ANN KIDDER
HW #8

11.8

1) $f(x,y,z) = 3x - y - 3z$ $\quad g(x,y,z) = x + y - z$

$h(x,y,z) = x^2 + 2z^2 - 1$

$\frac{df}{dx} = 3$ $\quad \frac{dg}{dx} = 1$ $\quad \frac{dh}{dx} = 2x$ $\quad$ ① $3 = \lambda + 2x\mu$

$\frac{df}{dy} = -1$ $\quad \frac{dg}{dy} = 1$ $\quad \frac{dh}{dy} = 0$ $\quad$ ② $-1 = \lambda$

$\frac{df}{dz} = -3$ $\quad \frac{dg}{dz} = -1$ $\quad \frac{dh}{dz} = 4z.$ $\quad$ ③ $-3 = -\lambda + 4z\mu$

② → ③

$-3 = -(-1) + 4z\mu$

$-4 = 4z\mu$

$-1 = z\mu$

$\frac{-1}{\mu} = z$

② → ①

$3 = -1 + 2x\mu$

$4 = 2x\mu$

$\frac{2}{\mu} = x$

plug into original eq $x^2 + 2z^2 = 1$

$\left(\frac{2}{\mu}\right)^2 + 2\left(\frac{-1}{\mu}\right)^2 = 1$

$\frac{4}{\mu^2} + \frac{2}{\mu^2} = 1$

$6 = \mu^2$

$\mu = \pm\sqrt{6}$

$\left(\frac{2}{\sqrt{6}}\right) + y - \left(\frac{-1}{\sqrt{6}}\right) = 0$

$\frac{3}{\sqrt{6}} + y = 0$

$y = -\frac{3}{\sqrt{6}}$

so $x = \frac{2}{\sqrt{6}}$

$y = \frac{-3}{\sqrt{6}}$

$z = -\frac{1}{\sqrt{6}}$

$f\left(\frac{2}{\sqrt{6}}, \frac{-3}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right) = 3\left(\frac{2}{\sqrt{6}}\right) - \left(-\frac{3}{\sqrt{6}}\right) - 3\left(\frac{-1}{\sqrt{6}}\right)$

$= \frac{6}{\sqrt{6}} + \frac{3}{\sqrt{6}} + \frac{3}{\sqrt{6}}$

$= \frac{12}{\sqrt{6}} = \frac{\cancel{12}^{2}\sqrt{6}}{\cancel{6}} = 2\sqrt{6}$

and

$x = -\frac{2}{\sqrt{6}}$

$y = \frac{3}{\sqrt{6}}$

$z = \frac{1}{\sqrt{6}}$

$f\left(\frac{-2}{\sqrt{6}}, \frac{3}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right) = 3\left(\frac{-2}{\sqrt{6}}\right) - \frac{3}{\sqrt{6}} - \frac{3}{\sqrt{6}}$

$= -\frac{12}{\sqrt{6}} = -2\sqrt{6}$

$2\sqrt{6}$ is the maximum
$-2\sqrt{6}$ is the minimum

2) First check critical points of f inside the bounded area.

$\frac{df}{dx} = 4x-4 \quad = 0 \quad x=1$

$\frac{df}{dy} = 6y \quad = 0 \quad y=0$

(1,0) is the only critical point in the interior.

$f(1,0) = 2(1)^2+3(0)^2-4(1)-5$
$= 2-9$
$= -7$

Use Lagrange to find extrema on the boundary.

$g(x,y) = x^2+y^2-16$

$\frac{dg}{dx} = 2x \qquad 4x-4 = 2x\lambda \qquad 4x-2x\lambda = 4 \qquad 2x(2-\lambda)=4 \qquad x = \frac{2}{2-\lambda}$

$\frac{dg}{dy} = 2y \qquad 6y = 2y\lambda. \qquad y=0 \text{ or } \lambda=3$

set $=16$ b/c we are looking for extremes.

