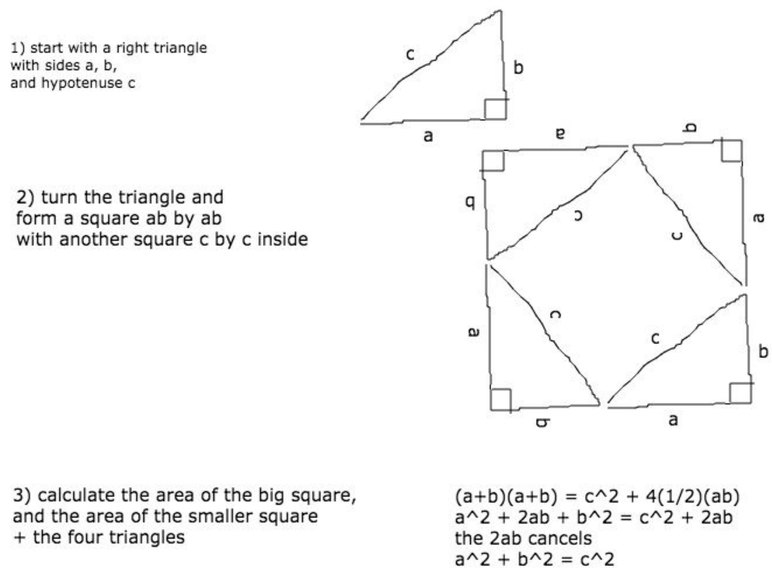
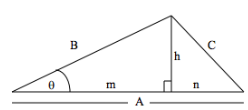
1 Basic Concept Problems





Given a triangle with sides A, B, and C with height h and an angle θ
we know that the left triangle will have the property $m^2 + h^2 = B^2$
and the right triangle will have the property $h^2 + n^2 = C^2$
by the pythagorean theorem. Therefore:
$n = A - m$
and $h^2 + (A - m)^2 = C^2$
$B^2 - m^2 + A^2 - 2Am + m^2 = C^2$
Furthermore, we know $cos\theta = \dfrac{m}{B}$
Therefore:
$B^2 + A^2 - 2ABcos\theta = C^2$

9.3

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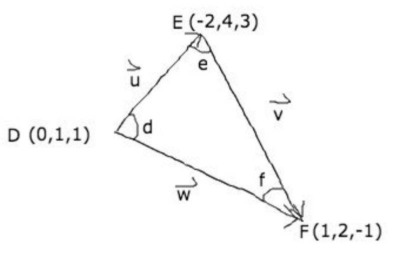

$$\vec{u} =< -2, 3, 2 >$$
$$\vec{v} =< 3, -2, -4 >$$
$$\vec{w} =< 1, 1, -2 >$$

The angle at D:

$$< -2, 3, 2 > \cdot < 1, 1, -2 > = \| < -2, 3, 2 > \|\| < 1, 1, -2 > \| cosd$$
$$-3 = \sqrt{17}\sqrt{6}cosd$$
$$cos^{-}1(\frac{-3}{\sqrt{102}}) = d$$
$$d \approx 107.28°$$

The angle at E:

$$-(< -2, 3, 2 >) \cdot < 3, -2, -4 > = \| < -2, 3, 2 > \|\| < 3, -2, -4 > \| cose$$
$$20 = \sqrt{17}\sqrt{29}cose$$
$$cos^{-}1(\frac{20}{\sqrt{493}}) = e$$
$$e \approx 25.74°$$

The angle at F:

$$-(< 1, 1, -2 >) \cdot -(< 3, -2, -4 >) = \| < 1, 1, -2 > \|\| < 3, -2, -4 > \| cosf$$
$$9 = \sqrt{6}\sqrt{29}cosf$$
$$cos^{-}1(\frac{9}{\sqrt{174}}) = f$$
$$f \approx 46.98°$$

