

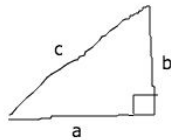
Homework 2

Ann Kidder

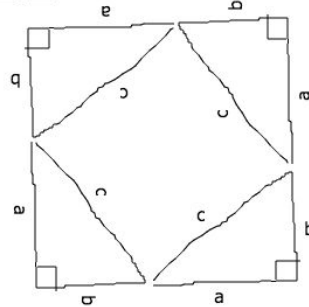
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1 Basic Concept Problems

1) start with a right triangle
with sides a , b ,
and hypotenuse c

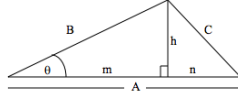


2) turn the triangle and
form a square ab by ab
with another square c by c inside



3) calculate the area of the big square,
and the area of the smaller square
+ the four triangles

$$\begin{aligned}(a+b)(a+b) &= c^2 + 4\left(\frac{1}{2}\right)(ab) \\ a^2 + 2ab + b^2 &= c^2 + 2ab \\ \text{the } 2ab \text{ cancels} \\ a^2 + b^2 &= c^2\end{aligned}$$



Given a triangle with sides A, B, and C with height h and an angle θ

we know that the left triangle will have the property $m^2 + h^2 = B^2$

and the right triangle will have the property $h^2 + n^2 = C^2$

by the pythagorean theorem. Therefore:

$$n = A - m$$

$$\text{and } h^2 + (A - m)^2 = C^2$$

$$B^2 - m^2 + A^2 - 2Am + m^2 = C^2$$

$$\text{Furthermore, we know } \cos\theta = \frac{m}{B}$$

Therefore:

$$B^2 + A^2 - 2AB\cos\theta = C^2$$

9.3

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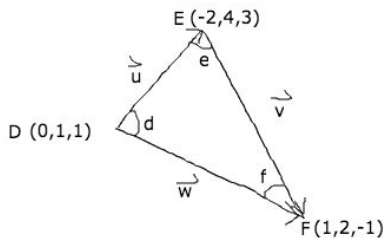
$$\|a\| = \sqrt{5}$$

$$\text{comp}_a b = \frac{a \cdot b}{\|a\|} = \frac{-4 + 2}{\sqrt{5}} = \frac{-2}{\sqrt{5}}$$

The vector projection is:

$$\text{proj}_a b = \frac{-2}{\sqrt{5}} \frac{\langle 1, 2 \rangle}{\sqrt{5}} = \frac{-2}{5} \langle 1, 2 \rangle$$

$$\text{proj}_a b = \langle \frac{-2}{5}, \frac{-4}{5} \rangle$$



$$\vec{u} = \langle -2, 3, 2 \rangle$$

$$\vec{v} = \langle 3, -2, -4 \rangle$$

$$\vec{w} = \langle 1, 1, -2 \rangle$$

The angle at D:

$$\langle -2, 3, 2 \rangle \cdot \langle 1, 1, -2 \rangle = \|\langle -2, 3, 2 \rangle\| \|\langle 1, 1, -2 \rangle\| \cos d$$

$$-3 = \sqrt{17}\sqrt{6}\cos d$$

$$\cos^{-1}\left(\frac{-3}{\sqrt{102}}\right) = d$$

$$d \approx 107.28^\circ$$

The angle at E:

$$-\langle -2, 3, 2 \rangle \cdot \langle 3, -2, -4 \rangle = \|\langle -2, 3, 2 \rangle\| \|\langle 3, -2, -4 \rangle\| \cos e$$

$$20 = \sqrt{17}\sqrt{29}\cos e$$

$$\cos^{-1}\left(\frac{20}{\sqrt{493}}\right) = e$$

$$e \approx 25.74^\circ$$

The angle at F:

$$-\langle 1, 1, -2 \rangle \cdot \langle 3, -2, -4 \rangle = \|\langle 1, 1, -2 \rangle\| \|\langle 3, -2, -4 \rangle\| \cos f$$

