Math E-21a HW #3

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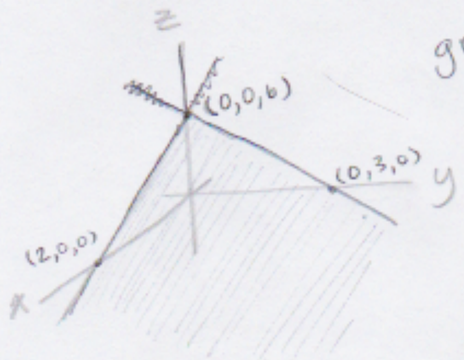
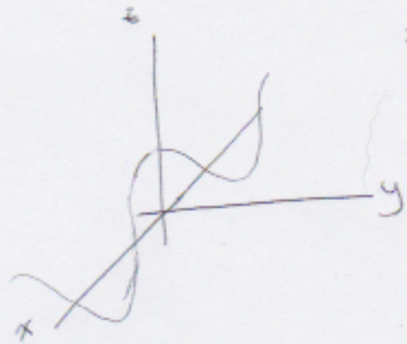
9.22.17

11.



graph of $f(x,y) = 6-3x-2y$ forms a plane

12.



$z = \cos(x)$ is the cos wave function on the x-z plane.

8. $(a \times b)\cdot[(b\times c)\times(c\times a)] = [a\cdot(b\times c)]^2$

$(a\times b)\cdot[c((b\times c)\cdot a) - a((b\times c)\cdot c)] = [a\cdot(b\times c)]^2$

$(a\times b)\cdot[c((b\times c)\cdot a - a(\underbrace{b\cdot(c\times c)}_{0})] =$

$(a\times b)\cdot c[(b\times c)\cdot a] =$

$[a\cdot(b\times c)][(b\times c)\cdot a] =$

$[a\cdot(b\times c)]^2 = [a\cdot(b\times c)]^2$

10. $\vec{AB} = \langle 1,3,-1\rangle$
$\vec{AC} = \langle -2,1,3\rangle$
$\vec{AD} = \langle -1,3,1\rangle$

$V = |\vec{AB}\cdot(\vec{AC}\times\vec{AD})|$
$= |\langle 1,3,-1\rangle\cdot\langle 10,-1,7\rangle|$
$= |-6|$
$\boxed{V=6}$

25a) $3x+y-4z=2$ and $3x+y-4z=24$.
$P(0,2,0)$

$a=3$
$b=1$
$c=-4$
$d=-24$

$$\text{distance} = \frac{|3(0)+1(2)+-4(0)+-24|}{\sqrt{3^2+1^2+(-4)^2}}$$

$$= \frac{|-22|}{\sqrt{26}}$$

$$\boxed{\text{distance} = \frac{22}{\sqrt{26}}}$$

b) $\text{distance} = \sqrt{(1+t)^2+(2-t)^2+(-1+2t)^2}$
$= \sqrt{1+2t+t^2+4-4t+t^2+1-4t+4t^2}$
$= \sqrt{6+-6t+6t^2}$
$= \sqrt{6(1-t+t^2)}$ to find the minimum we take the derivative & set it to 0.

$\frac{d}{dx} = \frac{1}{2}(6(1-t+t^2))^{-\frac{1}{2}}(6)(-1+2t)$
$= 3\frac{-1+2t}{\sqrt{6(1-t+t^2)}}$ is 0 when $t=\frac{1}{2}$

so the distance from $(0,0,0)$ to the line is shortest when $t=\frac{1}{2}$
and the distance is:

$= \sqrt{(\frac{3}{2})^2+(\frac{3}{2})^2+(0)^2} = \sqrt{\frac{18}{4}} = \boxed{\frac{3}{2}\sqrt{2}}$