$\left(\frac{2}{2-\lambda}\right)^2 + (0)^2 = 16$. (with $y=0$) or $\left(\frac{2}{2-3}\right)^2 + y^2 = 16$ (with $\lambda=3$)

$\frac{4}{(2-\lambda)^2} = 16$

$(2-\lambda)^2 = \frac{1}{4}$

$2-\lambda = \pm\frac{1}{2}$

$\lambda = \frac{3}{2}$ or $\lambda = \frac{5}{2}$

$x = \frac{2}{2-\frac{3}{2}} \qquad x = \frac{2}{2-\frac{5}{2}}$

$x = 4 \qquad x = -4$

$(-2)^2 + y^2 = 16$

$4 + y^2 = 16$

$y^2 = 12$

$y = \pm 2\sqrt{3}$ and $x = \frac{2}{2-3} = -2$

$f(-2, 2\sqrt{3}) = 2(4)+3(4\cdot3)-(-8)-5 = 8+36+8-5 = 47$

$f(-2,-2\sqrt{3}) = 2(4)+36+8-5 = 47$

$f(4,0) = 2(16)+0-16-5 = 11$

$f(-4,0) = 2(16)+0+16-5 = 43$

The max value is 47 at $(-2, \pm 2\sqrt{3})$
The min value is -7 at $(1,0)$

3a)

$P(L,K) = bL^aK^{1-a}$ $\qquad$ $g(L,K) = mL + nK - p = 0$

$\frac{dP}{dL} = baL^{a-1}K^{1-a}$ $\qquad$ $\frac{dg}{dL} = m$ $\qquad$ $baL^{a-1}K^{1-a} = m\lambda$ $\qquad$ $\lambda = \frac{baL^{a-1}K^{1-a}}{m}$

$\frac{dP}{dK} = bL^a(1-a)K^{1-a-1}$ $\qquad$ $\frac{dg}{dK} = n.$ $\qquad$ $bL^a(1-a)K^{-a} = n\lambda.$ $\qquad$ $\lambda = \frac{b(1-a)L^a}{K^a n}.$

$= bL^a(1-a)K^{-a}$

$$\frac{baL^{a-1}K^{1-a}}{m} = \frac{b(1-a)L^a}{K^a n}.$$

$$\cancel{b}a\frac{L^{a-1}}{L^a}K^{1-a}K^a = \frac{\cancel{b}(1-a)m}{n}.$$

$$a\frac{K}{L}n = (1-a)m.$$

$$Kn = \frac{(1-a)mL}{a}$$

plug into equation for constraint.

$$mL + \frac{(1-a)mL}{a} = p.$$

$$\frac{amL}{\cancel{a}} + \frac{mL}{a} - \frac{mLa}{a} = p$$

☆ $\frac{mL}{a} = p \iff \boxed{L = \frac{pa}{m}}$

plug back into equation above for K.

$$Kn = \frac{m\left(\frac{pa}{m}\right)}{a} - \frac{a\cancel{m}\left(\frac{pa}{\cancel{m}}\right)}{\cancel{a}} = p - pa$$

☆ $\boxed{K = \frac{p(1-a)}{n}.}$

3b) $P(L,K) = bL^aK^{1-a} - Q.$ $\quad C(L,K) = mL + nK$

$$\lambda baL^{a-1}K^{1-a} = m \quad \to \quad \lambda = \frac{m}{baL^{a-1}K^{1-a}}$$

$$\lambda bL^a(1-a)K^{-a} = n. \quad \lambda = \frac{n}{bL^a(1-a)K^{-a}} = \frac{nK^a}{bL^a(1-a)}$$

$$\frac{m}{\cancel{b}aL^{a-1}K^{1-a}} = \frac{nK^a}{\cancel{b}L^a(1-a)}$$

$$m\left(\frac{L^a}{L^{a-1}}\right)\left(\frac{1}{a}\right) = nK^aK^{1-a}$$

$$\left(\frac{m}{a}\right)\left(\frac{1}{L}\right) = nK. \Rightarrow K = \frac{m}{aLn}.$$

$$bL^a\left(\frac{m}{aLn}\right)^{1-a} = Q.$$

$$\frac{bL^{a-1+a}m^{1-a}}{a^{1-a}L^{1-a}n^{1-a}} = Q.$$

$$\frac{bm^{1-a}}{L(an)^{1-a}} = Q \longrightarrow \frac{b}{Q}\left(\frac{m}{an}\right)^{1-a} = L.$$

$$K = \frac{m}{a\left(\frac{b}{Q}\left(\frac{m}{an}\right)^{1-a}\right)n} = \frac{\cancel{m}Q(an)^{1-a-1}}{abn(m)^{1-a}} = \frac{Q}{b}m^{-a}a^{-a}n^{-a} = \frac{Q}{b(man)^a}$$

$$\boxed{L = \frac{b}{Q}\left(\frac{m}{an}\right)^{1-a} \qquad K = \frac{Q}{b(man)^a}}$$

4) $A(x,y,z)=[s(s-x)(s-y)(s-z)]^{\frac{1}{2}}$ where $s=\frac{p}{2}$ and $x+y+z=p$ (constraint)

