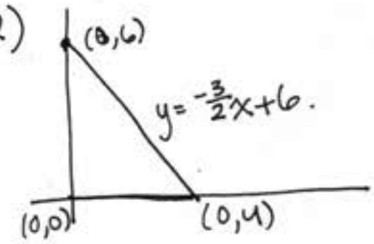
ANN KIDDER
HW #13

2)



$\frac{dz}{dx}=-2$

$\frac{dz}{dy}=5$

$0 \le x \le 4$

$0 \le y \le -\frac{3}{2}x+6.$

$$A(S)=\iint \sqrt{1+(-2)^2+(5)^2}\,dA.$$

$$=\int_0^4 \int_0^{-\frac{3}{2}x+6} \sqrt{30}\,dy\,dx$$

$$=\int_0^4 \sqrt{30}\left(6-\tfrac{3}{2}x\right)dx$$

$$=\sqrt{30}\left[6x-\frac{3x^2}{4}\right]\Big|_0^4$$

$$=\sqrt{30}\,[24-12]$$

$$\boxed{=12\sqrt{30}}$$

4). $\vec{r}_u=\langle 1,-3,1\rangle$

$\vec{r}_v=\langle 1,0,-1\rangle$

$\frac{d(u+v)}{du}=1$ $\quad \frac{d(2-3u)}{du}=-3$ $\quad \frac{d(1+u-v)}{du}=1$

$\frac{d(u+v)}{dv}=1$ $\quad \frac{d(2-3u)}{dv}=0$ $\quad \frac{d(1+u-v)}{dv}=-1$

$\vec{r}_u \times \vec{r}_v = \langle 3,2,3\rangle$

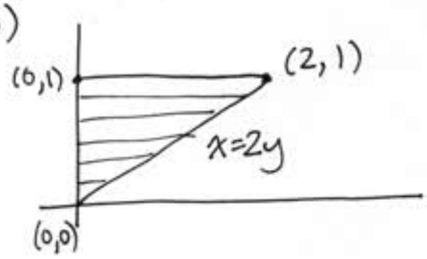
$$A(S)=\int_0^2\int_{-1}^1 |\langle 3,2,3\rangle|\,dv\,du=\int_0^2\int_{-1}^1 9+4+9\,dv\,du=\int_0^2\int_{-1}^1 22\,dv\,du.$$

$$=\int_0^2 22v\Big|_{-1}^1\,du=\int_0^2\left[22(1)-22(-1)\right]du=\int_0^2 44\,dv$$

$$=44v\Big|_0^2$$

$$\boxed{=88}$$

6)



$0 \le y \le 1$

$0 \le x \le 2y.$

$\frac{dz}{dx} = 3$

$\frac{dz}{dy} = 4y$

$$A(S) = \int_0^1 \int_0^{2y} \sqrt{1+9+16y^2}\, dx\, dy.$$

$$= \int_0^1 2y\sqrt{10+16y^2}\, dy.$$

$u = 10+16y^2$

$du = 32y\,dy.$

$\frac{1}{16}\, du = 2y\,dy.$

$$= \int_{y=0}^{y=1} \frac{1}{16}\sqrt{u}\; du$$

$$= \left(\frac{1}{16}\right)\left(\frac{2}{3}\right) u^{\frac{3}{2}} \Big|_{y=0}^{y=1}$$

$$= \frac{1}{24}\left(10+16y^2\right)^{\frac{3}{2}} \Big|_0^1$$

$$\boxed{= \frac{1}{24}\left[(26)^{\frac{3}{2}} - 10^{\frac{3}{2}}\right]}$$

10)

$x = y^2+z^2$ $\quad y = r\cos\theta$

$x = r^2$ $\quad z = r\sin\theta.$

$y^2+z^2 = 9 \quad r=3 \quad 0 \le r \le 3.$

full cylinder $0 \le \theta \le 2\pi$

$\vec{r}(r,\theta) = \langle r^2, r\cos\theta, r\sin\theta \rangle$

$\vec{r}_r = \langle 2r, \cos\theta, \sin\theta \rangle$

$\vec{r}_\theta = \langle 0, -r\sin\theta, r\cos\theta \rangle.$

$\vec{r}_r \times \vec{r}_\theta = \langle r\cos^2\theta + r\sin^2\theta, -2r^2\cos\theta, -2r^2\sin\theta \rangle = \langle r, -2r^2\cos\theta, -2r^2\sin\theta \rangle$

