ANN KIDDER
HW #5

11.2

10) $$\lim_{(0,y)\to(0,0)} \frac{6(0)^3 y}{2(0)^4+y^4} = 0$$ (try approaching along y-axis)

$$\lim_{(y,y)\to(0,0)} \frac{6y^4}{2y^4+y^4} = \frac{6\cancel{y^4}}{3\cancel{y^4}} = 2$$ (try approaching along the line $x=y$)

$0 \neq 2$ so the limit does not exist

14) $$\lim_{(0,y)\to(0,0)} \frac{(0)y^4}{0+y^8} = 0$$

$$\lim_{(y^4,y)\to(0,0)} \frac{y^8}{y^8+y^8} = \frac{1}{2}$$

$0 \neq \frac{1}{2}$, therefore the limit does not exist

11.4

6) $z=\ln(x-2y)$ at $(3,1,0)$

$\frac{dz}{dx} = \frac{1}{x-2y}$ at $(3,1,0)$ $\frac{dz}{dx} = \frac{1}{3-2} = \frac{1}{1} = 1$

$\frac{dz}{dy} = \frac{-2}{x-2y}$ at $(3,1,0)$ $\frac{dz}{dy} = \frac{-2}{3-2} = \frac{-2}{1} = -2$

$$z-0 = 1(x-3)+(-2)(y-1)$$

$$\boxed{z = x-2y-1}$$

33) diameter = 8cm, radius = 4cm, height = 12cm.

$V=\pi r^2 h$.

$\frac{dV}{dr} = 2\pi r h dr$

$\frac{dV}{dh} = \pi r^2 dh$

$dV = 2\pi r h dr + \pi r^2 dh$, $dr = -.04$ and $dh = -.08$

$dV = 2\pi(4)(12)(-.04) + \pi(4)^2(-.08)$

$dV \approx -16.08$.

the tin takes up a volume of $\boxed{\approx 16.08 \text{ cm}^3}$

37) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1R_2R_3}$

$$R = \frac{R_1R_2R_3}{R_1R_2+R_1R_3+R_2R_3}$$

$$\frac{dR}{dR_1} = \frac{R_2R_3(R_1R_2+R_1R_3+R_2R_3) - R_1R_2R_3(R_2+R_3)}{(R_1R_2+R_1R_3+R_2R_3)^2}$$

$$= \frac{R_2^2R_3^2}{(R_1R_2+R_1R_3+R_2R_3)^2}$$

$= \frac{R^2}{R_1^2}$ → this pattern will hold for any R_i so, $\frac{dR}{dR_i} = \frac{R^2}{R_i^2}$

we also know $\frac{1}{R} = \frac{1}{25} + \frac{1}{40} + \frac{1}{50} = \frac{200}{17}$ and $dR_i = .005 R_i$

Therefore, $dR = \frac{\left(\frac{200}{17}\right)^2}{(25)^{\cancel{2}}}(.005)(\cancel{25}) + \frac{\left(\frac{200}{17}\right)^2}{(40)^{\cancel{2}}}(.005)(\cancel{40}) + \frac{\left(\frac{200}{17}\right)^2}{(50)^2}(.005)(\cancel{50})$

$$= \left(\frac{200}{17}\right)^2\left(\frac{1}{200}\right)\left(\frac{1}{25} + \frac{1}{40} + \frac{1}{50}\right)$$

$$= \frac{\cancel{200}}{17^{\cancel{2}}}\left(\frac{17\cancel{85}}{\cancel{1000}}\right) = \frac{1}{17^{\cancel{2}}}\left(\frac{\cancel{17}}{1}\right)$$

$dR \boxed{= \frac{1}{17}}$ is the maximum error in R

44) To find the equation of the plane we need two vectors that are on the plane. The velocity vectors (tangent to surface at P) are on the tangent plane.

P(2,1,3) is at t=0 and u=1.

$v_1(t) = \langle 3, -2t, -4+2t \rangle$ $\quad v_2(u) = \langle 2u, 6u^2, 2 \rangle$

$v_1(0) = \langle 3, 0, -4 \rangle$ $\quad v_2(1) = \langle 2, 6, 2 \rangle$

$\vec{n} = \langle 3,0,-4 \rangle \times \langle 2,6,2 \rangle = \langle 24, -14, 18 \rangle \Rightarrow \langle 12, -7, 9 \rangle$. (equivalent, just scaled down by $\frac{1}{2}$)

$12(x-2) - 7(y-1) + 9(z-3) = 0$.

