12.9

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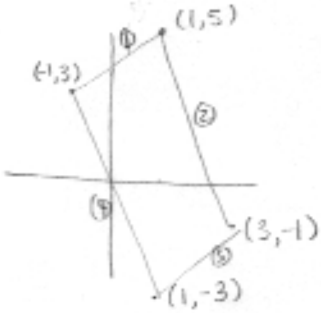
HW #11

16.



$\iint \cancel{4}\left(\frac{1}{\cancel{4}}(u+v)\right) + 8\left(\frac{1}{\cancel{4}}(v-3u)\right) dA$

$\iint u+v+2v-6u\,dA$

$\iint 3v-5u\,dA$

① $y-5 = \frac{2}{2}(x-1)$

$y = x+4 \longrightarrow u=-4$

② $y-5 = -\frac{\cancel{6}3}{\cancel{2}}(x-1)$

$y=-3x+8 \longrightarrow v=8$

③ $y+3 = \frac{2}{2}(x-1)$

$y = x-4 \longrightarrow u=4$

④ $y-3 = -3(x+1)$

$y = 3x \longrightarrow v=0$

$\int_0^8 \int_{-4}^4 3v-5u\left(\frac{1}{4}\right)du\,dv$

inner: $3vu - 5\frac{u^2}{2}\Big|_{-4}^{4}$

$= 12v - 40 - (-12v-40)$

$= 24v$

outer: $24\frac{v^2}{2}\Big|_0^8 = 768$

$\frac{768}{4} = \boxed{192}$

$\frac{dx}{du} = \frac{1}{4}$	$\frac{dx}{dv}\ \frac{1}{4}$
$\frac{dy}{du} = -\frac{3}{4}$	$\frac{dy}{dv} = \frac{1}{4}$

$\frac{1}{16} + \frac{3}{16} = \frac{4}{16} = \frac{1}{4}$

17.

$\frac{dx}{du} = 2$	$\frac{dx}{dv} = 0$
$\frac{dy}{du} = 0$	$\frac{dy}{dv} = 3$

6.

$\iint 4u^2(6)\,dA$

$24\int_0^{2\pi}\int_0^1 r^2\cos^2\theta\, r\,dr\,d\theta$

inner: $\frac{r^4}{4}\cos^2\theta\Big|_0^1 = \frac{1}{4}\cos^2\theta$

outer: $6\int_0^{2\pi}\frac{1}{2}(1+\cos 2\theta)\,d\theta$

$= 3\int_0^{2\pi}(1+\cos 2\theta)\,d\theta$

$= 3\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{2\pi}$

$= 3[2\pi]$

$\boxed{= 6\pi}$

$9(4u^2) + 4(9v^2) = 36$

$u^2 + v^2 = 1$

$u = r\cos\theta$

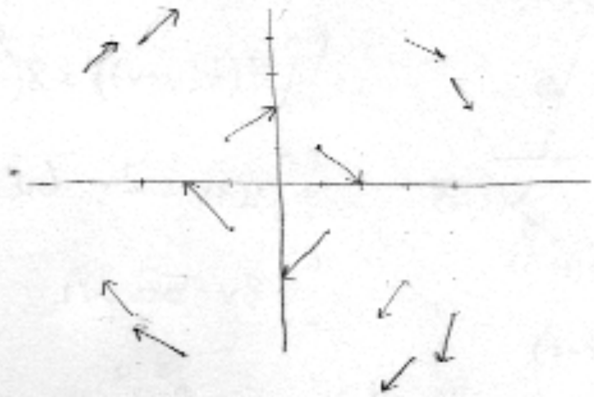
$v = r\sin\theta$

$0 \le r \le 1 \quad 0 \le \theta \le 2\pi$

13.1

6)