$|\vec{r}_r \times \vec{r}_\theta| = \sqrt{r^2 + 4r^4\cos^2\theta + 4r^4\sin^2\theta} = \sqrt{r^2+4r^4} = r\sqrt{1+4r^2}$

$u = 1+4r^2$

$du = 8r\,dr$

$\frac{1}{8}du = r\,dr.$

$$A(S) = \int_0^{2\pi}\int_0^3 r\sqrt{1+4r^2}\, dr\, d\theta.$$

$$= \int_0^{2\pi}\int_{r=0}^{r=3} \frac{1}{8}\sqrt{u}\; dr\, d\theta.$$

$$= \int_0^{2\pi} \frac{1}{8}\cdot\frac{2}{3} u^{\frac{3}{2}} \Big|_{r=0}^{r=3} d\theta = \int_0^{2\pi} \frac{1}{12}\left(1+4r^2\right)^{\frac{3}{2}} \Big|_0^3 d\theta = \int_0^{2\pi} \left(\frac{1}{12}\left[37^{\frac{3}{2}} - 1\right]\right) d\theta$$

$$\boxed{= \frac{\pi}{6}\left(37^{\frac{3}{2}} - 1\right)}$$

22) $\vec{r} = (b\cos\theta + a\cos\alpha\cos\theta)\vec{i} + (b\sin\theta + a\cos\alpha\sin\theta)\vec{j} + a\sin\alpha\vec{k}$

$= \cos\theta(b+a\cos\alpha)\vec{i} + \sin\theta(b+a\cos\alpha)\vec{j} + a\sin\alpha\vec{k}$

$\frac{d\vec{r}}{d\theta} = -\sin\theta(b+a\cos\alpha)\vec{i} + \cos\theta(b+a\cos\alpha)\vec{j} + 0\vec{k}$

$\frac{d\vec{r}}{d\alpha} = -a\sin\alpha\cos\theta\vec{i} - a\sin\alpha\sin\theta\vec{j} + a\cos\alpha\vec{k}$

$|\vec{r}_\theta \times \vec{r}_\alpha| = [a\cos\alpha\cos\theta(b+a\cos\alpha) - 0]\vec{i}$

$+ [0 + a\cos\alpha\sin\theta(b+a\cos\alpha)]\vec{j}$

$+ [a\sin\alpha\underbrace{\sin^2\theta(b+a\cos\alpha) + a\sin\alpha\cos^2\theta}_{=1}(b+a\cos\alpha)]\vec{k}$

$|\vec{r}_\theta \times \vec{r}_\alpha| = \sqrt{(a^2\cos^2\alpha\underbrace{\cos^2\theta(b+a\cos\alpha)^2 + a^2\cos^2\alpha\sin^2\theta}_{=1}(b+a\cos\alpha)^2 + a^2\sin^2\alpha(b+a\cos\alpha)^2}$

$= \sqrt{a^2\underbrace{\cos^2\alpha(b+a\cos\alpha)^2 + a^2\sin^2\alpha}_{=1}(b+a\cos\alpha)^2}$

$= \sqrt{a^2(b+a\cos\alpha)^2}$

$= a(b+a\cos\alpha)$

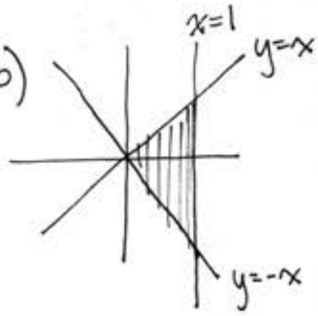
$A = \int_0^{2\pi}\int_0^b a(b+a\cos\alpha)\,da\,d\alpha \quad 0 \le a \le b. \quad 0 \le \alpha \le 2\pi$