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$$\|a\| = \sqrt{5}$$
$$comp_a b = \frac{a \cdot b}{\|a\|} = \frac{-4+2}{\sqrt{5}} = \frac{-2}{\sqrt{5}}$$

The vector projection is:

$$proj_a b = \frac{-2}{\sqrt{5}} \frac{< 1, 2 >}{\sqrt{5}} = \frac{-2}{5} < 1, 2 >$$
$$proj_a b =< \frac{-2}{5}, \frac{-4}{5} >$$

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$$a =< 2, -1, 4 > \text{ and } b =< 0, 1, \frac{1}{2} >$$
$$\|a\| = \sqrt{4+1+16} = \sqrt{21}$$
$$comp_a b = \frac{a \cdot b}{\sqrt{21}} = \frac{1}{\sqrt{21}}$$

The vector projection is:

$$proj_a b = \frac{1}{\sqrt{21}} \frac{< 2, -1, 4 >}{\sqrt{21}} = \frac{1}{21} < 2, -1, 4 >$$
$$proj_a b =< \frac{2}{21}, \frac{-1}{21}, \frac{4}{21} >$$

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Let all sides of the cube be length = 1, and let the edges lie along the x, y, z axis

The diagonal vector, $\vec{d}$ from (0,0,0) to (1,1,1) = <1,1,1>

Let the unit vector $\vec{k} =< 0, 0, 1 >$ be a side.

The angle between $\vec{d}$ and $\vec{k}$ is:

$$
\begin{aligned}
cos\theta &= \frac{< 0,0,1 > \cdot < 1,1,1 >}{\|u\|\|v\|} \\
&= \frac{0+0+1}{\sqrt{1}\sqrt{3}} \\
&= \frac{1}{\sqrt{3}} \\
\theta &= cos^{-}1\frac{1}{\sqrt{3}} \\
&= 54.74^{\circ}
\end{aligned}
$$

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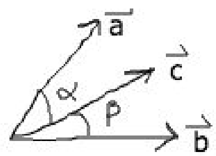

$$\begin{aligned}
cos\alpha &= \frac{a \cdot c}{\|a\|\|c\|} \\
&= \frac{a \cdot (\|a\|b + \|b\|a))}{\|a\|\|c\|} \\
&= \frac{a \cdot \|a\|a \cdot b + \|b\|a \cdot a}{\|a\|\|c\|} \\
&= \frac{\|a\|a \cdot b + \|b\|\|a\|^2}{\|a\|\|c\|} \\
&= \frac{a \cdot b + \|b\|\|a\|}{\|c\|} \\
cos\beta &= \frac{b \cdot c}{\|b\|\|c\|} \\
&= \frac{b \cdot (\|a\|b + \|b\|a))}{\|b\|\|c\|} \\
&= \frac{\|a\|b \cdot b + \|b\|a \cdot b}{\|b\|\|c\|} \\
&= \frac{\|a\|\|b\|^2 + \|b\|a \cdot b}{\|b\|\|c\|} \\
&= \frac{\|a\|\|b\| + a \cdot b}{\|c\|}
\end{aligned}$$

Now we can see that $cos\alpha = cos\beta$

$$\frac{a \cdot b + \|b\|\|a\|}{\|c\|} = \frac{\|a\|\|b\| + a \cdot b}{\|c\|}$$

9.4

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The cross product is orthogonol to both i+j+k and 2i+k:

$$i + j + k = < 1, 1, 1 > \text{ and } 2i + k = < 2, 0, 1 >$$
$$< 1, 1, 1 > \times < 2, 0, 1 > = (1 - 0)\vec{i} + (2 - 1)\vec{j} + (0 - 2)\vec{k}$$
$$< 1, 1, 1 > \times < 2, 0, 1 > = < 1, 1, -2 >$$

Divide by the magnitude to find the unit vector:

$$\begin{aligned}\text{unit vector } &= \frac{<1,1,-2>}{\sqrt{1+1+4}}\\ &=< \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}} >\end{aligned}$$