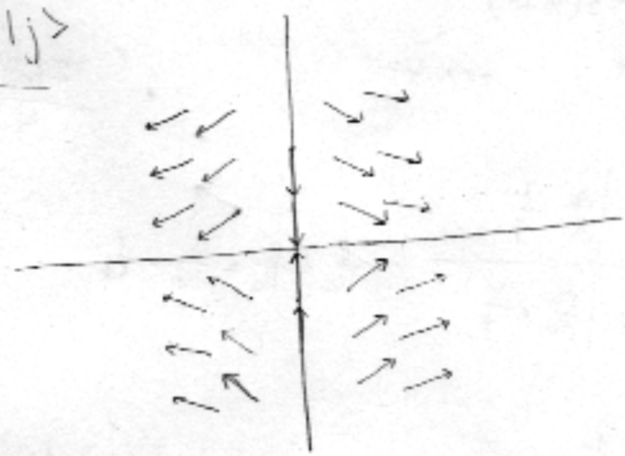
(x,y)	$F(x,y)$
$(1,1)$	$\langle 1, -1 \rangle$
$(4,3)$	$\langle \frac{3}{5}, -\frac{4}{5} \rangle$
$(3,4)$	$\langle \frac{4}{5}, -\frac{3}{5} \rangle$
$(-1,-1)$	$\langle -1, 1 \rangle$
$(1,-1)$	$\langle -1, -1 \rangle$
$(-1,1)$	$\langle 1, 1 \rangle$
$(3,-2)$	$\langle \frac{-2}{\sqrt{13}}, \frac{-3}{\sqrt{13}} \rangle$
$(4,-3)$	$\langle \frac{-3}{5}, \frac{-4}{5} \rangle$
$(-3,4)$	$\langle \frac{4}{5}, \frac{3}{5} \rangle$
$(-3,-4)$	$\langle \frac{-4}{5}, \frac{3}{5} \rangle$



25) $f_x = 2x$

$f_y = -1$

$\nabla f(x,y) = \langle 2xi, -1j \rangle$



13.2

2) $\int_C$ $ds = \int_0^1 t^2(2t)\sqrt{4t^2+4}\,dt$

$\frac{dx}{dt} = 2t$

$\frac{dy}{dt} = 2$

$= \int_0^1 2t^3\sqrt{4t^2+4}\,dt.$

$t = \tan(u)$

$dt = \sec^2 u\,du$

$1 = \tan(\frac{\pi}{4})$

$0 = \tan(0)$

$= \int_{t=0}^{t=1} 4t^3\sqrt{t^2+1}\,dt.$

$\int_0^{\pi/4} 4\tan^3 u\sqrt{\tan^2 u+1}\,\sec^2 u\,du$

$= \int_{u=0}^{\pi/4} 4\tan^3 u\sqrt{\sec^2 u}\,\sec^2 u\,du$

$= 4\int_0^{\pi/4} \tan^3 u \sec^3 u\,du$

$= 4\int_0^{\pi/4} \frac{\sin^3 u}{\cos^6 u}\,du. =$ inner: $\frac{\sin(1-\cos^2 u)}{\cos^6 u} = \frac{\sin u}{\cos^6 u} - \frac{\sin u \cancel{\cos^2 u}}{\cos^{\cancel{6}4} u}$

$\cos^{-5}$

$-5\cos^{-6}\sin$

$= 4\int_0^{\frac{\pi}{4}} \frac{\sin u}{\cos^6 u} - \frac{\sin u}{\cos^4 u}\,du.$

$= 4\left[\frac{1}{5}\cos^{-5} u - \frac{1}{3}\cos^{-3} u\right]_0^{\frac{\pi}{4}}.$

$= 4\left[\left[\left(\frac{1}{5}\right)\left(\frac{1}{\cos^5(\frac{\pi}{4})}\right) - \left(\frac{1}{3}\right)\frac{1}{\cos(\frac{\pi}{4})}\right] - \left[\left(\frac{1}{5}\right)\left(\frac{1}{\cos^5(0)}\right) - \left(\frac{1}{3}\right)\left(\frac{1}{\cos(0)}\right)\right]\right]$

$= 4\left[\left(\frac{4\sqrt{2}}{5} - \frac{4\sqrt{2}}{3}\right) - \left(\frac{1}{5} - \frac{1}{3}\right)\right]$

$= 4\left[\frac{12\sqrt{2} - 20\sqrt{2}}{15} - \frac{-2}{15}\right] = \boxed{4\left(\frac{-8\sqrt{2}+2}{15}\right)}$

