Linear Model

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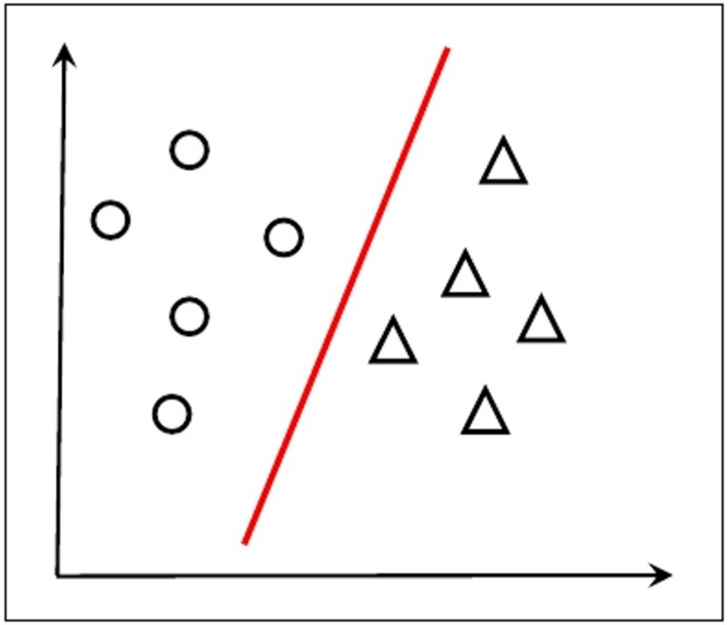
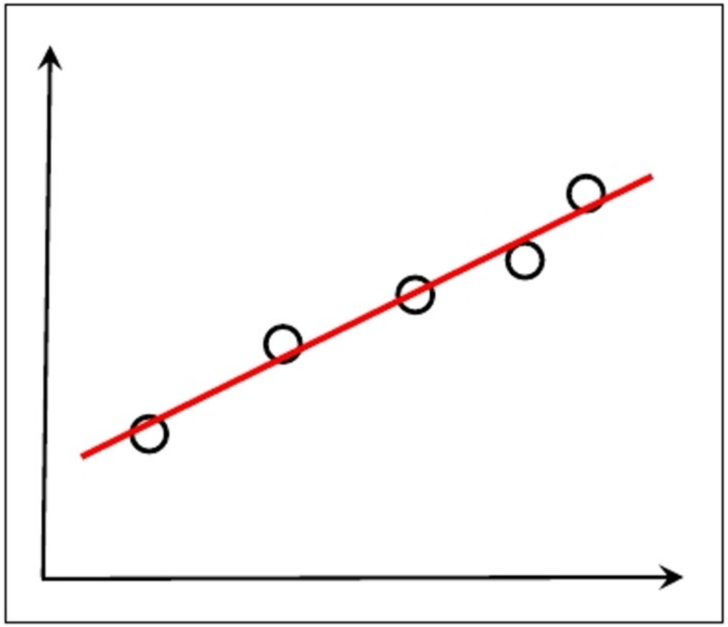
Linear Model

Classification



Regression



A linear model specifies a linear relationship between a dependent variable (Y/f(x)) and *d* independent variables (X):

$$f(\boldsymbol{x}) = w_1 x_1 + w_2 x_2 + \ldots + w_d x_d + b$$

Vectorization: $$f(\boldsymbol{x}) = \boldsymbol{w}^{\mathrm{T}} \boldsymbol{x} + b$$

Linear Regression

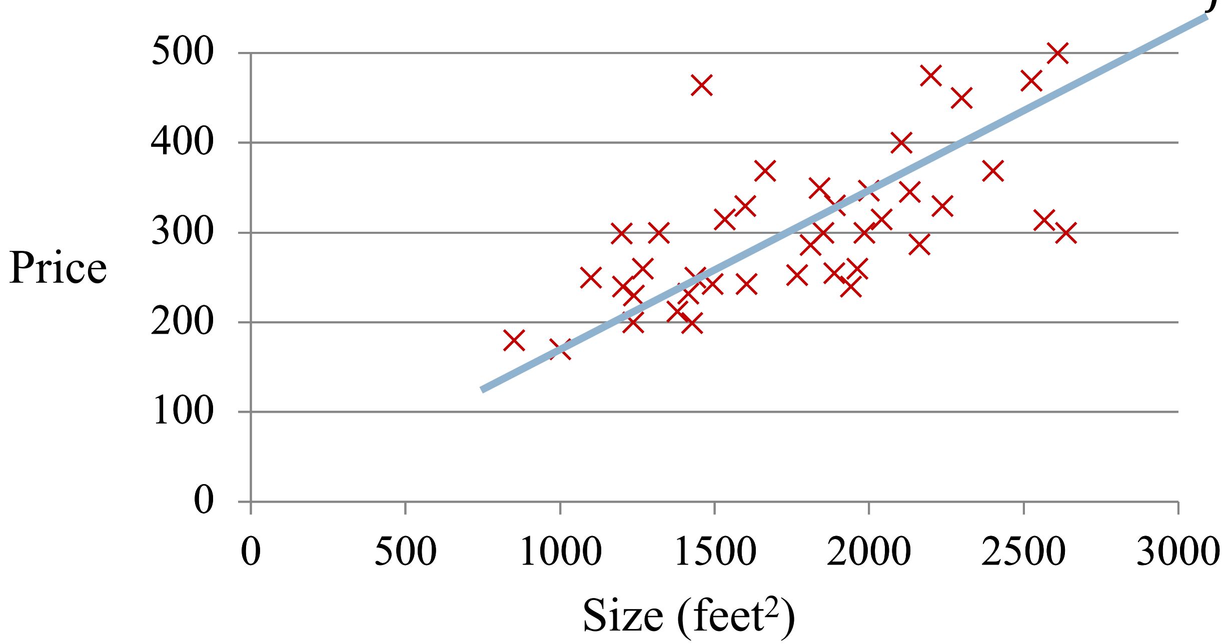
Trainning set

Size in feet² (x)	**Price ($) in 1000's (y)**
2104	460
1416	232
1534	315
852	178
…	…

Supervised Learning

Given the "real value" for each example.

$f(x) = w_1 x_1 + b$



Regression Problem

Predict real-valued output

Linear Regression with One Variable

Size in feet^2 (x)	**Price ($) in 1000's (y)**
2104	460
1416	232
1534	315
852	178
…	…

Hypothesis/Model: $y \approx f_\theta(x) = w_1 x_1 + b$

Parameters are: $\theta = \{w_1,\ b\}$

How to choose θ ?

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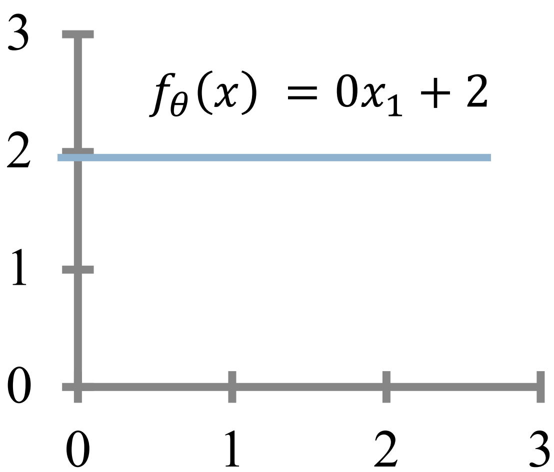
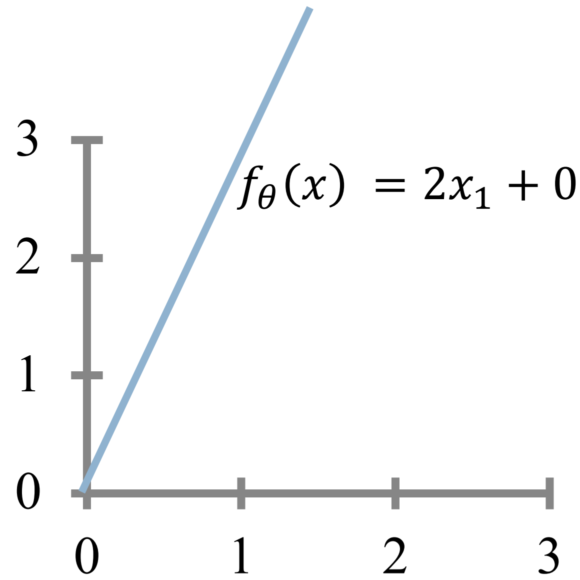
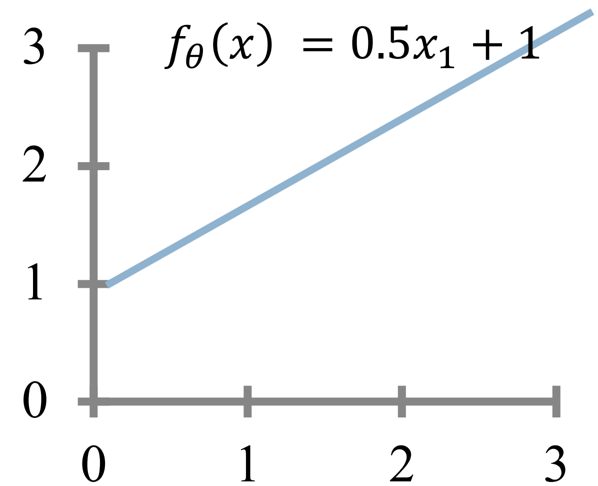
Linear Regression: Example

Hypothesis/Model: $f_\theta(x) = w_1 x_1 + b$ Parameters: $\theta = \{w_1, b\}$







$w_1 = 0$
$b = 2$

$w_1 = 2$
$b = 0$

$w_1 = 0.5$
$b = 1$

Idea: Choose $\theta = \{w_1, b\}$ so that $f_\theta(x)$ is close to y for our training examples *(x, y)* .

Linear Regression: Example

Hypothesis/Model: $f_\theta(x) = w_1 x_1 + b$

Size in feet² (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

m

Parameters: $\theta = \{w_1,\ b\}$

Cost Function:

$$E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(f_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