$g(x,y,z)=x+y+z-p$

$$\frac{dA}{dx}=\frac{1}{2}[s(-1)(s-y)(s-z)]^{-\frac{1}{2}} \qquad \frac{dg}{dx}=1 \qquad \frac{1}{2\sqrt{-s(s-y)(s-z)}}=\lambda$$

$$\frac{dA}{dy}=\frac{1}{2}[s(s-x)(-1)(s-z)]^{-\frac{1}{2}} \qquad \frac{dg}{dy}=1 \qquad \frac{1}{2\sqrt{-s(s-x)(s-z)}}=\lambda$$

$$\frac{dA}{dz}=\frac{1}{2}[s(s-x)(s-y)(-1)]^{-\frac{1}{2}} \qquad \frac{dg}{dz}=1 \qquad \frac{1}{2\sqrt{-s(s-x)(s-y)}}=\lambda$$

$$\text{①}\ \frac{1}{2\sqrt{-s(s-y)(s-z)}}=\text{②}\ \frac{1}{2\sqrt{-s(s-x)(s-z)}}=\text{③}\ \frac{1}{2\sqrt{-s(s-x)(s-y)}}$$

①+②

$$1^2=\left(\frac{\sqrt{-s(s-y)(s-z)}}{\sqrt{-s(s-x)(s-z)}}\right)^2$$

$$1=\frac{-\cancel{s}(s-y)\cancel{(s-z)}}{-\cancel{s}(s-x)\cancel{(s-z)}}$$

$$1=\frac{s-y}{s-x}$$

$$\cancel{s}-x=\cancel{s}-y$$

$$x=y$$

②+③

$$1^2=\left(\frac{\sqrt{-s(s-x)(s-z)}}{\sqrt{-s(s-x)(s-y)}}\right)^2$$

$$1=\frac{-\cancel{s}\cancel{(s-x)}(s-z)}{-\cancel{s}\cancel{(s-x)}(s-y)}$$

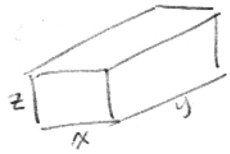
$$1=\frac{s-z}{s-y}$$

$$\cancel{s}-y=\cancel{s}-z$$

$$y=z$$

by the transitive property of equality $x=y=z$

5)



$V = xyz$

$SA \Rightarrow 2xy + 2xz + 2yz = 1500$

$xy + xz + yz = 750$

$EL \Rightarrow 4x + 4y + 4z = 200$

$x + y + z = 50$

$z = 50 - x - y$

plug into V.

$xy + x(50-x-y) + y(50-x-y) = 750$

$\cancel{xy} + 50x - x^2 - \cancel{xy} + 50y - xy - y^2 = 750$

$V = xy(50-x-y)$

$V(x,y) = 50xy - x^2y - xy^2$

$S(x,y) = 50x + 50y - x^2 - y^2 - xy - 750$

$\frac{dV}{dx} = 50y - 2xy - y^2$

$\frac{dS}{dx} = 50 - 2x - y$

$\frac{dV}{dy} = 50x - x^2 - 2xy$

$\frac{dS}{dy} = 50 - 2y - x$

① $50y - 2xy - y^2 = (50 - 2x - y)\lambda$

② $50x - x^2 - 2xy = (50 - 2y - x)\lambda$

① $\frac{y\cancel{(50-2x-y)}}{\cancel{50-2x-y}} = \lambda$

② $\frac{x\cancel{(50-x-2y)}}{\cancel{50-2y-x}} = \lambda$

$y = \lambda$ if $50 - 2x - y \neq 0$

$x = \lambda$ if $50 - 2y - x \neq 0$

if $x = y \neq 0$.

$50x + \quad - x^2 - x^2 - x^2 - 750 = 0$

$100x - 3x^2 - 750 = 0$

$x = \frac{-100 \pm \sqrt{100^2 - 4(-3)(-750)}}{-6}$

$= \frac{+100 \pm 10\sqrt{10}}{+6} = \frac{100 \pm 10\sqrt{10}}{6}$

if $50 - 2x - y = 0$.

$y = 50 - 2x$

or $50 - 2y - x = 0$

$x = 50 - 2y$

plug into constraint.