7) C_1: $x = 2t$ $\quad dx = 2dt$
$y = 0$ $\quad dy = 0$

$$\int_0^1 (2+t)(2t)(2dt) + (2t-0)(0)\,dt = \boxed{0}$$

C_2: $x = 2+t$ $\quad dx = dt$
$y = 2t$ $\quad dy = 2dt$

$(2,0)$ $\langle 1,2 \rangle$

$$\int_0^1 [(2+t)(2t) + (2+t-2t)(2)]dt$$

$$= \int_0^1 4t + 2t^2 + 4 - 2t\,dt$$

$$= 2\int_0^1 t^2 + t + 2\,dt$$

$$= 2\left[\frac{t^3}{3} + \frac{t^2}{2} + 2t\right]_0^1 = 2\left[\frac{1}{3} + \frac{1}{2} + 1\right] = \frac{11}{3}$$

$$\boxed{C_1 = 0 \quad C_2 = \frac{11}{3}}$$

10) $x = -1 + 2t$ $\quad dx = 2dt$
$y = 5 + t$ $\quad dy = 1dt$
$z = 4t$ $\quad dz = 4dt$

$$\int_0^1 (-1+2t)(5+t)(4t)^2 \sqrt{4+1+16}\,dt$$

$$= \int_0^1 (-5 + 10t - t + 2t^2)(16t^2)\sqrt{21}\,dt$$

$$= \sqrt{21}\int_0^1 (-80t^2 + 144t^3 + 32t^4)\,dt$$

$$= \sqrt{21}\left[-80\frac{t^3}{3} + 144\frac{t^4}{4} + 32\frac{t^5}{5}\right]_0^1 = \sqrt{21}\left(\frac{-80}{3} + \frac{144}{4} + \frac{32}{5}\right)$$

$$\boxed{= \frac{236\sqrt{21}}{15}}$$

20).

$x = t^2$ $\quad dx = 2t\,dt$

$y = t^3$ $\quad dy = 3t^2\,dt$

$z = t^2$ $\quad dz = 2t\,dt$.

$$\int_0^1 (t^2+t^3)(2t) + (t^3-t^2)(3t^2) + (t^4)(2t)\,dt$$

$$= \int_0^1 2t^3 + 2t^4 + 3t^5 - 3t^4 + 2t^5\,dt$$

$$= \int_0^1 2t^3 - t^4 + 5t^5\,dt$$

$$= \frac{2t^4}{4} - \frac{t^5}{5} + 5\frac{t^6}{6}\Big|_0^1 = \frac{2}{4} - \frac{1}{5} + \frac{5}{6}$$

$$\boxed{= \frac{17}{15}}$$

28)

$x = t$ $\quad dx = dt$

$y = 1+t^2$. $\quad dy = 2t\,dt$.

$$\int_{-1}^1 \frac{t}{\sqrt{t^2+(1+t^2)^2}} + \frac{(1+t^2)2t}{\sqrt{t^2+(1+t^2)^2}}\,dt$$

$$= \int_{-1}^1 \frac{t+2t+2t^3}{\sqrt{t^2+1+2t^2+t^4}}\,dt$$

$$= \int_{-1}^1 \frac{3t+2t^3}{\sqrt{1+3t^2+t^4}}\,dt$$

$u = 1+3t^2+t^4$

$du = 6t+4t^3\,dt$

$\frac{1}{2}du = 3t+2t^3\,dt$.

$$= \frac{1}{2}\int_{t=-1}^{t=1} \frac{1}{\sqrt{u}}\,du$$

$$= \frac{1}{2}\left(2\sqrt{1+3t^2+t^4}\right)\Big|_{-1}^1$$

$$\boxed{= \sqrt{5} - \sqrt{5} = 0}$$

34) 

$x = a\cos t \quad dx = -a\sin t \quad 0 \le t \le \frac{\pi}{2}$

$y = a\sin t \quad dy = a\cos t$

$$m = \int_0^{\frac{\pi}{2}} k(a\cos t)(a\sin t)\sqrt{a^2\sin^2 t + a^2\cos^2 t}\, dt$$

$$\sqrt{a^2\underbrace{(\sin^2 t + \cos^2 t)}_{=1}}$$

$$= ka^3\int_0^{\frac{\pi}{2}} \cos t \sin t \, dt$$

$2\sin t \cos t = \sin(2t)$

$$= ka^3\int_0^{\frac{\pi}{2}} \tfrac{1}{2}\sin(2t)\, dt$$

$\frac{1}{2}\sin 2t$

$$\frac{ka^3}{4}\left(-\cos(2t)\right)\Big|_0^{\frac{\pi}{2}}$$

$$\frac{ka^3}{4}(1+1) = \frac{2ka^3}{4} = \frac{ka^3}{2}$$

$$\bar{y} = \frac{2}{\cancel{ka^3}}\left[\cancel{ka^3}\int_0^{\frac{\pi}{2}} \cos t \sin t (a\sin t)\, dt\right]$$