$\int_0^{2\pi}\int_0^b ab + a^2\cos\alpha\,da\,d\alpha.$

$\int_0^{2\pi} \frac{a^2b}{2} + \frac{a^3}{3}\cos\alpha\Big|_0^b d\alpha = \int_0^{2\pi} \frac{b^3}{2} + \frac{b^3}{3}\cos\alpha\,d\alpha$

$= \frac{b^3}{2}\alpha + \frac{b^3}{3}\sin\alpha\Big|_0^{2\pi} = (\pi b^3 + 0) - (0+0)$

$\boxed{= \pi b^3}$

26) 

$0 \le x \le 1$
$-x \le y \le x$

$f(x,y) = \sqrt{1-x^2}$ $\quad f_x(x,y) = \frac{-x}{\sqrt{1-x^2}}$ $\quad f_y = 0$

$A = \sqrt{1 + \frac{x^2}{1-x^2}}$

$$A = 8\int_0^1 \int_{-x}^{x} \sqrt{1+\frac{x^2}{1-x^2}}\, dy\,dx.$$

$\xrightarrow{\text{inside}} \sqrt{\frac{1-\cancel{x^2}+\cancel{x^2}}{1-x^2}} = \sqrt{\frac{1}{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$

$$= 8\int_0^1 \left[x\frac{1}{\sqrt{1-x^2}} + x\frac{1}{\sqrt{1-x^2}}\right]dx = 8\int_0^1 \frac{2x}{\sqrt{1-x^2}}\,dx$$

$u = 1-x^2$
$du = -2x\,dx$
$-du = 2x\,dx$

$$= 8\int_{x=0}^{x=1} -\frac{1}{\sqrt{u}}\,du$$

$$= 8\left(-2u^{\frac{1}{2}}\right)\Big|_{x=0}^{x=1}$$

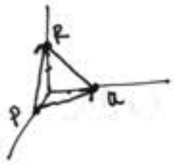
$$= 16(1-x^2)^{\frac{1}{2}}\Big|_0^1$$

$$= 16\left[(0^{\frac{1}{2}}) - (1^{\frac{1}{2}})\right]$$

$$= 16[-1]$$

$$\boxed{= -16.}$$

13.6

10) 

$(z=xy)$

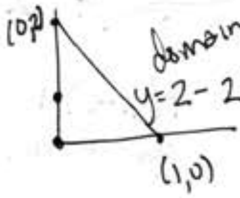
$z=2-2x-y=g(x,y)$.

$\iint_D f(x,y,g(x,y))\sqrt{1+(\frac{dz}{dx})^2+(\frac{dz}{dy})^2}\,dA$

$\frac{dz}{dx}=-2$

$\frac{dz}{dy}=-1$

domain in x-y plane



$0 \le x \le 1$.
$0 \le y \le 2-2x$.

$\iint xy\sqrt{6}\;dy\,dx$.

$\int_0^1\int_0^{2-2x} xy\sqrt{6}\,dy\,dx$.

$=\int_0^1 x\frac{y^2}{2}\sqrt{6}\Big|_0^{2-2x} dx$

$=\sqrt{6}\int_0^1 x\frac{(2-2x)^2}{2}$ $=$ $4-8x+4x^2$

$=\frac{4x-8x^2+4x^3}{2}$

$\frac{\sqrt{6}}{2}\left(\frac{4x^2}{2}-\frac{8x^3}{3}+\frac{4x^4}{4}\right)\Big|_0^1$

$=\frac{\sqrt{6}}{2}\left(2-\frac{8}{3}+1\right)=\frac{\sqrt{6}}{2}\left(\frac{6}{3}-\frac{8}{3}+\frac{3}{3}\right)=\boxed{\frac{\sqrt{6}}{6}.}$

