ANN KIDDER
HW #4.

10.2

16) $r(t)=4\sqrt{t}\,i+t^2 j+tk$

$r'(t)=2\frac{1}{\sqrt{t}}i+2tj+k$

$r'(1)=2i+2j+k$

$|r'(1)|=\sqrt{4+4+1}=\sqrt{9}=3$

unit tangent vector $=\frac{2i+2j+k}{3}=\boxed{\frac{2}{3}i+\frac{2}{3}j+\frac{1}{3}k}$

20) $r(t)=\langle \cos t, 3\sin t, 4t\rangle$

$r'(t)=\langle -\sin t, 3\cos t, 4\rangle$

$r'(0)=\langle 0, 3, 4\rangle$

$|r'(0)|=\sqrt{9+16}=\sqrt{25}=5$

$\boxed{T(0)=\langle 0, \frac{3}{5}, \frac{4}{5}\rangle}$

$r''(t)=\langle -\cos t, -3\sin t, 0\rangle$

$r'(t)\times r''(t)=\langle 12\sin t, -4\cos t, 3(\sin^2 t+\cos^2 t)\rangle$ but $\sin^2 t+\cos^2 t=1$ therefore

$=\langle 12\sin t, -4\cos t, 3\rangle$

52) If $r(t)$ is always perpendicular to $r'(t)$, then $r(t)\cdot r'(t)=0$.

$r(t)=\langle x(t), y(t), z(t)\rangle$

$|r(t)|=\sqrt{(x(t))^2+(y(t))^2+(z(t))^2}$

$|r(t)|^2=(x(t))^2+(y(t))^2+(z(t))^2=\underline{r(t)\cdot r(t)}$

$\frac{d}{dt}(r(t)\cdot r(t))=2x(t)x'(t)+2y(t)y'(t)+2z(t)z'(t)$

$=2(x(t)x'(t)+y(t)y'(t)+z(t)z'(t))$

$=2(r(t)\cdot r'(t))$ we are given $r(t)\cdot r'(t)=0$, therefore,

$=0$

Therefore $r(t)\cdot r(t)$ must be a constant since the derivative $=0$

This means $x(t)^2+y(t)^2+z(t)^2=c$ which is the equation for a sphere.

10.3

4). $x(t)=\cos(t)$
$y(t)=\sin(t)$ $\quad 0\le t\le \frac{\pi}{4}$
$z(t)=\ln(\cos(t))$

$x'(t)=-\sin(t)$
$y'(t)=\cos(t)$
$z'(t)=-\frac{\sin t}{\cos t}=-\tan(t)$

$$\text{length}=\int_0^{\frac{\pi}{4}}\sqrt{(-\sin(t))^2+(\cos t)^2+(-\tan(t))^2}\,dt=\int_0^{\frac{\pi}{4}}\sqrt{\underbrace{\sin^2 t+\cos^2 t}_{=1}+\tan^2 t}\,dt$$

$$=\int_0^{\frac{\pi}{4}}\sqrt{1+\tan^2 t}\,dt=\int_0^{\frac{\pi}{4}}\frac{1}{\cos(t)}\,dt=\int_0^{\frac{\pi}{4}}\sec(t)\,dt$$

$$=\ln|\sec t+\tan t|\Big|_0^{\frac{\pi}{4}}=\ln|\sec(\tfrac{\pi}{4})+\tan(\tfrac{\pi}{4})|-\ln\underbrace{|\sec(0)+\tan(0)|}_{=0}$$

$$\text{length}=\ln\left|\frac{2}{\sqrt{2}}+1\right|\ \boxed{\approx .881}$$

6). $x(t)=12t$
$y(t)=8t^{\frac{3}{2}}$ $\quad 0\le t\le 1$
$z(t)=3t^2$

$x'(t)=12$
$y'(t)=8(\frac{3}{2})t^{\frac{1}{2}}=12\sqrt{t}$
$z'(t)=6t$

$$\text{length}=\int_0^1\sqrt{144+144t+36t^2}\,dt=\int_0^1\sqrt{36(4+4t+t^2)}\,dt=6\int_0^1\sqrt{(t+2)^2}\,dt=6\int_0^1 t+2\,dt.$$