26a) $\vec{AB} = \langle -3, -2, 9 \rangle$

$\vec{AC} = \langle -1, 2, -5 \rangle$

$\vec{n} = \langle -8, -24, -8 \rangle$

$$-8(x-2) - 24(y-1) - 8(z-1) = 0$$
$$-8x + 16 - 24y + 24 - 8z + 8 = 0$$
$$-8x - 24y - 8z + 48 = 0$$
$$\boxed{8x + 24y + 8z = 48}$$

b) $B(-1, -1, 10)$

$$\vec{r} = \langle -1, -1, 10 \rangle + t\langle -8, -24, -8 \rangle$$
$$\vec{r} = \langle -8 - 8t, -1 - 24t, 10 - 8t \rangle$$
$$x = -8 - 8t \rightarrow t = \frac{x+8}{-8}$$
$$y = -1 - 24t \rightarrow t = \frac{y+1}{-24}$$
$$z = 10 - 8t \rightarrow t = \frac{z-10}{-8}$$

$$\boxed{\frac{x+8}{-8} = \frac{y+1}{-24} = \frac{z-10}{-8}}$$

c)

$$\cos\theta = \frac{\langle -8, -24, -8 \rangle \cdot \langle 2, -4, -3 \rangle}{\sqrt{(-8)^2 + (-24)^2 + (-8)^2}\sqrt{2^2 + (-4)^2 + (-3)^2}}$$
$$= \frac{104}{8\sqrt{11}\sqrt{29}}$$
$$\theta = \cos^{-1}\left(\frac{13}{\sqrt{11}\sqrt{29}}\right)$$
$$\boxed{\theta \approx 43^\circ}$$

d)

$$\begin{matrix} \langle -8, -24, -8 \rangle \\ \times \\ \langle 2, -4, -3 \rangle \end{matrix} = \langle 40, -40, 80 \rangle$$

$*|\ 8x + 24y + 8z = 48 \xrightarrow{z=0} 8x + 24y = 48$

and

$$2(x-2) - 4y - 3(z-4) = 0$$
$$2x - 4 - 4y - 3z + 12 = 0$$

$*|\ 2x - 4y - 3z = -8 \xrightarrow{z=0} 2x - 4y = -8$

26d con't)

$$8x+24y=48$$
$$\underline{-4(2x-4y=-8)}$$

$$\Downarrow$$

$$8x+24y=48$$
$$\underline{+-8x+16y=32}$$
$$40y=80$$
$$y=2$$

$\Rightarrow 2x-4(2)=-8$

$2x=0$

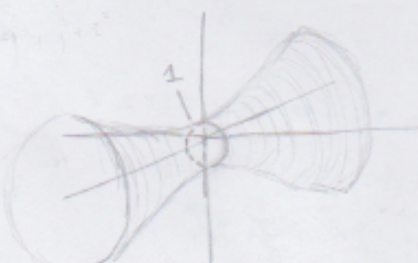
$x=0.$

$P(0,2,0)$ and $\langle 40,-40,80\rangle$

$$\boxed{\begin{aligned} x&=40t\\ y&=2-40t\\ z&=80t. \end{aligned}}$$

36) $y^2+z^2=1+x^2$

$-x^2+y^2+z^2=1$ this will be a hyperboloid of 1 sheet centered around the x-axis



1. the cross section at $x=0$ is a circle of radius 1

Homework 3

Ann Kidder

September 22, 2017

1 Chapter 9

1.1 True/False

1. True. $u \cdot v$ results in a scalar so the order of multiplication does not matter.
2. False. $< 1, 2, 3 > \times < 4, 5, 6 >=< -3, 6, -3 >$ However, $< 4, 5, 6 > \times < 1, 2, 3 >=< 3, -6, 3 >$ Thus $u \times v = -v \times u$
3. True. Though the signs are opposite for $u \times v$ and $v \times u$, once the components are squared they will be the same so the magnitudes are equal.
4. True. $u \cdot v$ will be a scalar so it does not matter what order k and u and v are multipled.
5. True. It does not matter what order the scalar is multiplied with the vector.
6. True. $|u \times w|$ is the area of a parallelogram with sides u and w. If you connect this to the parallelogram with sides v and w by the shared side w, you will get a total area of u+v by w, which is the same as $(u + v) \times w$
7. True. The final result is a scalar, so it does not matter what order the components are multipled by the cross and dot product.
8. False. The cross product of two vectors is another vector so order of operations matters. $u \times (v \times w) = v(u \times w) - w(u \times v)$
9. True. Because this can be rewritten as $v \cdot (u \times u)$ and $u \times u = 0$ this is true.
10. True. $(u + v) \times v = u \times v + v \times v$ and $v \times v = 0$ so this is true.
11. True. The cross product of two unit vectors will have a magnitude of one, so it will also be a unit vector.
12. False. It represents a line only if one and only one coefficient is 0.