$$9 = \sqrt{6}\sqrt{29}\cos f$$

$$\cos^{-1}\left(\frac{9}{\sqrt{174}}\right) = f$$

$$f \approx 46.98^\circ$$

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$$a = \langle 2, -1, 4 \rangle \text{ and } b = \langle 0, 1, \frac{1}{2} \rangle$$

$$\|a\| = \sqrt{4 + 1 + 16} = \sqrt{21}$$

$$\text{comp}_a b = \frac{a \cdot b}{\sqrt{21}} = \frac{1}{\sqrt{21}}$$

The vector projection is:

$$\text{proj}_a b = \frac{1}{\sqrt{21}} \frac{\langle 2, -1, 4 \rangle}{\sqrt{21}} = \frac{1}{21} \langle 2, -1, 4 \rangle$$

$$\text{proj}_a b = \langle \frac{2}{21}, \frac{-1}{21}, \frac{4}{21} \rangle$$

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Let all sides of the cube be length = 1, and let the edges lie along the x, y, z axis

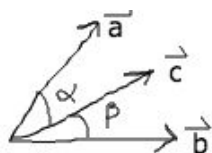
The diagonal vector, \vec{d} from (0,0,0) to (1,1,1) = $\langle 1,1,1 \rangle$

Let the unit vector $\vec{k} = \langle 0,0,1 \rangle$ be a side.

The angle between \vec{d} and \vec{k} is:

$$\begin{aligned} \cos \theta &= \frac{\langle 0,0,1 \rangle \cdot \langle 1,1,1 \rangle}{\|\vec{u}\| \|\vec{v}\|} \\ &= \frac{0+0+1}{\sqrt{1}\sqrt{3}} \\ &= \frac{1}{\sqrt{3}} \\ \theta &= \cos^{-1} \frac{1}{\sqrt{3}} \\ &= 54.74^\circ \end{aligned}$$

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$$\begin{aligned}
\cos\alpha &= \frac{a \cdot c}{\|a\|\|c\|} \\
&= \frac{a \cdot (\|a\|b + \|b\|a)}{\|a\|\|c\|} \\
&= \frac{a \cdot \|a\|a \cdot b + \|b\|a \cdot a}{\|a\|\|c\|} \\
&= \frac{\|a\|a \cdot b + \|b\|\|a\|^2}{\|a\|\|c\|} \\
&= \frac{a \cdot b + \|b\|\|a\|}{\|c\|} \\
\cos\beta &= \frac{b \cdot c}{\|b\|\|c\|} \\
&= \frac{b \cdot (\|a\|b + \|b\|a)}{\|b\|\|c\|} \\
&= \frac{\|a\|b \cdot b + \|b\|a \cdot b}{\|b\|\|c\|} \\
&= \frac{\|a\|\|b\|^2 + \|b\|a \cdot b}{\|b\|\|c\|} \\
&= \frac{\|a\|\|b\| + a \cdot b}{\|c\|}
\end{aligned}$$

Now we can see that $\cos\alpha = \cos\beta$

$$\frac{a \cdot b + \|b\|\|a\|}{\|c\|} = \frac{\|a\|\|b\| + a \cdot b}{\|c\|}$$

9.4

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The cross product is orthogonal to both $i+j+k$ and $2i+k$:

$$\begin{aligned}
i + j + k &= \langle 1, 1, 1 \rangle \text{ and } 2i + k = \langle 2, 0, 1 \rangle \\
\langle 1, 1, 1 \rangle \times \langle 2, 0, 1 \rangle &= (1-0)\vec{i} + (2-1)\vec{j} + (0-2)\vec{k} \\
\langle 1, 1, 1 \rangle \times \langle 2, 0, 1 \rangle &= \langle 1, 1, -2 \rangle
\end{aligned}$$

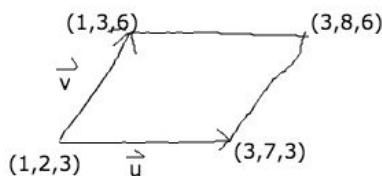
Divide by the magnitude to find the unit vector:

$$\begin{aligned}\text{unit vector} &= \frac{\langle 1, 1, -2 \rangle}{\sqrt{1+1+4}} \\ &= \langle \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}} \rangle\end{aligned}$$