Goal: minimize $E(\theta)$

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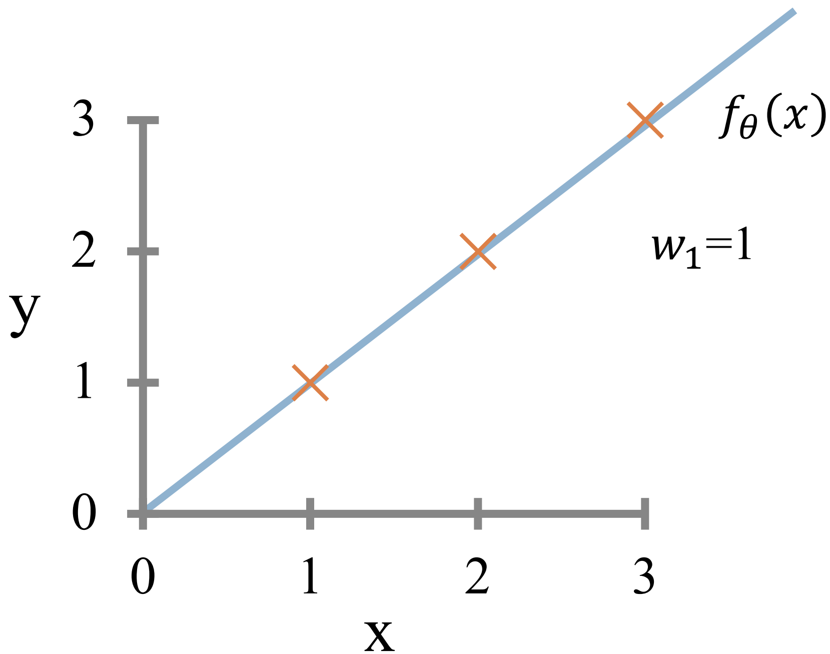
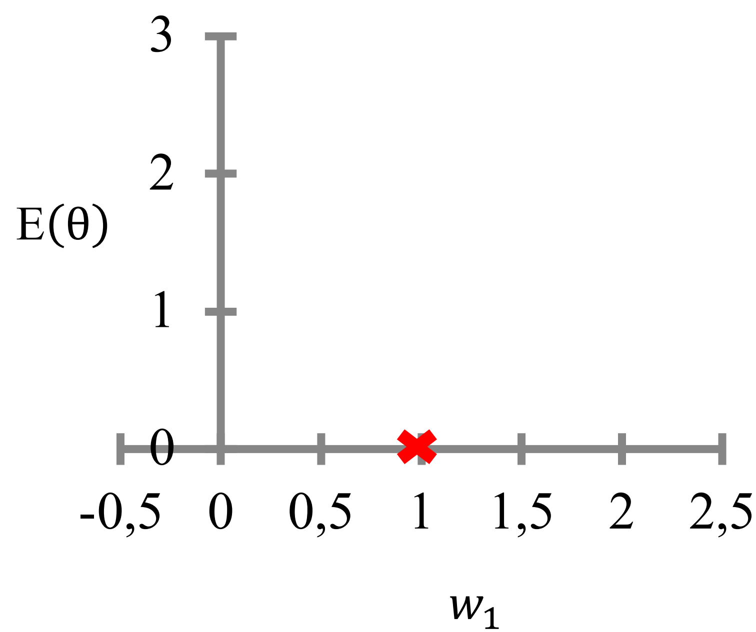
$f_\theta(x)$ Vs. $E(\theta)$

Simple hypothesis : $f_\theta(x) = w_1 x_1$
(ignoring parameter b)

$$\mathrm{E}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(f_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$



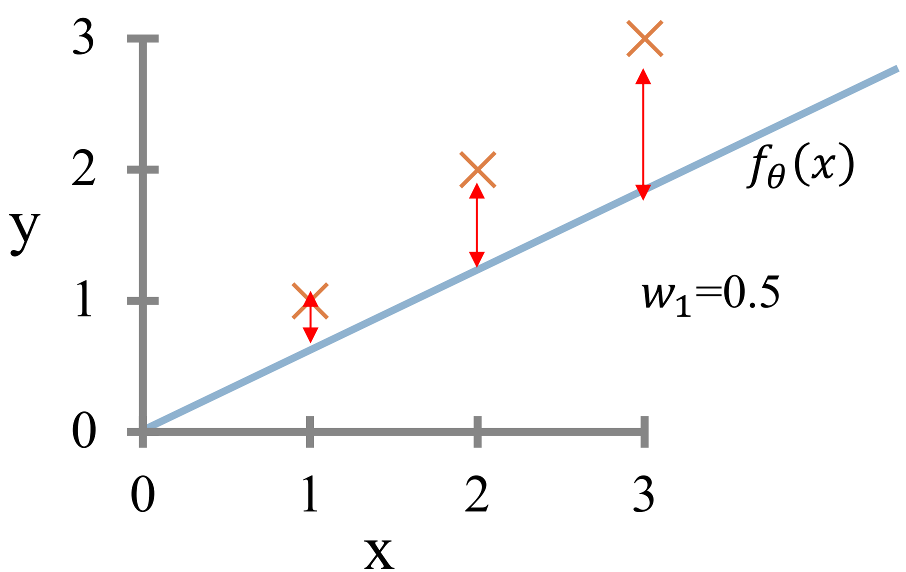


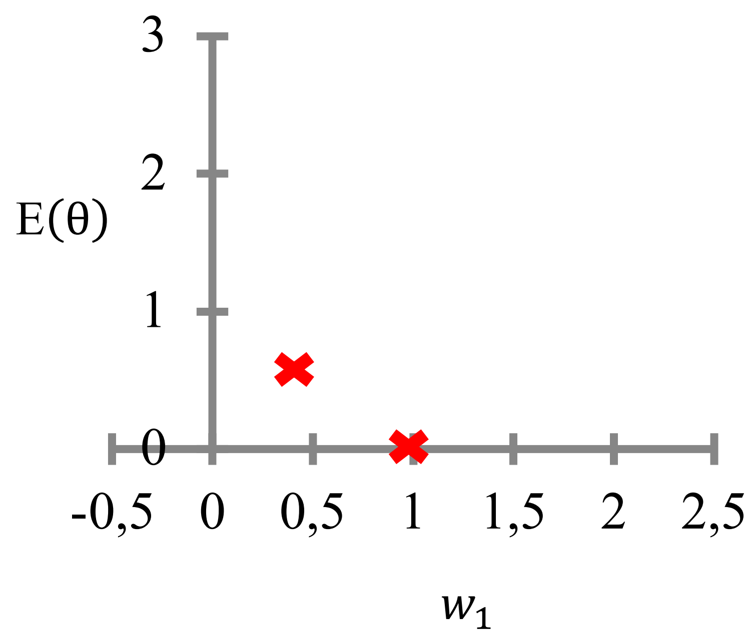
$\mathrm{E}(w_1{=}1) = \frac{1}{2m}[(1-1)^2 + (2-2)^2 + (3-3)^2] = 0$

$f_\theta(x)$ Vs. $E(\theta)$

Simple hypothesis : $f_\theta(x) = w_1 x_1$
(ignoring parameter b)

$$E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(f_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$



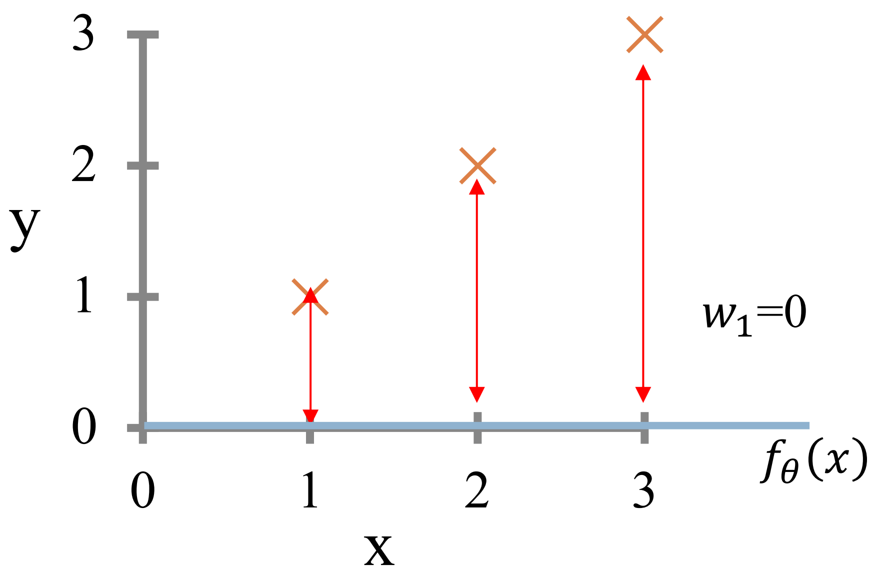


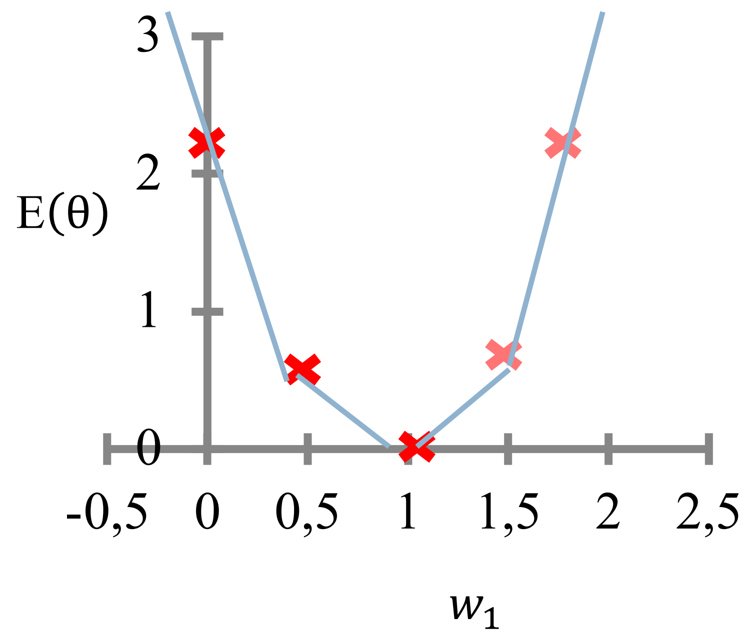
$E(w_1{=}0.5) = \frac{1}{2m}[\,(0.5-1)^2+(1-2)^2+(1.5-3)^2] =$
0.548

$f_\theta(x)$ Vs. $E(\theta)$

Simple hypothesis : $f_\theta(x) = w_1 x_1$
(ignoring parameter b)

$$E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(f_\theta(x^{(i)}) - y^{(i)})^2$$

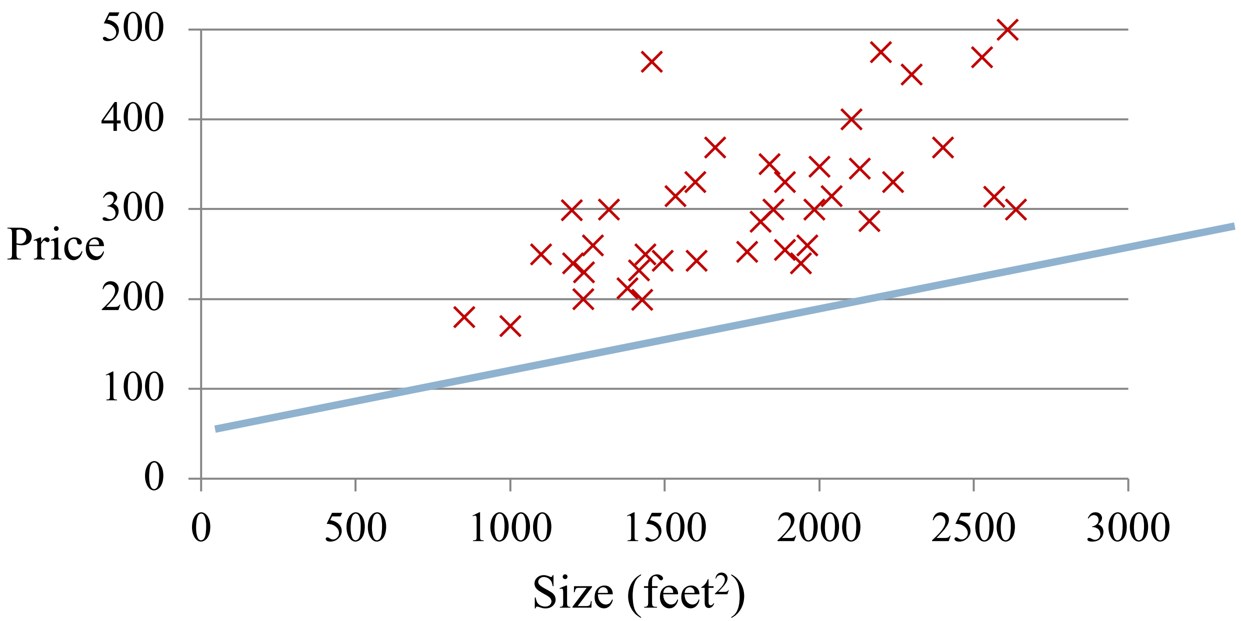




$$E(w_1{=}0) = \frac{1}{2m}[\,(0-1)^2+(0-2)^2+(0-3)^2] \approx 2.3$$

Choose parameter θ that minimizes cost function E(θ), which corresponds to finding a straight line that fits the data well.