The second unit vector would be the negative of the first:

$$=< \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}} >$$

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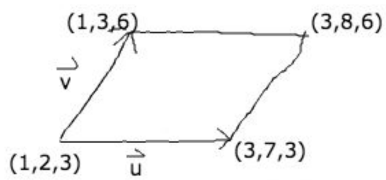

$$\begin{aligned}\vec{u} =< 2,5,0 > \text{ and } \vec{v} =< 0,1,3 >\\ A &= \| <2,5,0> \times <0,1,3> \|\\ A &= \| < -15, 6, -2 > \|\\ A &= \sqrt{265}\end{aligned}$$

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a)

$$\begin{aligned}\vec{PQ} =< 1,2,1 > \text{ and } \vec{PR} =< 5,0,-2 >\\ \text{The vector orthogonol to the plane} = \vec{PQ} \times \vec{PR}\\ \vec{PQ} \times \vec{PR} =< -5,7,-10 >\end{aligned}$$

b)

$$A = \frac{\| < -5, 7, -10 > \|}{2}$$
$$A = \frac{\sqrt{174}}{2} \approx 6.6$$

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a)

$$\begin{aligned} d &= \|b\| sin\theta \\ &= \frac{\|a\|}{\|a\|} \|b\| sin\theta \\ &= \frac{\|a\| \|b\| sin\theta}{\|a\|} \\ &= \frac{\|a \times b\|}{\|a\|} \end{aligned}$$

b)

$\vec{a} =< -1, -2, -1 >$ and $\vec{b} =< 1, -5, -7 >$

$$\begin{aligned} d &= \frac{\| < -1, -2, -1 > \times < 1, -5, -7 > \|}{\sqrt{1+4+1}} \\ &= \frac{\| < 9, -15, 7 > \|}{\sqrt{6}} \\ &= \frac{\sqrt{355}}{\sqrt{6}} \\ d &\approx 7.69 \end{aligned}$$

9.5

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$$\begin{aligned} \text{vector equation } &=< 6, -5, 2 > +t < 1, 3, \frac{-2}{3} > \\ &=< 6 + t, -5 + 3t, 2 + \frac{-2}{3}t > \end{aligned}$$

The parametric equations are:

$$\begin{aligned} x(t) &= 6 + t \\ y(t) &= -5 + 3t \\ z(t) &= 2 + \frac{-2}{3}t \end{aligned}$$

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$$\begin{aligned} x(t) &= 1 + t \\ y(t) &= 3t \\ z(t) &= 6 + t \end{aligned}$$

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$$\vec{n_1} =< 1, 2, 3 > \text{ and } \vec{n_2} =< 1, -1, 1 >$$

$< 1, 2, 3 > \times < 1, -1, 1 >$ is a line parallel to the line of intersection.

$$< 1, 2, 3 > \times < 1, -1, 1 >=< 5, 2, -3 >$$

Plug in z=0 to solve x, y and find a point on the line of intersection

$$x + 2y = 1 \text{ and } x - y = 1$$

This gives us $3y = 0$

Therefore (1,0,0) lies on the line of intersection and the symmetric equations are:

$$\frac{x-1}{5} = \frac{y}{2} = \frac{-z}{3}$$

And the parametric equations are:

$$\begin{aligned} x(t) &= 1 + 5t \\ y(t) &= 2t \\ z(t) &= -3t \end{aligned}$$

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Check ratio of coefficients to test if parallel:

$$\frac{1}{-1} = \frac{3}{1} = \frac{-1}{3}$$

$-1 \neq 3 \neq -3$therefore the lines are NOT parallel

Solve system of equations to test if intersecting:

$$1 + 2t = -1 + s$$
$$3t = 4 + s$$
$$2 - t = 1 + 3s$$

Solve the first equation for t:

$$2t = -2 + s$$
$$t = -1 + \frac{s}{2}$$

Plug into the second equation:

$$3(-1 + \frac{s}{2}) = 4 + s$$
$$-3 + \frac{3s}{2} = 4 + s$$
$$\frac{s}{2} = 7$$
$$s = 14$$

Plug s=14 this into the equation for t:

$$t = -1 + \frac{14}{2} = -6$$

Plug s=14 and t=-6 into the third equation:

$$2 + -6 = 1 + 3(14)$$

$-4 \neq 43$therefore the lines do not intersect.