$50x + 50(50-2x) - x^2 - (50-2x)^2 - x(50-2x) - 750 = 0$

$\cancel{50x} + 2500 - 100x - x^2 - (2500 - 200x + 4x^2) - \cancel{50x} + 100x^2 - 750$

$100x - 3x^2 - 750 = 0$

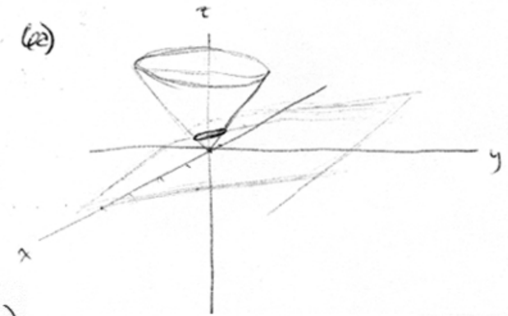
same as earlier.

$x = y = 21.937$ and $z = 50 - 2(21.937) = 6.126$

or $x = y = 11.396$ and $z = 50 - 2(11.396) = 27.208$

$$\boxed{\begin{array}{l} \min\ V(21.937, 21.937, 6.126) = 2948.03 \\ \max\ V(11.396, 11.396, 27.208) = 3533.47 \end{array}}$$

(6a)



$f(x,y,z) = z$ constraints: $g(x,y,z) = 4x-3y+8z-5$

$h(x,y,z) = x^2+y^2-z^2$

$P(x,y,z)$ is a point on the ellipse.

(6b)

$\frac{df}{dx} = 0 \quad \frac{dg}{dx} = 4 \quad \frac{dh}{dx} = 2x \rightarrow$ ① $0 = 4\lambda + 2x\mu$

$\frac{df}{dy} = 0 \quad \frac{dg}{dy} = -3 \quad \frac{dh}{dy} = 2y \rightarrow$ ② $0 = -3\lambda + 2y\mu \rightarrow \lambda = \frac{2y\mu}{3}$

$\frac{df}{dz} = 1 \quad \frac{dg}{dz} = 8 \quad \frac{dh}{dz} = -2z. \rightarrow$ ③ $1 = 8\lambda - 2z\mu$

① $-4\lambda = 2x\mu \rightarrow x = \frac{-4\lambda}{2\mu} = \frac{-2\lambda}{\mu}$

② $3\lambda = 2y\mu \rightarrow y = \frac{3\lambda}{2\mu}$

③ $8\lambda - 1 = 2z\mu \rightarrow z = \frac{8\lambda-1}{2\mu}.$

plug into constraint equations:

$4\left(\frac{-2\lambda}{\mu}\right) - 3\left(\frac{3\lambda}{2\mu}\right) + 8\left(\frac{8\lambda-1}{2\mu}\right) = 5$

$\frac{\overset{16}{-8}\lambda}{2\mu} - \frac{9\lambda}{2\mu} + \frac{64\lambda - 8}{2\mu} = 5$

$\frac{39\lambda - 8}{2\mu} = 5$

$\frac{39\lambda - 8}{10} = \mu.$

$\left(\frac{-2\lambda}{\mu}\right)^2 + \left(\frac{3\lambda}{2\mu}\right)^2 - \left(\frac{8\lambda-1}{2\mu}\right)^2 = 0$

$\frac{\overset{16}{4}\lambda^2}{4\mu^2} + \frac{9\lambda^2}{4\mu^2} - \frac{(64\lambda^2 - 16\lambda + 1)}{4\mu^2} = 0$

$\frac{16\lambda^2 + 9\lambda^2 - 64\lambda^2 + 16\lambda - 1}{4\mu^2} = 0.$

$\frac{-39\lambda^2 + 16\lambda - 1}{4\mu^2} = 0.$

if $\mu \neq 0$, $\lambda = \frac{-16 \pm \sqrt{16^2 - 156}}{-78}.$

$= \frac{-16 \pm \sqrt{100}}{-78} = \frac{-16 \pm 10}{-78}.$

$\lambda = \frac{1}{13}$ or $\frac{1}{3}$

$\mu = \frac{39\left(\frac{1}{13}\right) - 8}{10}$ or $\mu = \frac{39\left(\frac{1}{3}\right) - 8}{10}.$

$= \frac{-5}{10} = -\frac{1}{2} \qquad = \frac{5}{10} = \frac{1}{2}.$

plug back into equations for $x, y, \& z$.