Example: House price prediction



Hypothesis: $f_\theta(x) = 0.05x_1 + 50$

How to plot the E(θ) ?

Parameters: $\theta = \{w_1,\ b\}$

Cost Function: $E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(f_\theta(x^{(i)}) - y^{(i)})^2$

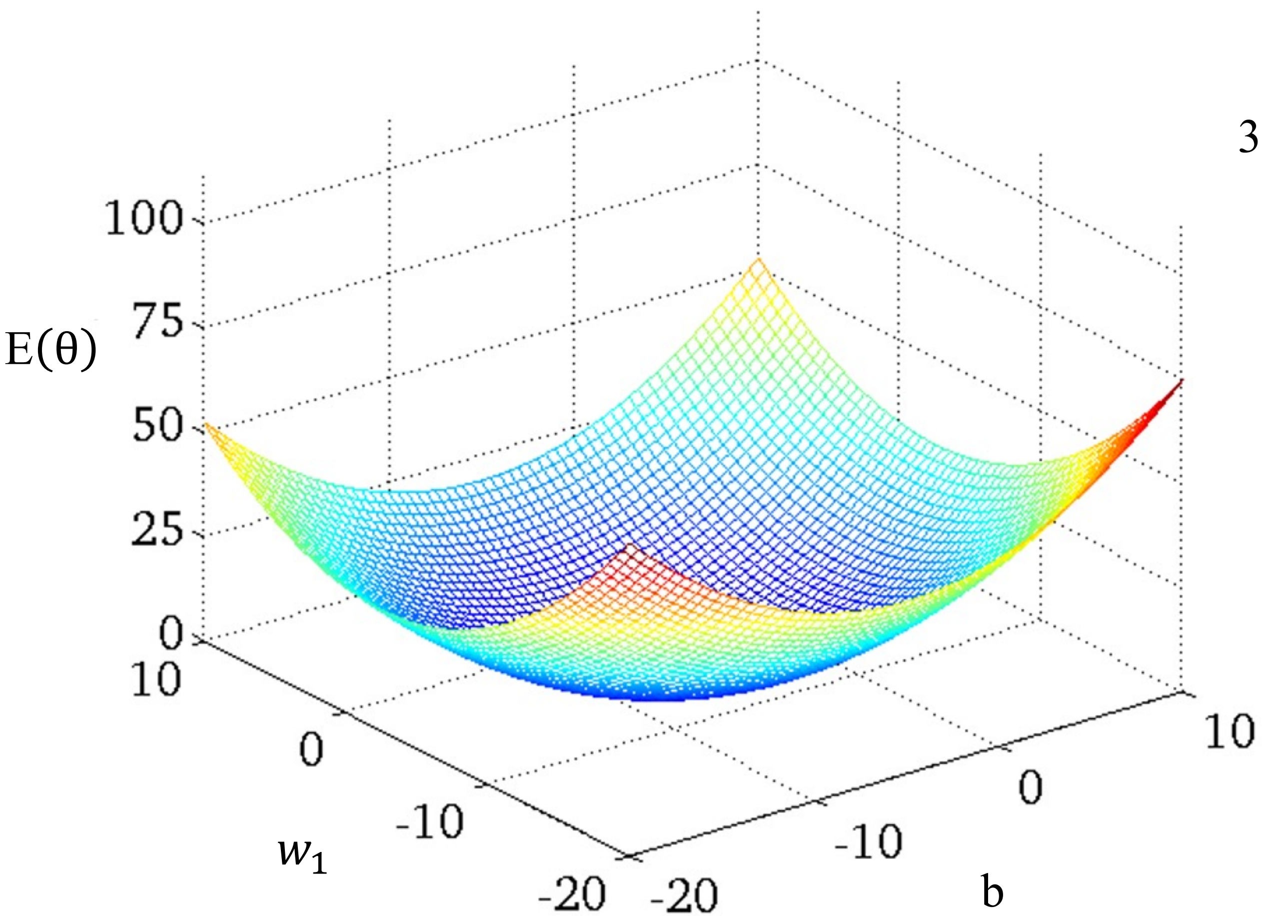
Goal: minimize $E(\theta)$

Hypothesis: $f_\theta(x) = w_1 x_1 + b$

Parameters: $\theta = \{w_1,\ b\}$

Cost Function: $E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(f_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$

Goal: minimize $E(\theta)$



3-d surface plot

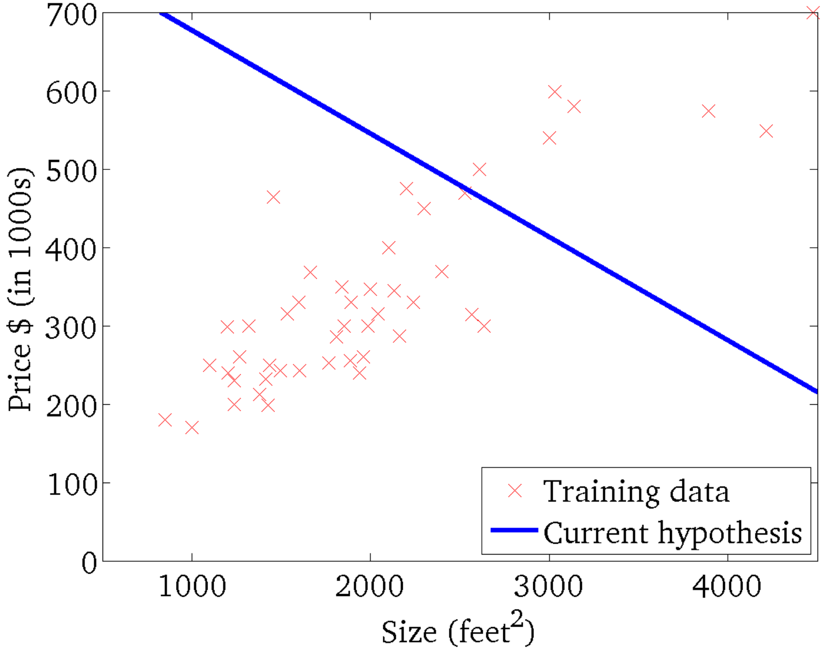
Contour figure

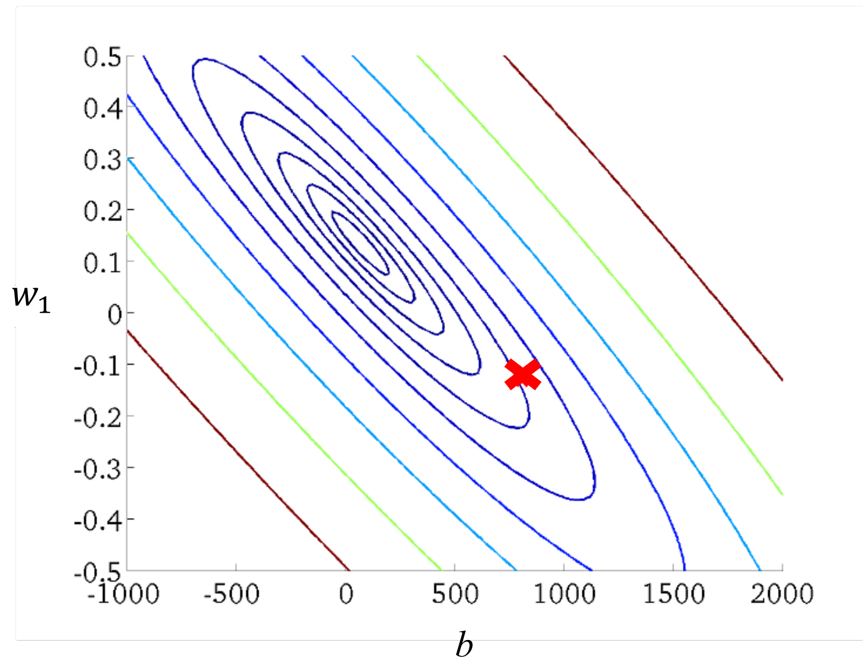
$f_\theta(x) = w_1 x_1 + b$

(for fixed $\theta = \{w_1,\ b\}$, this is a function of x)

$E(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (f_\theta(x^{(i)}) - y^{(i)})^2$

(function of the parameters $\theta = \{w_1,\ b\}$)





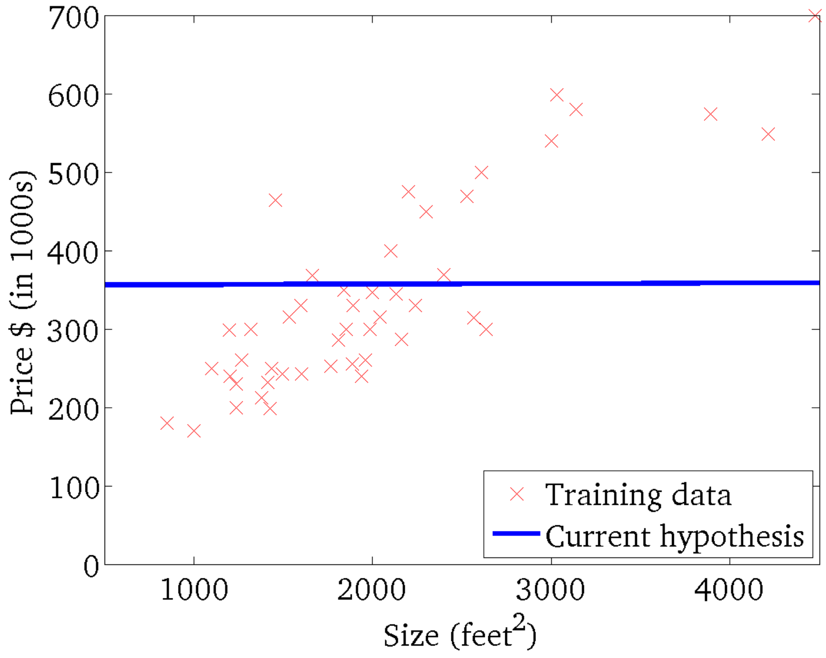
Contour figure

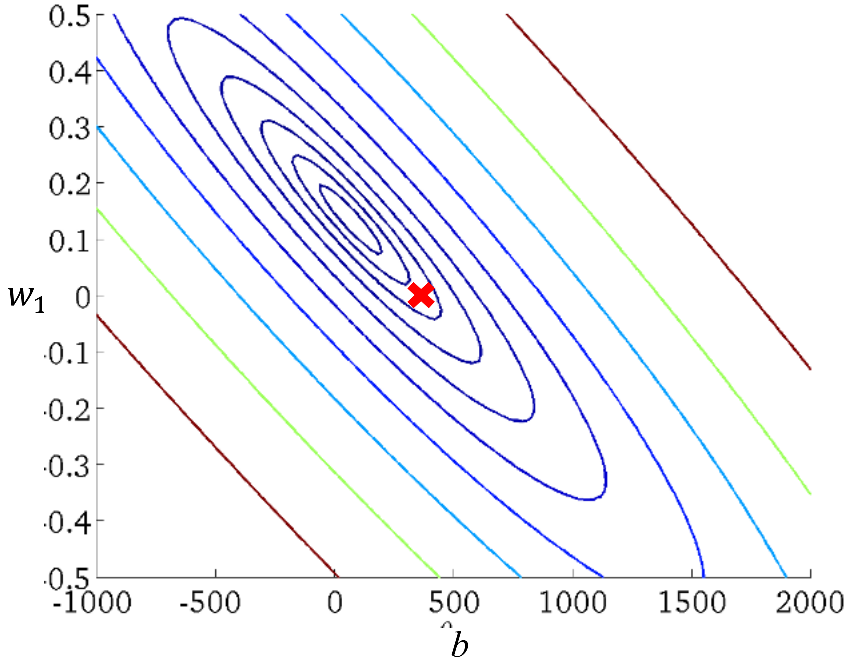
$$f_\theta(x) = w_1 x_1 + b$$

(for fixed $\theta = \{w_1,\ b\}$, this is a function of x)

$$\mathrm{E}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(f_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