Since the lines are not parallel and do not intersect, they must be skew.

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$$P(0,1,1) \text{ and } Q(1,0,1) \text{ and } R(1,1,0)$$
$$\vec{PQ} =< 1-0, 0-1, 1-1 > =< 1,-1,0 >$$
$$\vec{PR} =< 1-0, 1-1, 0-1 > =< 1,0,-1 >$$
Take the cross product to get the coefficients of the plane equation:
$$< 1,-1,0 > \times < 1,0,-1 >=< 1,1,1 >$$
Use P as the point for the equation of the plane:
$$1(x-0)+1(y-1)+1(z-1)=0$$
$$x+y+z=2$$

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The points P(6,0,-2) and Q(4,3,7) are on the plane.

Set t=1 to get a third point on the plane, R(2,8,11)
$$\vec{PQ} =< 4-6, 3-0, 7-(-2) >=< -2,3,9 >$$
$$\vec{PR} =< 2-6, 8-0, 11-(-2) >=< -4,8,13 >$$
Take the cross product to get the coefficients of the plane equation:
$$< -2,3,9 > \times < -4,8,13 >=< -33,-10,-4 >$$
Use P as the point for the equation of the plane:
$$-33(x-6)+-10(y-0)+-4(z+2)=0$$
$$-33x+198-10y-4z-8=0$$
$$-33x-10y-4z=-190$$

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Let Q be the plane that we are looking for.

Set z=0 to get a point on the line of intersection:

$$(1,3,0)$$

To get a normal vector for Q, take the cross product of two vectors parallel to Q.

The normal vector of the plane perpendicular to Q is parallel to Q:

$$< 1, 1, -2 >$$

Get a second vector parallel to Q by taking the cross product of the normal vectors of the two planes that form the line of intersection that Q passes through.

$$< 1, 0, -1 > \times < 0, 1, 2 > =< 1, -2, 1 >$$

Now take the cross product of these two vectors:

$$< 1, 1, -2 > \times < 1, -2, 1 > =< -3, -3, -3 >$$

Plug these into the plane equation:

$$\begin{aligned} -3(x-1) - 3(y-3) - 3(z-0) &= 0 \\ -3x + 3 - 3y + 9 - 3z &= 0 \\ -3x - 3y - 3z &= -12 \\ -3(x+y+z) &= -12 \\ x + y + z &= 4 \end{aligned}$$

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$$\begin{aligned} d &= \frac{|1(6) + (-2)(0) + (-4)(-2) + (-8)|}{\sqrt{1^2 + (-2)^2 + (-4)^2}} \\ &= \frac{|6 + 8 - 8|}{\sqrt{21}} \\ &= \frac{6}{\sqrt{21}} \\ d &\approx 1.31 \end{aligned}$$

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To find the distance we need a point on one plane.

Plug in z=0 to the first equation.

$$
\begin{aligned}
0 &= 4y - 2x \\
y &= \frac{1}{2}x
\end{aligned}
$$

Therefore the point (2,1,0) is on the first plane.

Now we can use the distance formula.

$$
\begin{aligned}
d &= \frac{|3(2) + -6(1) + 9(0) + -1|}{\sqrt{3^2 + (-6)^2 + 9^2}} \\
&= \frac{|6 - 6 - 1|}{\sqrt{126}} \\
&= \frac{1}{\sqrt{126}} \\
d &\approx .09
\end{aligned}
$$