The second unit vector would be the negative of the first:

$$= \langle \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \rangle$$

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$$\vec{u} = \langle 2, 5, 0 \rangle \text{ and } \vec{v} = \langle 0, 1, 3 \rangle$$

$$A = \| \langle 2, 5, 0 \rangle \times \langle 0, 1, 3 \rangle \|$$

$$A = \| \langle -15, 6, -2 \rangle \|$$

$$A = \sqrt{265}$$

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a)

$$\vec{PQ} = \langle 1, 2, 1 \rangle \text{ and } \vec{PR} = \langle 5, 0, -2 \rangle$$

The vector orthogonal to the plane = $\vec{PQ} \times \vec{PR}$

$$\vec{PQ} \times \vec{PR} = \langle -5, 7, -10 \rangle$$

b)

$$A = \frac{\| \langle -5, 7, -10 \rangle \|}{2}$$

$$A = \frac{\sqrt{174}}{2} \approx 6.6$$

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a)

$$\begin{aligned} d &= \|b\| \sin \theta \\ &= \frac{\|a\|}{\|a\|} \|b\| \sin \theta \\ &= \frac{\|a\| \|b\| \sin \theta}{\|a\|} \\ &= \frac{\|a \times b\|}{\|a\|} \end{aligned}$$

b)

$$\vec{a} = \langle -1, -2, -1 \rangle \text{ and } \vec{b} = \langle 1, -5, -7 \rangle$$

$$\begin{aligned} d &= \frac{\| \langle -1, -2, -1 \rangle \times \langle 1, -5, -7 \rangle \|}{\sqrt{1+4+1}} \\ &= \frac{\| \langle 9, -15, 7 \rangle \|}{\sqrt{6}} \\ &= \frac{\sqrt{355}}{\sqrt{6}} \\ d &\approx 7.69 \end{aligned}$$

9.5

2

$$\begin{aligned}\text{vector equation} &= \langle 6, -5, 2 \rangle + t \langle 1, 3, \frac{-2}{3} \rangle \\ &= \langle 6 + t, -5 + 3t, 2 + \frac{-2}{3}t \rangle\end{aligned}$$

The parametric equations are:

$$\begin{aligned}x(t) &= 6 + t \\ y(t) &= -5 + 3t \\ z(t) &= 2 + \frac{-2}{3}t\end{aligned}$$

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$$\begin{aligned}x(t) &= 1 + t \\ y(t) &= 3t \\ z(t) &= 6 + t\end{aligned}$$

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$\vec{n}_1 = \langle 1, 2, 3 \rangle$ and $\vec{n}_2 = \langle 1, -1, 1 \rangle$
 $\langle 1, 2, 3 \rangle \times \langle 1, -1, 1 \rangle$ is a line parallel to the line of intersection.

$$\langle 1, 2, 3 \rangle \times \langle 1, -1, 1 \rangle = \langle 5, 2, -3 \rangle$$

Plug in $z=0$ to solve x, y and find a point on the line of intersection

$$x + 2y = 1 \text{ and } x - y = 1$$

This gives us $3y = 0$

Therefore $(1, 0, 0)$ lies on the line of intersection and the symmetric equations are:

$$\frac{x-1}{5} = \frac{y}{2} = \frac{-z}{3}$$

And the parametric equations are:

$$\begin{aligned}x(t) &= 1 + 5t \\ y(t) &= 2t \\ z(t) &= -3t\end{aligned}$$

Check ratio of coefficients to test if parallel:

$$\frac{1}{-1} = \frac{3}{1} = \frac{-1}{3}$$

$-1 \neq 3 \neq -3$ therefore the lines are NOT parallel

Solve system of equations to test if intersecting:

$$1 + 2t = -1 + s$$

$$3t = 4 + s$$

$$2 - t = 1 + 3s$$

Solve the first equation for t:

$$2t = -2 + s$$

$$t = -1 + \frac{s}{2}$$

Plug into the second equation:

$$3\left(-1 + \frac{s}{2}\right) = 4 + s$$

$$-3 + \frac{3s}{2} = 4 + s$$

$$\frac{s}{2} = 7$$

$$s = 14$$

Plug s=14 this into the equation for t:

$$t = -1 + \frac{14}{2} = -6$$

Plug s=14 and t=-6 into the third equation:

$$2 + -6 = 1 + 3(14)$$

$-4 \neq 43$ therefore the lines do not intersect.