$x = \frac{-2\left(\frac{1}{13}\right)}{\left(-\frac{1}{2}\right)}$ and $x = \frac{-2\left(\frac{1}{3}\right)}{\left(\frac{1}{2}\right)} \rightarrow x = \frac{4}{13}$ and $-\frac{4}{3}$

$y = \frac{3\left(\frac{1}{13}\right)}{2\left(-\frac{1}{2}\right)}$ and $y = \frac{3\left(\frac{1}{3}\right)}{2\left(\frac{1}{2}\right)} \qquad y = -\frac{3}{13}$ and 1

$z = \frac{8\left(\frac{1}{13}\right) - 1}{2\left(-\frac{1}{2}\right)}$ and $z = \frac{8\left(\frac{1}{3}\right) - 1}{2\left(\frac{1}{2}\right)} \qquad z = \frac{5}{13}$ and $\frac{5}{3}$

Highest at $\left(-\frac{4}{3}, 1, \frac{5}{3}\right)$

Lowest at $\left(\frac{4}{13}, -\frac{3}{13}, \frac{5}{13}\right)$

7a) $f(x,y) = \sum_{i=1}^{n} x_i y_i$ $\qquad g(x,y) = \sum x_i^2 - 1$

$h(x,y) = \sum y_i^2 - 1$

$f_x = (1)n \qquad g_x = 2\sum x_i \qquad h_x = 0 \quad \rightarrow \quad n = 2\lambda \sum x_i$

$f_y = (1)n \qquad g_y = 0 \qquad h_y = 2\sum y_i \quad \rightarrow \quad n = 2\mu \sum y_i$

$$\sum x_i = \frac{n}{2\lambda}$$

$$\sum y_i = \frac{n}{2\mu}.$$

plug into constraints:

$\left(\frac{n}{2\lambda}\right)^2 = 1 \qquad \left(\frac{n}{2\mu}\right)^2 = 1$

$\frac{n^2}{4\lambda^2} = 1 \qquad \frac{n^2}{4\mu^2} = 1$

$n^2 = 4\lambda^2 \qquad n^2 = 4\mu^2$

$\frac{n^2}{4} = \lambda^2 \qquad \mu^2 = \frac{n^2}{4}$

$\mu = \sqrt{\frac{n^2}{4}}$

$\lambda = \pm\frac{n}{2} \qquad \mu = \pm\frac{n}{2}.$

so $\sum x_i = \frac{n}{2\left(\frac{\pm n}{2}\right)} = \frac{\pm n}{n} = \pm 1 \qquad \sum y_i = \frac{n}{2\left(\frac{\pm n}{2}\right)} = \pm 1.$

$\boxed{\begin{array}{l} \text{max at } (\sum x_i = 1 \text{ and } \sum y_i = 1) \text{ and } (\sum x_i = -1 \text{ and } \sum y_i = -1) \\ \text{min at } (\sum x_i = -1 \text{ and } \sum y_i = 1) \text{ and } (\sum x_i = 1 \text{ and } \sum y_i = -1) \end{array}}$

7b) $\sum \frac{a_i}{\sqrt{\sum a_j^2}} = \pm 1 \qquad \sum \frac{b_i}{\sqrt{\sum b_j^2}} = \pm 1.$

since $\sqrt{\sum a_j^2}$ and $\sqrt{\sum b_j^2}$ are always positive because the (1) sum of positive numbers is a positive, and (2) the square of a number is positive, (3) and the square root of a positive number is positive.

Therefore $\sum a_i = \pm 1$ and $\sum \sqrt{\sum a_j^2} = 1$ and $\sum b_i = \pm 1$ and $\sum \sqrt{\sum b_j^2} = 1$ so,

Therefore

$$\sum a_i \leq \sum \sqrt{\sum a_j^2} \text{ and } \sum b_i \leq \sum \sqrt{\sum b_j^2}$$

By the product rule

$$\boxed{\sum a_i b_i \leq \sum \sqrt{\sum a_j^2} \sqrt{\sum b_j^2}}$$

8) 

$V = \pi r^2 h$

$36 = 2\pi rh + \pi r^2 = SA$.

$\frac{dV}{dr} = 2\pi rh$. $\quad \frac{dSA}{dr} = 2\pi h + 2\pi r$

$\frac{dV}{dh} = \pi r^2 \quad \frac{dSA}{dh} = 2\pi r$

$2\pi rh = (2\pi h + 2\pi r)\lambda$.

$\cancel{2\pi} rh = (h+r)\cancel{2\pi}\lambda$

$\frac{rh}{h+r} = \lambda$.

$\pi r^2 = 2\pi r\left(\frac{rh}{h+r}\right)$

$= \frac{2\pi r^2 h}{h+r}$

$\cancel{\pi} r^2 h + \cancel{\pi} r^3 = 2\cancel{\pi} r^2 h$

$\pi r^3 = 2\pi r^2 h - \pi r^2 h$

$\cancel{\pi} r^3 = \cancel{\pi} r^2 h$.