(function of the parameters $\theta = \{w_1,\ b\}$)





$$f_\theta(x) = 0x_1 + 370$$

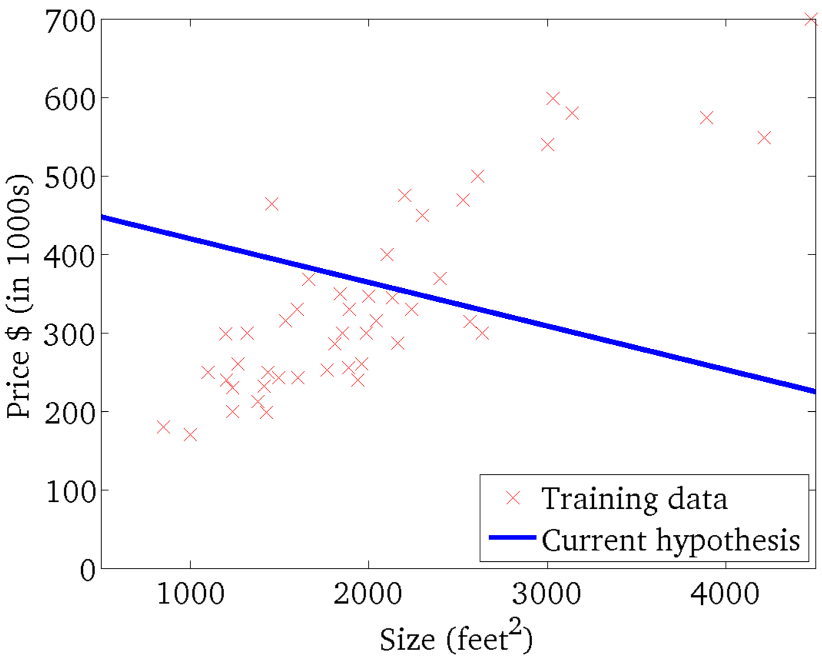
Contour figure

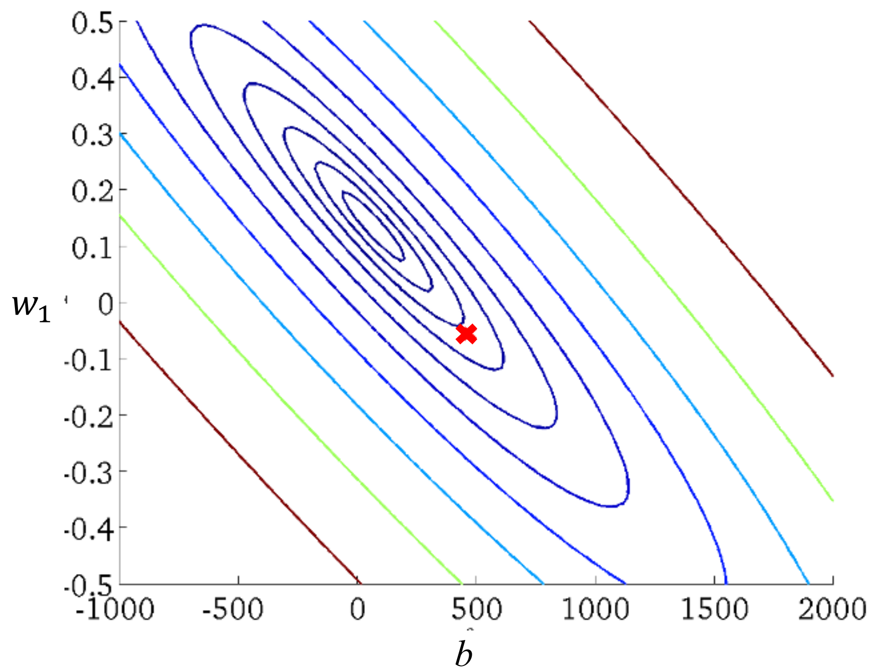
$$f_\theta(x) = w_1 x_1 + b$$

(for fixed $\theta = \{w_1,\ b\}$, this is a function of x)

$$E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(f_\theta(x^{(i)}) - y^{(i)})^2$$

(function of the parameters $\theta = \{w_1,\ b\}$)





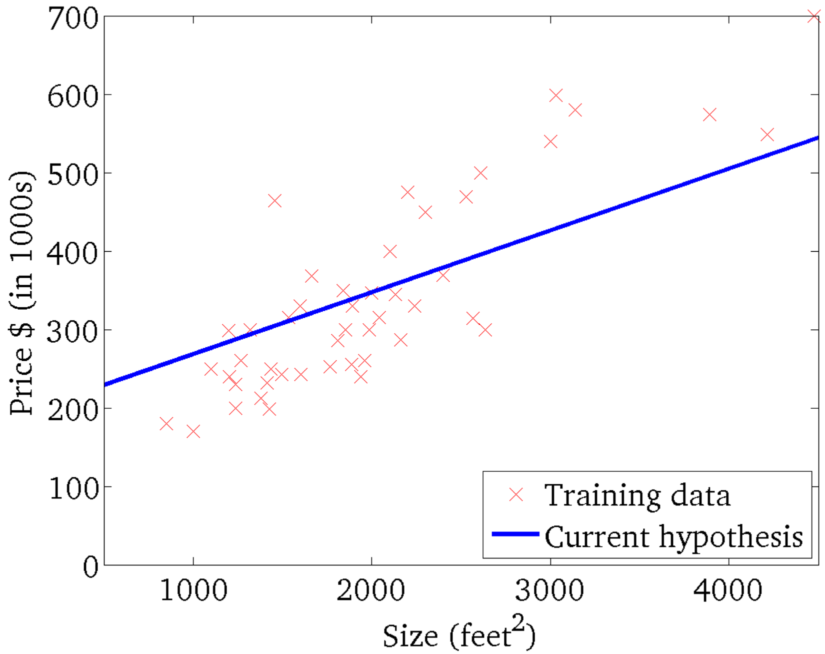
Contour figure

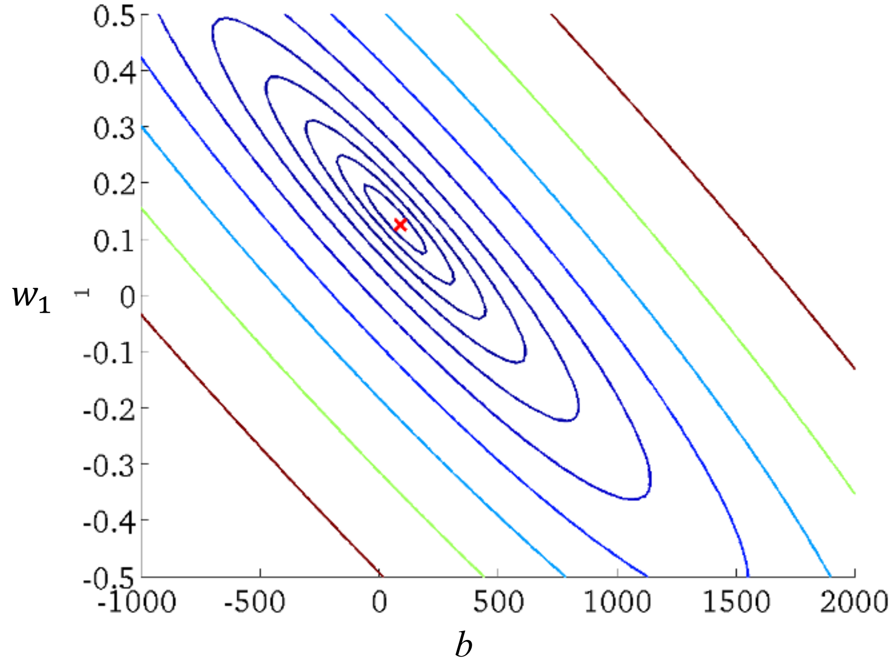
$$f_\theta(x) = w_1 x_1 + b$$

(for fixed $\theta = \{w_1,\ b\}$, this is a function of x)

$$\mathrm{E}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(f_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

(function of the parameters $\theta = \{w_1,\ b\}$)





Gradient descent

Hypothesis: $f_\theta(x) = w_1 x_1 + b$

Parameters: $\theta = \{w_1,\ b\}$

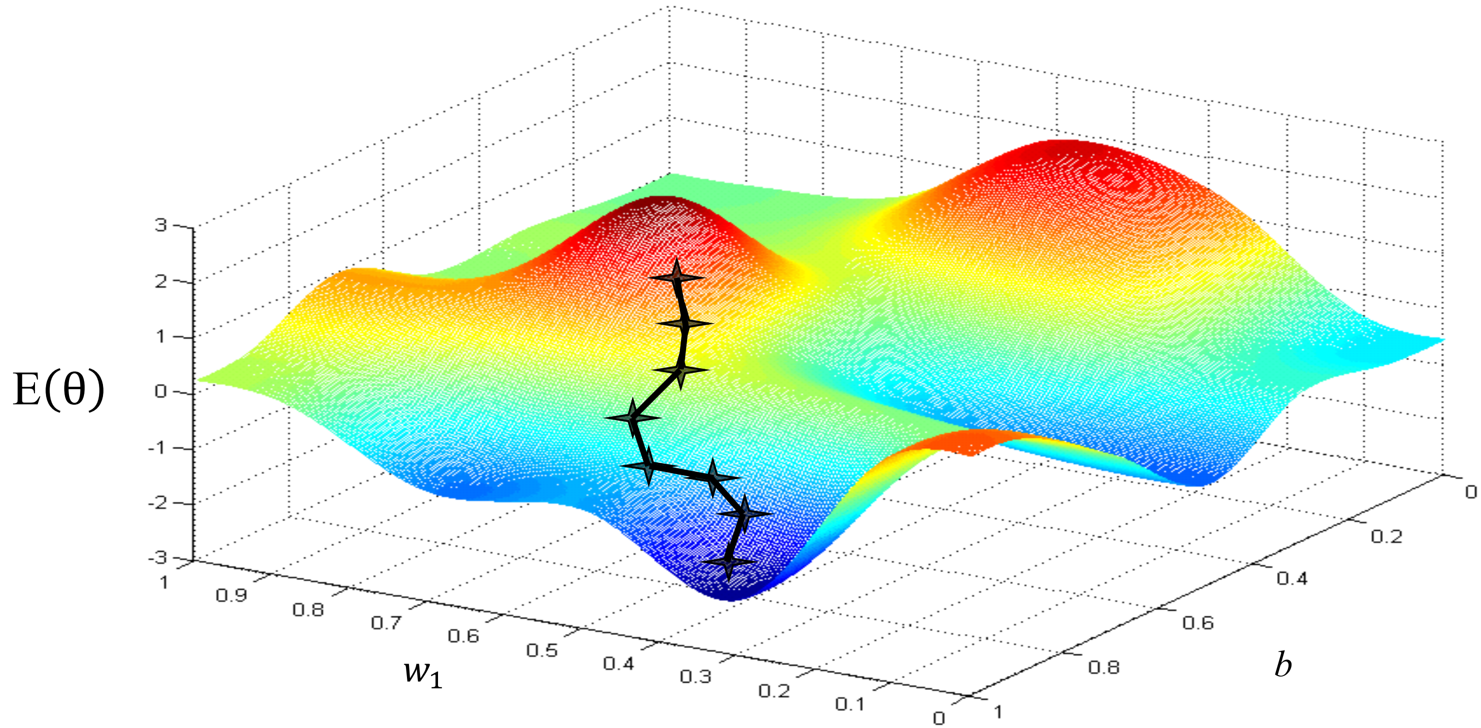
Cost Function: $E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(f_\theta\left(x^{(i)}\right) - y^{(i)})^2$