Since the lines are not parallel and do not intersect, they must be skew.

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$P(0, 1, 1)$ and $Q(1, 0, 1)$ and $R(1, 1, 0)$

$$\vec{PQ} = \langle 1 - 0, 0 - 1, 1 - 1 \rangle = \langle 1, -1, 0 \rangle$$

$$\vec{PR} = \langle 1 - 0, 1 - 1, 0 - 1 \rangle = \langle 1, 0, -1 \rangle$$

Take the cross product to get the coefficients of the plane equation:

$$\langle 1, -1, 0 \rangle \times \langle 1, 0, -1 \rangle = \langle 1, 1, 1 \rangle$$

Use P as the point for the equation of the plane:

$$1(x - 0) + 1(y - 1) + 1(z - 1) = 0$$

$$x + y + z = 2$$

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The points $P(6, 0, -2)$ and $Q(4, 3, 7)$ are on the plane.

Set $t=1$ to get a third point on the plane, $R(2, 8, 11)$

$$\vec{PQ} = \langle 4 - 6, 3 - 0, 7 - (-2) \rangle = \langle -2, 3, 9 \rangle$$

$$\vec{PR} = \langle 2 - 6, 8 - 0, 11 - (-2) \rangle = \langle -4, 8, 13 \rangle$$

Take the cross product to get the coefficients of the plane equation:

$$\langle -2, 3, 9 \rangle \times \langle -4, 8, 13 \rangle = \langle -33, -10, -4 \rangle$$

Use P as the point for the equation of the plane:

$$-33(x - 6) + -10(y - 0) + -4(z + 2) = 0$$

$$-33x + 198 - 10y - 4z - 8 = 0$$

$$-33x - 10y - 4z = -190$$

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Let Q be the plane that we are looking for.

Set $z=0$ to get a point on the line of intersection:

$$(1, 3, 0)$$

To get a normal vector for Q, take the cross product of two vectors parallel to Q.

The normal vector of the plane perpendicular to Q is parallel to Q:

$$\langle 1, 1, -2 \rangle$$

Get a second vector parallel to Q by taking the cross product of the normal vectors of the two planes that form the line of intersection that Q passes through.

$$\langle 1, 0, -1 \rangle \times \langle 0, 1, 2 \rangle = \langle 1, -2, 1 \rangle$$

Now take the cross product of these two vectors:

$$\langle 1, 1, -2 \rangle \times \langle 1, -2, 1 \rangle = \langle -3, -3, -3 \rangle$$

Plug these into the plane equation:

$$-3(x - 1) - 3(y - 3) - 3(z - 0) = 0$$

$$-3x + 3 - 3y + 9 - 3z = 0$$

$$-3x - 3y - 3z = -12$$

$$-3(x + y + z) = -12$$

$$x + y + z = 4$$

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$$\begin{aligned} d &= \frac{|1(6) + (-2)(0) + (-4)(-2) + (-8)|}{\sqrt{1^2 + (-2)^2 + (-4)^2}} \\ &= \frac{|6 + 8 - 8|}{\sqrt{21}} \\ &= \frac{6}{\sqrt{21}} \\ d &\approx 1.31 \end{aligned}$$

To find the distance we need a point on one plane.

Plug in $z=0$ to the first equation.

$$0 = 4y - 2x$$

$$y = \frac{1}{2}x$$

Therefore the point $(2,1,0)$ is on the first plane.

Now we can use the distance formula.

$$\begin{aligned} d &= \frac{|3(2) + -6(1) + 9(0) + -1|}{\sqrt{3^2 + (-6)^2 + 9^2}} \\ &= \frac{|6 - 6 - 1|}{\sqrt{126}} \\ &= \frac{1}{\sqrt{126}} \\ d &\approx .09 \end{aligned}$$