_s\tau = (T_e m_i/e^2B_0^2)^{1/2}$ is the ion grain gyro radius.

By using equations (7) and (8) and the dimensionless parameters, the Poisson equation can be expressed as

$$\partial_\xi^2\eta = M/u_z - \exp(-\eta). \tag{12}$$

N_e, N_i, η and $\vec{u}$ can be determined from equations (7)–(12) as a function of depth. The numerical results are listed in the next section.

2.1 Bohm's criterion

Here we will obtain Bohm's criterion for a collisional magnetized plasma sheath. By this criterion, the ion critical Mach number can be determined, which is the condition that the ion can enter the collisional magnetized plasma sheath. In order to find the Mach number we integrate equation (12), which leads to:

$$(\partial_\xi\eta)^2 + 2V = (\partial_\xi\eta)^2\Big|_{\xi=0}, \tag{13}$$

where

$$V = 1 - e^{-\eta} - \int_0^\eta \frac{M}{u_z}d\xi. \tag{14}$$

From equation (14) we have

$$\partial_\eta^2 V = \frac{M}{u_z^2}\frac{\partial_\xi u_z}{\partial_\xi\eta} - e^{-\eta} = -\frac{dN_i}{d\xi} - e^{-\eta}. \tag{15}$$

The Mach number can be found by adopting $\partial_\eta^2 V\big|_{\xi=0} \leqslant 0$ which corresponds to the maximum value point of the Sagdeev potential at the sheath edge [20,23]. Using $\partial_\eta^2 V\big|_{\xi=0} \leqslant 0$, we get

$$[dN_i/d\eta + \exp(-\eta)]_{\xi=0} \geqslant 0. \tag{16}$$

By using equations (8) and (11), the first term of the inequality (16) can be expressed as:

$$\left.\frac{dN_i}{d\eta}\right|_{\xi=0} = \frac{1}{M^2}\left(\frac{1}{\eta_0'}(\gamma\sin\theta u_{y0} + \alpha u_0^{(p+1)}M) - 1\right), \tag{17}$$

where $\eta_0' = d\eta/d\xi|_{\xi=0}$. Using equation (17) and inequality (16) we can get the critical Mach number as follows:

$$M^2 + \frac{\alpha}{\eta_0'}M(M^2 + u_{y0}^2 + u_{x0}^2)^{(p+1)/2} + \left(\frac{\gamma\sin\theta u_{y0}}{\eta_0'} - 1\right) \geqslant 0. \tag{18}$$

In specific cases we have:

(a) for a collisionless non-magnetised plasma sheath:

$$\alpha = 0\,,\, \gamma = 0 \rightarrow M^2 \geqslant 1, \tag{19}$$

i.e. the ion velocity at the sheath edge must be greater than the ion sound speed as reported in reference [7];

(b) for a collisionless magnetised plasma sheath:

$$\alpha = 0\ \rightarrow M^2 \geqslant 1 - \frac{\gamma \sin\theta u_{0y}}{\eta_0'}. \tag{20}$$

Relation (20) is the same as equation (19) for $u_{y0} = 0$. This means that if the velocity of ions at the sheath edge is embedded in the xz-plane, then Bohm's criterion for the collisionless magnetised plasma sheath will be the same as the non-magnetised sheath [6];

(c) for a collisional non-magnetised plasma sheath:

$$\gamma = 0 \rightarrow M^2 + \frac{\alpha}{\eta_0'} M(M^2 + u_{y0}^2 + u_{x0}^2)^{(p+1)/2} - 1) \geqslant 0, \tag{21}$$

which for the entrance of ions normal to the sheath boundary and in two specific cases (constant collisional mobility ($p = 0$) and constant collision frequency ($p = -1$)) leads to

$$p = 0 \rightarrow M \geqslant \sqrt{\frac{\eta'_0}{\alpha + \eta_0'}}$$
$$p\ =\ -1 \rightarrow M \geqslant \sqrt{1 + \frac{\alpha^2}{4\eta_0'^2}} - \frac{\alpha}{2\eta_0'}, \tag{22}$$

which are the same as reference [20].