$r = h$. is a critical point.

plug into SA:

$2\pi(h)h + \pi(h)^2 = 36$

$2\pi h^2 + \pi h^2 = 36$

$3\pi h^2 = 36$

$h^2 = \frac{12}{\pi}$

$h = \pm\sqrt{\frac{12}{\pi}}$ but must be $h>0$ so

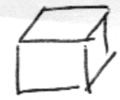
$r = h = 2\sqrt{\frac{3}{\pi}}$

$V = \pi\left(2\sqrt{\frac{3}{\pi}}\right)^2\left(2\sqrt{\frac{3}{\pi}}\right)$

$= \pi\, 4\left(\frac{3}{\pi}\right)\left(2\sqrt{\frac{3}{\pi}}\right)$

$= 24\sqrt{\frac{3}{\pi}}$.

$\boxed{V \approx 23.45}$ for the cylinder.



$V = xyz$

$SA = 36 = xy + 2xz + 2yz$

$\frac{dV}{dx} = yz \quad \frac{dSA}{dx} = y + 2z$

$\frac{dV}{dy} = xz \quad \frac{dSA}{dy} = x + 2z$

$\frac{dV}{dz} = xy \quad \frac{dSA}{dz} = 2x + 2y$

$x(yz = (y+2z)\lambda) \Rightarrow xyz = (y+2z)\lambda x$

$y(xz = (x+2z)\lambda) \Rightarrow xyz = (x+2z)\lambda y$

$z(xy = (2x+2y)\lambda) \Rightarrow xyz = (2x+2y)\lambda z$

divide each equation by λ, set equal

$\frac{xyz}{\lambda} = xy + 2xz = xy + 2zy = 2xz + 2yz$.

① $2xz = 2zy$

$x = y$ if $z \neq 0$

② $xy + \cancel{2xz} = \cancel{2xz} + 2yz$

$xy = 2yz$

$x = 2z$ if $y \neq 0$.

③ $xy + 2zy = 2xz + 2zy$

$xy = 2xz$

$y = 2z$ if $x \neq 0$.

$x = y = 2z$ if $x, y, z \neq 0$.

$x^2 + 2x\left(\frac{x}{2}\right) + 2x\left(\frac{x}{2}\right) = 36$

$x^2 + x^2 + x^2 = 36$

$x = 2\sqrt{3}$

$V = (2\sqrt{3})(2\sqrt{3})\left(\frac{2\sqrt{3}}{2}\right)$

$= 12\sqrt{3}$

$\boxed{V \approx 20.78.}$ for the box

$\boxed{\text{The cylinder is the greater volume by } \approx 2.67 \text{ units}^3}$

9a). $d_1 = \sqrt{(0-x)^2+(7-y)^2}$

$d_2 = \sqrt{(-5-x)^2+(-3-y)^2}$

$d_3 = \sqrt{(8-x)^2+(0-y)^2}$

sum of squares $= (-x)^2+(7-y)^2+(-5-x)^2+(-3-y)^2+(8-x)^2+(-y)^2$

$= x^2+49-14y+y^2+25+10x+x^2+9+6y+y^2+64-16x+x^2+y^2$

$D = 3x^2+3y^2-6x-8y+147$.

$D_x = 6x-6 = 0 \rightarrow x=1$

$D_y = 6y-8 = 0 \rightarrow y=\frac{4}{3}$

$(1,\frac{4}{3})$ minimizes the sum of the square of the distances from all three bones

9b) The dog should still sit at $(1,\frac{4}{3})$, this is within the circle delineated by the leash

9c) constraint $g(x,y) = x^2+y^2-1$. (circle of radius 1 centered at the origin).

$g_x = 2x \rightarrow 6x-6 = 2x\lambda \rightarrow x = \frac{6}{6-2\lambda}$

$g_y = 2y \rightarrow 6y-8 = 2y\lambda \rightarrow y = \frac{8}{6-2\lambda}$

$$\left(\frac{6}{6-2\lambda}\right)^2 + \left(\frac{8}{6-2\lambda}\right)^2 = 1$$

$$\frac{36+64}{36-24\lambda+4\lambda^2} = 1.$$

$100 = 36-24\lambda+4\lambda^2$

$0 = -64-24\lambda+4\lambda^2$

$\lambda^2-6\lambda-16 = 0.$

$\lambda = \frac{6\pm\sqrt{36+64}}{2}.$

$\lambda = \frac{16}{2}$ or $-\frac{4}{2}$

$\lambda = 8$ or -2.