Goal: minimize $E(\theta)$

Steps:

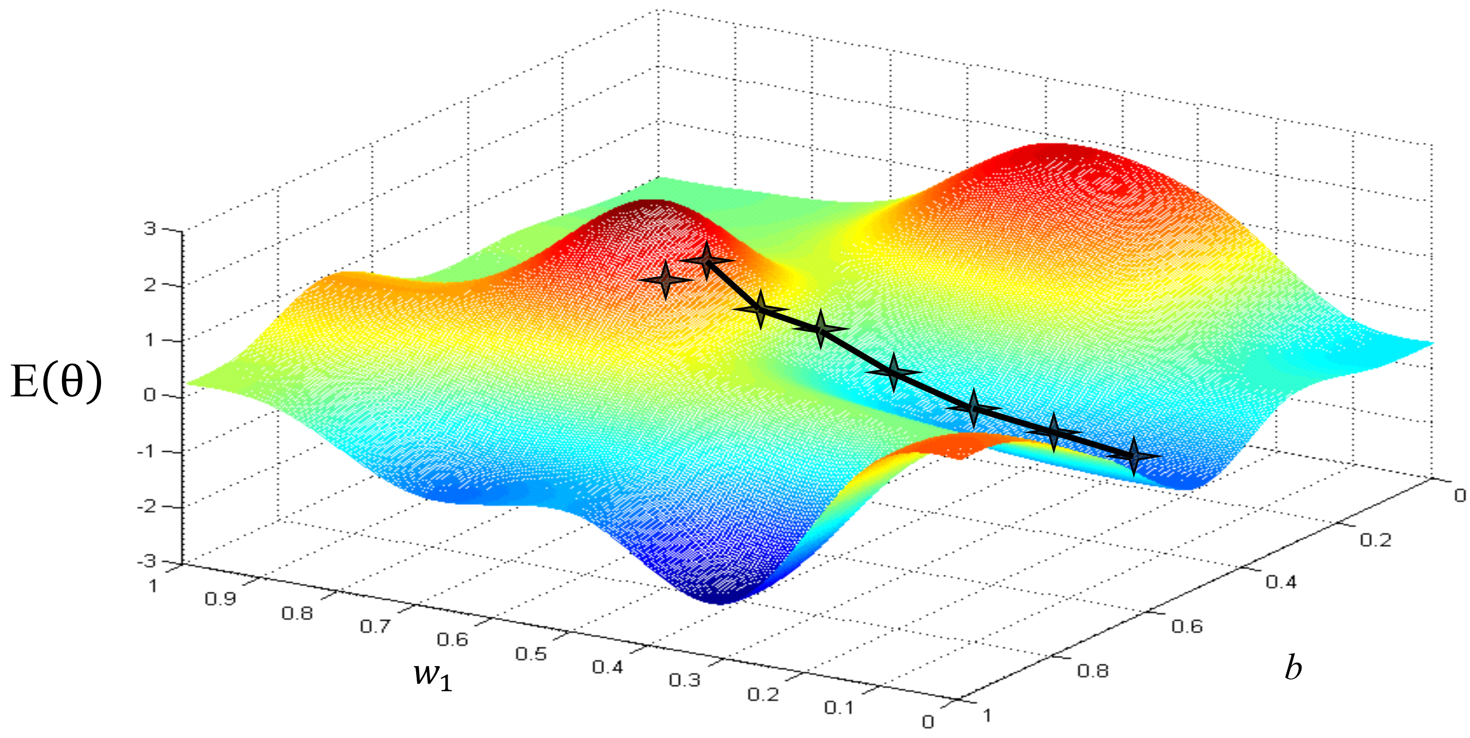
- Start with some $\theta = \{w_1,\ b\}$; *e.g.* $w_1 = 0,\ \ b = 0.$
- Keep changing θ to reduce $E(\theta)$ until end up at a minimum.

Intuition picture of gradient descent



Starting at some points on the surface of this fucntion.
Take a step in the direction of steepest descent.
Each step changes parameter θ to reduce E(θ) until end up at a local minimum.

Intuition picture of gradient descent



Starting at another point on the surface of this fucntion.
Each step changes parameter θ to reduce E(θ) until end up at a local minimum.
There are many local minimums.

Gradient descent algorithm

- Repeat until convergence

$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial \theta_j} E(\theta)} \qquad \theta = \{\theta_0 = w_1, \theta_1 = b\}$$

Learning rate

Gradient

- Simultaneous update
- Learning rate
  - α determines the step size at each iteration while moving toward a minimum of a cost function.
  - If α is too small, gradient descent can be slow;
  - If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.
- Gradient descent can converge to a local minimum, even with the learning rate α fixed.

Gradient descent for linear regression

Hypothesis: $\boxed{f(x) = wx_i + b \qquad f(x_i) \simeq y_i}$

Cost Function:
$$(w^*, b^*) = \underset{(w,b)}{\arg\min} \sum_{i=1}^{m} (f(x_i) - y_i)^2$$
$$= \underset{(w,b)}{\arg\min} \sum_{i=1}^{m} (y_i - wx_i - b)^2$$

Minimize:

$$\boldsymbol{E}(\boldsymbol{\theta}) = E_{(w,b)} = \sum_{i=1}^{m} (y_i - wx_i - b)^2$$

Derivates

Finding partial derivates of function $E_{(w,b)}$ with two variables: w and b

$$\begin{aligned}
\frac{\partial E_{(w,b)}}{\partial w} &= \frac{\partial}{\partial w}\left[\sum_{i=1}^{m}(y_i - wx_i - b)^2\right] \\
&= \sum_{i=1}^{m}\frac{\partial}{\partial w}\left[(y_i - wx_i - b)^2\right] \\
&= \boxed{\sum_{i=1}^{m}[2\cdot(y_i - wx_i - b)\cdot(-x_i)]} \\
&= \sum_{i=1}^{m}\left[2\cdot\left(wx_i^2 - y_ix_i + bx_i\right)\right] \\
&= 2\cdot\left(w\sum_{i=1}^{m}x_i^2 - \sum_{i=1}^{m}y_ix_i + b\sum_{i=1}^{m}x_i\right) \\
&= 2\left(w\sum_{i=1}^{m}x_i^2 - \sum_{i=1}^{m}(y_i - b)\,x_i\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E_{(w,b)}}{\partial b} &= \frac{\partial}{\partial b}\left[\sum_{i=1}^{m}(y_i - wx_i - b)^2\right] \\
&= \sum_{i=1}^{m}\frac{\partial}{\partial b}\left[(y_i - wx_i - b)^2\right] \\
&= \boxed{\sum_{i=1}^{m}[2\cdot(y_i - wx_i - b)\cdot(-1)]} \\
&= \sum_{i=1}^{m}[2\cdot(b - y_i + wx_i)] \\
&= 2\cdot\left[\sum_{i=1}^{m}b - \sum_{i=1}^{m}y_i + \sum_{i=1}^{m}wx_i\right] \\
&= 2\left(mb - \sum_{i=1}^{m}(y_i - wx_i)\right)
\end{aligned}$$

Gradient descent for linear regression

Hypothesis: $f_\theta(x) = w_1 x_1 + b$

Parameters: $\theta = \{w_1,\ b\}$

Cost Function: $E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(f_\theta(x^{(i)}) - y^{(i)})^2$

Goal: minimize $E(\theta)$

$\frac{\partial}{\partial \theta_j} E(\theta)$

- Repeat until convergence

$$w_1 := w_1 - \alpha \boxed{\frac{1}{m}\sum_{i=1}^{m}\left[f_\theta(x^{(i)}) - y^{(i)}\right] x^{(i)}}$$

$$b := b - \alpha \frac{1}{m}\boxed{\sum_{i=1}^{m}\left[f_\theta(x^{(i)}) - y^{(i)}\right]}$$

update w_1 and b simultaneously

Linear Regression: Example

Size in feet² (x)	**Price ($) in 1000's (y)**
2104	460
1416	232
1534	315
852	178

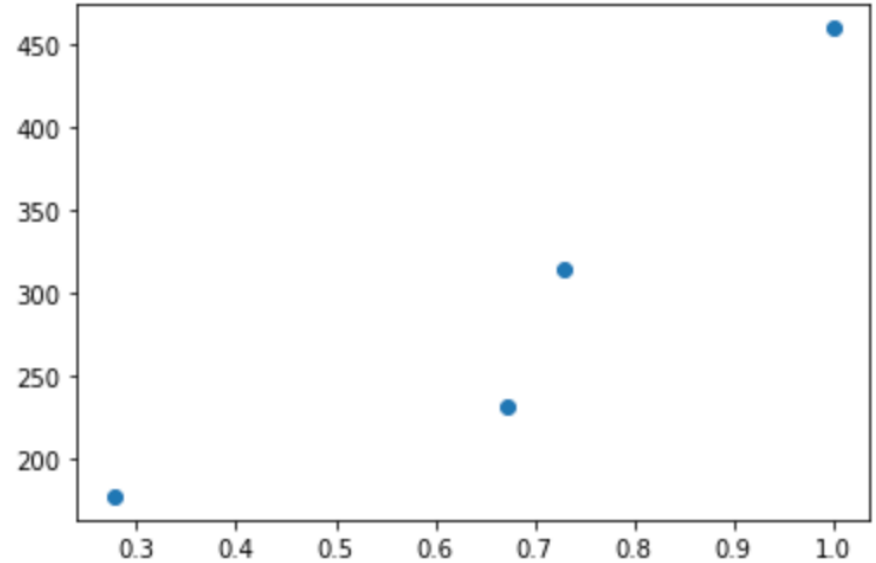
starting point(can be random):
$w = 0, \quad b = 0$

learning rate $a = 0.5$

max_inter = 10

Normalizing & add a column with value one

```
array([[1.        , 1.        ],
       [1.        , 0.6730038 ],
       [1.        , 0.72908745],
       [1.        , 0.27661597]])
```



Iteration 1

Hypothesis: $f_\theta(x) = w_1 x_1 + b$

Parameters: $\theta = \{w_1,\ b\}$

Cost Function: $E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(f_\theta(x^{(i)}) - y^{(i)})^2$

Estimation: (w = 0, b = 0)

$$f_\theta(1) = (0 * 1 + 0) = 0$$

$$f_\theta(0.67) = (0 * 67 + 0) = 0$$

$$f_\theta(0.72) = (0 * 0.72 + 0) = 0$$

$$f_\theta(0.27) = (0 * 0.27 + 0) = 0$$

Normalized input x

```
array([[1.        , 1.        ],
       [1.        , 0.6730038 ],
       [1.        , 0.72908745],
       [1.        , 0.27661597]])
```

Cost: $E(\theta) = \frac{1}{2*4}[(0-460)^2 + (0-232)^2 + (0-315)^2 + (0-178)^2] \approx 49541.62$