$$= 2a\int_0^{\frac{\pi}{2}} \cos t \sin^2 t \, dt$$

$$2a\left[\frac{\sin^3 t}{3}\right]\Big|_0^{\frac{\pi}{2}} = 2a\left[\frac{1}{3}\right] = \frac{2a}{3}$$

$$\bar{x} = \frac{2}{\cancel{ka^3}}\left[\cancel{ka^3}\int_0^{\frac{\pi}{2}} \cos t \sin t (a\cos t)\, dt\right]$$

$$= 2a\int_0^{\frac{\pi}{2}} \sin t \cos^2 t \, dt$$

$$2a\left[\frac{-\cos^3 t}{3}\right]_0^{\frac{\pi}{2}} = 2a\left[0 - -\frac{1}{3}\right] = \frac{2a}{3}$$

mass is $\frac{ka^3}{2}$

center of mass is $\bar{x} = \frac{2a}{3}$, $\bar{y} = \frac{2a}{3}$

13.3

8) $\frac{dF_1}{dy} = -4y$ $\qquad \frac{dF_2}{dx} - \frac{dF_1}{dy} = 4y + 4y = 8y$

$\frac{dF_2}{dx} = 4y$

$\boxed{F(x,y) \text{ is not a conservative vector field}}$

10) $\frac{dF_1}{dy} = x\cos xy + x^2 y\sin(xy) + x\cos xy$

$= 2x\cos(xy) - x^2 y\sin(xy)$

$\frac{dF_2}{dx} = 2x\cos xy + x^2 \sin(xy)y$

$\frac{dF_2}{dx} - \frac{dF_1}{dy} = 0$ and $F(x,y)$ is continuous so

$\boxed{F(x,y) \text{ is a conservative vector field}}$

$f_x(x,y) = xy\cos xy + \sin xy \xrightarrow{\int} x\sin(xy)$

$f_y(x,y) = x^2\cos xy \xrightarrow{\int} x\sin(xy)$

$\boxed{f(x,y) = x\sin(xy) + C}$

12) $f_x(x,y) = x^2 \xrightarrow{\int} \frac{x^3}{3}$

$f_y(x,y) = y^2 \xrightarrow{\int} \frac{y^3}{3}$ $\qquad f(x,y) = \frac{x^3}{3} + \frac{y^3}{3} + C$

$f(2,8) - f(-1,2) = \left(\frac{2^3}{3} + \frac{8^3}{3}\right) - \left(\frac{-1}{3} + \frac{2^3}{3}\right)$

$= \cancel{\frac{8}{3}} + \frac{512}{3} + \frac{1}{3} - \cancel{\frac{8}{3}}$

$\boxed{= \frac{513}{3}}$

16) $f_x(x,y,z) = 2xz + y^2 \xrightarrow{\int} x^2z + y^2x$

$f_y(x,y,z) = 2xy \xrightarrow{\int} xy^2$

$f_z(x,y,z) = x^2 + 3z^2 \xrightarrow{\int} x^2z + z^3$

$$f(x,y,z) = x^2z + xy^2 + z^3 + C$$

$x = t^2$
$y = t+1$
$z = 2t-1$
$0 \le t \le 1$

$$(t^2)^2(2t-1) + t^2(t+1)^2 + (2t-1)^3 \Big|_0^1$$

$$= [1(1) + 1(4) + (1)] - [0 + 0 + -1]$$

$$= 6 + 1 = \boxed{7}$$

20) $\frac{F_1}{dy} = -e^{-x}$ $\quad$ $-e^{-x} = -e^{-x}$ and $F(x,y)$ is continuous from $(0,1)$ to $(1,2)$

$\frac{F_2}{dx} = -e^{-x}$ $\quad$ So the line integral is independent of path.

$f_x(x,y) = 1 - ye^{-x} \xrightarrow{\int} x + ye^{-x}$

$f_y(x,y) = e^{-x}$

$$f(x,y) = x + ye^{-x} + C$$

$$f(1,2) - f(0,1) = [1 + 2e^{-1}] - [0 + e^0]$$

$$= \cancel{1} + \frac{2}{e} - \cancel{1}$$

$$\boxed{= \frac{2}{e}}$$