3 Numerical calculation

Here we solve the basic equations of the model, equations (7)–(12), numerically. Some typical dimensionless parameters of the numerical calculation are employed as follows: $\eta(z = 0) = 0\,,\, \eta'(z = 0) = 0.01$ and $u_{y0} = 0$. Considering $u_{y0} = 0$ enables us to fix the Mach number in all numerical calculations as $M = 1$, which satisfies the inequality (18). Other parameters are mentioned in each numerical calculation.

Fluctuations of the ion flow velocity in the magnetic sheath have been investigated in some correlative works in the case of normal entrance of the ions to the sheath boundary. Figure 2a shows these fluctuations versus the depth direction and the ion velocity in the x-direction at the sheath edge in a collisionless magnetised plasma sheath. Here we have adopted $\theta = 40$, $\gamma = 5$ and $\alpha = 0$. As presented in the figure, the fluctuations diminish with increasing u_{x0}. The variations of electrostatic potential in this case are shown in Figure 2b. The figure shows that the electrostatic potential decreases smoothly with increasing u_{x0}. These behaviours are reasonable due to the fact that with increasing magnitude of u_{x0}, the angle between the magnetic field and the ion flow velocity at the sheath edge decreases, which leads to a reduction of the effect of the magnetic field on the ion characteristics [7].

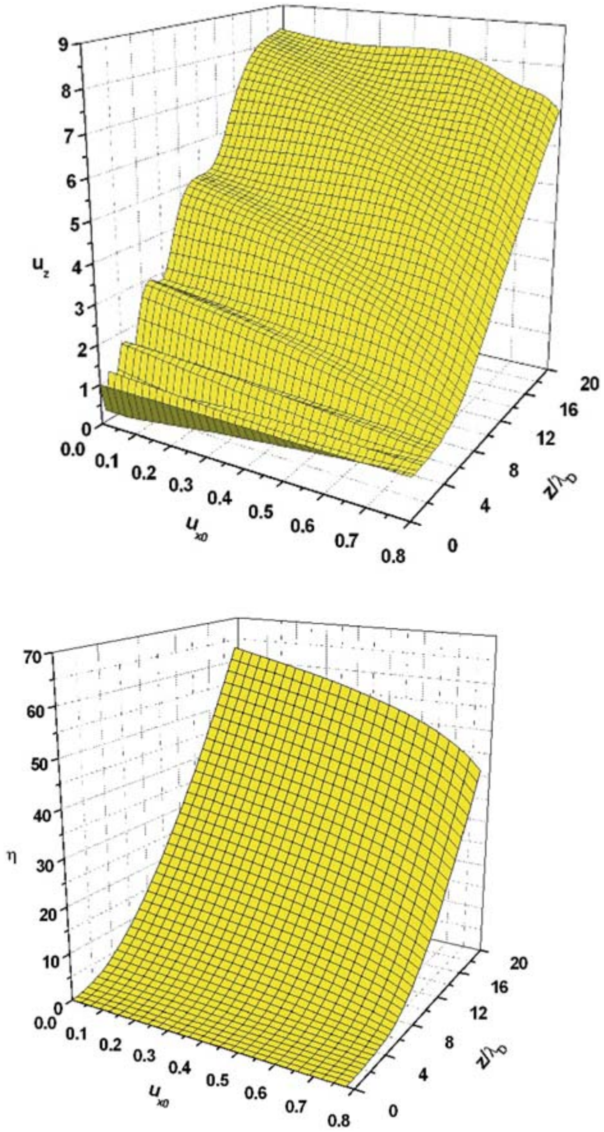

Fig. 2. (Color online) The ion flow velocity in the depth direction (u_z), and the electrostatic potential (η) in a collisionless plasma sheath for different values of u_x at the sheath edge ($\theta = 40°$, $\gamma = 5$, $M = 1$ and $u_{y0} = 0$).

Figure 3 depicts the effects of ion-neutral collisions on the characteristics of the ions that enter the sheath with an oblique incident angle $\mathbf{u}_0 = (0.2,\ 0,\ 1)$. These effects have been investigated for normal entrance of the ions in several works such as [9,10,22]. The results show that the effect of collisions on the dynamics of the ions that enter the sheath with an oblique incident angle is the same as the case in which the ions enter the sheath perpendicular to the sheath boundary. The collision damps the fluctuations in the ion velocity, and the fluctuations rapidly diminish with increasing distance from the sheath edge. As the figure shows, the effects of collisions are more apparent for a constant collision cross section.

