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Correlated fast ion stopping in magnetized classical plasma

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Abstract

The results of a theoretical investigation on the stopping power of an ion pair in a magnetized electron plasma are presented, with particular emphasis on the two-ion correlation effects. The analysis is based on the assumptions that the magnetic field is classically strong ($\lambda_B \ll a_c \ll \lambda_D$, where λ_B, a_c and λ_D are respectively the electron de Broglie wavelength, Larmor radius and Debye length) and that the velocity of the two ions is identical and fixed. The stopping power and *vicinage function* in a plasma are computed by retaining two-ion correlation effects and is compared with the results of the individual-projectile approximation. © 1998 Elsevier Science B.V.

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The problem of the interaction of a beam of fast charged particles with a plasma has been attracting more and more interest in the last years in connection with the proposed scheme of inertial confinement fusion based on the use of heavy-ion beams to drive the *DT* target towards the ignition conditions [1].

Several physical situations can be conceived in which the beam ions are closely spaced so that their stopping in the ionized medium is not exactly that of charged particles whose dynamics is independent of the presence of each other, but suffers from mutual correlations [2,3]. It can be the case of very high-density ion beams or, more realistically, when ion clusters are to be used instead of standard ion beams [4].

The nature of experimental plasma physics is such that experiments are usually performed in the presence of magnetic fields, and consequently it is of interest to investigate the effects of a magnetic field on the stopping power. The strong magnetic fields used in laboratory investigations of plasmas can appreciably influence the processes determined by Coulomb collisions [5]. This influence is even more important in white dwarfs and in neutron stars, the magnetic fields on the surfaces of which can attain strengths up to 10^5–10^{10} kG.

The stopping of uncorrelated charged particles in a magnetized plasma has been the subject of several papers [6–9]. The stopping of a fast test particle moving with a velocity u much higher than the electron thermal velocity v_T was studied in Refs. [6,8]. The energy loss of a charged particle moving with arbitrary velocity was studied in Ref. [7]. The expression derived there for the Coulomb logarithm corresponds to the classical description of collisions.

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In Ref. [9] expressions were derived describing the stopping power of a slow charged particle in a Maxwellian plasma with a classically strong (but not quantizing) magnetic field ($\lambda_B \ll a_c \ll \lambda_D$, where λ_B, a_c and λ_D are respectively the electron de Broglie wavelength, Larmor radius and Debye length), under conditions such that the scattering processes must be described quantum-mechanically.

Here we present an evaluation of the stopping power of an ion pair moving in a warm collisionless plasma placed in a classical strong homogeneous magnetic field, with the aim to show under which conditions correlation effects can be important and how they modify the single-particle expression of the stopping power.

A uniform plasma is considered in the presence of a homogeneous magnetic field B_0 which is assumed sufficiently strong so that $\lambda_B \ll a_c \ll \lambda_D$. From these conditions we can obtain $3\times 10^{-6} n_0^{1/2} < B_0 < 10^5$ T (n_0 and T are respectively the unperturbed number density and temperature of the plasma), where n_0 is measured in cm^{-3}, T is measured in eV and B_0 in kG. Because of this assumption, the perpendicular cyclotron motion of the ions and plasma electrons is neglected.

Let us analyze the interaction process of a two-ion system moving in plasma. The interionic vector is $\boldsymbol{l}$. Also, due to the high frequencies involved, the very weak response of the plasma ions is neglected and the Vlasov–Poisson equations to be solved for the perturbation to the electron distribution function, f_1, and the potential φ, are as follows,

$$\left(\frac{\partial}{\partial t} + (\boldsymbol{b}\cdot\boldsymbol{v})(\boldsymbol{b}\cdot\nabla)\right) f_1(\boldsymbol{r},\boldsymbol{v},t) = -\frac{e}{m}(\boldsymbol{b}\cdot\nabla\varphi)\left(\boldsymbol{b}\cdot\frac{\partial f_0(v)}{\partial \boldsymbol{v}}\right), \tag{1}$$

$$\nabla^2\varphi = -4\pi e\,[Z_1\delta(\boldsymbol{r}-\boldsymbol{l}-\boldsymbol{u}t) + Z_2\delta(\boldsymbol{r}-\boldsymbol{u}t)] + 4\pi e\int \mathrm{d}\boldsymbol{v}\, f_1(\boldsymbol{r},\boldsymbol{v},t), \tag{2}$$

where $\boldsymbol{b}$ is the unit vector parallel to B_0, f_0 is the unperturbed electron distribution function which is taken to be uniform and Maxwellian, and $\boldsymbol{u}$ is the velocity of two ions. Here, $Z_1 e$ and $Z_2 e$ are the effective charges of the two ions, assumed to be constant throughout the slowing-down process.