13. True. Because $z^2 = 0$ the set of all points described is a circle in the xy-plane

14. False. $u \cdot v = u_1 v_1 + u_2 v_2$, which is a scalar, not a vector.

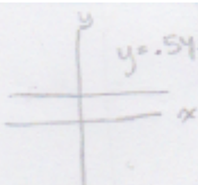
15. False. $< 1, 1, 0 > \cdot < -1, 1, 0 >= 0$

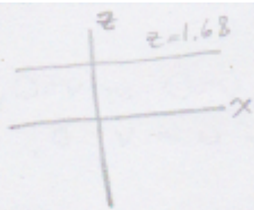
16. False. If u and v are non-zero, parallel vectors then $u \times v = 0$ Example, $< 1, 1, 0 > \times < 2, 2, 0 >=< 0, 0, 0 >$

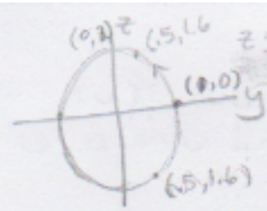
17. True. If $u \cdot v = 0$ that means either u or v is zero.

18. True. $|u{\cdot}v| = \sqrt{u_1 v_1^2 + u_2 v_2^2 + u_3 v_3^2}$ and $||u||||v|| = \sqrt{u_1^2 + u_2^2 + u_3^2}\sqrt{v_1^2 + v_2^2 + v_3^2}$. If you square both sides and multiply out the right, you see that the highest multiple of the left is 2 while the right is 4

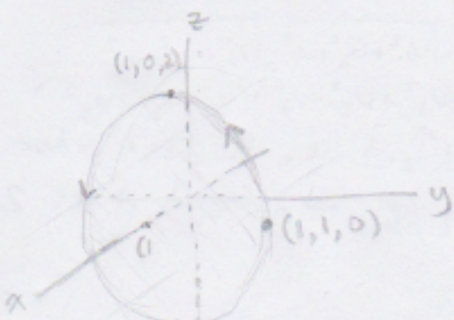
9) $x=1$
$y=\cos t$
$z=2\sin t$





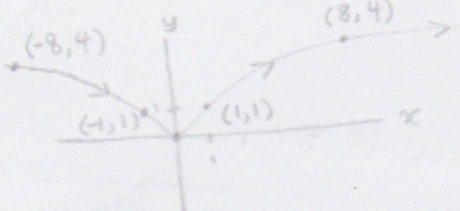


$z=\pm 2\sin(\cos^{-1} y)$

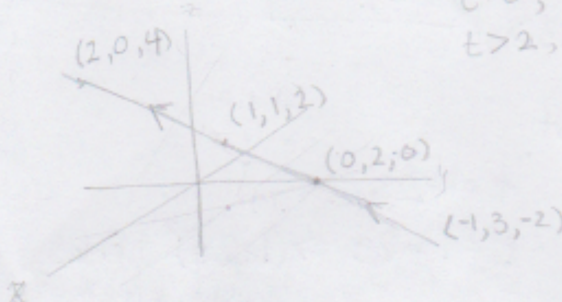


10.1

6). $x = t^3$
$y = t^2$

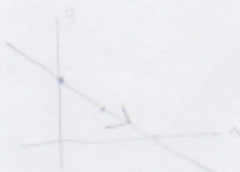


7). $x = t$
$y = 2-t$
$z = 2t$



$t=0$, $(0,2,0)$
$t>2$, positive

xy $y = 2-x$



yz $y = 2 - \frac{z}{2}$

xz $z = 2x$

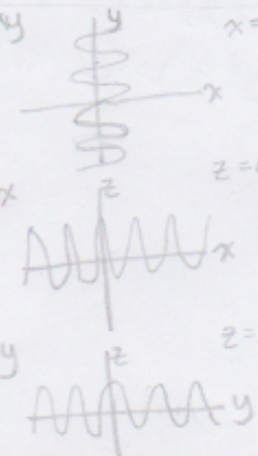
8) $x = \sin(\pi t)$
$y = t$
$z = \cos(\pi t)$