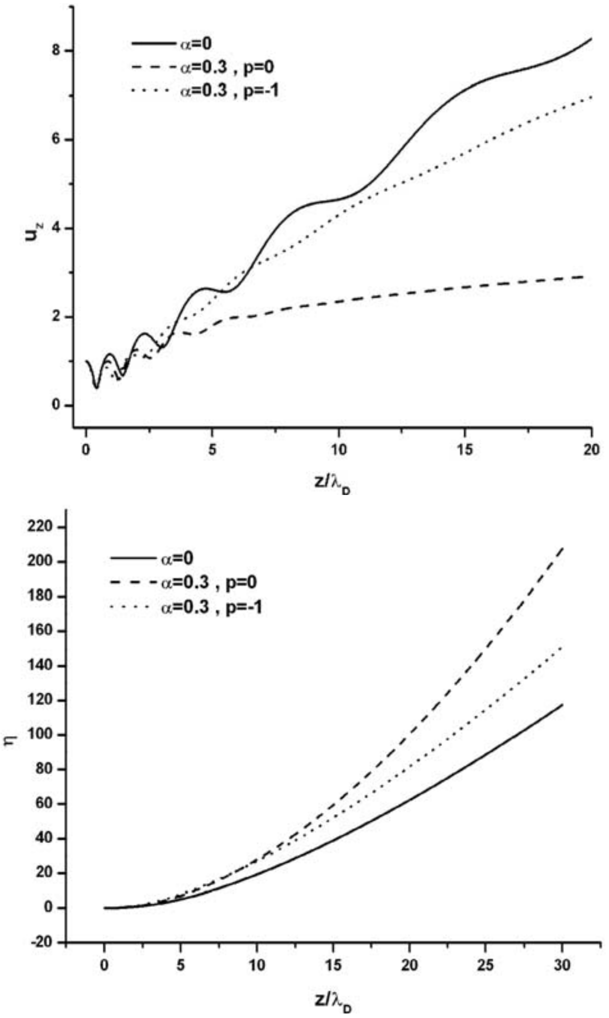

Fig. 3. The effects of ion-neutral collisions on ion characteristics for oblique entrance of ions into the sheath ($u_{x0} = 0.2$, $u_{y0} = 0$, $u_{z0} = 1$, $\gamma = 5$, $\theta = 40$).