By solving Eqs. (1) and (2) in space-time Fourier components, we obtain the following expression for the electrostatic potential,

$$\varphi(\boldsymbol{r},t) = \frac{4\pi e}{(2\pi)^3}\int \mathrm{d}\boldsymbol{k}\,\frac{\exp[\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{u}t)]}{k^2\varepsilon(\boldsymbol{k},\boldsymbol{k}\cdot\boldsymbol{u})} \times [Z_2 + Z_1\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{l})], \tag{3}$$

where

$$\varepsilon(\boldsymbol{k},\omega) = 1 + \frac{1}{k^2\lambda_D^2} W\left(\frac{\omega}{|\boldsymbol{k}\cdot\boldsymbol{b}|v_T}\right) \tag{4}$$

is the longitudinal dielectric function for a Maxwellian plasma placed in a strong magnetic field and $W(\xi) = A(\xi) + \mathrm{i}B(\xi)$ is the plasma dispersion function [10].

The stopping power of the ion pair can be computed by summing up the forces acting on two ions, due to the electric field induced in the plasma; it reads

$$S = -\frac{\mathrm{d}W}{\mathrm{d}z} = (Z_1^2 + Z_2^2)S_{\mathrm{ind}} + 2Z_1Z_2S_{\mathrm{corr}}(\boldsymbol{l}), \tag{5}$$

where

$$S_{\mathrm{ind}} = \frac{4\pi e^2}{(2\pi)^3 u}\int \mathrm{d}\boldsymbol{k}\,\frac{\boldsymbol{k}\cdot\boldsymbol{u}}{k^2}\,\mathrm{Im}\,\frac{-1}{\varepsilon(\boldsymbol{k},\boldsymbol{k}\cdot\boldsymbol{u})}, \tag{6}$$

$$S_{\mathrm{corr}} = \frac{4\pi e^2}{(2\pi)^3 u}\int \mathrm{d}\boldsymbol{k}\,\frac{\boldsymbol{k}\cdot\boldsymbol{u}}{k^2}\,\mathrm{Im}\,\frac{-1}{\varepsilon(\boldsymbol{k},\boldsymbol{k}\cdot\boldsymbol{u})}\cos(\boldsymbol{k}\cdot\boldsymbol{l}). \tag{7}$$

There are two contributions to the stopping power of two ions. The first one is the uncorrelated particle contribution and represents the energy loss of the two projectiles due to the coupling with collective plasma modes (the first term in Eq. (5)). The second contribution is responsible for the correlated motion of two ions by means of the resonant interaction with the excited plasma waves (the second term in Eq. (5)). Both terms are responsible of the irreversible transfer of the two-ion energy to plasma through resonant electrons.

The stopping properties of the medium can be also conveniently described in terms of the *partial range* defined as follows,

$$R(E) = \frac{E_{\mathrm{in}} - E}{S(E_{\mathrm{in}})}, \tag{8}$$

where E_{in} is the injection ion energy and E is the actual value of the ion energy during the slowing-down process. For $E \simeq 0$, $R(0)$ gives the penetration depth (*range*) of the incident particle.

Let us begin with the evaluation of the stopping power of two ions with the same effective charge Ze. For an arbitrary relative position of the two test ions, the expressions of the uncorrelated (proportional to S_{ind}) and correlated (proportional to S_{corr}) stopping powers of the ion pair becomes

$$S_{\mathrm{ind}}(\lambda) = \frac{e^2}{2\pi\lambda_D^2}\left[\frac{B(\lambda)}{2}\ln\frac{B^2(\lambda)+[s^2+A(\lambda)]^2}{B^2(\lambda)+A^2(\lambda)} - A(\lambda)\left(\arctan\frac{s^2+A(\lambda)}{B(\lambda)} - \arctan\frac{A(\lambda)}{B(\lambda)}\right)\right], \quad (9)$$

$$S_{\mathrm{corr}}(l,\vartheta,\lambda) = \frac{2e^2}{\pi\lambda_D^2}B(\lambda)\int_0^s \frac{k^3\mathrm{d}k}{[k^2+A(\lambda)]^2+B^2(\lambda)} \times Q(kL\cos\vartheta; kL\sin\vartheta)\,. \quad (10)$$