14) $x = y + 2z^2$

$\frac{dx}{dy} = 1$

$\frac{dx}{dz} = 4z.$

$$S = \int_0^1\int_0^1 z\sqrt{1+1+16z^2}\,dz\,dy$$

$$= \int_0^1\int_0^1 z\sqrt{2+16z^2}\,dz\,dy$$

$u = 2+16z^2$

$du = 32z\,dz$

$\frac{1}{32}du = z\,dz$

$$= \int_0^1\int_{z=0}^{z=1} \sqrt{u}\left(\frac{1}{32}\right) du\,dy$$

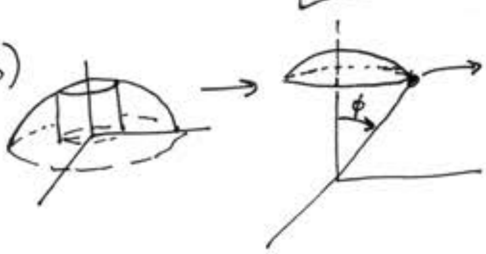
$$= \frac{1}{32}\int_0^1 \frac{2}{3}u^{\frac{3}{2}}\Big|_{z=0}^{z=1} dy$$

$$= \frac{1}{32}\int_0^1 \frac{2}{3}(2+16z^2)^{\frac{3}{2}}\Big|_0^1 dy$$

$$= \frac{1}{\cancel{32}_{16}}\int_0^1 \frac{\cancel{2}}{3}\left[18^{\frac{3}{2}} - 2^{\frac{3}{2}}\right] dy$$

$$= \frac{1}{48}\left(18^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) y\Big|_0^1$$

$$\boxed{= \frac{1}{48}\left(18^{\frac{3}{2}} - 2^{\frac{3}{2}}\right)}$$

16) 

$x^2+y^2=1$ and $x^2+y^2+z^2=4$ intersect.

$1+z^2=4$

$z^2=3.$

$z=\sqrt{3}$

$2\cos\phi = \sqrt{3}$

$\cos\phi = \frac{\sqrt{3}}{2}$

$\phi = \frac{\pi}{6}.$

$0 \le \phi \le \frac{\pi}{6}.$

$0 \le \theta \le 2\pi.$

$r(\theta,\phi) = 2\sin\phi\cos\theta\,\vec{i} + 2\sin\phi\sin\theta\,\vec{j} + 2\cos\phi\,\vec{k}$

$r_\theta = -2\sin\phi\sin\theta\,\vec{i} + 2\sin\phi\cos\theta\,\vec{j}$

$r_\phi = 2\cos\phi\cos\theta\,\vec{i} + 2\cos\phi\sin\theta\,\vec{j} - 2\sin\phi\,\vec{k}$

$r_\theta \times r_\phi = (-4\sin^2\phi\cos\theta)\vec{i} + (-4\sin^2\phi\sin\theta)\vec{j} + (-4\sin\phi\cos\phi\,\underline{\sin^2\theta} - 4\sin\phi\cos\phi\,\underline{(\cos^2\theta)})\vec{k}$

$= -4\sin^2\phi\cos\theta\,\vec{i} - 4\sin^2\phi\sin\theta\,\vec{j} - 4\sin\phi\cos\phi\,\vec{k}$

$|r_\theta \times r_\phi| = \sqrt{16\sin^4\phi\,\underline{\cos^2\theta} + 16\sin^4\phi\,\underline{\sin^2\theta} + 16\sin^2\phi\cos^2\phi}$

$= 4\sqrt{\sin^4\phi + \sin^2\phi\cos^2\phi} = 4\sqrt{\sin^2\phi(\sin^2\phi + \cos^2\phi)} = 4\sin\phi.$

con't $\rightarrow$

16 con't) $\displaystyle\int_0^{\frac{\pi}{6}}\int_0^{2\pi} (2\sin\phi\sin\theta)^2(4\sin\phi)\,d\theta\,d\phi$

$$= 4\int_0^{\frac{\pi}{6}}\int_0^{2\pi} \sin^3\phi\,\sin^2\theta\,d\theta\,d\phi$$

inner: $\sin^3\phi\left(\frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right)\Big|_0^{2\pi} = \sin^3\phi\,(\pi - 0) = \pi\sin^3\phi.$

outer: $4\pi\displaystyle\int_0^{\frac{\pi}{6}} \sin^3\phi\,d\phi$

$$4\pi\left[-\cos\phi + \frac{\cos^3\phi}{3}\right]\Big|_0^{\frac{\pi}{6}}$$

$$= 4\pi\left(\left[-\frac{\sqrt{3}}{2} + \frac{\cancel{3}\sqrt{3}}{8}\cdot\frac{1}{\cancel{3}}\right] - \left[-1 + \frac{1}{3}\right]\right)$$

$$= 4\pi\left(\left(-\frac{4\sqrt{3}}{8} + \frac{\sqrt{3}}{8}\right) - \left(-\frac{2}{3}\right)\right)$$