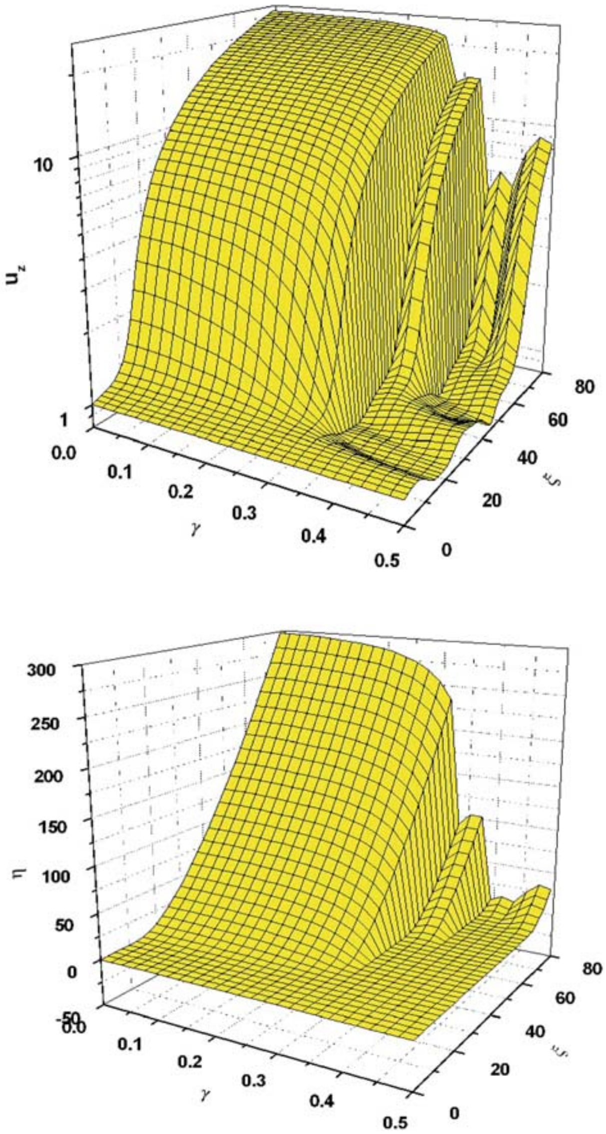

Fig. 4. (Color online) The ion flow velocity in the depth direction and the electrostatic potential for various strengths of magnetic field when ions enter the sheath with an inclined angle $\mathbf{u} = (0.2,\ 0,\ 1)$ ($\theta = 10^\circ$ and $\alpha = 0$).

The results of Figures 2 and 3 show that for strong magnetic fields in the x-direction, which causes fluctuations in the ion flow velocity, the ion characteristics in the sheath are the same for small and zero values of u_{x0}.

Now we are going to investigate the ion characteristics in weak magnetic fields in the x-direction. We have considered two cases: small θ angle and small values of γ. It must be mentioned that the dynamics of the ions that enter the sheath perpendicular to its boundary are the same as those entering the non-magnetised plasma sheath.

Figure 4 shows the ion velocity and electrostatic potential with depth direction versus different magnitudes of magnetic field for $u_{x0} = 0.2$, $u_{y0} = 0$, $u_{z0} = 1$, $\theta = 10^\circ$ and $\alpha = 0$. As illustrated in the figure, for small values of γ the ion characteristics are the same as those for $u_{x0} = 0$. However, for specific values of γ (approximately $\gamma > 0.3$) the dynamics of ions are completely different in comparison with the normal entrance of the ions. In these values, the ion flow velocity in the depth direction fluctuates around the boundary value for $\zeta = 0 - \zeta \cong 50$, i.e. the weak magnetic field prevents the increasing of ion velocity. Considering relation (8), it means that for specific values of u_{x0}, θ and γ, the ions gather and the sheath distance increases.

Figure 5 depicts the ion flow velocity in the depth direction for a specific magnetic field ($\gamma = 0.5$ and $\theta = 10$) and different values of u_{x0}. The figure shows that the fluctuations of u_z around the boundary value, complicatedly depend on the boundary value of the ion velocity and the magnitude and the orientation of the magnetic field.

In numerical calculations, which are shown in Figures 4 and 5, we have considered an inclined entrance of ions into a collisionless plasma sheath by adopting $\alpha = 0$. These properties for the inclined entrance of ions into a collisional plasma sheath are investigated in Figure 6. The figure shows the ion velocity in the depth direction in a

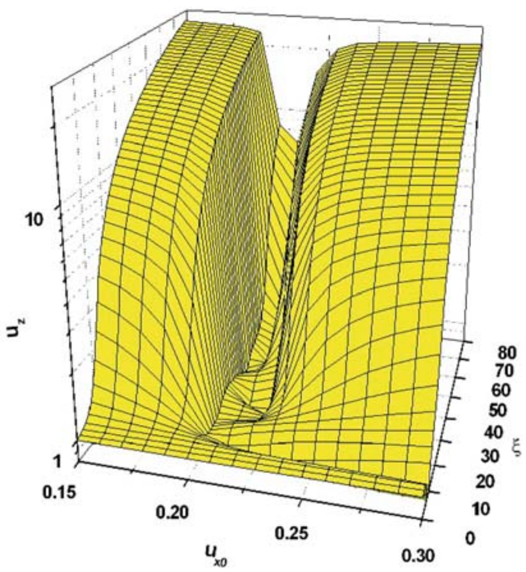

Fig. 5. (Color online) The variations of u_z versus depth direction and different values of u_{x0} ($\gamma = 0.5$, $\theta = 10$ and $\alpha = 0$).

collisional magnetised plasma sheath for $\gamma = 0.02$, $\theta = 20°$ and $\alpha = 0.05$ for normal and inclined entrance of the ions. The results show that the gathering of ions does not occur for a weak collision frequency (corresponding to α value approximately greater than 0.002). Similar numerical calculations have shown that the ion dynamics for an inclined entrance of ions into a collisional plasma sheath are the same as that of a normal entrance. To have a better understanding of this fact, we have calculated the ion velocity for an inclined entrance, $u_0 = (0.3,\ 0,\ 1)$, and normal entrance, $u_0 = (0,\ 0,\ 1)$, versus various α values. Figures 6a and 6b show that the ion velocity in the collisionless sheath is completely different for the normal and inclined entrance of ions. However, for weak ion collision frequency, corresponding to $\alpha > 0.002$, the ion velocity in the depth direction for an inclined entrance is the same as that of the normal entrance.