Here

$$Q(a;b) = \int_0^1 \cos(ax)J_0(b\sqrt{1-x^2})x\,\mathrm{d}x\,, \quad (11)$$

$J_0(z)$ is the Bessel function of the zero order, ϑ is the angle between the interionic vector $\boldsymbol{l}$ and the velocity vector $\boldsymbol{u}$, $L = l/\lambda_D$, $\lambda = u/v_T$, and $s = k_{\max}\lambda_D$ with $k_{\max} = 1/r_{\min}$, where $r_{\min}$ is the effective minimum impact parameter. Here $k_{\max}$ has been introduced to avoid the divergence of the integral caused by the incorrect treatment of the short-range interactions between the ion pair and the plasma electrons within the linearized Vlasov theory. The value of $k_{\max}$ will be $1/a_c$ for fusion plasmas, since the magnetized plasma approximation which neglects the perpendicular motion of the electrons ceases to be valid for collision parameters less than a_c.

Consistently with the notation introduced above we separate the single-particle contribution from the correlated one of the stopping power. Then, defining the *interference* or *vicinage function* $\chi(\boldsymbol{l})$ as [2,3]

$$\chi(l,\vartheta,\lambda) = \frac{S_{\mathrm{corr}}(l,\vartheta,\lambda)}{S_{\mathrm{ind}}(\lambda)}\,, \quad (12)$$

expression (5) can be put in the form

$$S = 2Z^2 S_{\mathrm{ind}}(\lambda)[1+\chi(\boldsymbol{l})]\,. \quad (13)$$

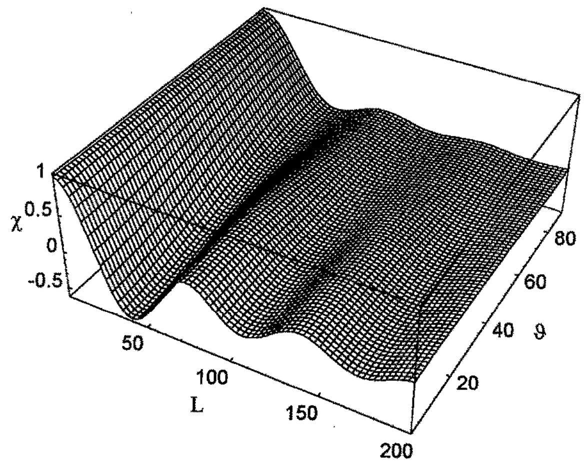

Fig. 1. The *vicinage* function χ versus $L = l/\lambda_D$ and ϑ (degrees). The parameters are $\lambda = 10$, $s = 10$.

Here, χ describes the intensity of correlation effects with respect to uncorrelated situation. In the case of the fast ions ($\lambda \gg 1$), from expressions (9) and (10), we obtain

$$\chi(l,\vartheta,\lambda) \simeq 2Q\left(\frac{L}{\lambda}\cos\vartheta; \frac{L}{\lambda}\sin\vartheta\right). \quad (14)$$

For two values of the orientation angle $\vartheta = 0^\circ$ and $\vartheta = 90^\circ$, expression (14) for the *vicinage* function becomes [11]

$$\chi(l,0,\lambda) = 2\left(\frac{\sin(L/\lambda)}{L/\lambda} - \frac{1-\cos(L/\lambda)}{(L/\lambda)^2}\right), \quad (15)$$

$$\chi(l,\pi/2,\lambda) = \frac{2}{L/\lambda}J_1(L/\lambda)\,, \quad (16)$$

where $J_1(z)$ is the Bessel function of the first order.