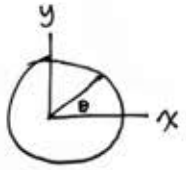
$$= 4\pi\left(\frac{-3\sqrt{3}}{8} + \frac{2}{3}\right)$$

$$= 4\pi\left(\frac{-9\sqrt{3}}{24} + \frac{16}{24}\right)$$

$$= \cancel{4}\pi\left(\frac{16-9\sqrt{3}}{\cancel{24}\,6}\right)$$

$$\boxed{= \frac{(16-9\sqrt{3})\pi}{6}}$$

24)



$z = \sqrt{x^2+y^2}$

$z = \sqrt{r^2}$

$z = r.$

$x = r\cos\theta$ $\quad 0 \le \theta \le 2\pi$

$y = r\sin\theta$ $\quad 0 \le r \le 1.$

$z = r.$

$\vec{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, r\rangle$

$\vec{r}_r = \langle \cos\theta, \sin\theta, 1\rangle$

$\vec{r}_\theta = \langle -r\sin\theta, r\cos\theta, 0\rangle.$

$\vec{r}_r \times \vec{r}_\theta = \langle -r\cos\theta, -r\sin\theta, r\rangle$

since S has a downward orientation x-1.

$\vec{n}\,dS = \langle r\cos\theta, r\sin\theta, -r\rangle\,dS.$

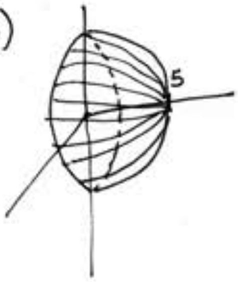
$$\int_0^{2\pi}\int_0^1 \langle r\cos\theta, r\sin\theta, r^4\rangle \cdot \langle r\cos\theta, r\sin\theta, -r\rangle \, dr\,d\theta.$$

$$= \int_0^{2\pi}\int_0^1 r^2\underbrace{\cos^2\theta + r^2\sin^2\theta}_{=1} - r^5 \, dr\,d\theta.$$

$$= \int_0^{2\pi}\int_0^1 r^2 - r^5 \, dr\,d\theta.$$

$$= \int_0^{2\pi} \frac{r^3}{3} - \frac{r^6}{6}\Big|_0^1 d\theta = \int_0^{2\pi} \frac{1^2}{3} - \frac{1}{6}\, d\theta$$

$$= \theta\left(\frac{1}{6}\right)\Big|_0^{2\pi} \quad \frac{2\pi}{6} = \boxed{\frac{\pi}{3}}$$

26) 

$F(x,y,z) = xz\hat{\imath} + x\hat{\jmath} + y\hat{k}$ S is $x^2+y^2+z^2=25$ $y \geq 0$.

$\hookrightarrow r=5$

translating to spherical coordinates:

$x = 5\sin\phi\cos\theta$
$y = 5\sin\phi\sin\theta$
$z = 5\cos\phi.$

$\vec{r}(\phi,\theta) = \langle 5\sin\phi\cos\theta, 5\sin\phi\sin\theta, 5\cos\phi \rangle$