$$=\frac{1}{2}t^2+2t\Big|_0^1$$
$$=(\tfrac{1}{2}+2)-(0+0)$$

$$\text{length}\ \boxed{=\frac{3}{2}}$$

20a) $r(t) = \langle t, \frac{1}{2}t^2, t^2 \rangle$

$r'(t) = \langle 1, t, 2t \rangle$

$\frac{d}{dt}(1+5t^2)^{-\frac{1}{2}}$

$= -\frac{1}{2}(1+5t^2)^{-\frac{3}{2}}(10t^2)$

$\frac{d}{dt}\left(\frac{t}{\sqrt{1+5t^2}}\right)$ quotient rule

$= \dfrac{\sqrt{1+5t^2} - \frac{5t^2}{\sqrt{1+5t^2}}}{1+5t^2}$

$= \dfrac{\frac{1+5t^2-5t^2}{\sqrt{1+5t^2}}}{1+5t^2} = \dfrac{t}{\sqrt{1+5t^2}}$

$\frac{d}{dt}\left(\frac{2t}{\sqrt{1+5t^2}}\right)$ quotient rule

$= \dfrac{\sqrt{1+5t^2} - \frac{10t^2}{\sqrt{1+5t^2}}}{1+5t^2}$

$= \dfrac{2}{(1+5t^2)(\sqrt{1+5t^2})}$

$T(t) = \dfrac{\langle 1, t, 2t \rangle}{\sqrt{1+t^2+4t^2}} = \dfrac{\langle 1, t, 2t \rangle}{\sqrt{1+5t^2}}$

$T(t) = \left\langle \dfrac{1}{\sqrt{1+5t^2}}, \dfrac{t}{\sqrt{1+5t^2}}, \dfrac{2t}{\sqrt{1+5t^2}} \right\rangle$

$\boxed{T'(t) = \left\langle -\dfrac{5t}{(1+5t^2)^{\frac{3}{2}}}, \dfrac{1}{(1+5t^2)\sqrt{1+5t^2}}, \dfrac{2}{(1+5t^2)\sqrt{1+5t^2}} \right\rangle}$

magnitude $= \sqrt{\dfrac{25t^2}{(1+5t^2)^3} + \dfrac{1}{(1+5t^2)^2(1+5t^2)} + \dfrac{4}{(1+5t^2)^2(1+5t^2)}}$

$= \sqrt{\dfrac{25t^2+5}{(1+5t^2)^3}}$

$= \sqrt{5\left(\dfrac{\cancel{5t^2+1}}{(5t^2+1)^{\cancel{3}\,2}}\right)}$

$= \sqrt{5\left(\dfrac{1}{(5t^2+1)^2}\right)}$

$= \dfrac{\sqrt{5}}{5t^2+1}.$

$N'(t) = \left\langle \left(\dfrac{-5t}{(1+5t^2)^{\frac{3}{2}}}\right)\left(\dfrac{5t^2+1}{\sqrt{5}}\right), \left(\dfrac{1}{(\cancel{1+5t^2})\sqrt{1+5t^2}}\right)\left(\dfrac{\cancel{5t^2+1}}{\sqrt{5}}\right), \left(\dfrac{2}{(\cancel{1+5t^2})\sqrt{1+5t^2}}\right)\left(\dfrac{\cancel{5t^2+1}}{\sqrt{5}}\right) \right\rangle$

$\boxed{N(t) = \dfrac{1}{\sqrt{5}}\left\langle \dfrac{-5t}{\sqrt{1+5t^2}}, \dfrac{1}{\sqrt{1+5t^2}}, \dfrac{2}{\sqrt{1+5t^2}} \right\rangle}$