$12x - 24 - 7y + 7 + 9z - 27 = 0$.

$\boxed{12x - 7y + 9z = 44}$

11.5

2) $\frac{dz}{dx} = -\sin(x+4y)$

$\frac{dz}{dy} = -\sin(x+4y)(4) = -4\sin(x+4y)$

$\frac{dx}{dt} = 20t^3$

$\frac{dy}{dt} = -t^{-2}$

$$\frac{dz}{dt} = (-\sin(x+4y))(20t^3) + (-4\sin(x+4y))(-t^{-2})$$

$$\boxed{= -20t^3\sin(x+4y) + \frac{4\sin(x+4y)}{t^2}}$$

36) $V = \pi r^2 \frac{h}{3}$

$\frac{dV}{dr} = 2\pi r \frac{h}{3}$ $\qquad \frac{dr}{dt} = 1.8 \frac{m}{s}$

$\frac{dV}{dh} = \frac{\pi r^2}{3}$ $\qquad \frac{dh}{dt} = -2.5 \frac{m}{s}$

$$\frac{dV}{dt} = 2\pi r \frac{h}{3}(1.8) + \frac{\pi r^2}{3}(-2.5) \quad \text{at } r = 120\text{m and } h = 140\text{ m}$$

$$= 2\pi(120)\frac{(140)}{3}(1.8) + \frac{\pi(120)^2}{3}(-2.5)$$

$$\boxed{\frac{dV}{dt} = 8160\pi}$$

38) $V = IR$

$\frac{dV}{dI} = R$

$\frac{dV}{dR} = I$

$$\frac{dV}{dt} = R\left(\frac{dI}{dt}\right) + I\left(\frac{dR}{dt}\right)$$

$$-0.01\frac{V}{s} = (400)\left(\frac{dI}{dt}\right) + .08(.03)$$

$$\frac{dI}{dt}\boxed{= -.000031 \frac{A}{s}}$$

11.6

8). $f(x,y) = \frac{y^2}{x}$

$\frac{df}{dx} = y^2(-1)(x^{-2}) = \frac{-y^2}{x^2}$

$\frac{df}{dy} = \frac{2y}{x}$

a) gradient is $\nabla f(x,y) = \left\langle \frac{-y^2}{x^2}, \frac{2y}{x} \right\rangle$

$\nabla f(1,2) = \left\langle \frac{-(2^2)}{1^2}, \frac{2\cdot 2}{1} \right\rangle = \left\langle \frac{-4}{1}, \frac{4}{1} \right\rangle = \boxed{\langle -4, 4 \rangle}$

b) at (1,2) $\nabla f(1,2) = \langle -4, 4 \rangle$

c) $D_u f(x,y) = \frac{2}{3}\left(\frac{-y^2}{x^2}\right) + \frac{\sqrt{5}}{3}\left(\frac{2y}{x}\right)$

$D_u f(1,2) = \frac{2}{3}\left(\frac{-4}{1}\right) + \frac{\sqrt{5}}{3}\left(\frac{4}{1}\right) = -\frac{8}{3} + \frac{4\sqrt{5}}{3} = \frac{4\sqrt{5}-8}{3} \approx \boxed{.315}$

10 a) $f(x,y,z) = \sqrt{x+yz} = (x+yz)^{\frac{1}{2}}$

$\frac{df}{dx} = \frac{1}{2\sqrt{x+yz}}$

$\frac{df}{dy} = \frac{z}{2\sqrt{x+yz}}$

$\frac{df}{dz} = \frac{y}{2\sqrt{x+yz}}$

$\nabla f(x,y,z) = \boxed{\left\langle \frac{1}{2\sqrt{x+yz}}, \frac{z}{2\sqrt{x+yz}}, \frac{y}{2\sqrt{x+yz}} \right\rangle}$

b) $\nabla f(1,3,1) = \left\langle \frac{1}{2\sqrt{4}}, \frac{1}{2\sqrt{4}}, \frac{3}{2\sqrt{4}} \right\rangle = \boxed{\left\langle \frac{1}{4}, \frac{1}{4}, \frac{3}{4} \right\rangle}$

c) rate of change at P ⇒ $\vec{v} = \left\langle \frac{-1}{3}, \frac{-2}{3}, \frac{2}{3} \right\rangle$