$0 \leq \phi \leq \pi$
$0 \leq \theta \leq \pi$

$\vec{r}_\phi = \langle 5\cos\phi\cos\theta, 5\cos\phi\sin\theta, -5\sin\phi \rangle$
$\vec{r}_\theta = \langle -5\sin\phi\sin\theta, 5\sin\phi\cos\theta, 0 \rangle$
$\vec{r}_\phi \times \vec{r}_\theta = \langle 25\sin^2\phi\cos\theta, 25\sin^2\phi\sin\theta, 25\sin\phi\cos\phi \rangle$

hemisphere

$$\int_0^\pi \int_0^\pi \langle 25\sin\phi\cos\phi\cos\theta, 5\sin\phi\cos\theta, 5\sin\phi\sin\theta \rangle \cdot \langle 25\sin^2\phi\cos\theta, 25\sin^2\phi\sin\theta, 25\sin\phi\cos\phi \rangle d\phi d\theta$$

$$= \int_0^\pi \int_0^\pi 625\sin^3\phi\cos\phi\cos^2\theta + 125\sin^3\phi\cos\theta\sin\theta + 125\sin^2\phi\cos\phi\sin\theta \, d\phi d\theta$$

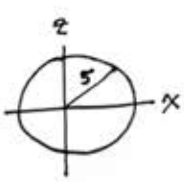
inner: $\underbrace{\frac{625}{4}\sin^4\phi\cos^2\theta}_{=0} + 125\left(\frac{\cos^3\phi}{3} - \cos\phi\right)\cos\theta\sin\theta + \underbrace{\frac{125}{3}\sin^3\phi\sin\theta}_{=0.}\Big|_0^\pi$

$\left[125\left(-\frac{1}{3}+1\right)\cos\theta\sin\theta\right] - \left[125\left(\frac{1}{3}-1\right)\cos\theta\sin\theta\right] = 125\frac{4}{3}\cos\theta\sin\theta$

outer: $\frac{500}{3}\int_0^\pi \cos\theta\sin\theta \, d\theta$

$= \frac{500}{3} \frac{\sin^2\theta}{2}\Big|_0^\pi$

$= 0.$

disk 

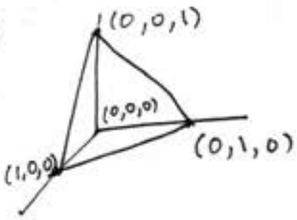
$x^2+z^2=5$, $y=0$

$r=5$ $0 \leq \theta \leq 2\pi.$

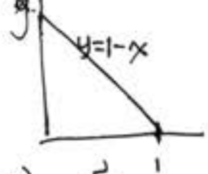
$\vec{r}(\theta) = \langle 5\cos\theta, 0, 5\sin\theta \rangle$
$\vec{r}'(\theta) = \langle -5\sin\theta, 0, 5\cos\theta \rangle$
$\vec{r}(\theta)\cdot\vec{r}'(\theta) = -25\cos\theta\sin\theta + 0 + 25\sin\theta\cos\theta.$
$= 0.$

$$\boxed{\iint_S F \cdot dS = 0}$$

30)

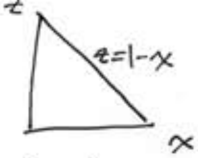


S_{xy} $z=0$



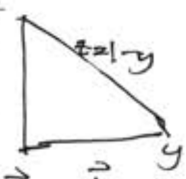
$\vec{n} = -\vec{k}$

S_{xz} $y=0$



$\vec{n} = -\vec{j}$

S_{yz} $x=0.$



$\vec{n} = -\vec{i}$

$$\iint_{S_{xy}} F \cdot -\vec{k}\, dS$$
$$= \iint_{S_{xy}} -x\, dS$$
$$= \int_0^1\int_0^{1-x} -x\sqrt{1+0+0}\, dy\, dx$$
$$= \int_0^1\int_0^{1-x} -x\, dy\, dx$$
$$= \int_0^1 -x(1-x)\, dx$$
$$= \int_0^1 -x + x^2\, dx$$
$$= -\frac{x^2}{2} + \frac{x^3}{3}\Big|_0^1$$
$$= -\frac{1}{2} + \frac{1}{3}$$
$$= -\frac{3}{6} + \frac{2}{6}$$
$$= -\frac{1}{6}$$