$|T'(t)| = \sqrt{\left(\dfrac{1}{(1+5t^2)^{\frac{3}{2}}}\right)^2(25t^2+1+4)}$

$= \sqrt{\dfrac{5(1+5t^2)}{(1+5t^2)^3}}$

$= \sqrt{\dfrac{5}{(1+5t^2)}}$

$= \dfrac{\sqrt{5}}{1+5t^2}$

$|r'(t)| = \sqrt{1+t^2+4t^2}$

$= \sqrt{1+5t^2}$

$k(t) = \dfrac{\frac{\sqrt{5}}{1+5t^2}}{\sqrt{1+5t^2}} = \boxed{\dfrac{\sqrt{5}}{(1+5t^2)^{\frac{3}{2}}}}$

22) $r(t) = ti + t^2 j + e^t k$

$r'(t) = i + 2tj + e^t k$

$r''(t) = 2j + e^t k$

$$\|\underbrace{\langle 1, 2t, e^t\rangle}_{r'(t)} \times \underbrace{\langle 0, 2, e^t\rangle}_{r''(t)}\| = \|\langle 2te^t - 2e^t, -e^t, 2\rangle\|$$

$$= \sqrt{4e^{2t}(t-1)^2 + e^{2t} + 4}$$

$$= \sqrt{4e^{2t}(t^2 - 2t + 1) + e^{2t} + 4}$$

$$= \sqrt{4e^{2t}t^2 - 8e^{2t}t + 4e^{2t} + e^{2t} + 4}$$

$$\|r'(t) \times r''(t)\| = \sqrt{4e^{2t}t^2 - 8e^{2t}t + 5e^{2t} + 4}$$

$$\|r'(t)\|^3 = \left(\sqrt{1 + 4t^2 + e^{2t}}\right)^3$$

$$= (1 + 4t^2 + e^{2t})^{\frac{3}{2}}$$

$$k(t) = \frac{\sqrt{4e^{2t}t^2 - 8e^{2t}t + 5e^{2t} + 4}}{(1 + 4t^2 + e^{2t})^{\frac{3}{2}}}$$

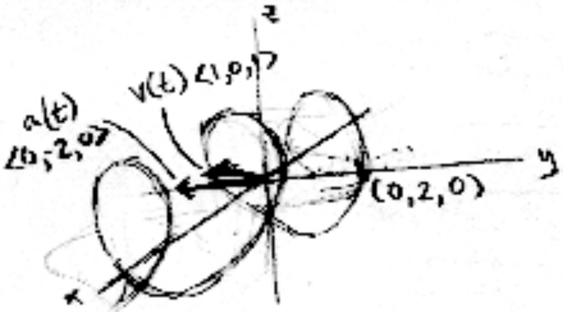
10.4

8) $r(t) = t\,i + 2\cos t\,j + \sin t\,k, \quad t=0$

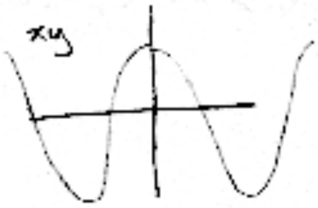
velocity $r'(t) = i + (-2\sin t)j + \cos t\,k \quad$ at $t=0$ $\langle 1, 0, 1\rangle$

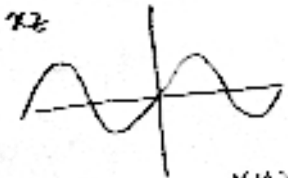
accel. $r''(t) = -2\cos t\,j - \sin t\,k \quad$ at $t=0$ $\langle 0, -2, 0\rangle$

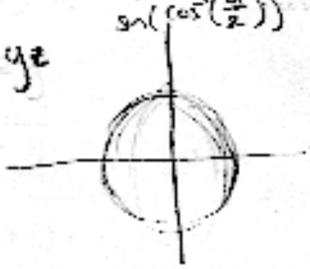
speed $= |v(t)| = |r'(t)| = \sqrt{1+4\sin^2 t+\cos^2 t}$ at $t=0 \rightarrow \sqrt{1+0+1} = \sqrt{2}$.