Fig. 1 shows χ as a function of L and ϑ for $\lambda = 10$ and $s = 10$. It is shown that χ can decrease or increase for a large ϑ-value, depending on the interionic distance l (see also expressions (15) and (16)). Meanwhile, the correlation effects decrease for a large ϑ-value in plasma in the absence of magnetic field [2]. It should be clear that this effect is accounted for by the character of electrostatic potential of test charged particles in magnetized plasma. As shown in Ref. [12], in the frame of the test particle, moving in a plasma placed in the strong magnetic field ($a_c < \lambda_D$), the part of the spatially oscillatory potential has spherical

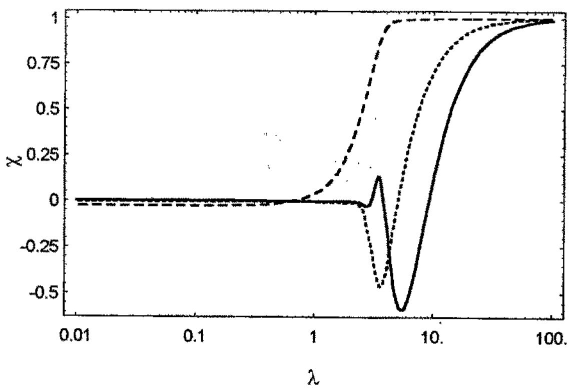

Fig. 2. The *vicinage* function χ versus λ for $\vartheta = 0^\circ$ and $s = 10$. Dashed line: $L = 1$; dotted line: $L = 11$; solid line: $L = 21$.

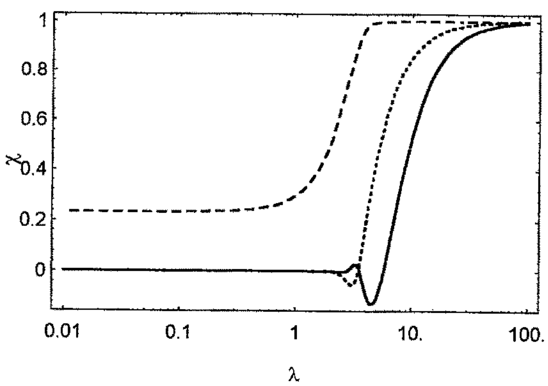

Fig. 3. The *vicinage* function χ versus λ for $\vartheta = 90^\circ$ and $s = 10$. Dashed line: $L = 1$; dotted line: $L = 11$; solid line: $L = 21$.

symmetry over the hemisphere behind the particle and is zero ahead of the particle. The second part has a different character, which makes the potential continuous at the plane containing the particle, is oscillatory in the radial direction, but decreases almost monotonically in the axial direction.

Figs. 2 and 3 show χ as a function of λ, in the cases of $\vartheta = 0^\circ$ and $\vartheta = 90^\circ$, respectively, for different values of L. As expected, correlation effects increase in the high-velocity limit or for small L-values. In the cases of $\vartheta = 15^\circ$, $\vartheta = 45^\circ$ and $\vartheta = 75^\circ$, the correlation effect between two protons is shown in Fig. 4, where the total range $R(0)$ is shown as a function of L. The parameters are $E_{\rm in} = 9.2$ MeV, $T = 100$ eV, $n_0 = 10^{22}$ cm^{-3}, $B_0 = 10^6$ kG (such conditions are

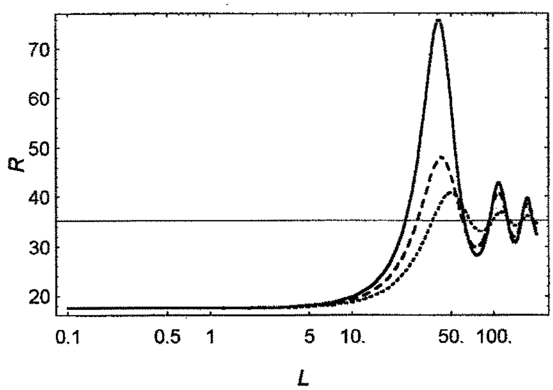

Fig. 4. Total range R (cm) versus L for two protons ($E_{\rm in} = 9.2$ MeV) moving in a magnetized electron plasma ($T = 100$ eV, $n_0 = 10^{22}$ cm^{-3}, $B_0 = 10^6$ kG). Solid line: $\vartheta = 15^\circ$; dashed line: $\vartheta = 45^\circ$; dotted line: $\vartheta = 75^\circ$; horizontal solid line: uncorrelated protons.

possible on the surface of a neutron star). The halving of $R(0)$ (corresponding to the doubling of the stopping power), with respect to the case of two uncorrelated protons, is shown. It should be clear that test-ion densities corresponding to $l \simeq u/\omega_p$ are unrealistic for conventional ion beams. However, small interionic distances ($\simeq 10^{-8}$ cm) occur in clusters [4].

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