Gradient:

w' = $\frac{\partial E(\theta)}{\partial w} = \frac{1}{4}[(0-460)*1 + (0-232)*0.67 + (0-315)*0.72 + (0-178)*0.27] \approx$ - 222.575

b' = $\frac{\partial E(\theta)}{\partial b} = (\frac{1}{4})[(0-460) + (0-232) + (0-315) + (0-178)] \approx$- 296.25

Update:

$$w_1 := w_1 - \alpha * w' = 0 - 0.5 * (-222.575) \approx 111.2875$$
$$b := b - \alpha * b' = 0 - 0.5 * (-296.25) \approx 148.125$$

New parameters

Iteration 2

Hypothesis: $f_\theta(x) = w_1 x_1 + b$

Parameters: $\theta = \{w_1,\ b\}$

Cost Function: $E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(f_\theta(x^{(i)}) - y^{(i)})^2$

Estimation: (w = 111.28, b = 148.12)

$$f_\theta(1) = (111.28 * 1 + 148.12) \approx 260$$

$$f_\theta(0.67) = (111.28 * 67 + 148.12) \approx 223$$

$$f_\theta(0.72) = (111.28 * 0.72 + 148.12) \approx 229$$

$$f_\theta(0.27) = (111.28 * 0.27 + 148.12) \approx 179$$

Normalized input x

```
array([[1.        , 1.        ],
       [1.        , 0.6730038 ],
       [1.        , 0.72908745],
       [1.        , 0.27661597]])
```

Cost: $E(\theta) = (\frac{1}{2*4})[(260-460)^2 + (223-232)^2 + (229-315)^2 + (179-178)^2] \approx 5918.73$

Gradient:

w' = $\frac{\partial E(\theta)}{\partial w} = (\frac{1}{4})\ [(260-460) * 1 + (223-232) * 0.67 + (229-315) * 0.72 + (179-178) * 0.27] \approx -66.92$

b' = $\frac{\partial E(\theta)}{\partial b} = (\frac{1}{4})[(260-460) + (223-232) + (229-315) + (179-178)] \approx -73.5$

Update:

$$w_1 := w_1 - \alpha\ * w' = 0 - 0.5\ * (-66.92) \approx 184.72$$
$$b := b - \alpha\ * b' = 0 - 0.5\ * (-73.5) \approx\ 145.33$$

New parameters

Continue…

Iteration 3

```
iter: 3 cost: 2778.85472457512
prediction: [330.06397483 282.53897763 290.69006727 224.92873394]
gradient: [-14.19456158 -25.16648269]
parameters: [191.8231828  157.92131416]
```

Iteration 4

```
iter: 4 cost: 2487.2205920097786
prediction: [349.74449696 298.10482769 306.96163143 235.50674024]
gradient: [  1.32942408 -13.9300234 ]
parameters: [191.15847076 164.88632586]
```

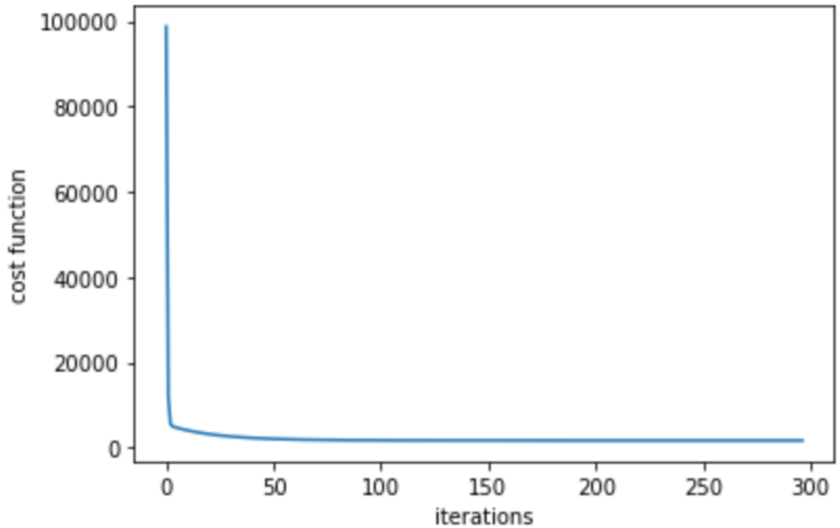
Iteration 5

```
iter: 5 cost: 2398.9324937073943
prediction: [356.04479662 302.12759501 311.37502203 236.76866166]
gradient: [  5.32901883 -10.78641023]
parameters: [188.49396135 170.27953098]
```

…..

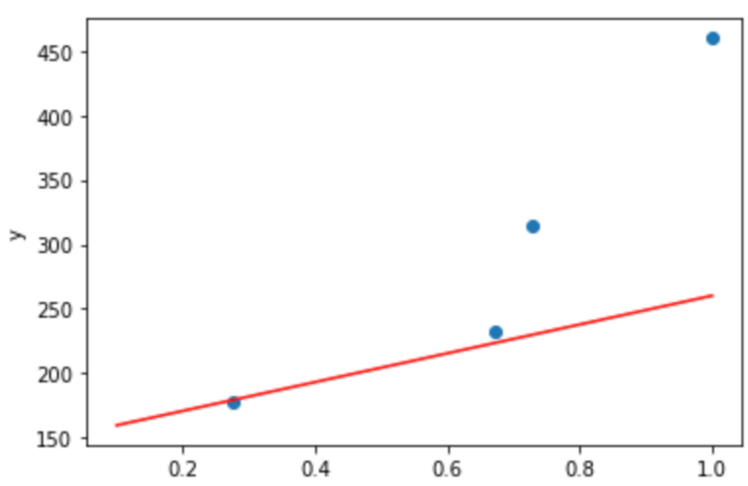
Until it converges

Linear Regression: Example

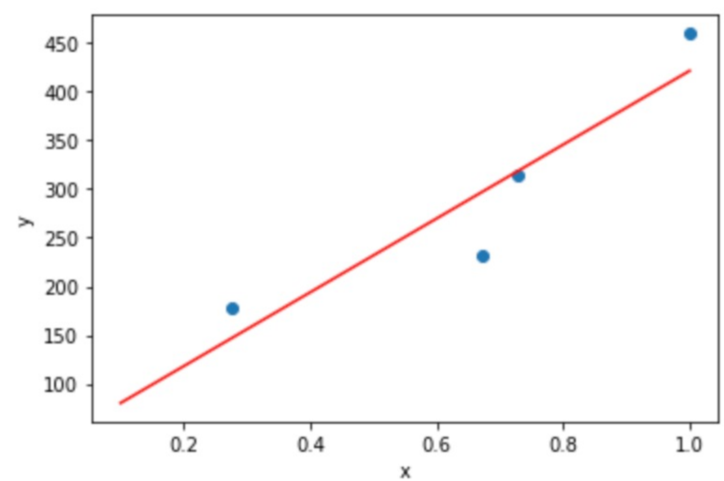


learning_rate = 0.5

```
iter: 0 cost: 98663.42912068815
```

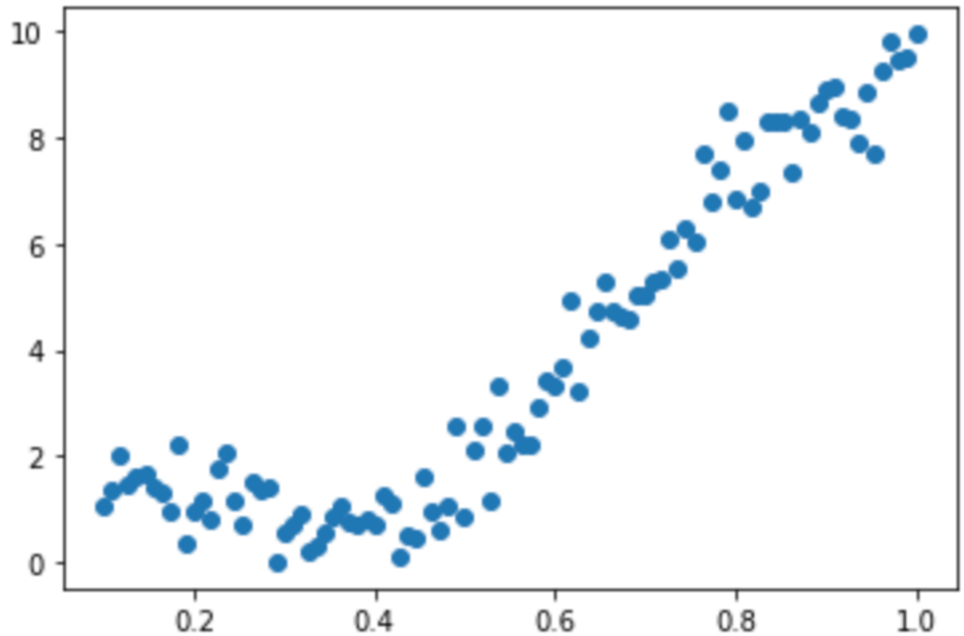



```
iter: 296 cost: 1684.027676592667 8
```