$x = t$
$y = 2\cos t$
$z = \sin t$







36) $r(t) = t\,i + t^2 j + 3t\,k$.

$v(t) = r'(t) = i + 2t\,j + 3k$.

$a(t) = r''(t) = 2j$

$$a_T = \frac{r'(t)\cdot r''(t)}{\|r'(t)\|} = \frac{\langle 1, 2t, 3\rangle \cdot \langle 0, 2, 0\rangle}{\sqrt{1+4t^2+9}}$$

$$\boxed{a_T = \frac{4t}{\sqrt{10+4t^2}}}$$

$$a_N = \frac{\|r'(t)\times r''(t)\|}{\|r'(t)\|} = \frac{\langle 1, 2t, 3\rangle \times \langle 0, 2, 0\rangle}{\sqrt{10+4t^2}}$$

$\langle 1 \quad 2t \quad 3\rangle$
$\times$
$\langle 0 \quad 2 \quad 0\rangle$
$=$
$\langle -6, 0, 2\rangle$

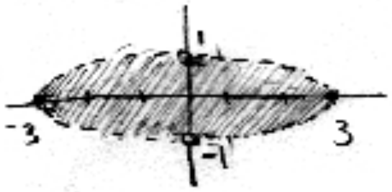
$$a_N = \frac{\langle -6, 0, 2\rangle}{\sqrt{10+4t^2}}$$

$$\boxed{a_N = \left\langle \frac{-6}{\sqrt{10+4t^2}}, 0, \frac{2}{\sqrt{10+4t^2}} \right\rangle}$$

11.1

5) $9-x^2-9y^2>0$ b/c $\ln(\_\_\_) > 0$

range goes from $(-\infty, \ln(9)]$



← wide ellipse

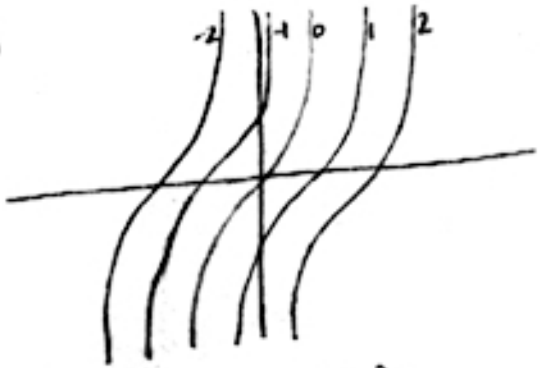
6) $f(x,y)=\sqrt{y}+\sqrt{25-x^2-y^2}$

$0 \le y \le 5$

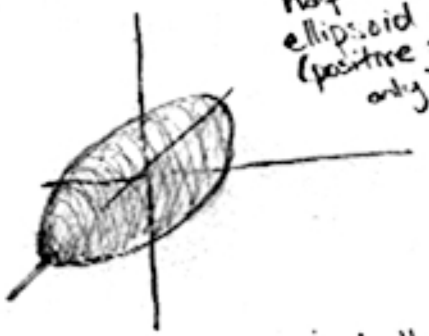
$-5 \le x \le 5$

← semi-circle

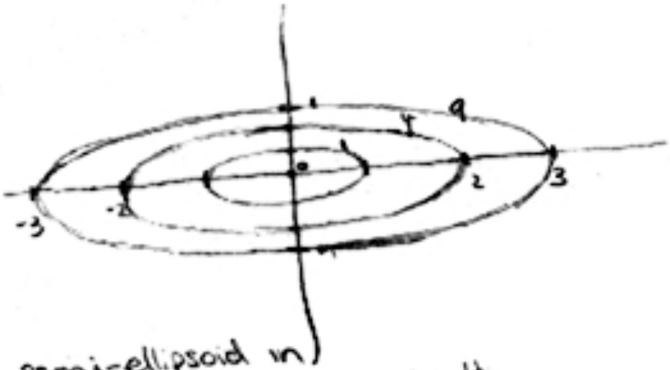
20)



27)



half-ellipsoid (positive z only)



It is difficult to represent the semi-ellipsoid in the 3D-diagram. It looks fully 3D around the x-axis. It is clearer that it is only for positive z-values on the contour graph.