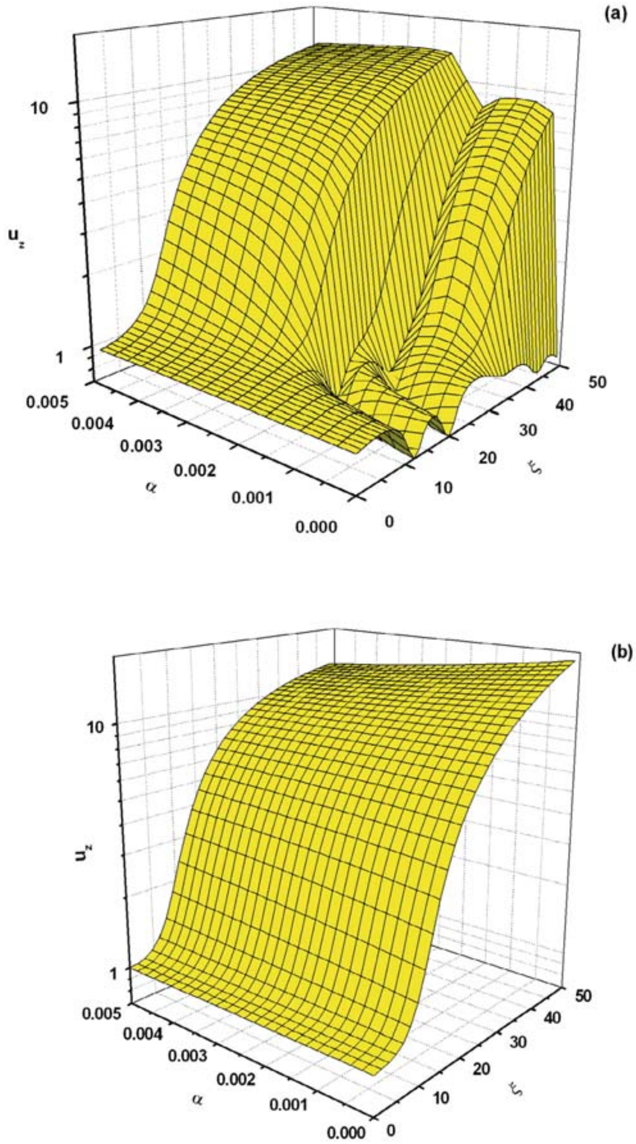

Fig. 6. (Color online) The ion velocity in the depth direction for entrance of ions with an oblique incident angle, $u_0 = (0.3,\ 0,\ 1)$; (a) normal to the sheath boundary, $u_0 = (0,\ 0,\ 1)$; (b) versus different α values (γ=0.2, $\theta = 10°$ and $p = 0$).

4 Conclusion

The electrodynamic properties of a magnetised plasma sheath were calculated numerically by using the fluid model for a collisional and collisionless plasma sheath. We investigated the dynamics of ions that enter the magnetised sheath with an oblique incident angle and compared the results with the normal entrance of ions. The numerical results were as follows:

- For a strong magnetic field, the oblique angle of velocity at the sheath edge causes the fluctuations in the ion velocity to be damped.
- For a collisionless weak magnetised plasma sheath, the ions that enter the sheath obliquely behave completely differently in comparison with a normal entrance. These complicated behaviours occur for special values of the strength and orientation of the weak magnetic field and the ion velocity at the sheath edge. In these values, the velocity of the ion does not increase and fluctuates around the boundary value.
- For a weak magnetic field in the case of a collisional sheath, the dynamics of ions that enter the weak magnetised plasma sheath with an oblique incident angle are the same as that of those entering the sheath perpendicular to the boundary, even for a weak collision frequency.

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