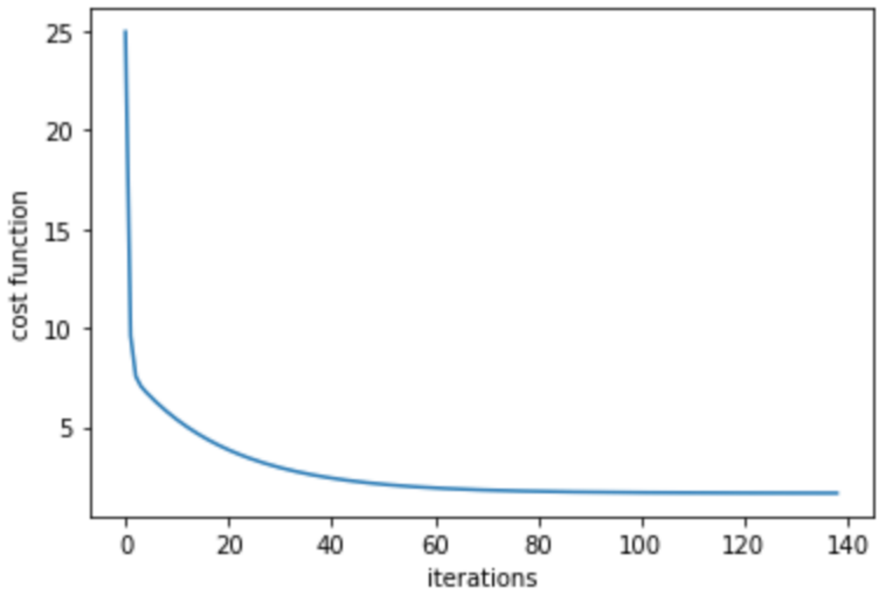

```
Converged.
```

Linear Regression: Example-2

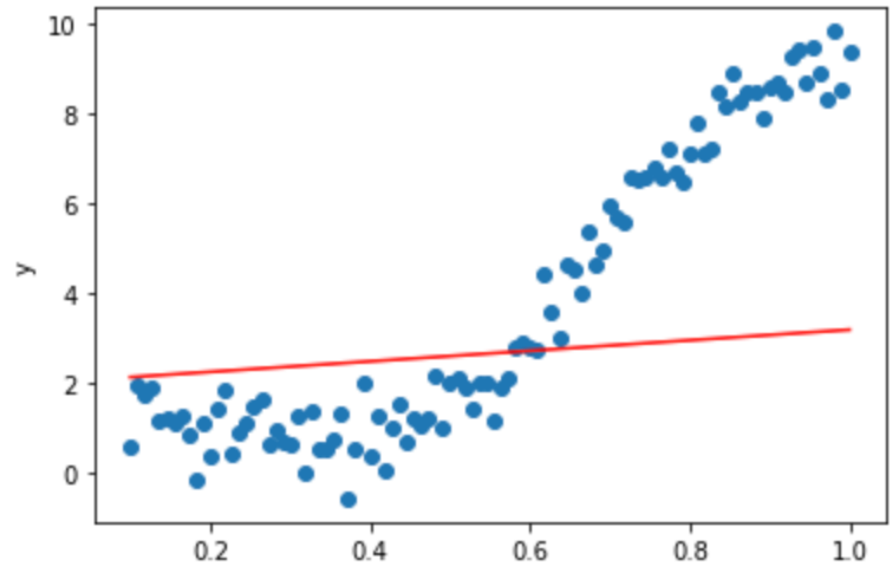


N = 100 samples

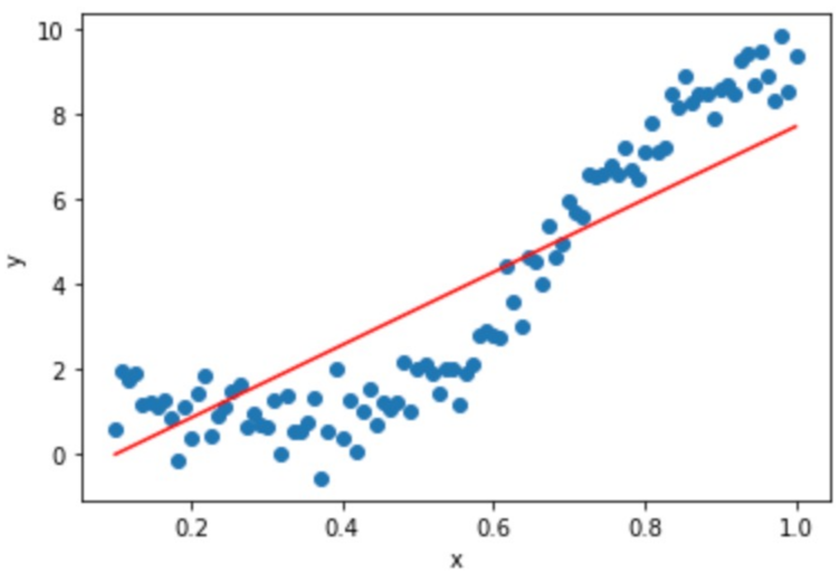
learning_rate = 0.5



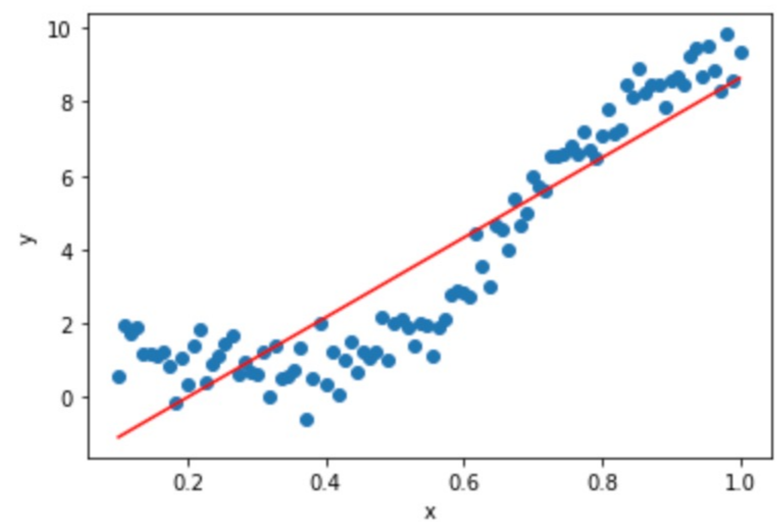

```
iter: 0 cost: 24.973176452747104
```




```
iter: 50 cost: 2.0982659858775956
```




```
iter: 138 cost: 1.6546338133174023
```




```
Converged.
```

Multivariate linear regression

Size in feet² (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178

$$f_\theta(x) = w_1 x_1 + b$$

linear regression with one variable

Size (feet²) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Price ($1000) y
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178
…	…	…	…	…

linear regression with multiple variables

n – is number of features

$$f_\theta(x) = w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + \cdots + w_n x_n + b$$

$x_0 = 1$

Hypothesis: $f_\theta(x) = \theta^T x = w_0 x_0 + w_1 x_1 + \cdots + w_n x_n$

Parameters: $\theta = \{w_0, \ldots, w_n\}$

Cost Function: $E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(f_\theta(x^{(i)}) - y^{(i)})^2$

Goal: minimize $E(\theta)$

- Repeat until convergence

$$w_0 := w_0 - \alpha \frac{1}{m}\sum_{i=1}^{m}[f_\theta(x^{(i)}) - y^{(i)}]x_0^{(i)}$$

$$w_1 := w_1 - \alpha \frac{1}{m}\sum_{i=1}^{m}[f_\theta(x^{(i)}) - y^{(i)}]x_1^{(i)}$$

update θ simultaneously

$$w_2 := w_1 - \alpha \frac{1}{m}\sum_{i=1}^{m}[f_\theta(x^{(i)}) - y^{(i)}]x_2^{(i)}$$

……

Feature scaling

Size (feet²) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Price ($1000) y
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178
…	…	…	…	…

Get every feature into approximately a $-1 \leq x_i \leq 1$ range.

Normalization:

$$x_i = \frac{x_i - \mu_i}{S_i}$$

Standard deviation:

$$S_i = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu_i)^2}$$

For example:

$$x_1 = \frac{size - 1000}{2000}$$

$$x_2 = \frac{\#bedrooms - 2}{4}$$

…

Multivariate linear regression: Vectorization

	Size (feet²)	Number of bedrooms	Number of floors	Age of home (years)	Price ($1000)
x_0	x_1	x_2	x_3	x_4	y
1	2104	5	1	45	460
1	1416	3	2	40	232
1	1534	3	2	30	315
1	852	2	1	36	178
…	…	…	…	…	…

$$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix} \qquad y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

$$\theta = (X^T X)^{-1} X^T y$$

Normal equation

$(X^T X)^{-1}$ is inverse of matrix $X^T X$.

Logistic Regression

Binary Classification

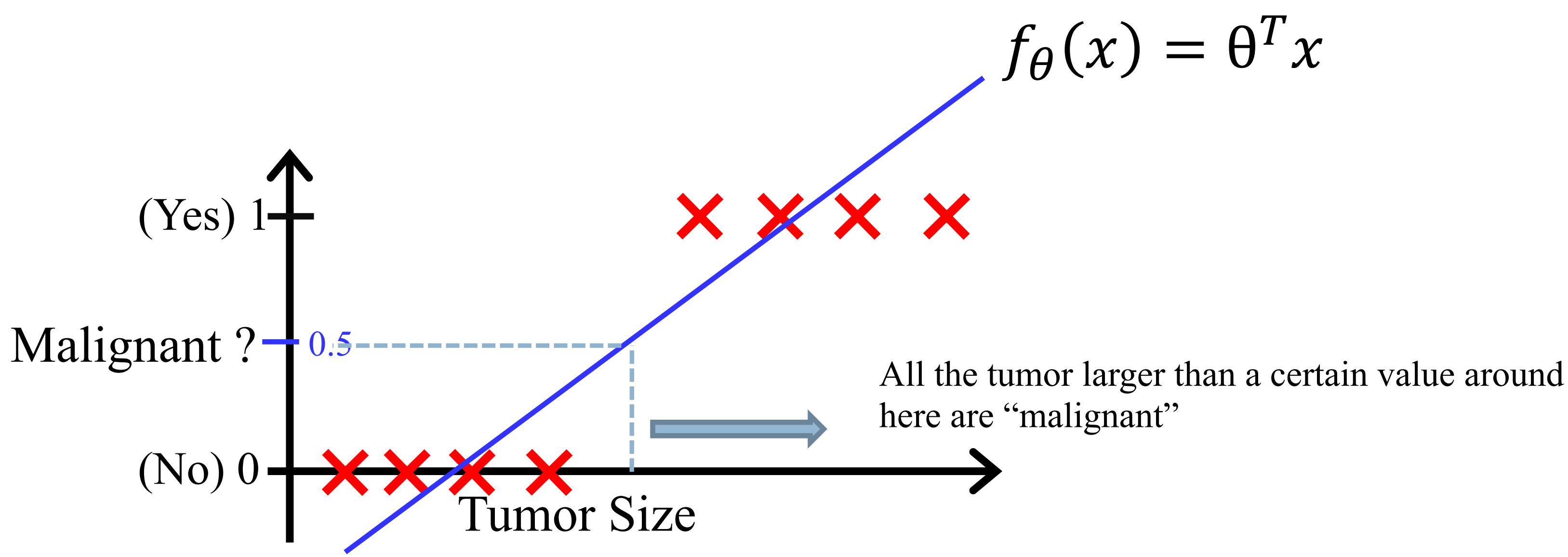
Email: Spam / Not Spam?
Watermelon: Good / not?
Tumor: Malignant / Benign ?

e.g. "tumor"

$$x \longrightarrow \boxed{\text{Model}} \longrightarrow y \in \{0, 1\}$$

0: "Negative Class" (e.g., benign tumor)
1: "Positive Class" (e.g., malignant tumor)

Classification with linear regression?



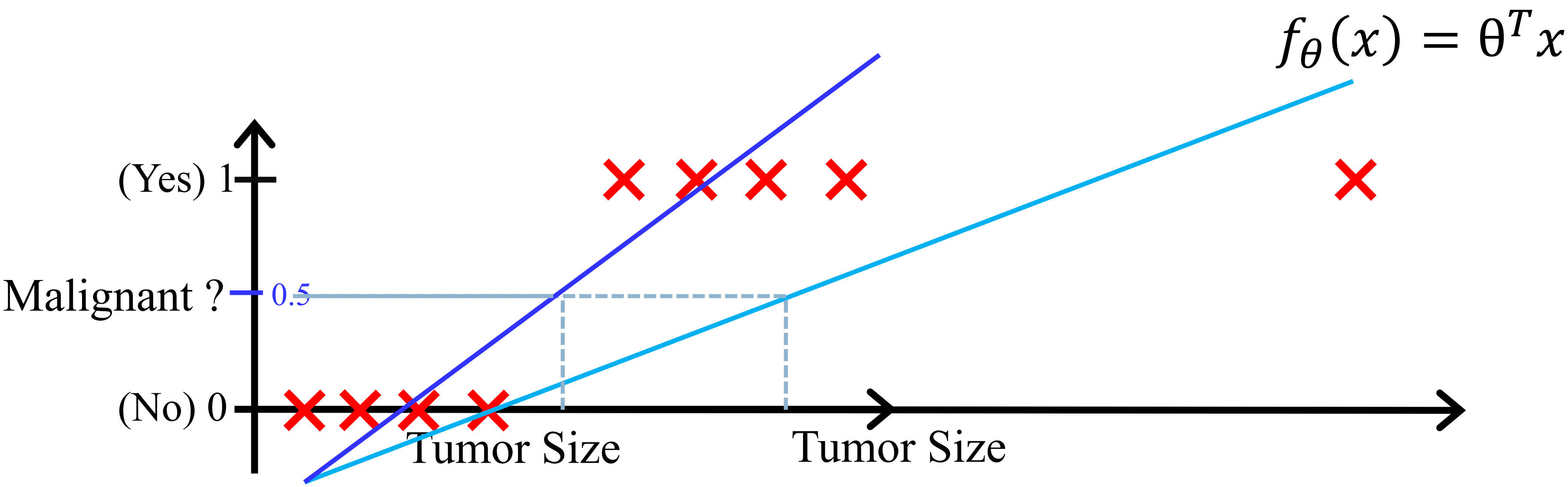
Hypothesis with linear regression : $f_\theta(x) = \theta^T x$

Threshold = 0.5

If $f_\theta(x) \geq 0.5$, output "Yes"

If $f_\theta(x) \leq 0.5$, output "No"

Classification with linear regression?