11.3

16) $f(x,y) = x^4y^3 + 8x^2y$

$$\frac{df}{dx} = 3x^3y^3 + 16xy$$

$$\frac{df}{dy} = 3x^4y^2 + 8x^2$$

20) $z = \tan xy$

$$\frac{dz}{dx} = y\sec^2(xy)$$

$$\frac{dz}{dy} = x\sec^2(xy)$$

25) $f(r,s) = r\ln(r^2+s^2)$

$$\frac{df}{dr} = \ln(r^2+s^2) + r\,\frac{2r}{r^2+s^2} = \boxed{\ln(r^2+s^2) + \frac{2r^2}{r^2+s^2}}$$

$$\frac{df}{ds} = r\,\frac{2s}{r^2+s^2} = \boxed{\frac{2sr}{r^2+s^2}}$$

36) $f(x,y,z,t) = \frac{xy^2}{t+2z} = xy^2(t+2z)^{-1}$

$$\frac{df}{dx} = \frac{y^2}{t+2z}$$

$$\frac{df}{dy} = 2y\left(\frac{x}{t+2z}\right) = \frac{2xy}{t+2z}$$

$$\frac{df}{dz} = xy^2(-1)(t+2z)^{-2}(2) = \frac{-2xy^2}{(t+2z)^2}$$

$$\frac{df}{dt} = xy^2(-1)(t+2z)^{-2}(1) = \frac{-xy^2}{(t+2z)^2}$$

39) $f(x,y) = \ln\left(x + (x^2+y^2)^{\frac{1}{2}}\right)$; $f_x(3,4)$.

$$f_x(x,y) = \frac{1}{x+\sqrt{x^2+y^2}}\left(1 + \cancel{\tfrac{1}{2}}(x^2+y^2)^{-\frac{1}{2}}(\cancel{2}x)\right)$$

$$= \left(\frac{1}{\cancel{x+\sqrt{x^2+y^2}}}\right)\left(\frac{\cancel{\sqrt{x^2+y^2}+x}}{\sqrt{x^2+y^2}}\right)$$

$$= \frac{1}{\sqrt{x^2+y^2}}$$

$$f_x(3,4) = \frac{1}{\sqrt{9+16}} = \boxed{\frac{1}{5}}$$

40) $f(x,y) = \arctan\left(\frac{y}{x}\right)$; $f_x(2,3)$

$$f_x(x,y) = \frac{1}{1+\left(\frac{y}{x}\right)^2}(y)(-1)\left(\frac{1}{x^2}\right)$$

$$= \frac{\frac{-y}{x^2}}{\frac{x^2}{x^2}+\frac{y^2}{x^2}} = \frac{\frac{-y}{x^2}}{\frac{x^2+y^2}{x^2}}$$

$$= \frac{-y}{x^2+y^2}$$

$$f_x(2,3) = \frac{-3}{4+9} = \boxed{\frac{-3}{13}}$$

76) $P(L,K) = bL^{\alpha}K^{\beta}$

$$\frac{dP}{dL} = bK^{\beta}\alpha L^{\alpha-1}$$

$$\frac{dP}{dK} = bL^{\alpha}\beta K^{\beta-1}$$

plug into the equation: $L\frac{dP}{dL} + K\frac{dP}{dK}$.

$$L(bK^{\beta}\alpha L^{\alpha-1}) + K(bL^{\alpha}\beta K^{\beta-1})$$

$$= \underline{bK^{\beta}}\alpha \underline{L^{\alpha}} + b\underline{L^{\alpha}}\beta \underline{K^{\beta}}$$

$$= bL^{\alpha}K^{\beta}(\alpha+\beta)$$

$= P(\alpha+\beta)$ ✓ equation is satisfied.