$x = \frac{6}{-10}$ or $\frac{6}{10}$

$y = \frac{8}{-10}$ or $\frac{8}{10}$

critical points at $(\frac{-6}{10}, -\frac{8}{10})$ and $(\frac{6}{10}, \frac{8}{10})$

$d = 3(-(\frac{6}{-10}))^2 + 3(\frac{8}{-10})^2 - 6(\frac{6}{-10}) - 8(\frac{8}{-10}) + 147 = 160$

$d = 3(\frac{6}{10})^2 + 3(\frac{8}{10})^2 - 6(\frac{6}{10}) - 8(\frac{8}{10}) + 147 = 140$

The dog should sit at $(\frac{6}{10}, \frac{8}{10})$ to minimize the sum of the square of the distances

10a) $p(x,y)=500x^{.7}y^{.5}$

$p(x,y)=500x^{.7}y^{.5}-40{,}000$ (constraint) $\qquad C(x,y)=35x+16y$.

$C_x=35 \qquad P_x=500(.7)x^{-.3}y^{.5} \rightarrow 35=350x^{-.3}y^{.5}\lambda$

$C_y=16 \qquad P_y=500(.5)x^{.7}y^{-.5} \rightarrow 16=250x^{.7}y^{-.5}\lambda$.

set λ in each equation equal:

$$\frac{1}{10x^{-.3}y^{.5}}=\frac{8}{125x^{.7}y^{-.5}}$$

$$125x^{.7}y^{-.5}=80x^{-.3}y^{.5}$$

$$125\frac{x^{.7}}{x^{-.3}}=80\frac{y^{.5}}{y^{-.5}}$$

$$125x=80y.$$

$$y=\frac{25}{16}x.$$

plug into constraint:

$$40{,}000=500x^{.7}\left(\frac{25}{16}x\right)^{.5}$$

$$80=x^{.7}\frac{5}{4}x^{.5}$$

$$64=x^{1.2}=x^{\frac{6}{5}}$$

$$64^{\frac{5}{6}}=x$$

min x & y. $\left[\begin{array}{l} x=32. \\ y=50 \end{array}\right.$

$C(32,50)=35(32)+16(50)$

$=1920\$$.

10b). $4800=35x+16y$ (constraint).

The derivatives are same as above.

$35\lambda=350x^{-.3}y^{.5} \qquad \lambda=10x^{-.3}y^{.5}$

$16\lambda=250x^{.7}y^{-.5} \qquad \lambda=\frac{125}{8}x^{.7}y^{-.5}$

$10x^{-.3}y^{.5}=\frac{125}{8}x^{.7}y^{-.5}$

$80y=125x$

$y=\frac{25}{16}x$. plug into constraint.

$4800=35x+16\left(\frac{25}{16}x\right)$

$=35x+25x$

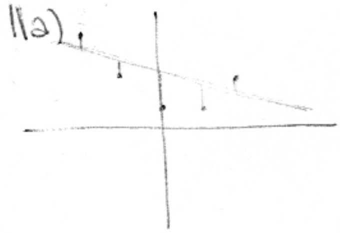
$=60x$

max x & y. $\left[\begin{array}{l} x=80 \\ y=125. \end{array}\right.$

$C(80,125)=35(80)+16(125)$

$=4800\$$.

11a).



$y = ax + b$.

$\text{error} = \sum_{i=1}^{5} (y_i - (ax_i + b))^2$ difference b/w actual y and predicted y squared

$f(a,b) = \sum y_i^2 - 2y_i(ax_i+b) + (ax_i+b)^2 = \sum y_i^2 - 2y_i a x_i - 2y_i b + a^2x_i^2 + 2abx_i + b^2$

$f_a = -2\sum y_i x_i + 2\sum x_i^2 a + 2b\sum x_i = 0 \rightarrow a\sum x_i^2 + b\sum x_i = \sum y_i x_i$

$f_b = -2\sum y_i + 2a\sum x_i + 2b = 0 \rightarrow a\sum x_i + 5b = \sum y_i$

$\sum_{i=1}^{5} b = b(5)$ b/c $i=1$ to $i=5$

$\sum x_i = -2 + -1 + 0 + 1 + 2 = 0$

$\sum y_i = 4 + 2 + 1 + 1 + 2 = 10$

$\sum y_i x_i = -8 + -2 + 0 + 1 + 4 = -5$

$\sum x_i^2 = 4 + 1 + 0 + 1 + 4 = 10$

plug in:

$a(10) + b(0) = -5$

$a(0) + b = 10$

$5b = 10 \quad b = \frac{10}{5} = 2$.