$$\iint_{S_{xz}} F \cdot -\vec{j}$$
$$= \iint_{S_{xz}} y - z\, dS$$
$$= \int_0^1\int_0^{1-x} y - z\sqrt{1+0+0} \quad \text{and } y=0$$
$$= \int_0^1\int_0^{1-x} -z\, dz\, dx$$
$$= \int_0^1 -\frac{z^2}{2}\Big|_0^{1-x} dx$$
$$= \int_0^1 -\frac{(1-x)^2}{2}\, dx$$
$$= \int_0^1 -\frac{(1-2x+x^2)}{2}\, dx$$
$$= -\frac{1}{2}\left[x - x^2 + \frac{x^3}{3}\right]\Big|_0^1$$
$$= -\frac{1}{2}\left[1 - 1 + \frac{1}{3}\right]$$
$$= -\frac{1}{6}.$$

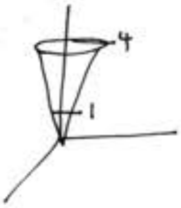
$$\iint_{S_{yz}} F \cdot -\vec{i}$$
$$= \iint_{S_{yz}} -y\, dS$$
$$= \int_0^1\int_0^{1-y} -y\, dz\, dy$$
$$= \int_0^1 -y(1-y)\, dy$$
$$= \int_0^1 -y + y^2\, dy$$
$$= -\frac{y^2}{2} + \frac{y^3}{3}\Big|_0^1$$
$$= -\frac{1}{2} + \frac{1}{3}$$
$$= -\frac{1}{6}.$$

S = surface of tetrahedron with vertices (1,0,0), (0,1,0), (0,0,1)

$z = 1-x-y.$ $\quad 0 \le x \le 1 \quad 0 \le y \le 1-x.$

$$\int_0^1\int_0^{1-x} 1-x-y\, dy\, dx.$$
$$\int_0^1 y - xy - \frac{y^2}{2}\Big|_0^{1-x} dx = \int_0^1 (1-x) - x(1-x) - \frac{(1-x)^2}{2}\, dx. = \int_0^1 1-x-x+x^2 - \frac{1-2x+x^2}{2}\, dx$$
$$= \int_0^1 \frac{2-4x+2x^2-1+2x-x^2}{2}\, dx = \frac{1}{2}\int_0^1 1-2x+x^2\, dx$$
$$= \frac{1}{2}\left[x - x^2 + \frac{x^3}{3}\right]\Big|_0^1$$
$$= \frac{1}{2}\left[\cancel{1} - \cancel{1} + \frac{1}{3}\right]$$
$$= \frac{1}{6}.$$
$$-\frac{1}{6} - \frac{1}{6} - \frac{1}{6} + \frac{1}{6} = \boxed{-\frac{1}{3}}$$

38)



$$\frac{dz}{dx} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{dz}{dy} = \frac{y}{\sqrt{x^2+y^2}}.$$

$1 \le r \le 4$

$0 \le \theta \le 2\pi.$

$z = \sqrt{x^2+y^2}$

$z = r.$

$$\iint_0 10 - z\sqrt{\left(\frac{x^2}{x^2+y^2}\right) + \left(\frac{y^2}{x^2+y^2}\right) + 1}\; dA.$$

$$= \int_0^{2\pi}\int_1^4 10 - r\sqrt{\frac{x^2+y^2+x^2+y^2}{x^2+y^2}}\; dA$$

$$= \int_0^{2\pi}\int_1^4 10 - r\sqrt{\frac{2r^2}{r^2}}\; r\,dr\,d\theta.$$

$$= \sqrt{2}\int_0^{2\pi}\int_1^4 (10-r)\,r\,dr\,d\theta = \sqrt{2}\int_0^{2\pi}\int_1^4 10r - r^2\,dr\,d\theta.$$

$$= \sqrt{2}\int_0^{2\pi} 10\frac{r^2}{2} - \frac{r^3}{3}\Big|_1^4 d\theta$$

$$= \sqrt{2}\int_0^{2\pi}\left(80 - \frac{64}{3}\right) - \left(5 - \frac{1}{3}\right) d\theta. = \sqrt{2}\int_0^{2\pi} 54\, d\theta.$$

$$= \sqrt{2}\,(54)(2\pi)$$

$$\boxed{= 108\pi\sqrt{2}}$$