Hypothesis with linear regression : $f_\theta(x) = \theta^T x$

Threshold = 0.5

If $f_\theta(x) \geq 0.5$, output "Yes"

If $f_\theta(x) \leq 0.5$, output "No"

Add extra samples, linear regression may give you a wrose hypothesis.
Applying linear regression to a classification **is not a good idea**.

Logistic regression model

Linear regression model: $f_\theta(x) = \theta^T x$

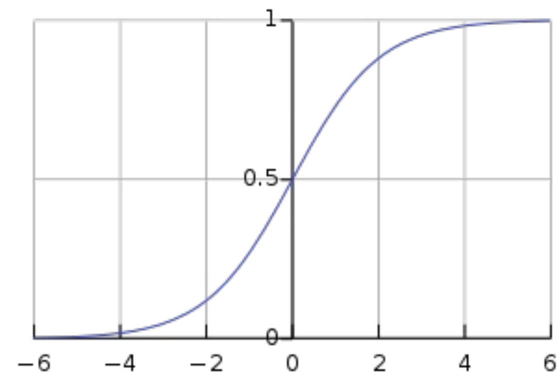
Linear regression can be: $f_\theta(x) > 1 \ or. \ f_\theta(x) < 0$

Want: $0 \leq f_\theta(x) \leq 1$

$$f_\theta(x) = g(\theta^T x)$$

where,

$$g(z) = \frac{1}{1 + e^{-z}}$$



$$f_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

Sigmoid/Logistic function

m examples

Training set: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \cdots, (x^{(m)}, y^{(m)})\}$

$$x \in \begin{bmatrix} x_0 \\ x_1 \\ \cdots \\ x_n \end{bmatrix} \qquad x_0 = 1, y \in \{0, 1\}$$

Hypothesis: $f_\theta(x) = \dfrac{1}{1 + e^{-\theta^T x}}$ where, $\theta^T x = w_0 x_0 + w_1 x_1 + \cdots + w_n x_n$

What cost function we choose?

How to choose parameters θ ?

Cost function

- Chosee cost function same with linear regression

Cost Function: $E(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(f_\theta\left(x^{(i)}\right) - y^{(i)})^2$

But the hypothesis looks like: $f_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$

Problem: it is non-convex function, there are many local minums. Gradient descent algorithm not guranteed to find a good local minma.

Want: the cost function should be convex.

Hypothesis: $f_\theta(x) = \dfrac{1}{1 + e^{-\theta^T x}}$ where, $\theta^T x = w_0 x_0 + w_1 x_1 + \cdots + w_n x_n$

Parameters: $\theta = \{w_0, \dots, w_n\}$ $y \in \{0, 1\}$ Real-value from data set

Cost function: $E(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log f_\theta(x^{(i)}) + (1 - y^{(i)}) \log(1 - f_\theta(x^{(i)}))]$

Corss entroy lost function

if $y^{(i)} = 0$:

$$E(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} (1 - y^{(i)}) \log(1 - f_\theta(x^{(i)}))]$$

if $y^{(i)} = 1$:

$$E(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log f_\theta(x^{(i)})]$$

Goal: minimize $E(\theta)$

Logistic regression: Gradient Descent

Cost function: $E(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log f_\theta(x^{(i)}) + (1 - y^{(i)})\log(1 - f_\theta(x^{(i)})]$

Goal: minimize $E(\theta)$

Repeat {

$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial \theta_j} E(\theta)}$$

(simultaneously update all θ)

Learning rate

Gradient

}

Logistic regression: Gradient Descent

Cost function: $E(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log f_\theta\left(x^{(i)}\right) + \left(1 - y^{(i)}\right)\log(1 - f_\theta\left(x^{(i)}\right)]$

Goal: minimize $E(\theta)$

$$f_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

Repeat {

$$\theta_j := \theta_j - \alpha \frac{1}{m}\sum_{i=1}^{m}\left[f_\theta\left(x^{(i)}\right) - y^{(i)}\right] x^{(i)}$$

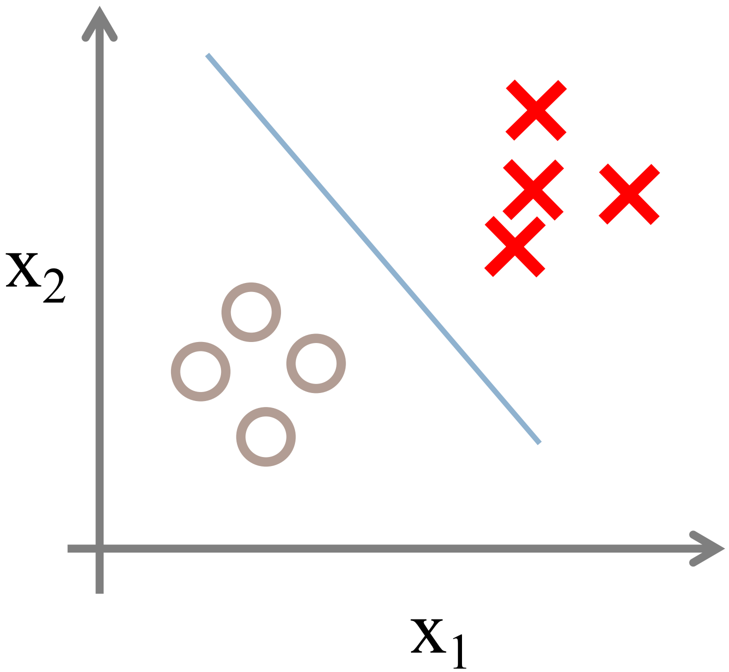
}

(simultaneously update all θ)

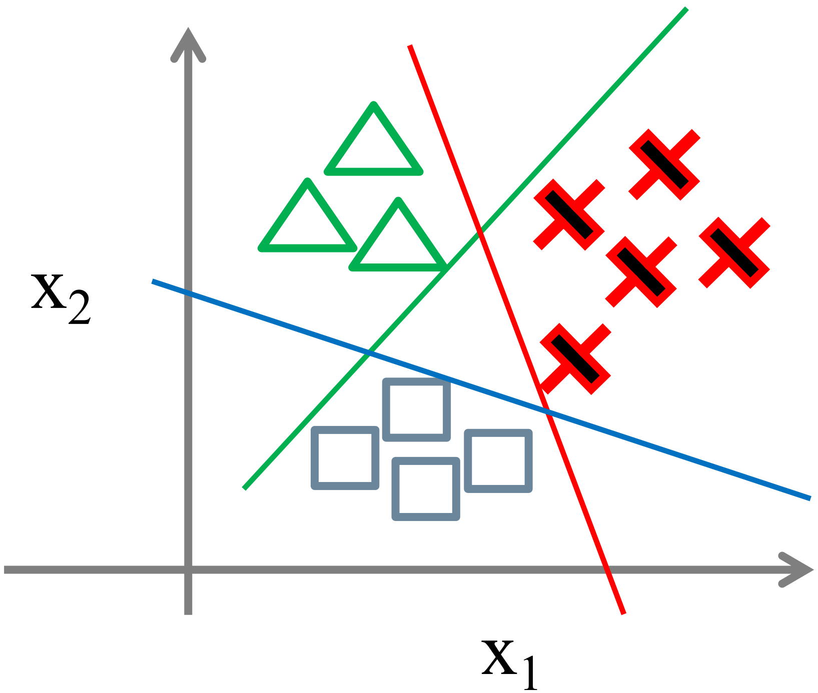
Similar with linear regression.

Multiclass classification

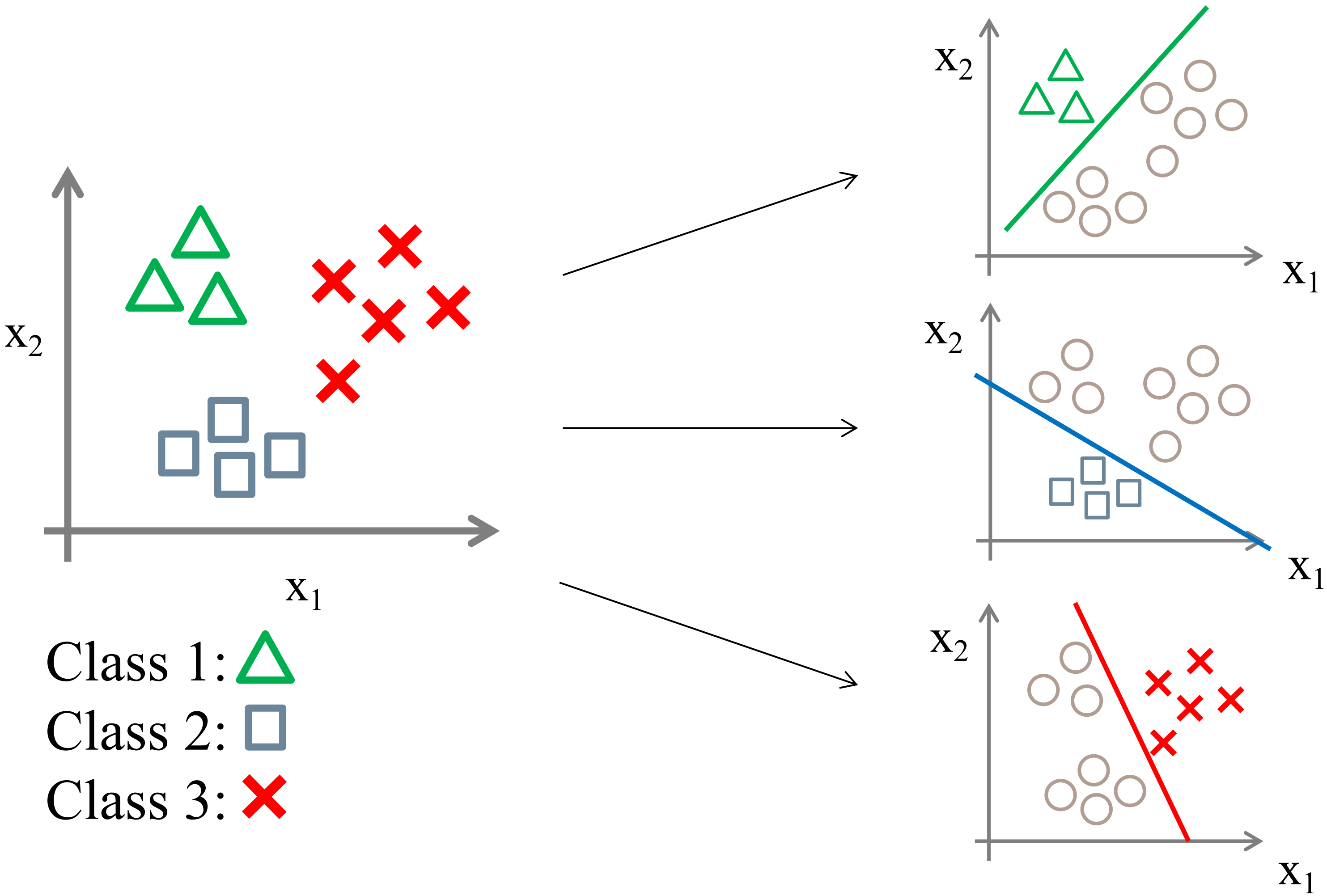
Binary classification:



Multi-class classification:



One-vs-rest



- Thank you!