Corkscrew shape

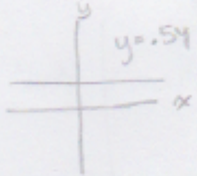
xy $x = \sin \pi y$

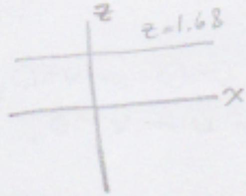
zx $z = \cos(\pi \sin(\pi t))$

zy $z = \cos \pi y$



9) $x=1$
$y=\cos t$
$z=2\sin t$





$z=2\sin(\cos^{-1}y)$

44) $t=1+2t$.
$t=-1$ If they collide it will be at $t=-1$
$\vec{r}_1(-1)=\langle -1,1,-1\rangle$
$\vec{r}_2(-1)=\langle -1,-5,-13\rangle$
$\boxed{\text{No, they do not collide}}$

is there a time such that
$\vec{r}_1(t)=\vec{r}_2(s)$?

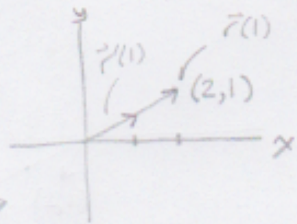
$t=1+2s$.
$t^2=1+6s \Rightarrow (1+2s)^2=1+6s$
$1+4s+4s^2=1+6s$
$4s^2-2s=0$
$2s(2s-1)=0$
So $s=0$ and $s=\frac{1}{2}$.
and $t=1$ and $t=2$.

$\boxed{\begin{matrix}\vec{r}_1(1)=\langle 1,1,1\rangle \\ \vec{r}_2(0)=\langle 1,1,1\rangle\end{matrix} > \text{paths intersect}}$

$\boxed{\begin{matrix}\vec{r}_1(2)=\langle 2,4,8\rangle \\ \vec{r}_2(\frac{1}{2})=\langle 2,4,8\rangle\end{matrix} > \text{paths intersect}}$

10.2

4a) $x = 1+t$
$y = \sqrt{t}$ $t=1$



$\vec{r}(1) = \langle 2, 1 \rangle$

$\vec{r}'(1) = \langle 1, \frac{1}{2} \rangle$

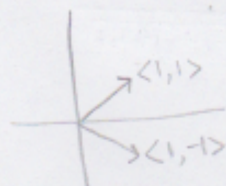
$r'(t) = \langle 1, \frac{1}{2\sqrt{t}} \rangle$

6a) $x = e^{t}$
$y = e^{-t}$ at $t=0$

$y = e^{-\ln(x)}$
$y = \frac{x}{e}$

$\vec{r}'(t) = \langle e^{t}, -e^{-t} \rangle$

$\vec{r}(0) = \langle 1, 1 \rangle$
$\vec{r}'(0) = \langle 1, -1 \rangle$



24) $\vec{r}'(t) = \langle \frac{1}{t}, \frac{1}{\sqrt{t}}, 2t \rangle$

$0 = \ln t$
$2 = 2\sqrt{t}$ at $t=1$ so
$1 = t^2$

$\vec{r}'(1) = \langle 1, 1, 2 \rangle$

$$\boxed{\begin{aligned} x &= t \\ y &= 2+t \\ z &= 1+2t \end{aligned}}$$

32) $t = 3 - s$

$1 - t = s - 2 \rightarrow t = 3 - s$

$3 + t^2 = s^2$

$3 + (3-s)^2 = s^2$

$3 + 9 - 6s + s^2 = s^2$

$12 - 6s = 0$

$12 = 6s$

$s = 2$ and $t = 1$

The curves intersect at $(1, 0, 4)$

$r_1'(t) = \langle 1, -1, 2t \rangle$

$r_2'(s) = \langle -1, 1, 2s \rangle$

$$\cos\theta = \frac{\langle 1, -1, 1 \rangle \cdot \langle -1, 1, 4 \rangle}{\sqrt{1^2 + (-1)^2 + 1^2}\sqrt{(-1)^2 + 1^2 + 4^2}}$$

$$= \frac{2}{\sqrt{3}\sqrt{18}}$$

$$= \frac{2}{3\sqrt{3}\sqrt{2}}$$

$$\theta = \cos^{-1}\left(\frac{2}{3\sqrt{3}\sqrt{2}}\right)$$

$$\boxed{\theta \approx 74^\circ}$$