$10a = -5$

$a = \frac{-5}{10} = -\frac{1}{2}$.

line of best fit is $\boxed{y = -\frac{1}{2}x + 2}$

11b). $y = ax^2 + bx + c$.

$\text{error} = \sum_{i=1}^{5} (y_i - (ax_i^2 + bx_i + c))^2$

$f(a,b,c) = \sum y_i^2 - 2\sum y_i(ax_i^2 + bx_i + c) + \sum(ax_i^2 + bx_i + c)^2$

$= \sum y_i^2 - 2a\sum y_i x_i^2 - 2b\sum y_i x_i - 2c\sum y_i + a^2\sum x_i^4 + 2ab\sum x_i^3 + 2ac\sum x_i^2 + 2bc\sum x_i + c^2 n + b^2\sum x_i^2$. $n=5$

① $f_a = -2\sum y_i x_i^2 + 2a\sum x_i^4 + 2b\sum x_i^3 + 2c\sum x_i^2$

② $f_b = -2\sum y_i x_i + 2a\sum x_i^3 + 2c\sum x_i + 2b\sum x_i^2$

③ $f_c = -2\sum y_i + 2a\sum x_i^2 + 2b\sum x_i + 10c$.

$\sum x_i^4 = 34$

$\sum x_i^3 = 0$

$\sum x_i^2 = 10$

$\sum x_i = 0$

$\sum y_i = 10$

$\sum x_i y_i = -5$

$\sum y_i x_i^2 = 27$

① $f_a \Rightarrow -2(27) + 2a(34) + 2b(0) + 2c(10) = 0$

② $f_b \Rightarrow -2(-5) + 2a(0) + 2c(0) + 2b(10) = 0$

③ $f_c \Rightarrow -2(10) + 2a(10) + 2b(0) + 10c$

① $68a + 20c = 54 \rightarrow c =$

② $20b = -10 \rightarrow b = -\frac{1}{2}$

③ $20a + 10c = 20 \rightarrow a = \frac{2-c}{2}$

plug into ①

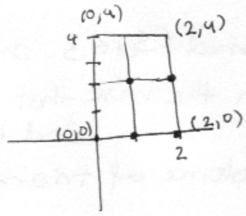
$68\left(\frac{2-c}{2}\right) + 20c = 54 \rightarrow 136 - 28c = 108 \rightarrow \boxed{c=1}$

plug into ③

$a = \frac{2-1}{2} = \frac{1}{2}$.

$\boxed{y = \frac{1}{2}x^2 - \frac{1}{2}x + 1}$

12a)



lower right corners: ① $(1,0)$
② $(2,0)$
③ $(1,2)$
④ $(2,2)$

$\Delta A = 2$.

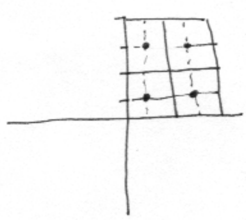
Riemann sum approximation is:

$= f(1,0)\cdot 2 + f(2,0)\cdot 2 + f(1,2)\cdot 2 + f(2,2)\cdot 2$ and $f(x,y) = x + 2y^2$.

$= 2 + 4 + 18 + 20$

$\boxed{= 44}$

12b)



midpoints: ① $(\frac{1}{2}, 1)$
② $(\frac{3}{2}, 1)$
③ $(\frac{1}{2}, 3)$
④ $(\frac{3}{2}, 3)$

$\Delta A = 2$.

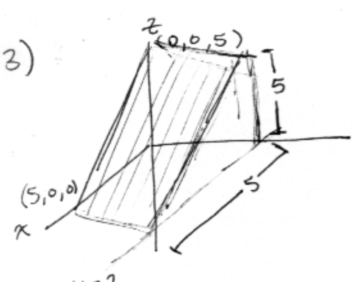
Riemann sum approx is:

$= (\frac{1}{2} + 2(1)^2)\cdot 2 + (\frac{3}{2} + 2(1)^2)\cdot 2 + (\frac{1}{2} + 2(3)^2)\cdot 2 + (\frac{3}{2} + 2(3)^2)\cdot 2$

$= 5 + 7 + 37 + 39$

$\boxed{= 88}$

13)



The solid delineated by $z = 5 - x$ and $0 \le y \le 3$ and $0 \le x \le 5$ is a solid triangular cylinder. with the base that is $\times 5$ and height :.

The volume is. $\frac{1}{2}$(base)(height) − volume of triangle over the height. $= \frac{1}{2}(25)(3) = \frac{75}{2}$.