$\hookrightarrow = \left(\frac{1}{4}\right)\left(-\frac{1}{3}\right) + \left(\frac{1}{4}\right)\left(-\frac{2}{3}\right) + \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)$

$= \frac{-1 + -2 + 6}{12} = \boxed{\frac{1}{4}}$

12) $f(x,y)=\ln(x^2+y^2)$

$$\frac{df}{dx}=\frac{1}{x^2+y^2}(2x)$$

$$\frac{df}{dy}=\frac{1}{x^2+y^2}(2y)$$

unit vector from $\vec{v}=\langle -1,2\rangle$ is

$$\vec{u}=\left\langle \frac{-1}{\sqrt{5}},\frac{2}{\sqrt{5}}\right\rangle$$

$$=\left(\frac{2x}{x^2+y^2}\right)\left(\frac{-1}{\sqrt{5}}\right)+\left(\frac{2y}{x^2+y^2}\right)\left(\frac{2}{\sqrt{5}}\right)$$

$$=\left(\frac{4}{4+1}\right)\left(\frac{-1}{\sqrt{5}}\right)+\left(\frac{2}{4+1}\right)\left(\frac{2}{\sqrt{5}}\right)$$

$$=\frac{-4}{5\sqrt{5}}+\frac{4}{5\sqrt{5}}$$

$$\boxed{=0.}$$

16) $f(x,y,z)=\sqrt{xyz}=(xyz)^{\frac{1}{2}}$

$$\frac{df}{dx}=\frac{1}{2}(xyz)^{-\frac{1}{2}}(yz)=\frac{yz}{2\sqrt{xyz}}.$$

$$\frac{df}{dy}=\frac{1}{2}(xyz)^{-\frac{1}{2}}(xz)=\frac{xz}{2\sqrt{xyz}}.$$

$$\frac{df}{dz}=\frac{1}{2}(xyz)^{-\frac{1}{2}}(xy)=\frac{xy}{2\sqrt{xyz}}$$

unit vector from $\vec{v}=\langle -1,-2,2\rangle$ is

$$\vec{u}=\left\langle \frac{-1}{3},\frac{-2}{3},\frac{2}{3}\right\rangle$$

$$=\frac{yz}{2\sqrt{xyz}}\left(\frac{-1}{3}\right)+\frac{xz}{2\sqrt{xyz}}\left(\frac{-2}{3}\right)+\frac{xy}{2\sqrt{xyz}}\left(\frac{2}{3}\right)$$

$$=\frac{12}{2\sqrt{36}}\left(-\frac{1}{3}\right)+\frac{18}{2\sqrt{36}}\left(-\frac{2}{3}\right)+\frac{6}{2\sqrt{36}}\left(\frac{2}{3}\right)$$

$$=-\frac{1}{3}+(-1)+\frac{1}{3}$$

$$\boxed{=-1}$$

32 a) $z=1000-.005x^2-.01y^2$

$\frac{dz}{dx}=-.01x$

due south is $\langle 0,-1\rangle$

$\frac{dy}{dx}=-.02y$

$D_u(x,y)=(-.01)(x)(0)+(-.02)(y)(-1)$

$D_u(60,40)=.8$

Since the slope is positive you will ascend as you walk due south at a rate of .8

b) The unit vector for NW is $\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$

$D_u(60,40)=(-.01)(60)(-\frac{1}{\sqrt{2}})+(-.02)(40)(\frac{1}{\sqrt{2}})$

$=\frac{.6}{\sqrt{2}}-\frac{.8}{\sqrt{2}}=-\frac{.2}{\sqrt{2}}=\frac{-\sqrt{2}}{10}$

You will desend at a rate of $\frac{-\sqrt{2}}{10}$

c) $\nabla f(x,y)=\langle -.01x,-.02y\rangle$

$\nabla f(60,40)=\langle -.6,-.8\rangle$

$|\nabla f(60,40)|=\sqrt{(-.6)^2+(-.8)^2}=\sqrt{.36+.64}=\sqrt{1}=1$

The largest slope is in the direction $\langle -.6,-.8\rangle$. The rate of ascent is 1. which means the change in height & distance are equivalent so the angle is 45° above the horizontal